Fairness and Desert in Tournaments

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Abstract

We model the behavior of agents who, in competitive situations, care about receiving their "just deserts", i.e., what they feel they deserve. In particular we analyze the strategic behavior of two identical desert-motivated agents in a rank-order tournament. Each agent is assumed to be loss averse about an endogenous and meritocratically determined reference point that represents her perceived entitlement. Sufficiently strong desert concerns render the usual symmetric equilibrium unstable or non-existent and allow asymmetric desert equilibria to arise in which one agent works hard while the other slacks off. By pushing their effort levels apart, the agents end up closer to their reference points on average. As a result, agents may prefer competition for status or rank to a random allocation, even though the supply of status/rank is fixed. We also find that when employees are desert-motivated, but continue to play a symmetric equilibrium, an employer will prefer a tournament to a linear relative performance pay scheme if output noise is sufficiently fat-tailed.

Keywords: Desert, Equity, Tournament, Loss Aversion, Reference-Dependent Preferences, Status, Relative Performance Evaluation.
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1 Introduction

Rank-order tournaments, where agents compete for a fixed prize or set of prizes, are ubiquitous. Promotional tournaments are common in consulting, law partnerships, academia and industry. Firms frequently use bonus schemes based on relative performance. Sports contests, examinations, patent races, elections and competition for status can all be thought of as tournaments. We claim that in a competitive setting such as a tournament, agents often care about receiving their "just deserts". We adopt a meritocratic notion of desert according to which an agent's perceived entitlement will be sensitive to the effort she has exerted relative to her rival, who is otherwise identical: an agent feels she deserves more if and only if she works harder than the rival.[1] Desert-motivated agents will feel hard done by if they receive less than what they perceive to be their fair recompense, while feelings of elation or guilt are possible if they do better than they deserve. We analyze how the introduction of desert alters equilibrium play and payoffs in tournaments, and we apply our findings to analyze competition for status/rank and the design of incentive schemes.

While little work has been carried out to formally introduce desert concerns into a theoretical economic model,[2] there exists an empirical literature which supports the idea that people are indeed motivated by a meritocratic notion of desert. According to a review of relevant literature by Konow (2003), *"a common view is that differences owing to birth, luck and choice are all unfair and that only differences attributable to effort are fair"* (pp. 1207-1208). Furthermore, Konow (1996) distills an *accountability principle* from the responses to his attitude survey according to which a person's entitlement varies in direct proportion to the value of his relevant discretionary variables, relative to others (p.19). This is closely related to the claims of equity theory, a social psychological theory of fairness that has its origins in Aristotle's claim that the equitable ratio of outcomes is proportional to the ratio of inputs (Konow, 2003). In its modern manifestation equity theory goes back to Adams (1965), whose work led him to conclude that *"When [a person] finds that his outcomes and inputs are not in balance in relation to those of others, feelings of inequity result"* (p. 280) and that *"there can be little doubt that inequity results in dissatisfaction, in an unpleasant emotional state, be it anger or guilt"* (p. 283). The significance of equity theory is also noted by Rabin (1998) who writes that *"desert will obviously*

1 We leave open the question of what constitutes one's just deserts if some agents possess an advantage such as a higher innate ability or a lower cost of effort.

2 An exception is Konow (2000) who develops a model in which people are influenced by the departure of the allocation from a fair one which reflects the efforts exerted by all agents. Agents experience dissonance costs when the allocation departs from the one they believe to be fair and psychic costs of self-deception when this belief departs from the truly fair allocation. However, Konow considers only the optimal division of output by a dictator given efforts, and not the choice of effort levels by the agents. Varian (1974) analyzes whether allocations can be fair, in the sense that they are efficient and nobody envies anyone else's bundle, where agents can substitute between leisure and labour (i.e., exert effort).

be relevant in many situations - and the massive psychological literature on "equity theory" shows that people feel that those who have put more effort into creating resources have more claim on those resources" (p. 18).

Experimental economics provides further evidence in favor of the idea that people are sensitive to considerations of desert. In a bargaining experiment, Burrows and Loomes (1994) find that bargained outcomes tend to exhibit greater inequality, awarding higher final payoffs to the party that began with a greater initial endowment, when endowments were allocated according to parties' performances in a simple word game than when the endowments were allocated at random. They conclude that the results of their experiment are consistent with the proposition that *"many people believe that when different individuals have a similar ability and opportunity to put in effort, those that put in more effort should get a greater reward because they are relatively deserving"* (p. 220). Konow (2000) experimentally confirms his theoretical prediction that dictators with no stake in the allocation between a pair of agents distribute rewards in proportion to the ratio of efforts exerted in an envelope filling task. Frohlich et al. (2004) provide further experimental evidence that a notion of entitlement to one's "just deserts" motivates dictators' allocation decisions: where the amount to be allocated is determined by the dictator and recipient's efforts in a proof-reading task, the authors find that *"the just deserts response is modal"* (p. 109). In a setting similar to Frohlich et al.'s, but where "effort" is measured by investment of money, Cappelen et al. (2007) find that: *"the majority of the participants ... care about the investments made by the opponent when they decide how much to offer"* (p. 14).[3]

We suppose that two identical agents compete in a Lazear and Rosen (1981) type rank-order tournament and that each agent maximizes a utility function that comprises three components: a *material utility* component, which resembles a standard utility function, a *cost of effort* component, and a *desert utility* component, which depends on the departure of the agent's actual material utility from that associated with her deserved reference payoff. This reference point is endogenous and is given by the agent's expected payoff, given the chosen effort levels.[4] The idea is that, given the ex ante symmetry of the agents, an agent's average payoff is a reflection of the useful effort she has exerted relative to her rival and thus plausibly represents the proportion of the prize that she feels she deserves.

Moreover, we assume that desert utility is more steeply increasing in the loss domain than the gain domain. This captures the central stylized fact - loss aversion - that has emerged from the empirical literature on reference-dependent preferences more generally: losses relative to

[3] Güth (1988) surveys some earlier relevant experimental evidence.

[4] Experimental evidence from the psychology literature supports the thesis that agents' emotional responses when receiving the payoff associated with a lottery depend on expectations. See, for instance, Mellers (1999).

the reference point loom larger than corresponding gains (see Rabin (1998) for a survey of this literature). In the specific context of fairness judgments, Kahneman et al. (1986) find strong evidence for loss aversion. People apparently perceive that it is more important that firms avoid hurting customers relative to a "fair" reference transaction than that they attempt to increase customer surplus relative to this reference level.

As well as having empirical support, there are also a priori grounds to accept an assumption of loss aversion in this context. As one agent's undeserved bonus has meant another's undeserved loss, we might expect any sensation of elation the agent experiences as a result of having her reference point exceeded to be somewhat muted. Indeed, if the agent were sufficiently socially minded she would presumably feel an overall sensation of pain. We remain agnostic as to whether desert utility is rising or falling above the reference point. When it is rising we say that preferences exhibit *desert elation,* while when it is falling we say that preferences exhibit *desert guilt.*

If desert utility is in fact falling in the gain domain, then desert preferences will be structurally similar to the preferences of the inequity averse agents of Fehr and Schmidt (1999),[5] albeit defined with respect to a different reference point. Inequity averse agents exhibit a preference for equality of payoffs across agents in a reference group: when there are only two agents in the group, each agent's reference point is simply the payoff of the other agent. By contrast, in our setup reference points are functions of the effort levels. In this sense, our agents adopt a more sophisticated conception of fairness than those of Fehr and Schmidt. They care about the relationship between the distribution of material payoffs and the distribution of agents' efforts, not just about the brute distribution of material payoffs.[6]

The structure of our agents' preferences also clearly resembles that of other models of reference-dependent preferences such as Kahneman and Tversky's (1979) Prospect Theory. However, there are also important points of contrast. In particular, while the reference point of the value function in Kahneman and Tversky's (1979) Prospect Theory is exogenous, our reference points are endogenously determined (since they depend on agents' efforts). In this sense, our model has some of the flavor of Kőszegi and Rabin (2006) and Loomes and Sugden (1986) in which a loss corresponds to a situation in which an agent's endogenous expectations have been confounded eliciting disappointment, while a gain corresponds to a situation in which

[5] Bolton and Ockenfels (2000) develop a similar theory of inequity aversion.

[6] Desert utility can be understood as a kind of "inequity aversion" if the latter is taken to denote the preferences of agents who dislike departures from "fair" outcomes. We adopt the terminology "desert utility" because agents in our model are averse to a particular kind of unfairness - namely the unfairness associated with failing to receive one's just deserts, which depend on the vector of efforts but not on the monetary payoff of the other agent. Extending Fehr and Schmidt's notion of inequity aversion to allow aversion to differences in payoffs net of effort costs, as done by Demougin and Fluet (2003) in the context of tournaments, implies that, unlike in our set-up, the payoff of the other agent enters into the reference point (together with the difference in effort costs).

her expectations have been exceeded eliciting elation.

In contrast to both Kőszegi and Rabin (2006) and Loomes and Sugden (1986), who focus on the decision problem of a single agent, our focus is on a two-agent setting in which strategic considerations are operative. As in Loomes and Sugden's (1986) model, but in contrast to Kőszegi and Rabin (2006), our reference points are sensitive to agents' actual choices, rather than depending on agents' prior expectations about those actions.[7] Assuming desert elation rather than desert guilt, our model of preferences can therefore be interpreted as a model of disappointment aversion with strategic interactions. In this sense, our paper can be viewed as an exploration of further behavioral implications of disappointment aversion.

1.1 An Example

For concreteness, think of two salesmen of equal ability competing on a shop floor for a bonus paid to the salesman who sells the most units over a month. This bonus could for example be a direct monetary payment, a promotion, or the monetary value of the reputational effect linked to being publicly made salesman of the month. Suppose the two salesmen work equally hard, but fortuitously one happens to win because a few easy sales happened to come his way. Our thesis is that the loser suffers a psychological cost and that this cost is greater than any elation experienced by the winner. If the loser had worked less hard than his rival, we claim that his desert loss would have been lessened as he would have felt he deserved less than his rival, while if he had worked harder, his desert loss would have been even greater. Finally, we posit that the salesmen anticipate any psychological costs or benefits arising from desert when choosing how hard to work.

1.2 Summary of Findings

Lazear and Rosen (1981) found that given identical agents compete in the tournament there is a unique and symmetric Nash equilibrium. We find that weak desert concerns do not affect the equilibrium, so effort choices are unchanged, although desert does lower expected payoffs. However, sufficiently large desert concerns render the unique symmetric equilibrium that prevails in the absence of desert concerns either unstable or non-existent. Moreover, asymmetric desert equilibria can arise in which one agent exerts high effort and the other agent low effort even if agents are otherwise identically situated. Intuitively, when identical agents exert the same level of effort they have an equal chance of winning, but one ends up winning and the other losing

[7] Shalev (2000) also considers reference points which vary with the chosen action in a strategic setting. Shalev takes the reference point to be the expected utility of the action, *including* expected reference-dependent losses and gains.

even though neither is more deserving than the other. The greater the difference in the agents' efforts, the greater the probability that an outcome emerges in which the more hardworking and therefore more deserving agent wins and the closer on average are both agents' reference points to their actual payoffs. This means that in expectation the losses from desert are at a maximum when the effort difference is zero and are falling as the effort difference rises. In other words, desert concerns give agents an incentive to choose different levels of effort.

In the tournament context, desert concerns entail that an agent's objective function has a simple mean-variance form. It is important to note, however, that risk aversion (in the absence of desert concerns) will not generate such an objective function. Lazear and Rosen's (1981) finding that with standard agents there is a unique and symmetric Nash equilibrium holds even after introducing risk aversion over monetary payoffs.

We develop two applications of our model, the first using asymmetric equilibria and the second symmetric ones. First, we interpret our tournament as a zero-sum competition for status/rank (in which effort does not increase the size of the pie). We find that desert concerns can undermine the standard conclusion that competition for the fixed supply of status is socially wasteful. Competition can allow asymmetric equilibria, lowering desert losses, so the agents may in fact prefer to compete for a fixed supply of status rather than have that status allocated randomly.

Second, we turn to an employer's choice of relative performance compensation scheme. When desert concerns are weak, so employees continue to play a symmetric equilibrium, we find that an employer prefers a tournament to relative performance pay linear in the difference in employees' outputs when the distribution of output noise is sufficiently fat-tailed. In that case, desert losses under the linear scheme become too costly. Our theory of desert thus provides a new explanation for why employers might choose to use tournament-style incentive schemes.

