Sequential Innovation and the Duration of Technology Licensing

John Gordanier and Chun-Hui Miao*

Abstract

A large literature has examined the optimal payment scheme in technology licensing. In this paper, by assuming that technology transfer is not completely reversible, we consider the payment scheme and the duration of licensing contracts offered by an innovator with a sequence of possible innovations. We find that it may be optimal to license the innovation for less than the full length of the patent. We also propose a new rationale for the use of royalty contracts: they resolve a time-inconsistency problem faced by the innovator. Our results suggest that licensing contracts based on royalty have a longer duration than fixed-fee licenses and are more likely to be used in industries where sequential innovations are frequent. (JEL D21, D40, L13)

Keywords: Innovation, Licensing, Patent, Royalty, Technology Leakage, Time Consistency.

*Department of Economics, University of South Carolina, Columbia, SC 29208.
In this paper, we study an outsider innovator’s optimal licensing policy. In particular, we consider the optimal payment scheme and the duration of licensing contracts in a setting with potential sequential innovations and some irreversibility of technology transfers. Our main findings are:

1. It can be optimal to issue a license for less than the length of the patent;
2. Even under complete information and risk neutrality, royalty may be more profitable than fixed-fee licensing;
3. Licensing contracts based on royalty tend to have a longer duration;
4. Royalty contracts are more likely to be used in industries where sequential innovations are frequent and intellectual property protection is weak.

Technology transfer through licensing is a common method to utilize a patent. A large literature on technology licensing has studied the optimal payment scheme of selling a cost-reducing innovation (Arrow 1962, Kamien and Tauman 1984, 1986, Katz and Shapiro 1985, 1986, Kamien et al. 1992; see Kamien, 1992 for a survey). It has been shown that licensing by means of a royalty is inferior to that of a fixed-fee or an auction for an outside innovator, regardless of the industry size or the magnitude of the innovation.

Subsequent studies try to explain the wide prevalence of royalties in practice by examining the many variants of the standard model. These studies include models with asymmetric information (Gallini and Wright, 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992; Poddar and Sinha, 2002; Sen, 2005), variation in the quality of innovation (Rockett, 1990), product differentiation (Muto, 1993; Wang and Yang, 1999; Poddar and Sinha, 2004; Stamatopoulos and Tauman, 2003), moral hazard (Macho-Stadler et al., 1996; Choi, 2001), risk aversion (Bousquet et al., 1998), incumbent innovator (Shapiro, 1985; Wang, 1998,

---

1Rostoker (1984) finds that 39% of licensing contracts rely on royalties, while only 15% of them specify a fixed-fee. Macho-Stadler, Martinez-Giralt and Perez-Castrillo (1996) study a sample of 241 contracts. Nearly sixty percent of the contracts are totally based on royalty.
2002; Kamien and Tauman, 2002; Sen, 2002; Sen and Tauman, 2003), Stackelberg leadership (Filippini, 2001; Kabiraj, 2002, 2004) or strategic delegation (Saracho, 2002).

However, surprisingly few studies have examined the duration of technology licensing even though it is an important dimension of every contract. More concretely, should the innovator license the innovation for the entire length of the patent, or should a series of short-term contracts be used? While existing theoretical models implicitly assume that a license remains in effect for the duration of the patent, most actual contract agreements terminate before the underlying patents expire. Anand and Khanna (2000) study the structure of licensing contracts involving at least one US participant, signed during the period 1990-93, and find that no contract agreement lasts more than 10 years, even though the terms of patents range from 14 to 20 years in the US.²

A more interesting fact is the variation in the duration of licensing contracts. Macho-Stadler, Martinez-Giralt and Perez-Castrillo (1996) study a sample of 241 contracts between Spanish and foreign firms and find that contracts based on royalty tend to have a longer duration than fixed-fee contracts. Of the contracts containing fixed payments, 24.5% are one-year contracts, while this proportion falls to 6.2% in the set of contracts containing royalty payments. At the other extreme, 58% of the 174 contracts with royalty payments are long-term contracts (at least five years) while only 15% of the contracts with fixed payments had a duration of at least five years. Using the same dataset, Mendi (2005) studies the impact of contract duration in determining scheduled payments in technology transfer. He finds a positive relationship between contract duration and the probability of the parties including royalties in the first period of the agreement.³