By merging the literatures on tournaments, equity and loss aversion, we have been able to develop an empirically motivated meritocratic theory of desert in competitive situations. In doing so, we have contributed to the understanding of strategic behavior in the presence of loss aversion with endogenous reference points. Our novel insights into the behavior of desert-motivated agents are of interest in themselves, but also have important implications in any applied setting where the size of desert losses matter.

The paper proceeds as follows. Section 2 sets out the model. Section 3 derives general results, provides a simple analytical example and discusses experimental testing. Section 4 presents two applications, the first to status competition and the second to the design of incentive schemes. Section 5 concludes. The appendix presents omitted proofs.

2 Model

Two identical agents are competing to win a fixed prize of monetary value $v > 0$ in a Lazear and Rosen (1981) type rank-order tournament.[8] (Later, in Section 4.2, the prize will be chosen optimally by a firm.) The agents simultaneously choose effort levels $e_i \in [0, \infty)$ at a twice continuously differentiable cost $C(e_i)$, where $C(0) = 0$, $C'(0) \geq 0$, $C'(e_i) > 0$ for $e_i > 0$, and $C''(e_i) > 0$. Agent $i's$ output is given by $\psi_i = e_i + \epsilon_i$ where the noise term $\epsilon_i \backsim f(\epsilon_i)$ with $E[\epsilon_i] = 0$ and $Var[\epsilon_i] = \sigma^2$. The ϵ_i's are i.i.d. across the two agents. The agent with the higher ψ_i wins the prize. Let $P_i(e_i, e_j)$ represent i's probability of winning and let $\eta \equiv \epsilon_j - \epsilon_i$. As the ϵ_i's are i.i.d., $\eta \backsim q(\eta)$ with $E[\eta] = 0$, $Var[\eta] = 2\sigma^2$ and $q(\eta)$ is symmetric about zero. We assume that the c.d.f. of η, $Q(\cdot)$, is twice continuously differentiable, that $q(\eta) > 0 \ \forall \eta$, and that $vq(0) > C'(0)$. Then:

$$\begin{aligned} P_i(e_i, e_j) &= \Pr\left[\psi_i \geq \psi_j\right] = \Pr\left[e_i + \epsilon_i \geq e_j + \epsilon_j\right] = \Pr\left[e_i - e_j \geq \eta\right] = Q(e_i - e_j) \in (0, 1) \\ P_j(e_i, e_j) &= 1 - P_i(e_i, e_j) = 1 - Q(e_i - e_j) \end{aligned}$$

We capture the agents' desert concerns by supposing that each cares not only about her monetary payoff y_i, but also about the comparison of this payoff with a reference point r_i that represents the payoff that she perceives that she deserves. We assume that the agents feel they deserve their expected monetary payoff,[9] so:

$$r_i = E[y_i] = vP_i(e_i, e_j) = vQ(e_i - e_j)$$

Given the ex ante symmetry of the agents, an agent's expected payoff reflects the useful (to her) effort she has exerted relative to her rival and thus plausibly represents the proportion of the prize that she feels she deserves.[10] Our notion of desert is meritocratic: $e_i = e_j \Rightarrow P_i = P_j = \frac{1}{2}$ and $\frac{\partial vP_i(e_i,e_j)}{\partial e_i} = vq(e_i - e_j) > 0$ so an agent feels she deserves more than her rival if and only if she has put in more effort. The agents' notion of desert is also consistent in that $r_i + r_j = v$.

The agents are assumed loss averse around their endogenous reference points. In particular,

[8] Introducing a prize for the loser has no effect on the analysis, as all payoffs are increased by a constant. v would then be the difference between the winner's prize and the loser's prize.

[9] As noted in Section 1, the concept of a reference point given by endogenous expectations follows Kőszegi and Rabin (2006) and Loomes and Sugden (1986). The analysis would be unaltered if agents compared monetary payoffs net of effort costs to expectations net of effort costs, as the efforts costs would cancel in the comparison.

[10] Alternatives might be that $r_i = \frac{e_i}{e_i+e_j}v$ or $r_i = \frac{C(e_i)}{C(e_i)+C(e_j)}v$ (with $r_i = r_j = \frac{v}{2}$ when $e_i = e_j = 0$). However, both these formulations have the disadvantage of being discontinuous at zero. In particular if one agent slacked off completely and the second put in just a tiny bit of effort, the second would feel she deserved all of the prize. Also, these formulations take no account of the varying marginal impact of effort on winning probabilities - we find it plausible that the greater the impact of a unit of effort, the stronger the effect on feelings of desert.

agent i's instantaneous utility is assumed to be separable in money, desert payoff and effort cost as follows:

$$U_i\left(y_i, e_i, e_j\right) = y_i + D_i\left(y_i - r_i\right) - C\left(e_i\right)$$

where desert utility $D_i(x)$ has the following piecewise linear reference-dependent form:

$$D_i\left(x\right) = \begin{cases} gx & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ lx & \text{if } x < 0 \end{cases}$$

lx represents the utility associated with situations in which $y_i < r_i$ and the agent receives less than she deserves. In that case we say that the agent suffers a *desert loss* and we assume that such losses are unambiguously painful, so $l > 0$.

gx represents the utility associated with situations in which $y_i > r_i$ and the agent receives more than she deserves. g can be positive or negative depending on whether the agent's preferences exhibit *desert elation* or *desert guilt*. We restrict $g > -1$ to avoid giving the tournament winner an incentive to forgo material utility to reduce guilt (either by burning money or making a transfer to the loser).

Let $\lambda \equiv l - g$. The assumption of loss aversion implies that $\lambda > 0$, i.e., $l > g$, so desert losses resonate more strongly than any desert elation, as is consistent with Prospect Theory (see Kahneman and Tversky (1979), p. 279). The utility function is kinked at the reference point, but this kink is not essential for our results on tournaments (in the sense that there exist unkinked $D_i(x)'s$ under which our results continue to hold).[11]

The above entails the following formulation for expected utility:

$$\begin{aligned} EU_i\left(e_i, e_j\right) &= P_i\left[v + g\left(v - r_i\right)\right] + \left(1 - P_i\right)\left[0 - l\left(r_i - 0\right)\right] - C\left(e_i\right) \\ &= P_i\left[v + g\left(v - vP_i\right)\right] + \left(1 - P_i\right)\left(-lvP_i\right) - C\left(e_i\right) \\ &= vP_i - v\lambda P_i\left(1 - P_i\right) - C\left(e_i\right) \end{aligned} \quad (1)$$

(1) looks like a representation of mean-variance preferences, so we might suspect that desert introduces similar effects to risk aversion. However, in the absence of desert introducing risk aversion over monetary payoffs (as done by Nalebuff and Stiglitz (1983)) does not change qualitative behavior, so our simple tournament structure nicely disentangles the effects of desert

[11] Suppose that instead of linear desert preferences, agents exhibited an unkinked quadratic loss function with $D_i(x) = gx^2$ for $x > 0$, $D_i(x) = -l\left(-x\right)^2$ for $x \leq 0$ and $l = -g$. Then $EU_i = P_i\left[v + g\left(v - r_i\right)^2\right] + \left(1 - P_i\right)\left[0 - l\left(r_i - 0\right)^2\right] - C(e_i)$ which simplifies to $vP_i - v^2 lP_i\left(1 - P_i\right) - C(e_i)$. This has the same form as (1), replacing λ with vl. Thus, all our results on tournaments continue to hold (qualitatively), demonstrating that they do not depend on the kink.

from those of risk aversion.[12] A corollary is that if our identical agents were loss averse around a fixed reference point they would behave qualitatively as in the standard model, as such loss aversion can be represented by a risk averse concave utility function. Thus the endogeneity of our reference points is crucial to our results.

Notice that instead of thinking of each agent as having a fixed conception of how much she deserves equal to her expected payoff, we could instead model the agent as holding two reference points, corresponding to a win and a loss. The agent then compares her actual payoff to both reference points, weighting the desert payoffs in each case by the probability of each reference point. This transplants Kőszegi and Rabin (2006)'s reference lottery concept to our desert setting (see their equation (2)). As they put it: *"the sense of gain or loss from a given consumption outcome derives from comparing it with all outcomes possible under the reference lottery"* (p. 1137). Applying such a framework, we get:

$$\begin{aligned} U_i(Win) &= v + P_i\left[-l(v-v)\right] + (1-P_i)g(v-0) - C\left(e_i\right) \\ U_i(Loss) &= P_i\left[-l(v-0)\right] + (1-P_i)g(0-0) - C\left(e_i\right) \end{aligned}$$

Calculating the *ex ante* EU_i, we end up with (1) again, just as when the reference point is simply the expected payoff.

The game's payoff functions are common knowledge. Taking e_j as given, agent i chooses her effort to maximize her expected utility, $EU_i\left(e_i, e_j\right)$. We assume that the problem of moral hazard precludes the agents' use of insurance.[13] Having exerted their chosen effort levels, the agents receive their monetary payoffs and also observe the effort level exerted by the other agent.[14] The choice of effort affects EU_i in the standard way by altering expected material utility $E\left[y_i\right]$ and the cost of effort $C\left(e_i\right)$. Effort also affects utility via the desert function, first by altering the distribution of monetary payoffs, but also by changing the agent's reference point r_i. The more effort the agent puts in, the more she feels she deserves. Holding the distribution of monetary payoffs fixed, higher effort increases the scope for desert losses and reduces the scope for undeserved gains and hence the potential for desert elation or guilt.

[12] Assuming that $U_i = \phi(y_i) - C(e_i)$ with $\phi' > 0$ and $\phi'' < 0$, $EU_i = P_i\phi(v) + (1-P_i)\phi(0) - C\left(e_i\right)$ which equals $P_i\left[\phi(v) - \phi(0)\right] + \phi(0) - C\left(e_i\right)$. This utility function does not exhibit mean-variance preferences. The reason is that with only two possible outcomes, success or failure, all the payoff-relevant information is captured by P_i. Thus, in the absence of desert introducing risk aversion does not change the tournament analysis qualitatively: normalizing $\phi(0)$ to zero, we can simply replace v with $\phi(v)$.

[13] Furthermore, narrow framing of losses and gains (see Barberis et al.) may prevent the agents from diversifying the variability of their desert payoffs across multiple tournaments or other events.

[14] In the absence of such observability, the psychological foundation for desert payoffs would be undermined as payoffs would depend on uncertain conjectures about rivals' actions. Also, agents might attempt to infer information about e_j from y_i, complicating the problem. Finally, the impact of deviations on rival payoffs would be different, an issue if the game were repeated.

We call a Nash equilibrium of this game a *desert equilibrium,* and we restrict attention to pure-strategy equilibria. In a desert equilibrium, each agent's effort choice is optimal given the effort chosen by her rival. Note that in contrast to Rabin (1993)'s fairness equilibrium, our game is standard in the sense that payoffs depend purely on actions and not on beliefs about agents' actions or intentions. Thus, unlike Rabin, we do not have to rely on Geanakoplos et al. (1989)'s concept of a psychological equilibrium, and all the usual existence results apply to desert equilibria.

3 Results

Referring to (1), we can measure the *strength of desert* by λ. In the limit as $g \to l$, so $\lambda \to 0$, expected desert payoffs tend to zero and hence do not affect behavior. Note also that because $\lambda > 0$ and $P_i < 1$, expected desert payoffs are always strictly negative. Letting

$$\Omega_i\left(e_i - e_j\right) \equiv P_i\left(1 - P_i\right) = Q\left(e_i - e_j\right)\left(1 - Q\left(e_i - e_j\right)\right), \tag{2}$$

we call the expression $v\lambda\Omega_i\left(e_i - e_j\right)$, which is strictly positive, agent i's *desert deficit.* From (2) we can see that the desert deficit is a concave function of P_i that is maximized at $P_i = \frac{1}{2}$. It also follows that the desert deficit is strictly quasi-concave in the effort difference, and hence in each agent's effort, since

$$\frac{\partial\Omega_i(e_i-e_j)}{\partial e_i} = \frac{\partial\Omega_i(e_i-e_j)}{\partial(e_i-e_j)} = q\left(e_i - e_j\right)\left(1 - 2Q\left(e_i - e_j\right)\right) \tag{3}$$

which is strictly positive whenever $e_i < e_j$ since then $Q\left(e_i - e_j\right) < \frac{1}{2}$, strictly negative if $e_i > e_j$ and zero if $e_i = e_j$. Finally, note that $P_i = 1 - P_j$, so $P_i\left(1 - P_i\right) = P_j\left(1 - P_j\right)$, and the agents always face the same desert deficit, i.e., $\Omega_i\left(x\right) = \Omega_j\left(-x\right)$. By the symmetry of the agents, $\Omega_j\left(-x\right) = \Omega_i\left(-x\right)$. Putting these two facts together, we see that $\Omega_i\left(e_i - e_j\right)$ is symmetric about zero. These properties of the desert deficit are summarized in the following lemma.

Lemma 1 *Each agent's desert deficit is given by the function* $v\lambda\Omega_i\left(e_i - e_j\right)$ *where* $\Omega_i\left(x\right) \equiv Q\left(x\right)\left(1 - Q\left(x\right)\right)$. $\Omega_i\left(x\right)$ *is (i) strictly positive and strictly quasi-concave for all* x; *(ii) maximized at* $x = 0$ *where* $P_i = \frac{1}{2}$; *and (iii) symmetric about zero.*

Intuitively, when agents exert equal efforts and thus have equal chances of winning, both the winner and loser end up far from their common reference point. As one increases her effort above the other and so the chances of winning become less equal, the expected payoff of the favorite and the underdog become less equal, and it becomes more likely that the favorite wins.

Thus, the average departure between agents' monetary payoffs and their reference points falls, reducing the scope for both desert losses and desert elation or guilt. Since losses loom larger than any elation by assumption, the overall desert deficit falls for both agents.

Next we note the following:

$$\frac{\partial P_i(e_i,e_j)}{\partial e_i} = \frac{\partial Q(e_i-e_j)}{\partial(e_i-e_j)} = q\left(e_i-e_j\right) \tag{4}$$
$$\frac{\partial P_j(e_i,e_j)}{\partial e_j} = \frac{-\partial Q(e_i-e_j)}{\partial(e_i-e_j)}(-1) = q\left(e_i-e_j\right) \tag{5}$$

Thus, at any (e_i, e_j) pair, the agents face the same marginal effect of effort on the probability of winning. This is because an increase in e_i is equivalent to a decrease in e_j as winning probabilities depend on $e_i - e_j$, while the impact of e_i on P_i is the opposite of its impact on P_j as $P_i = 1 - P_j$.

Using (1), (4) and (5), the first order conditions (FOCs) are:

$$\frac{\partial EU_i}{\partial e_i} = vq\left(e_i-e_j\right) - v\lambda\left[\left(1-2P_i\right)q\left(e_i-e_j\right)\right] - C'\left(e_i\right) = 0 \tag{6}$$
$$\frac{\partial EU_j}{\partial e_j} = vq\left(e_i-e_j\right) - v\lambda\left[\left(1-2P_j\right)q\left(e_i-e_j\right)\right] - C'\left(e_j\right) = 0$$

while the second order conditions (SOCs) are:

$$\frac{\partial^2 EU_i}{\partial(e_i)^2} = v\frac{\partial q(e_i-e_j)}{\partial e_i} - v\lambda\left[\left(1-2P_i\right)\frac{\partial q(e_i-e_j)}{\partial e_i} - 2\left[q\left(e_i-e_j\right)\right]^2\right] - C''\left(e_i\right) \leq 0 \tag{7}$$
$$\frac{\partial^2 EU_j}{\partial(e_j)^2} = v\frac{-\partial q(e_i-e_j)}{\partial e_i} - v\lambda\left[\left(1-2P_j\right)\frac{-\partial q(e_i-e_j)}{\partial e_i} - 2\left[q\left(e_i-e_j\right)\right]^2\right] - C''\left(e_j\right) \leq 0$$

3.1 Symmetric Equilibria

As originally discovered by Lazear and Rosen (1981), in the absence of desert ($l = g = \lambda = 0$) the FOCs imply that, given the strict convexity of $C\left(e_i\right)$, any (pure-strategy) equilibrium must be symmetric and unique. Such an equilibrium will be given by $e_i^* = e_j^* = C'^{-1}\left(vq\left(0\right)\right)$, with $e_i^* = e_j^* > 0$.[15] Asymmetric equilibria are not possible, as at any (e_i, e_j) pair the agents' marginal impact of effort on winning probabilities $q\left(e_i - e_j\right)$ are identical, as explained above.[16] The symmetry of $q(\eta)$ about zero implies that $\frac{\partial q(0)}{\partial e_i} = 0$, so the SOCs in the absence of desert are satisfied as $C''\left(e_i\right) > 0$. As noted by Lazear and Rosen (1981) and by Nalebuff and Stiglitz (1983), even if the local SOCs are satisfied, a pure-strategy equilibrium may not exist if σ^2 is

[15] The assumption that $vq(0) > C'(0)$ ensures that if $e_i = 0$, e_j has a strict incentive to work, so all symmetric equilibrium efforts are strictly positive.

[16] Formally, suppose $e_i^* > e_j^* \geq 0$. $e_i^* > 0 \Rightarrow vq\left(e_i^* - e_j^*\right) = C'\left(e_i^*\right)$. But then $vq\left(e_i^* - e_j^*\right) > C'\left(e_j^*\right)$, so j has a strict incentive to work harder. Note that this argument does not depend on the assumption that $vq(0) > C'(0)$.

too low.[17] In what follows, we assume existence in the absence of desert.

In the presence of desert considerations, any symmetric equilibrium will be the same as without desert. Assuming symmetry, $e_i = e_j$ so $P_i = P_j = Q(0) = \frac{1}{2}$. Thus, $1 - 2P_i = 0$ and the desert term in the FOCs drop out. However for sufficiently strong desert, the SOCs will no longer be satisfied. Given symmetry, agent i's SOC reduces to $2v\lambda\left[q\left(0\right)\right]^2 \leq C''\left(C'^{-1}\left(vq\left(0\right)\right)\right)$, which only holds for λ sufficiently small.

On i's reaction function (RF), $d\frac{\partial EU_i}{\partial e_i} = \frac{\partial^2 EU_i}{\partial (e_i)^2} de_i^* + \frac{\partial^2 EU_i}{\partial e_j \partial e_i} de_j = 0$, so $\frac{de_i^*}{de_j} = -\frac{\frac{\partial^2 EU_i}{\partial e_j \partial e_i}}{\frac{\partial^2 EU_i}{\partial (e_i)^2}}$. The cross-derivative is:

$$\frac{\partial^2 EU_i}{\partial e_j \partial e_i} = v\frac{-\partial q(e_i - e_j)}{\partial e_i} - v\lambda\left[\left(1 - 2P_i\right)\frac{-\partial q(e_i - e_j)}{\partial e_i} + 2\left[q\left(e_i - e_j\right)\right]^2\right] \tag{8}$$

At a symmetric equilibrium, we found above that $\frac{\partial q(0)}{\partial e_i}$ and $1 - 2P_i = 0$, and the SOC must hold. Thus, at a symmetric equilibrium the slope of $i's$ RF is:[18]

$$\frac{de_i^*}{de_j} = \frac{-2v\lambda[q(0)]^2}{C''(C'^{-1}(vq(0))) - 2v\lambda[q(0)]^2} \leq 0$$

The RFs are weakly downwards sloping and have the same slope, so any symmetric equilibrium will be asymptotically stable[19] if and only if:

$$\frac{de_i^*}{de_j} > -1 \Leftrightarrow 4v\lambda\left[q\left(0\right)\right]^2 < C''\left(C'^{-1}\left(vq\left(0\right)\right)\right)$$

Absent desert, stability is automatic, but for sufficiently strong desert any symmetric equilibrium will be asymptotically unstable. The following proposition summarizes these findings.

Proposition 1

In the absence of desert considerations, the equilibrium is symmetric and unique. Any symmetric desert equilibrium must be the same as without desert.

For $\lambda \in \left[\frac{C''\left(C'^{-1}(vq(0))\right)}{4v[q(0)]^2}, \frac{C''\left(C'^{-1}(vq(0))\right)}{2v[q(0)]^2}\right]$, *such a symmetric desert equilibrium will be asymptotically unstable.*

For $\lambda > \frac{C''\left(C'^{-1}(vq(0))\right)}{2v[q(0)]^2}$, *i.e., for sufficiently strong desert, such a symmetric desert equilibrium cannot exist (as the second order conditions will be violated).*

[17] If σ^2 is too low, $vq\left(0\right)$ may be so high that at the local symmetric equilibrium the agents prefer to deviate to zero effort. Of course, global concavity of the objective function would guarantee existence.

[18] If $\lambda = \frac{C''\left(C'^{-1}(vq(0))\right)}{2v[q(0)]^2}$, i.e., the SOCs are satisfied with equality, it may be that no symmetric equilibrium exists. If one does, the slopes of the RFs are undefined. An infinitesimal change in e_j requires a discontinuous jump in e_i^*, so the equilibrium cannot be stable.

[19] See Fudenberg and Tirole (1991, pp. 23-25) for more on asymptotic stability and tâtonnement adjustment processes.

Similarly to the case without desert, the need for global optimality may rule out a symmetric desert equilibrium even if the local SOCs are satisfied. One might wonder whether this might make it impossible for unstable symmetric desert equilibria to exist, but we can show that for sufficiently high noise or convex costs, stable and unstable symmetric desert equilibria can indeed exist (see Appendix B, part (i) in Gill and Stone, 2006).

The result in Proposition 1 that in a symmetric desert equilibrium effort is the same as without desert is driven by the fact that at such a symmetric desert equilibrium, each agent has an equal chance of winning. As a result, Ω_i is maximized, and so from Lemma 1 the desert deficit is at its strongest. Because Ω_i is at an extremum, the effect of desert on *marginal* incentives is zero. The result should be contrasted with Grund and Sliwka's (2005) finding that in tournaments inequity averse agents put in more effort in equilibrium. Grund and Sliwka's agents care about the equity of outcomes irrespective of effort levels and hence any consideration of whether outcomes were deserved or not. Receiving more than the rival induces *compassion* and receiving less gives rise to *envy*. Agents want to work harder to avoid envy and less hard to avoid compassion, and because envy is assumed to be a stronger emotion than compassion, the agents work harder in a symmetric equilibrium.

Around a symmetric desert equilibrium, if e_j goes up slightly, P_i, and hence Ω_i, falls. Thus, agent i's incentive to exert effort is reduced compared to the no desert case. Increasing effort raises the desert deficit by making the expected winnings more symmetrical, and so the RFs become *strictly* downwards sloping rather than flat as in the no desert case. This means that, by contrast to the no desert case, if j can precommit to a level of effort before i chooses her effort, j will have a local strategic incentive to choose effort above the desert equilibrium level. With the power to precommit the derivative of j's utility with respect to her effort level is given by

$$\frac{dEU_j}{de_j} = \frac{\partial EU_j}{\partial e_j} + \frac{\partial EU_j}{\partial e_i}\frac{de_i^*}{de_j}$$

At the desert equilibrium, j's FOC implies that $\frac{\partial EU_j}{\partial e_j} = 0$, the slope of the RFs implies that $\frac{de_i^*}{de_j} < 0$, and $\frac{\partial EU_j}{\partial e_i} = -vq(0) + v\lambda\Omega_j'(0) < 0$ since $q(0) > 0$ and $\Omega_j'(0) = 0$. Thus, $\frac{dEU_j}{de_j} > 0$ and j would like to increase her effort above the equilibrium level. By contrast, as in Dixit (1987), without desert the RFs are flat, so $\frac{de_i^*}{de_j} = 0$ and therefore $\frac{dEU_j}{de_j} = 0$ at the equilibrium.[20]

With sufficiently strong desert, the RFs become sufficiently downwards sloping that in (e_i, e_j) space, RF$_j$ crosses RF$_i$ from above, and so any symmetric desert equilibrium becomes unstable. For very strong desert, the objective function becomes locally convex around the no desert

[20] Dixit (1987) also analyzes asymmetric tournaments in which the favorite (underdog) has a local incentive to precommit to a higher (lower) level of effort.

symmetric equilibrium as the agents have too strong an incentive to create an asymmetry in order to reduce the large desert deficit, so the SOCs no longer hold and there is no symmetric desert equilibrium.

3.2 Asymmetric Equilibria

The finding that when we introduce desert, the symmetric equilibrium might be unstable, or indeed no longer exist at all, leads one to ask whether asymmetric equilibria are possible with desert. We saw above that without desert asymmetric equilibria are impossible in this identical agent model. Furthermore, introducing inequity aversion while retaining symmetry of the agents does not alter this basic finding, as discovered by Grund and Sliwka (2005). In this section, we investigate whether asymmetric equilibria can arise with desert. We start by showing that for sufficiently weak desert concerns, there can be no asymmetric equilibrium.