²In the United States, under current patent law, the term of patent are: (1) For applications filed on or after June 8, 1995, the patent term is 20 years from the filing date of the earliest US application to which priority is claimed (excluding provisional applications). (2) For applications that were pending on and for patents that were still in force on June 8, 1995, the patent term is either 17 years from the issue date or 20 years from the filing date of the earliest US application to which priority is claimed (excluding provisional applications), the longer term applying. (3) Design patents, unlike utility patents, have a term of 14 years from the date of issue.

³The author also provides a theoretical model to explain his empirical finding. His model is different from ours in many aspects, but the crucial difference is that he takes the duration of contract as given, while in our model it is endogenous.
In this paper, we introduce a model of technology licensing that analyzes the duration of contracts as well as the optimal payment scheme. The model builds on two observations. First, technology advances are destructive. A new innovation often renders past innovations obsolete. This means that an innovator who engages in a series of innovations potentially faces a time-consistency problem in technology licensing: once a license is sold, the innovator may have an excessive incentive to invest in new technologies (Waldman 1996, Rey and Tirole 2007). This decreases the value of the initial innovation. At the same time, it may be costly to write a complete long-term contract in which license fees are contingent upon the outcome of risky investments for future improvements (Williamson 1975). Therefore, a long-term fixed-fee license can be sub-optimal.

Second, the transfer of knowledge is irreversible. Once transferred, it is difficult for the innovator to retract the knowledge from a licensee (Caves, Crookell and Killing 1983; Brousseau, Coeurderoy and Chaserant 2007). This means that a licensee may be able to utilize an innovation even after the license has expired. We call this "technology leakage" and model it as the licensee retaining a fraction of the cost savings of the initial innovation without renewing the license.¹ Conceptually, we can think of a technology as embodying both tangible assets and intangible know-how. While the termination of a license may stop the use of tangible assets by past licensees, it is difficult, if possible at all, to prevent them from utilizing the technology know-how. The existence of technology leakage creates a potential downside for short-term contracts.

In our model, there are two periods. An innovator sells licenses, which can last a single period or two periods, by either fixed fees or royalties. Whereas long-term fixed-fee contracts potentially prevent technology leakage, they distort the innovator’s incentive to invest in subsequent innovations. Short-term fixed-fee contracts or long-term fixed-fee contracts with opt-out clauses do better, but each is unable to entirely resolve the time-consistency problem.

¹Our use of the term "technology leakage" should be distinguished from the occasional uses in newspaper articles (e.g., "Expulsions Tied To Fear Of Technology Leaks", Philip Taubman, New York Times, April 24, 1983) that refer to the more blatant theft of technologies. In our model, technology leakage is not illegal and is present only because intellectual property protection is imperfect.
because of technology leakage. Long-term royalty contracts do not have a time-consistency problem, but royalty can lower the value of the initial innovation. We derive conditions under which it is optimal for the innovator to license the technology for less than the length of the patent and conditions under which the uses of royalty contracts are optimal.

To our knowledge, Gandal and Rockett (1995) and Antelo (2009) are the only theoretical papers that have examined the optimal duration of licensing contracts.\(^5\) The first paper focuses on fixed-fee licensing and derives conditions under which the innovator licenses the initial technology bundled with all future improvements and conditions under which licenses to each innovation are sold period-by-period. The other paper focuses on royalties in a model of asymmetric information, in which a licensee’s output in a short-term contract signals her cost. Neither paper compares different payment schemes, nor are they concerned with the innovator’s problems identified in this paper.\(^6\)