Proposition 2 *For strictly positive λ close enough to zero, no asymmetric desert equilibrium can exist.*

Proof. See Appendix. ■

Next we show that for sufficiently strong desert concerns, asymmetric equilibria can indeed exist. We prove the existence of equilibria in which one agent works hard while the other slacks off completely, but, depending on λ, the shape of the noise distribution and the cost of effort function, less extreme interior asymmetric equilibria may also exist.

Since the two agents are identical, they will have identical reaction functions. Let $e^*(e_j, \lambda)$ denote agent i's reaction function, or global optimum given e_j and λ.[21] Let $e^*(0, \lambda)$ denote the best response to an opponent exerting zero effort (throughout, if more than one exists, we take $e^*(0, \lambda)$ to refer to the lowest one). We first show that the best response to zero effort is strictly positive, $e^*(0, \lambda) > 0$, and then find that for λ sufficiently large $e^*(e^*(0, \lambda), \lambda)$ - the best response to $e^*(0, \lambda)$ - is equal to zero. Thus, asymmetric desert equilibria exist in which one agent exerts strictly positive effort $e^*(0, \lambda) > 0$ and the other agent slacks off completely.

We start by showing that $e^*(0, \lambda) > 0$. Because we have assumed $vq(0) > C'(0)$, even without desert the best response to no effort is strictly positive. Desert considerations simply increase the incentive to work when the rival slacks off, as doing so reduces the expected desert deficit. As λ rises, the desert deficit gets stronger for any difference in the efforts, so the agent has a stronger incentive to push effort up to reduce the desert deficit, i.e., $e^*(0, \lambda)$ goes up.

[21] vP_i and $-v\lambda P_i(1-P_i)$ are bounded, while $C''(e_i) > 0$ implies that $C(e_i)$ is unbounded. Thus, given the continuity of P_i and $C(e_i)$, a global optimum must exist, as i will not wish to raise e_i indefinitely. In general, more than one might exist.

Lemma 2 *(i)* $e^*(0,\lambda) > 0$*; (ii)* $e^*(0,\lambda)$ *is strictly increasing in* λ*; and (iii)* $e^*(0,\lambda)$ *is unbounded above as* λ *rises.*

Proof. See Appendix. ■

The proof of Proposition 3 also makes use of the following lemma, which follows from the fact that EU_i net of effort costs is a function of $e_i - e_j$, while effort costs depend on e_i. Thus for $e_j > 0$, agent i will wish to set $e_i - e_j$ lower than for $e_j = 0$, as for any $e_i - e_j$ the marginal impact of effort on utility net of costs is the same, but the marginal cost of effort is higher.

Lemma 3 *For* $e_j > 0$*, the difference between an agent's global optimum effort* $e^*(e_j,\lambda)$ *and her rival's effort* e_j *is always less than the best response to zero effort, i.e.,* $e^*(e_j,\lambda) - e_j < e^*(0,\lambda)$.

Proof. See Appendix. ■

By raising λ sufficiently, we can raise $e^*(0,\lambda)$ so high and make desert considerations so important that in response to $e^*(0,\lambda) > 0$ the rival wants to set zero effort to reduce the desert deficit as much as possible (without incurring the huge cost of working harder than her rival, by Lemma 3), so we get asymmetric equilibria. Asymptotic stability follows in non-pathological cases, as the slacker i's reaction function is locally vertical in (e_i, e_j) space, i.e., for small changes in e_j away from $e^*(0,\lambda)$, i wishes to remain at $e_i = 0$.[22]

Proposition 3 *For sufficiently large* λ*: (i) there exist two asymmetric desert equilibria in each of which one agent exerts strictly positive effort* $e^*(0,\lambda) > 0$ *and the other agent exerts zero effort as the unique best response; and (ii) such equilibria are asymptotically stable, so long as* $e^*(e_j,\lambda)$ *changes smoothly in* e_j *at* $e_j = 0$.

Proof. See Appendix. ■

Intuitively, in an asymmetric equilibrium in which i is exerting zero effort and j is exerting high effort $e^*(0,\lambda)$, j is more likely to win and feels that such a win is deserved while i is more likely to lose but feels that such a loss is deserved. If j lowers her effort or i increases hers then

[22] Stone (2004) and Ederer and Fehr (2006) also find that introducing loss aversion into a tournament set-up can allow ex ante identical agents to play asymmetric equilibria. In Stone's model winning probabilities are restricted to be linear and, unlike in this paper but as in Kőszegi and Rabin (2006), the agents take their reference points as fixed when they optimize. In a two-stage tournament with feedback on interim performance, Ederer and Fehr (2006) find that loss averse agents may play an asymmetric equilibrium at the interim stage. The interim leader's advantage gives him a higher reference point and so a greater incentive to exert effort. Cornes and Hartley (2003) introduce loss aversion into a Tullock-style rent-seeking contest. They focus on symmetric equilibria (which always exist), but note that asymmetric equilibria are also possible. Stone (2006) finds that agents with self-image concerns may play an asymmetric equilibrium to avoid revealing too much information to themselves about their own ability.

on average the departure between monetary payoffs and agents' reference points will increase, increasing the desert deficit (see Lemma 1). Thus, agents have an incentive not to reduce the difference in their efforts. For sufficiently large λ this force deters i from increasing her effort above zero, even if doing so would increase her probability of winning sufficiently for the increase in material utility to cover the increase in her effort costs. Given agent i exerts zero effort, j's material and desert payoffs are both increasing as she increases her effort, and she will increase her effort up until the point at which the marginal disutility of effort overwhelms the resulting marginal reduction of the desert deficit and increase in her expected monetary payoff.

Of course, the existence of such asymmetric equilibria raises the question of how the players could coordinate on one of them. Exiting from the strict confines of the game at hand, some form of pre-play communication might aid coordination, or, with repeated play, a tâtonnement adjustment process could lead to one of the equilibria (Proposition 3 tells us that the equilibria are stable to such a process once they are reached). Note also that if the game was played twice, one equilibrium of the repeated game would be for the agents to play an asymmetric equilibrium in each period, but reversing the role of the hard worker and slacker across the periods. In the absence of discounting, the payoffs would become symmetric across the two periods. Again, pre-play communication could help coordinate on the roles.

3.3 Example with Uniform Noise

In this subsection, we solve a simple example analytically to aid the understanding of the general results above. We assume the following.

Assumptions

(i) $\eta \backsim U[-\gamma, \gamma]$

(ii) $C(e_i) = \frac{ce_i^2}{2}$

(iii) $v < \frac{2\gamma^2 c}{1+\lambda}$

Under (i), $q(\eta) = \frac{1}{2\gamma}$ for $\eta \in [-\gamma, \gamma]$ and $Q(e_i - e_j) = \frac{e_i - e_j + \gamma}{2\gamma}$ for $|e_i - e_j| \leq \gamma$.[23] The γ parameter is a measure of noise - the more noise, the lower the marginal effect of effort on the probability of winning. The agents face a linear probability of winning function, up to a bound where $P_i = 1$ at $e_i = e_j + \gamma$ and down to a bound where $P_i = 0$ at $e_i = e_j - \gamma$. As we will now explain, assumption (iii) makes the model consistent with our general framework, in which $q(\eta) > 0$ $\forall \eta$ and $Q(e_i - e_j)$ is twice continuously differentiable. Restricting efforts to the range

[23] No standard underlying noise function that we are aware of would give η uniformly distributed. However, we have chosen a uniform distribution here for its analytical and pedagogical convenience. We can also think of the noise as arising from the measurement of the difference in efforts rather than of each agent's separate effort level.

$|e_i - e_j| \le \gamma$ and using (6) gives:

$$\begin{aligned} \frac{\partial EU_i}{\partial e_i} &= v\frac{1}{2\gamma} - v\lambda\left[\left(1 - 2\left(\frac{e_i - e_j + \gamma}{2\gamma}\right)\right)\frac{1}{2\gamma}\right] - ce_i \\ &= v\frac{1}{2\gamma} + v\lambda\left[\frac{e_i - e_j}{2\gamma^2}\right] - ce_i \\ \frac{\partial^2 EU_i}{\partial (e_i)^2} &= \frac{v\lambda}{2\gamma^2} - c \end{aligned}$$

Thus, the objective function is strictly globally concave (over $|e_i - e_j| \le \gamma$) for $v\lambda < 2\gamma^2 c$. We assume that $v < \frac{2\gamma^2 c}{1+\lambda}$, which implies global concavity, and also that $i's$ optimal effort $e_i^* < \gamma$ according to the following lemma.

Lemma 4 *Given* $v < \frac{2\gamma^2 c}{1+\lambda}$, $e_i^* < \gamma\ \forall e_j \in [0, \infty)$.

Proof. See Appendix. ∎

Lemma 4 allows us to restrict attention to $e_i \in [0, \gamma]$ when calculating equilibria. Thus $|e_i - e_j| \le \gamma$, so $Q(e_i - e_j)$ is twice continuously differentiable, $q(e_i - e_j) > 0$ and the objective function is strictly concave. The FOCs are satisfied where

$$v\gamma + v\lambda(e_i - e_j) = 2\gamma^2 ce_i \Leftrightarrow \frac{v[\gamma - \lambda e_j]}{2\gamma^2 c - v\lambda} = e_i$$

Note that $\frac{v[\gamma - \lambda e_j]}{2\gamma^2 c - v\lambda} < \gamma \Leftrightarrow v + v\lambda - \frac{v\lambda e_j}{\gamma} < 2\gamma^2 c$ which holds by assumption (iii). Therefore, we get the following linear reaction functions:

$$e_i^*(e_j) = \begin{cases} \frac{v[\gamma - \lambda e_j]}{2\gamma^2 c - v\lambda} \in (0, \gamma) & \text{if } e_j < \frac{\gamma}{\lambda} \\ 0 & \text{if } e_j \ge \frac{\gamma}{\lambda} \end{cases}$$

Now, $e_i^*(0) = \frac{v\gamma}{2\gamma^2 c - v\lambda} \lesseqqgtr \frac{\gamma}{\lambda} \Leftrightarrow v\gamma\lambda \lesseqqgtr 2\gamma^3 c - v\gamma\lambda \Leftrightarrow \lambda \lesseqqgtr \frac{\gamma^2 c}{v}$. Thus we get the following proposition, as illustrated in Figures 1 to 4.

Proposition 4

For $\lambda < \frac{\gamma^2 c}{v}$, *the unique desert equilibrium* $e_i^* = e_j^* = \frac{v}{2\gamma c}$ *is symmetric and stable.*
For $\lambda > \frac{\gamma^2 c}{v}$, *the set of desert equilibria is*

$$(e_i^*, e_j^*) \in \left\{\left(\frac{v\gamma}{2\gamma^2 c - v\lambda}, 0\right), \left(0, \frac{v\gamma}{2\gamma^2 c - v\lambda}\right), \left(\frac{v}{2\gamma c}, \frac{v}{2\gamma c}\right)\right\}.$$

The symmetric equilibrium remains, but is unstable, and there are now also two asymmetric stable desert equilibria.

For $\lambda = \frac{\gamma^2 c}{v}$, the reaction functions coincide, so there is a continuum of desert equilibria given by

$$\left(e_i^*, e_j^*\right) \in \left\{ \left(\frac{v\left[\gamma - \lambda e_j^*\right]}{2\gamma^2 c - v\lambda}, e_j^* \right) : e_j^* \in \left[0, \frac{\gamma}{\lambda}\right] \right\}.$$

As we would expect, effort in both the symmetric and asymmetric equilibria is increasing in v and decreasing in c and in the noise γ. In the asymmetric case, as predicted by Lemma 2 higher λ increases the agents' incentive to differentiate and so increases the hard worker's effort.

Figure 1: $\lambda < \frac{\gamma^2 c}{v}$ and $\frac{\gamma}{\lambda} \leq \gamma$

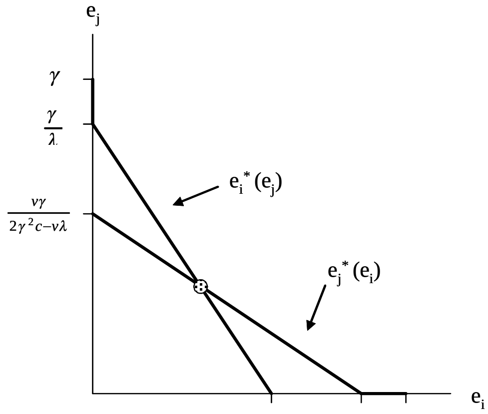


Figure 2: $\lambda < \frac{\gamma^2 c}{v}$ and $\frac{\gamma}{\lambda} > \gamma$

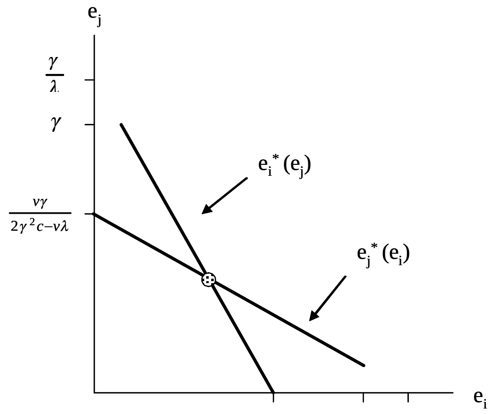


Figure 3: $\lambda > \frac{\gamma^2 c}{v}$

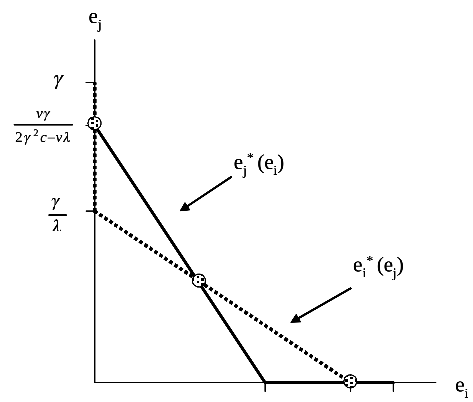
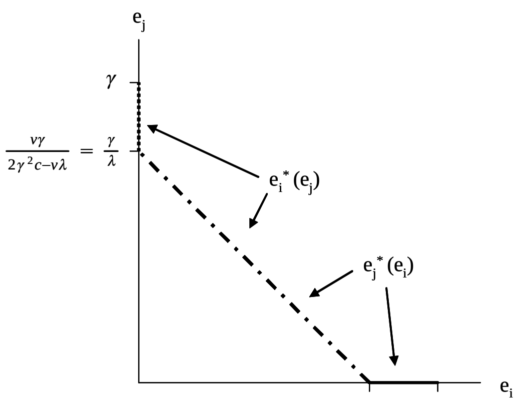


Figure 4: $\lambda = \frac{\gamma^2 c}{v}$



3.4 Experimental Evidence and Testing

The simple example from the previous subsection suggests an experimental test of our theory. In a laboratory setting, we can vary the value of $\frac{\gamma^2 c}{v}$. Our theory implies a specific prediction, which to the best of our knowledge is unique to our model, namely that as $\frac{\gamma^2 c}{v}$ changes from a high to a low value the agents should move from a symmetric equilibrium to an asymmetric one.

The existing experimental evidence on symmetric tournaments tends to report only the mean of effort choices across pairs of players (and not the variance between a pair's effort choices), which would mask any asymmetries. Nonetheless the finding of excess variability in such means (see for instance the classic paper by Bull et al. (1987) and, more recently, van Dijk et al. (2001)) is consistent with the agents attempting to coordinate on an asymmetric equilibrium.[24] Interestingly, in a setting with multiple prizes where the theoretical prediction was for the agents to all exert maximal effort, Harbring and Irlenbusch (2003) report commonly finding highly asymmetric behavior, with some agents in a particular group putting in maximal effort and the others exerting very low effort.

Schotter and Weigelt (1992) and Orrison et al. (2004) experimentally study asymmetric tournaments. In "unfair" tournaments, to win the disadvantaged agent $j's$ output needs to exceed that of the advantaged agent i by a strictly positive margin k. Theory predicts that the equilibrium remains symmetric, but advantaged agents have been found to work harder than their disadvantaged counterparts. In the unfair tournament, $P_i(e_i, e_j) = \Pr\left[e_i + \epsilon_i + k \geq e_j + \epsilon_j\right] = Q\left(e_i - e_j + k\right)$, giving the FOCs as in (6) replacing $q\left(e_i - e_j\right)$ with $q\left(e_i - e_j + k\right)$. Thus the no-desert equilibrium is symmetric with $e_i^* = e_j^* = C'^{-1}\left(vq\left(k\right)\right)$. With desert there can be no symmetric equilibrium as $e_i = e_j \Rightarrow P_i > P_j$. In fact, starting from the no-desert symmetric equilibrium effort pair $\frac{\partial EU_i}{\partial e_i} > 0$ and $\frac{\partial EU_j}{\partial e_j} < 0$, while starting from any symmetric effort pair $\frac{\partial EU_i}{\partial e_i} > \frac{\partial EU_j}{\partial e_j}$, which suggests that desert might help explain the asymmetric effort choices found in practice. This possibility warrants further theoretical and experimental investigation.

4 Applications

We now turn to two applications of our theory of desert, the first using asymmetric equilibria and the second symmetric ones.

First, we analyze competition for status in the presence of desert concerns. We show that when desert is strong enough that agents play an asymmetric equilibrium, competition for status dominates a random allocation of status / rank, even though effort is socially wasteful. Thus desert considerations can provide a psychological basis for allowing agents to compete for a fixed supply of status.

Second, we consider an employer's choice of relative performance compensation scheme. We show that when desert is not too strong, so employees play a symmetric equilibrium, an employer will choose a tournament over relative performance pay linear in the difference in the employees' outputs if output noise is sufficiently fat-tailed. Desert considerations thus provide a new explanation for why employers might choose to use tournament-style incentive schemes.

[24] Note that in most of the experimental literature, and in all the studies cited in this section, agent pairings or groupings are kept constant through multiple repetitions of the stage game.

4.1 Status Competition

Agents often compete for status within a group, where an agent's status is defined as her ordinal rank in the group. Within a group the supply of rank is fixed, which is why a number of authors have considered competition for status to be a socially wasteful zero-sum game. Recent notable examples of such a perspective are presented by Hopkins and Kornienko (2004) and Frank (2005), who consider agents competing for status by spending on positional goods:

"In the equilibrium, the additional expenditure on conspicuous consumption has no effect on the individual's position in the social hierarchy, and thus it is "wasteful" in the sense it leads to a Pareto-inferior outcome" (Hopkins and Kornienko, pp. 1091-1092).

"expenditure arms races focused on positional goods... divert resources from non-positional goods, causing welfare losses" (Frank p. 137).

We can interpret our tournament as a competition for status. Agents care about their relative rank in the distribution of ψ, with ψ_i distributed as before, and they value a higher rank at v. Much of the literature has focused on status as determined by spending on positional goods, and we can think of ψ as such spending, where agents exert effort to increase the budget they can spend on such goods. Our model can also incorporate many other types of status concerns. For example, we can think of winning the tournament as being allocated a position of higher importance in an organization.

In the absence of desert concerns, the zero-sum nature of the final status allocation does indeed lead to the conclusion that competition over status is socially wasteful. In the unique equilibrium of the game, the agents exert the same level of effort so each has an equal opportunity of winning and losing. Thus, the agents would be better off if they could somehow enforce an equal reduction in their efforts, since this would reduce wasteful expenditure on effort while leaving the winning probabilities unchanged. It follows that both agents would be better off if competition was banned and social rank was instead determined randomly.

The conclusion that banning competition is good for welfare may be undermined when agents have desert concerns.[25] This is true whenever asymmetric equilibria can arise. An asymmetric equilibrium has the feature that, more often than not, it will allocate the higher status to the agent who exerted higher effort and therefore deserves it more and the lower status to the agent who deserves it less. By contrast, when status is allocated randomly, as when competition is banned or when competing agents exert identical efforts, the outcome is less satisfactory in desert terms: although neither agent is more deserving than the other, one agent is always deemed the winner and the other the loser at the end of the tournament. Thus, on average,

[25] Of course, the conclusion is also undermined if positive externalities from effort are sufficiently strong. We show that even absent any such externalities, desert considerations can reverse the standard argument.

the discrepancy between what an agent receives ex post and what she deserves given agents' efforts is larger than in any asymmetric equilibrium - as shown in Lemma 1, the desert deficit is always lower the more unequal the winning probabilities. When competition is not permitted, the higher desert deficit can overwhelm the benefit of lower effort, compared to an asymmetric equilibrium under competition.

To see this more formally, we take the analytical example from Section 3.3 in which the probability of winning functions are linear in effort. If $\lambda < \frac{\gamma^2 c}{v}$ then the unique desert equilibrium is symmetric. Thus, randomly allocating status via a coin flip clearly increases welfare as effort costs go down, but the desert deficit each agent faces remains unchanged at $\frac{-v\lambda}{4}$. On the other hand, if $\lambda > \frac{\gamma^2 c}{v}$ then only asymmetric desert equilibria are stable. Assuming such a stable equilibrium to be played, denoting the high effort agent by H and the low effort agent by L, and letting $t \equiv 2\gamma^2 c - v\lambda$, we know from Proposition 4 that $e_H = \frac{v\gamma}{t} \in (0, \gamma)$ and $e_L = 0$. Thus:

$$
\begin{aligned}
P_H &= \frac{\frac{v\gamma}{t}+\gamma}{2\gamma} = \frac{1}{2} + \frac{v}{2t} \in (0,1) \\
P_L &= \frac{\frac{-v\gamma}{t}+\gamma}{2\gamma} = \frac{1}{2} - \frac{v}{2t} \in (0,1)
\end{aligned}
$$

Letting $\Delta_i \equiv EU_i\left(e_i^*, e_j^*\right) - EU_i\left(0,0\right)$ denote the difference between utility with competition and without, and using (1) along with the fact that $P_H = 1 - P_L$:

$$
\begin{aligned}
\Delta_H &= v\left(\tfrac{1}{2} + \tfrac{v}{2t}\right) - v\lambda\left(\tfrac{1}{4} - \left(\tfrac{v}{2t}\right)^2\right) - \tfrac{c}{2}\left(\tfrac{v\gamma}{t}\right)^2 - \left[\tfrac{v}{2} - \tfrac{v\lambda}{4}\right] \\
\Delta_L &= v\left(\tfrac{1}{2} - \tfrac{v}{2t}\right) - v\lambda\left(\tfrac{1}{4} - \left(\tfrac{v}{2t}\right)^2\right) - \left[\tfrac{v}{2} - \tfrac{v\lambda}{4}\right]
\end{aligned}
$$

Thus, using the fact that $t > 0$ in the example,

$$
\begin{aligned}
\sum \Delta_i &= 2v\lambda\left(\tfrac{v}{2t}\right)^2 - \tfrac{c}{2}\left(\tfrac{v\gamma}{t}\right)^2 > 0 \\
&\Leftrightarrow \; v\lambda > \gamma^2 c \Leftrightarrow \lambda > \tfrac{\gamma^2 c}{v}
\end{aligned}
$$

so we get the following.

Proposition 5 *Under the assumptions of Section 3.3, when* $\lambda > \frac{\gamma^2 c}{v}$ *(so only the asymmetric desert equilibria are stable) welfare is lower if competition is banned and status is allocated randomly. Thus even when the supply of status is fixed, competition for status is not always socially wasteful.*

Note that an immediate corollary is that the asymmetric equilibria are more efficient than the unstable symmetric one, as the unstable symmetric equilibrium has even worse welfare properties than imposing zero effort.

Although total utility is higher with competition, the slacker will prefer competition to be banned for $\lambda \in \left(\frac{\gamma^2 c}{v}, \frac{4}{3}\frac{\gamma^2 c}{v}\right)$:[26]

$$\begin{aligned} \Delta_L &= -v\left(\tfrac{v}{2t}\right) + v\lambda\left(\tfrac{v}{2t}\right)^2 < 0 \\ &\Leftrightarrow \lambda < \tfrac{2t}{v} = \tfrac{4\gamma^2 c - 2v\lambda}{v} \Leftrightarrow \lambda < \tfrac{4}{3}\tfrac{\gamma^2 c}{v} \end{aligned}$$

In this range, preventing her rival from competing increases the slacker's desert deficit, but also increases her probability of winning sufficiently to compensate. If desert is too strong, even the slacker prefers the competitive set-up. On the other hand, despite the effort cost the hardworking agent prefers competition for all $\lambda > \frac{\gamma^2 c}{v}$:

$$\begin{aligned} \Delta_H &= v\left(\tfrac{v}{2t}\right) + v\lambda\left(\tfrac{v}{2t}\right)^2 - \tfrac{c}{2}\left(\tfrac{v\gamma}{t}\right)^2 > 0 \\ &\Leftrightarrow v\left(\tfrac{2t}{v}\right) + v\lambda - 2\gamma^2 c > 0 \Leftrightarrow t > 0 \end{aligned}$$

In conclusion, desert considerations can provide a psychological basis for preferring competition, even in what appears to be a zero-sum game such as a competition for status: such competition is not necessarily socially wasteful.

4.2 Relative Performance Pay Schemes

Firms commonly use tournament-type incentive schemes to motivate staff. (For some recent evidence see DeVaro (2006). See also Conyon et al. (2001), Prendergast (1999) and the references therein.) In this section, we analyze how desert preferences impact on the design of relative performance incentive schemes, thus contributing to the literature which considers when firms might choose to implement tournaments and to the rapidly-growing behavioral contract literature.[27] Throughout, we assume that from the perspective of firms efforts are either unobservable or unverifiable, so compensation must be based on output.

A number of papers have compared tournaments to individual performance-based compensation (such as piece rates). A first strand of literature emphasizes that tournaments iron out common output shocks that are unobservable to the employer. In the presence of risk aversion they will thus be preferred if the variance of the common shock is sufficiently large (see for instance Green and Stokey, 1983). However, tournaments are not the only incentive scheme

[26] In the example, $v < \frac{2\gamma^2 c}{1+\lambda}$ so $\lambda < \frac{2\gamma^2 c - v}{v}$. Note that $\frac{2\gamma^2 c - v}{v} > \frac{4}{3}\frac{\gamma^2 c}{v} \Leftrightarrow \frac{\gamma^2 c}{v} > \frac{3}{2}$, so for $\frac{\gamma^2 c}{v} \leq \frac{3}{2}$, the slacker prefers competition to be banned for all the relevant range of λ, i.e., for all $\lambda \in \left(\frac{\gamma^2 c}{v}, \frac{2\gamma^2 c - v}{v}\right)$.

[27] For a survey of the literature on behavioral contract theory more generally, see Englmaier (2005). Bartling and von Siemens (2004), Itoh (2004) and Rey Biel (2007) all analyze how inequity aversion over monetary payoffs net of effort costs impact on compensation design in very simple frameworks with binary effort choices.

which iron out such shocks: other schemes which base payments on the relative output of workers can also do the trick. A second strand of literature points out that tournaments fix total payments in advance, so employers do not have the incentive to under-report performance that occurs with individual performance schemes (see Malcomson, 1984). Yet again, however, not just tournaments but any relative performance scheme with fixed total payments solves the problem.

We compare how our tournament (which is the simplest rank-based incentive scheme) fares relative to the simplest incentive scheme which takes the magnitude of the difference in outputs into account, namely pay linear in the output difference of the workers. We call the latter linear relative performance pay (LRPP). Both schemes iron out common additive output shocks[28] and entail a fixed total wage bill. In the absence of desert, we find that the employer is indifferent between the two schemes, while in the presence of desert considerations the shape of the noise distribution has a fundamental impact on an employer's choice between these two relative performance pay schemes.[29]

We assume that an employer is designing an incentive scheme for two identical workers with weak desert preferences (who therefore play a symmetric equilibrium under the tournament). The employer is required to induce a total effort $\widehat{e} > 0$. However, our results are robust to allowing the employer to choose productive effort to maximize profits (see Gill and Stone (2006), Corollary 1, p. 24). Under the tournament, the employer chooses a fixed payment of F_T to each worker and a prize v. Under LRPP, the employer pays each worker a wage w_i linear in the difference in their outputs. The employer chooses the strength of incentives α plus a fixed payment F_{LRPP}:

$$w_i = \alpha \left(\psi_i - \psi_j\right) + F_{LRPP}$$

The employer must design the scheme to satisfy the workers' participation constraint. In particular each worker's expected utility must cover her outside option $\overline{U}$. For simplicity we assume that the workers face no ex post credit constraints.[30] We compare wage costs $W(\widehat{e})$ of

[28] We abstract from such shocks in the analysis below, but introducing an additive common output shock does not alter any of our results.

[29] Nalebuff and Stiglitz (1983, pp. 36-37) analyzed LRPP, comparing it to the use of piece rates. The only existing comparison of tournaments and LRPP that we are aware of occurs in McLaughlin (1988, p.235), who claims to find that in a free entry model with risk aversion and normal noise, the tournament can induce greater effort.

[30] In particular, the workers can absorb unlimited ex post penalties. The penalties for performing worse than the rival need not be monetary. Non-monetary payoffs could include making the poor performance public knowledge, assigning the employee to less interesting tasks, giving the employee less responsibility or writing a bad reference at the end of the employment contract. A more realistic scheme might put bounds on the linearity, so for $\left|\psi_i - \psi_j\right|$ greater than this bound, pay is no longer increasing or decreasing in the difference in outputs. However, this substantially complicates the analysis and implies two countervailing effects on the value of the scheme to the employer. First, for a given α such a scheme reduces desert losses, so the fixed payments fall. However, α needs to rise to induce the same level of effort as the expected impact of effort on pay is lowered.

inducing the target from using the tournament to those from using LRPP.

4.2.1 Wage Costs: LRPP

As in the tournament, under LRPP each worker's reference point is taken to be her expected monetary payoff,[31] so:

$$\begin{aligned} r_i &= E\left[\alpha\left(\psi_i-\psi_j\right)+F_{LRPP}\right]=\alpha\left(e_i-e_j\right)+F_{LRPP} \\ w_i-r_i &= \alpha\left[\left(\psi_i-e_i\right)-\left(\psi_j-e_j\right)\right]=\alpha\left[\epsilon_i-\epsilon_j\right]=-\alpha\eta \\ EU_i &= \alpha\left(e_i-e_j\right)+F_{LRPP}+\int_{-\infty}^{0} g\left[-\alpha x\right]q(x)dx+\int_{0}^{\infty} l\left[-\alpha x\right]q(x)dx-C(e_i) \end{aligned}$$

As $q\left(\eta\right)$ is symmetric about zero, $\int_0^\infty xq(x)dx=-\int_{-\infty}^0 xq(x)dx$, and we get

$$EU_i=\alpha\left(e_i-e_j\right)-\lambda\alpha\int_0^\infty xq(x)dx-C(e_i)+F_{LRPP}$$

EU_i is strictly concave, and the first order condition is $\alpha=C'(e_i^*)$, so each worker setting $e_i^*=C'^{-1}\left(\alpha\right)$ is the unique Nash equilibrium.[32] Note that this is independent of λ, as given α the expected desert loss is the same for any e_i-e_j. However, desert will affect how much the employer needs to pay his workers to satisfy the participation constraint.

To induce total effort of $\widehat{e}$, the employer sets $\alpha=C'(\frac{\widehat{e}}{2})$, so

$$EU_i=-\lambda C'(\tfrac{\widehat{e}}{2})\int_0^\infty xq(x)dx-C(\tfrac{\widehat{e}}{2})+F_{LRPP}$$

To satisfy the participation constraint, he must set

$$F_{LRPP}\left(\widehat{e}\right)=\lambda C'(\tfrac{\widehat{e}}{2})\int_0^\infty xq(x)dx+C(\tfrac{\widehat{e}}{2})+\overline{U} \tag{9}$$

The total wage costs $W_{LRPP}\left(\widehat{e}\right)$ are simply $2F_{LRPP}\left(\widehat{e}\right)$. The employer needs to compensate the workers both for the cost of effort and for the expected desert loss. As $\widehat{e}$ rises, so higher powered incentives are required, both costs to the employer are increasing. Wage costs are also increasing in λ.

4.2.2 Wage Costs: Tournament

We assume here that $\widehat{e}$ is induced by a symmetric desert equilibrium in the tournament, i.e., λ is not too high (see Propositions 1 and 2). Under the tournament, the monetary payoff y_i and

[31] As in the tournament, the reference point reflects the marginal impact of effort on the expected payment and is meritocratic and consistent.

[32] Of course, if $\alpha\leq C'(0)$ we have a corner solution at $e_i^*=0$.

the reference point r_i are both increased by F_T, so the desert deficit term is unaffected. Thus, as under the LRPP scheme, conditional on the participation constraint being satisfied behavior is unaltered by the fixed payments. From Section 3.1, to induce total effort $\widehat{e}$ the employer sets $v = \frac{C'(\frac{\widehat{e}}{2})}{q(0)}$, so using (1):

$$EU_i = \frac{v}{2} - \frac{v\lambda}{4} - C(\tfrac{\widehat{e}}{2}) + F_T = \frac{C'(\frac{\widehat{e}}{2})}{2q(0)} - \frac{\lambda C'(\frac{\widehat{e}}{2})}{4q(0)} - C(\tfrac{\widehat{e}}{2}) + F_T$$

To satisfy the participation constraint, he must set

$$F_T(\widehat{e}) = -\frac{C'(\frac{\widehat{e}}{2})}{2q(0)} + \frac{\lambda C'(\frac{\widehat{e}}{2})}{4q(0)} + C(\tfrac{\widehat{e}}{2}) + \overline{U}$$

and wage costs $W_T(\widehat{e})$ are $v + 2F_T(\widehat{e})$ so

$$W_T(\widehat{e}) = \frac{C'(\frac{\widehat{e}}{2})}{q(0)} + 2\left[-\frac{C'(\frac{\widehat{e}}{2})}{2q(0)} + \frac{\lambda C'(\frac{\widehat{e}}{2})}{4q(0)} + C(\tfrac{\widehat{e}}{2}) + \overline{U}\right] = \frac{\lambda C'(\frac{\widehat{e}}{2})}{2q(0)} + 2C(\tfrac{\widehat{e}}{2}) + 2\overline{U} \qquad (10)$$

Just as for LRPP, under the tournament the employer has to compensate the workers both for the cost of effort and for the expected desert loss, and again costs are rising in $\widehat{e}$ and λ.

4.2.3 Wage Costs: Comparison

We now compare the wage costs under LRPP to those under a tournament. Using (9) and (10):

$$W_T(\widehat{e}) \lesseqqgtr W_{LRPP}(\widehat{e}) \Leftrightarrow 2\lambda C'(\tfrac{\widehat{e}}{2})\int_0^\infty xq(x)dx \gtreqqless \frac{\lambda C'(\frac{\widehat{e}}{2})}{2q(0)}$$

First, note that in the absence of desert ($l = g = \lambda = 0$), LRPP and the tournament have the same cost. In each case, the workers need to be compensated just for their effort costs (plus the outside option). However, for $\lambda > 0$, they also need to be compensated for expected desert losses. With $\lambda > 0$, the tournament is cheaper if and only if $4q(0)\int_0^\infty xq(x)dx \geq 1$. Because $q(\eta)$ is symmetric about zero,

$$E[|\eta|] = \int_0^\infty xq(x)dx + \int_{-\infty}^0 |x|q(x)dx = \int_0^\infty 2xq(x)dx$$

and so we get the following proposition.

Proposition 6 *For λ low enough that target effort $\widehat{e}$ is induced by a symmetric desert equilibrium in a tournament, $W_T(\widehat{e}) \lesseqgtr W_{LRPP}(\widehat{e}) \Leftrightarrow 2q(0)E[|\eta|] \gtreqless 1$, i.e., wage costs are lower under the tournament than under linear relative performance pay if and only if $2q(0)E[|\eta|] \geq 1$. In the absence of desert, the two schemes cost the same.*

Remarkably, this result depends only on the shape of the noise distribution. In particular, it is independent of the size of the target $\widehat{e}$, and of the size of λ (within the relevant range). A higher $q(0)$ favors the tournament, as marginal incentives at the symmetric equilibrium are higher and so the prize required to induce a given level of effort is lower. In the absence of desert this would not make the tournament any cheaper, as the fixed payments would have to rise correspondingly to satisfy the participation constraint. However, in the presence of desert a lower prize lowers the desert deficit, as the workers' wages will be closer to their reference points on average, and so wage costs are lower. Under LRPP, on the other hand, $q(0)$ has no effect on marginal incentives.

Fatter tails as measured by $E[|\eta|]$ also favor the tournament. With fatter tails, η is more likely to be far from its mean, and hence under LRPP the workers are more likely to receive wages far from their reference point, increasing the fixed payments necessary to compensate them for their expected desert losses. Under a tournament, however, the desert deficit depends only on the prize and the probability of winning, which are independent of $E[|\eta|]$ for a given $q(0)$ at the symmetric equilibrium.

In order to see how Proposition 6 applies in practice, we compare the two schemes given specific noise distributions. Suppose first that $\epsilon_i \sim N[0, \sigma^2]$. The ϵ_i's are i.i.d., so $\eta = (\epsilon_j - \epsilon_i) \sim N[0, 2\sigma^2]$. Thus $q(x) = \frac{1}{\sqrt{2\sigma^2}\sqrt{2\pi}} \exp\left(\frac{-x^2}{2(2\sigma^2)}\right)$. To integrate, we use a change of variable, setting $a = \frac{x^2}{2(2\sigma^2)}$. Then $\frac{da}{dx} = \frac{2x}{2(2\sigma^2)}$, so $2(2\sigma^2)\, da = 2x dx$ and hence:

$$\begin{aligned} 2q(0)E[|\eta|] &= 2\frac{1}{\sqrt{2\sigma^2}\sqrt{2\pi}} \int_0^\infty \frac{2(2\sigma^2)}{\sqrt{2\sigma^2}\sqrt{2\pi}} \exp(-a) da \\ &= \tfrac{2}{\pi}\left[-\exp(-a)\right]_0^\infty = \tfrac{2}{\pi} < 1 \end{aligned}$$

Thus for normally distributed noise, LRPP is always strictly cheaper whatever the variance. As the variance rises, $q(0)$ falls, while $E[|\eta|]$ rises in an exactly compensating fashion. However, under a fatter-tailed distribution, the tournament can dominate. Suppose instead that η is distributed according to the Student's t-distribution with $z \in (1, \infty)$ degrees of freedom. Thus $q(x) = \frac{\Gamma\left(\frac{z+1}{2}\right)}{\sqrt{z\pi}\Gamma\left(\frac{z}{2}\right)} \left(1 + \frac{x^2}{z}\right)^{\frac{-1-z}{2}}$ where Γ is the gamma function.[33] To integrate, we use a change

[33] The variance is undefined for $z \in (1, 2]$, but none of our results are affected. Also note that the use of non-integer degrees of freedom is legitimate: see for instance Shaw (2006).

of variable, with $b = \frac{x^2}{z}$. Then $\frac{db}{dx} = \frac{2x}{z}$, so $2xdx = zdb$ and hence:

$$\begin{aligned} 2q(0)E\left[|\eta|\right] &= 2\frac{\Gamma\left(\frac{z+1}{2}\right)}{\sqrt{z\pi}\Gamma\left(\frac{z}{2}\right)}\int_0^\infty z\frac{\Gamma\left(\frac{z+1}{2}\right)}{\sqrt{z\pi}\Gamma\left(\frac{z}{2}\right)}\left(1+b\right)^{\frac{-1-z}{2}}db \\ &= 2z\left(\frac{\Gamma\left(\frac{z+1}{2}\right)}{\sqrt{z\pi}\Gamma\left(\frac{z}{2}\right)}\right)^2\left[\frac{2}{1-z}\left(1+b\right)^{\frac{1-z}{2}}\right]_0^\infty = \frac{4z}{z-1}\left(\frac{\Gamma\left(\frac{z+1}{2}\right)}{\sqrt{z\pi}\Gamma\left(\frac{z}{2}\right)}\right)^2 \end{aligned}$$

Figure 5 below plots this expression. We can see that $2q(0)E\left[|\eta|\right] \geq 1 \Leftrightarrow z \leq 2$.[34] As z gets large, the expression tends to $\frac{2}{\pi}$, its value under the normal, which is consistent with the fact that the Student's t-distribution approaches the standard normal as $z \to \infty$. Thus, for $z > 2$ degrees of freedom, the employer prefers LRPP, for $z = 2$, the employer is indifferent, and for $z \in (1, 2)$, the tournament dominates. Figure 6 below compares the density functions of a normal (full line) and a Student's t-distribution for $z = \frac{3}{2}$ (dotted line), calibrating the variance of the normal to give a common $q(0)$. The Student's t density has sufficiently fatter tails that the employer prefers to use a tournament.

Figure 5

$2q(0)E\left[|\eta|\right]$ for $z \in (1, 10)$

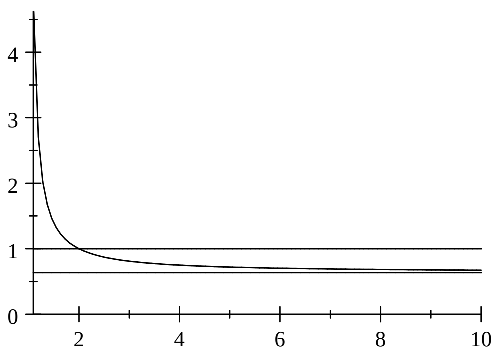
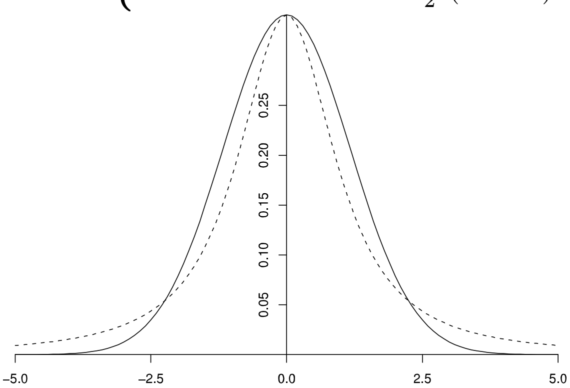


Figure 6

$q(x)$ for $\begin{cases} \text{Normal (full)} \\ \text{Student's t with } z = \frac{3}{2} \text{ (dotted)} \end{cases}$



The following corollary summarizes these findings. Gill and Stone (2006, pp. 25-26) provide a numerical example based on the Student's t-distribution to show that choosing the right compensation scheme can have a significant impact on the wage bill.

Corollary 1 (Corollary to Proposition 6) *For normally distributed noise,* $2q(0)E\left[|\eta|\right] = \frac{2}{\pi} < 1$ *so linear relative performance pay strictly dominates the tournament (for any variance). For noise* η *distributed according to the (fatter-tailed) Student's t-distribution, the tournament strictly dominates linear relative performance pay for* $z \in (1, 2)$ *degrees of freedom.*

We have found that wage costs are lower under a tournament than under LRPP if the distribution of noise is sufficiently fat-tailed, given that the required effort level is induced by

[34] We can confirm that $8\left(\frac{\Gamma\left(\frac{3}{2}\right)}{\sqrt{2\pi}\Gamma(1)}\right)^2 = 8\left(\frac{\frac{1}{2}\sqrt{\pi}}{\sqrt{2\pi}.1}\right)^2 = 1.$

a symmetric desert equilibrium in the tournament. Tournaments may also be preferred if the required effort can be induced by an asymmetric desert equilibrium or if the employer chooses to induce an asymmetric equilibrium when she selects effort optimally. Pushing the workers' effort levels apart lowers the desert deficit, which in turn lowers the fixed fee needed to satisfy the workers' participation constraint. Of course, this is counterbalanced by the fact that it is less efficient to make one worker take on the whole task: $C(\widehat{e}) > 2C(\frac{\widehat{e}}{2})$ by the convexity of costs. See Gill and Stone (2006, pp. 26-28) for an analysis of this issue.

5 Conclusion

In this paper, we have merged the literatures on tournaments, equity and loss aversion in order to model the behavior of desert-motivated agents in competitive situations. In line with existing psychological and experimental evidence, our agents adopt a meritocratic notion of desert. Our model has allowed us to develop novel conclusions about the play of identical agents in tournaments. In doing so, we have contributed to the understanding of strategic behavior in the presence of endogenous reference points. Applying our model has also permitted us to generate important insights into when competition for status might be socially beneficial and when employers might choose tournaments over other more continuous forms of relative performance pay.

Fruitful extensions to our framework might analyze situations with many agents and prizes or with asymmetric agents where some agents enjoy a productivity advantage (perhaps perceived to be unfair or undeserved). Our concept of desert could be applied to other strategic settings such as bargaining and the provision of public goods. Empirical and experimental evidence could also be collected to test whether agents behave according to the theoretical predictions of our model and so do in fact act as if they care about receiving their "just deserts". In particular, assuming desert preferences we have developed novel predictions about how agents move from symmetric to asymmetric equilibria in tournaments as the value of the prize rises and about how firms choose between tournaments and linear incentive schemes as the shape of the noise distribution changes.

Appendix

Proof of Proposition 2. Suppose that an asymmetric equilibrium $\left(e_i^*, e_j^*\right)$ exists with $e_i^* > e_j^*$. From the FOCs in (6), noting that $1 - 2P_j = 2P_i - 1$ and allowing for the possibility that $e_j^* = 0$,

$$\frac{\partial EU_i}{\partial e_i} - \frac{\partial EU_j}{\partial e_j} = 2v\lambda\left[(2P_i - 1)\, q\left(e_i^* - e_j^*\right)\right] - C'\left(e_i^*\right) + C'\left(e_j^*\right) \geq 0$$

Let $\overline{q} \equiv \sup q(\eta)$, which must exist as $q(\eta)$ is a density continuous on $(-\infty, \infty)$. Then $v\lambda\left[(2P_i - 1)\, q\left(e_i - e_j\right)\right] \leq v\lambda\overline{q}$, so $2v\lambda\overline{q} \geq C'\left(e_i^*\right) - C'\left(e_j^*\right)$. Now $C'\left(e_i^*\right) > C'\left(e_j^*\right)$ and as $\lambda \to 0$, $2v\lambda\overline{q} \to 0$. Thus as $\lambda \to 0$, $C'\left(e_i^*\right) - C'\left(e_j^*\right)$ goes to zero, so $e_i^* - e_j^*$ goes to zero. Thus from the FOC e_i^* goes to $C'^{-1}\left(vq(0)\right) > 0$ so e_j^* does so as well.

Let e_x be "near" to $C'^{-1}\left(vq(0)\right)$ iff

$$\left|e_x - C'^{-1}\left(vq(0)\right)\right| \leq \max\left\{\left|e_i^* - C'^{-1}\left(vq(0)\right)\right|, \left|e_j^* - C'^{-1}\left(vq(0)\right)\right|\right\}$$

We now consider the slope of the agents' reaction functions (RFs) for efforts near to $C'^{-1}\left(vq(0)\right)$. The slope of RF$_i$ is given by $-\frac{\frac{\partial^2 EU_i}{\partial e_j \partial e_i}}{\frac{\partial^2 EU_i}{\partial (e_i)^2}}$. As $\lambda \to 0$, (8)$\to 0$ and (7)$\to -C''(e_i)$ (remember $\frac{\partial q(0)}{\partial e_i} = 0$ and q' is continuous) for efforts near to $C'^{-1}\left(vq(0)\right)$, so the slope of the RFs go to zero. In (e_i, e_j) space, RF$_i$ tends to a vertical line while RF$_j$ tends to a horizontal line. Thus there exists a $\delta > 0$ such that for all $\lambda < \delta$, the RFs can only cross once near to $C'^{-1}\left(vq(0)\right)$, so there must be a single equilibrium near to $C'^{-1}\left(vq(0)\right)$. But by the symmetry of the problem there must be (at least) two asymmetric equilibria near to $C'^{-1}\left(vq(0)\right)$, so we get contradiction. ■

Proof of Lemma 2. Note first that given $e_j = 0$, $\Omega_i'(0) = 0$ at $e_i = 0$ and $\Omega_i'(e_i) < 0$ for $e_i > 0$ from Lemma 1.

(i) Suppose $e^*(0, \lambda) = 0$. $\Omega_i'(0) = 0$ and by assumption $vq(0) > C'(0)$. Thus, $\frac{\partial EU_i(e_i, 0, \lambda)}{\partial e_i} = vq(0) - C'(0) > 0$ at $e_i = 0$, so i has a strict incentive to increase effort, a contradiction.

(ii) We suppose that for $\lambda_2 > \lambda_1$, $e^*(0, \lambda_2) \leq e^*(0, \lambda_1)$ and find a contradiction.

Case (a): $e^*(0, \lambda_2) < e^*(0, \lambda_1)$. As $e^*(0, \lambda) > 0$ and $\Omega_i'(e_i) < 0$ for $e_i > 0$, $\Omega_i\left(e^*(0, \lambda_1)\right) - \Omega_i\left(e^*(0, \lambda_2)\right) < 0$. By definition of global optimality:

$$[EU_i(e^*(0, \lambda_1), 0, \lambda_1) - EU_i(e^*(0, \lambda_2), 0, \lambda_1)] + [EU_i(e^*(0, \lambda_2), 0, \lambda_2) - EU_i(e^*(0, \lambda_1), 0, \lambda_2)] \geq 0$$

But

$$\begin{aligned} EU_i(e^*(0,\lambda_1),0,\lambda_1) - EU_i(e^*(0,\lambda_1),0,\lambda_2) &= -(\lambda_1-\lambda_2)\,v\Omega_i(e^*(0,\lambda_1)) \\ EU_i(e^*(0,\lambda_2),0,\lambda_2) - EU_i(e^*(0,\lambda_2),0,\lambda_1) &= -(\lambda_2-\lambda_1)\,v\Omega_i(e^*(0,\lambda_2)) \end{aligned}$$

so we require that $(\lambda_2-\lambda_1)\,v\,[\Omega_i(e^*(0,\lambda_1)) - \Omega_i(e^*(0,\lambda_2))] \geq 0$, a contradiction.

Case (b): $e^*(0,\lambda_2) = e^*(0,\lambda_1)$. Given $e^*(0,\lambda_1) > 0$ the FOCs imply the following which, together with $\Omega_i'(e^*(0,\lambda_1)) < 0$, contradicts $\lambda_2 > \lambda_1$:

$$vq(e^*(0,\lambda_1)) - v\lambda_2\Omega_i'(e^*(0,\lambda_1)) = vq(e^*(0,\lambda_1)) - v\lambda_1\Omega_i'(e^*(0,\lambda_1))$$

(iii) Given $e_j = 0$ and $e_i > 0$, $\Omega_i'(e_i) < 0$. Thus for any $x > 0$, we can find a $\lambda > 0$ such that for $\forall e_i \in (0,x]$:

$$\tfrac{\partial EU_i(e_i,0,\lambda)}{\partial e_i} = vq(e_i) - v\lambda\Omega_i'(e_i) - C'(e_i) > 0$$

so $e^*(0,\lambda) > x$. ■

Proof of Lemma 3. We suppose that $e^*(e_j,\lambda) - e_j \geq e^*(0,\lambda)$ and find a contradiction.

Case (a): $e^*(e_j,\lambda) - e_j > e^*(0,\lambda)$. By definition of global optimality:

$$[EU_i(e^*(0,\lambda),0,\lambda) - EU_i(e^*(e_j,\lambda)-e_j,0,\lambda)] + [EU_i(e^*(e_j,\lambda),e_j,\lambda) - EU_i(e^*(0,\lambda)+e_j,e_j,\lambda)] \geq 0$$

But

$$\begin{aligned} EU_i(e^*(0,\lambda),0,\lambda) - EU_i(e^*(0,\lambda)+e_j,e_j,\lambda) &= -C(e^*(0,\lambda)) + C(e^*(0,\lambda)+e_j) > 0 \\ EU_i(e^*(e_j,\lambda),e_j,\lambda) - EU_i(e^*(e_j,\lambda)-e_j,0,\lambda) &= -C(e^*(e_j,\lambda)) + C(e^*(e_j,\lambda)-e_j) < 0 \end{aligned}$$

so we require that $C(e^*(0,\lambda)+e_j) - C(e^*(0,\lambda)) \geq C(e^*(e_j,\lambda)) - C(e^*(e_j,\lambda)-e_j)$. Because $C''(e_i) > 0$, an increase in effort of e_j from the higher base of $e^*(e_j,\lambda) - e_j$ increases costs by strictly more[35], so we have a contradiction.

Case (b): $e^*(e_j,\lambda) - e_j = e^*(0,\lambda)$. This implies that:

$$vq(e^*(e_j,\lambda)-e_j) - v\lambda\Omega_i'(e^*(e_j,\lambda)-e_j) = vq(e^*(0,\lambda)) - v\lambda\Omega_i'(e^*(0,\lambda))$$

so, given $e^*(0,\lambda) > 0$ from Lemma 2 and $C''(e_i) > 0$, from the FOCs we immediately get a contradiction. ■

[35] $\frac{d(C(x+y)-C(x))}{dx} = C'(x+y) - C'(x) > 0$ for $x \geq 0$ and $y > 0$.

Proof of Proposition 3. Let $e^{**}(\lambda) \equiv e^*(e^*(0,\lambda),\lambda)$. Let $\overline{\lambda}$ be the λ such that $\lambda[1-2Q(-e^*(0,\lambda))] = 1$. As $e^*(0,\lambda) > 0$ is increasing in λ from Lemma 2, $Q'(x) > 0$ and $Q(x) \in \left(0, \frac{1}{2}\right)$ for $x < 0$, we see that $1 - 2Q(-e^*(0,\lambda)) \in (0,1)$ and is increasing in λ, so such a $\overline{\lambda}$ exists and is unique. For $\lambda > \overline{\lambda}$, let $\widehat{e}(\lambda)$ be the e_i such that $\lambda[1-2Q(e_i - e^*(0,\lambda))] = 1$. Since $Q(x) \to \frac{1}{2}$ as $x \to 0$, $\widehat{e}(\lambda) \in (0, e^*(0,\lambda))$ and is unique. Further, $\widehat{e}(\lambda)$ is strictly increasing in λ and unbounded above: as λ goes up, so does $e^*(0,\lambda)$, so $\widehat{e}(\lambda)$ needs to rise by even more. Thus, as $e^*(0,\lambda)$ is unbounded, so is $\widehat{e}(\lambda)$.

(i) We will show that for λ large enough, neither (a) $e^{**}(\lambda) \geq \widehat{e}(\lambda)$ nor (b) $e^{**}(\lambda) \in (0, \widehat{e}(\lambda))$ is possible, so given a global optimum always exists (see footnote 21), $e^{**}(\lambda) = 0$.

Case (a): Suppose $e^{**}(\lambda) \geq \widehat{e}(\lambda)$. Letting $\Delta EU_i(e_i, e_j, \lambda) \equiv EU_i(e_i, e_j, \lambda) - EU_i(0, e_j, \lambda)$:

$$\begin{aligned} \Delta EU_i\left(e^{**}(\lambda), e^*(0,\lambda), \lambda\right) &= v\left[Q\left(e^{**}(\lambda) - e^*(0,\lambda)\right) - Q\left(-e^*(0,\lambda)\right)\right] \\ &\quad - v\lambda\left[\Omega_i\left(e^{**}(\lambda) - e^*(0,\lambda)\right) - \Omega_i\left(-e^*(0,\lambda)\right)\right] \\ &\quad - \left[C\left(e^{**}(\lambda)\right) - C(0)\right] \end{aligned} \tag{11}$$

The first term of (11) is bounded above by v since Q is a c.d.f. The second term is strictly negative, as $|e^{**}(\lambda) - e^*(0,\lambda)| < |-e^*(0,\lambda)|$ and Ω_i is strictly quasi-concave and symmetric about zero from Lemma 1. Where $e^{**}(\lambda) < e^*(0,\lambda)$, the inequality is automatic, while where $e^{**}(\lambda) \geq e^*(0,\lambda)$, $e^{**}(\lambda) - e^*(0,\lambda) < e^*(0,\lambda)$ by Lemma 3. Thus $\Delta EU_i(e^{**}(\lambda), e^*(0,\lambda), \lambda) < v - C(e^{**}(\lambda)) \leq v - C(\widehat{e}(\lambda))$. As $\widehat{e}(\lambda)$ is unbounded above as λ rises and $C''(e_i) > 0$, for sufficiently large λ $\Delta EU_i(e^{**}(\lambda), e^*(0,\lambda), \lambda) < 0$, a contradiction as i would then prefer to set zero effort.

Case (b): Suppose that $e^{**}(\lambda) \in [0, \widehat{e}(\lambda))$. Now, at $e_i = e^{**}(\lambda)$

$$\begin{aligned} \frac{\partial EU_i\left(e_i, e^*(0,\lambda), \lambda\right)}{\partial e_i} &= vq\left(e^{**}(\lambda) - e^*(0,\lambda)\right) - v\lambda\Omega_i'\left(e^{**}(\lambda) - e^*(0,\lambda)\right) - C'\left(e^{**}(\lambda)\right) \\ &= vq\left(e^{**}(\lambda) - e^*(0,\lambda)\right)\left\{1 - \lambda\left[1 - 2Q\left(e^{**}(\lambda) - e^*(0,\lambda)\right)\right]\right\} - C'\left(e^{**}(\lambda)\right) \end{aligned}$$

Since $e^{**}(\lambda) < \widehat{e}(\lambda)$, $\lambda[1 - 2Q(e^{**}(\lambda) - e^*(0,\lambda))] > 1$, so $\frac{\partial EU_i(e_i, e^*(0,\lambda),\lambda)}{\partial e_i} < 0$ at $e_i = e^{**}(\lambda)$, and hence we have a contradiction unless $e^{**}(\lambda) = 0$.

(ii) We start by showing that for small enough $\widetilde{e}_j > 0$, $e_i = 0$ remains the unique global optimum in response to $e_j = e^*(0,\lambda) \pm \widetilde{e}_j$, so where i is the slacker, her reaction function is locally vertical in (e_i, e_j) space. First, consider $e_i \geq \widehat{e}(\lambda)$. For large enough λ, $\Delta EU_i(e_i, e^*(0,\lambda), \lambda) < 0$ as $e^{**}(\lambda) = 0$ is the unique global optimum from above. Furthermore, $\max\limits_{e_i \geq \widehat{e}(\lambda)} \Delta EU_i(e_i, e^*(0,\lambda), \lambda)$ exists as $EU_i(e_i, e^*(0,\lambda), \lambda)$ is unbounded below as e_i goes up. Thus we can find a small enough $\widetilde{e}_j$ such that $\Delta EU_i(e_i, e^*(0,\lambda) \pm \widetilde{e}_j, \lambda) < 0$ $\forall e_i \geq \widehat{e}(\lambda)$. Second, consider $e_i < \widehat{e}(\lambda)$. From above, $\frac{\partial EU_i(e_i,e^*(0,\lambda),\lambda)}{\partial e_i} < -C'(e_i) \leq 0$. Now $\frac{\partial EU_i(e_i,e^*(0,\lambda),\lambda)}{\partial e_i} \leq \max_{e_i<\widehat{e}(\lambda)} \frac{\partial EU_i(e_i,e^*(0,\lambda),\lambda)}{\partial e_i} < 0$, so we can find a small enough $\widetilde{e}_j$ that $\frac{\partial EU_i(e_i,e^*(0,\lambda)\pm\widetilde{e}_j,\lambda)}{\partial e_i} < 0\ \forall e_i < \widehat{e}(\lambda)$. (If the slope is greater at $\widehat{e}(\lambda)$ than any $e_i < \widehat{e}(\lambda)$, no maximum will exist on our range, but then $\frac{\partial EU_i(e_i,e^*(0,\lambda),\lambda)}{\partial e_i} < -C'(\widehat{e}(\lambda)) < 0$.) Asymptotic stability follows immediately given the reaction function of the slacker i is locally vertical in (e_i, e_j) space and the high effort agent j's reaction function has a locally well-defined finite slope. This last follows from assuming $e^*(e_j,\lambda)$ changes smoothly in e_j at $e_j = 0$. ■

Proof of Lemma 4. Suppose first that $e_j - \gamma < \gamma$. If $\frac{\partial EU_i}{\partial e_i} < 0$ at $e_i = \gamma$, no $e_i \in [\gamma, \gamma + e_j]$ can be optimal by the strict concavity of the objective function, and $EU_i(e_i > \gamma + e_j, e_j) < EU_i(e_i = \gamma + e_j, e_j)$ as $P_i = 1$ in both cases, but effort costs are higher in the former. At $e_i = \gamma$, $e_i - e_j \leq \gamma$, so $\frac{\partial EU_i}{\partial e_i} < 0$ given

$$v\frac{1}{2\gamma} + v\lambda\left[\frac{\gamma}{2\gamma^2}\right] - c\gamma < 0 \Leftrightarrow v < \frac{2\gamma^2 c}{1+\lambda}$$

Suppose second that $e_j - \gamma \geq \gamma$. If $\frac{\partial EU_i}{\partial e_i} < 0$ at $e_i = e_j - \gamma$, then no $e_i > e_j - \gamma$ can be optimal by the strict concavity of the objective function. Furthermore, $EU_i(0, e_j) > EU_i(e_i \in (0, e_j - \gamma], e_j)$ as $P_i = 0\ \forall e_i \leq e_j - \gamma$ but effort costs are lowest at $e_i = 0$. The condition for $\frac{\partial EU_i}{\partial e_i} < 0$ is weaker in this case as at $e_i = e_j - \gamma$, $e_i - e_j < \gamma$ and $e_i \geq \gamma$. ■

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