Our conceptualization of technology leakage is most closely related to papers by Macho-Stadler et al. (1996) and Choi (2001), who have developed incomplete contract models of a licensing relationship that is susceptible to moral hazard. They find that royalties are more often used in licensing agreements embodying know-how than in contracts where no know-how is transferred. Royalty reduces the moral hazard temptation of not actually transferring all the know-how. The problem arises because there are costs of transferring the know-how and hence the innovator has an interest in not disclosing too much information to other firms. Royalties raise the licensor’s incentives to transfer all the know-how. While these papers and ours share the prediction that the use of royalty is positively correlated with the amount

\(^5\)Farrell and Shapiro (2008) consider a variable royalty rate, contingent upon the outcome of a court challenge of the validity of the patent. In their model, the innovator offers licenses to all downstream firms by assumption, therefore a fixed-fee license is offered only if the downstream firm has no competition. In an extension of their model, they consider short-term licenses, which are contracts that do not survive a finding of validity.

\(^6\)A number of papers study the optimal patent policy in markets with sequential innovation (Green and Scotchmer 1995, Scotchmer 1996, O’Donoghue, 1998, Besen and Maskin, 2000, Denicolo 2002). In these models, a sequence of innovations is undertaken by different firms rather than being concentrated in one firm and their focus is on the length and breadth of patents. Oster (1996) is the only other paper that considers the optimal licensing scheme under sequential innovation. By way of an example, she explores the strategic opportunities created by exclusive licensing in a research-intensive market with sequential innovations, but contracts are short-term by assumption in her model.
of know-how involved in technology transfer, there are subtle differences. They implicitly assume that an innovation can be transferred without transferring all necessary know-how, because transferring know-how is costly. Our paper complements theirs by assuming that technology know-how, once transferred, cannot be withdrawn even after the contractual relationship ends.

The remainder of the paper is organized as follows: Section I presents the environment and assumptions of our model of innovation and licensing. In section II, we consider a simple example to illustrate the basic intuition. In Section III, we solve the innovator’s problem in period 2. In Section IV, we find the optimal licensing scheme in period 1 and report comparative statics results. In Section V, we discuss the robustness of our results. Section VI concludes. Any formal proofs omitted from the main text are contained in the appendix.

I. The Model

We consider an industry consisting of \( n \geq 2 \) identical firms all producing the same good with a linear cost function, \( C(q) = c_0q \), where \( q \) is the quantity produced and \( c_0 > 0 \) is the constant marginal cost of production. The inverse demand function of the industry is \( p = \max\{0, a - Q\} \), where \( a > c_0 \) and \( Q \) is the total production level.\(^7\) In addition to the \( n \) firms, there is an innovator that engages in a series of innovations. She seeks to license the innovations to all or some of the \( n \) firms so as to maximize her profit.

The game lasts two periods. At the beginning of period 1, the innovator owns a patent on a cost-reducing innovation, which reduces the marginal cost of production from \( c_0 \) to \( c_1 \). The patent is valid for both periods. At the beginning of period 2, the innovator can make a further investment in R&D. If the new R&D effort is successful, then it will generate a second innovation that reduces the cost of production further to \( c_2 \). Hence \( c_2 < c_1 < c_0 \).

The probability of a successful second innovation is \( Pr \) and it increases with the amount

---

\(^7\)Only some of our results rely on the assumption of a linear demand, which is the most often used demand function in the technology licensing literature. Their reliances on this assumption will be indicated when we present those results.
of investment $I$. For ease of exposition, we assign a particular functional form to $Pr(I)$ such that it equals $2\sqrt{\rho I}$, where $\rho \leq [2\pi^M(c_2)]^{-1}$. The innovator stops all R&D activities after two periods and the game ends. We assume that the initial innovation is drastic, i.e., $p^M(c_1) < c_0$, but the second innovation can be drastic or non-drastic, i.e., $p^M(c_2)$ can be below or above $c_1$.

In order to model technology leakage, we assume that if a downstream firm that licenses the technology $c_1$ in period 1 does not license any innovation in period 2, its marginal cost of production in period 2 is $c' \in [c_1, c_0)$. According to this assumption, a licensee can retain some fraction of the cost saving from the initial technology transfer, even if he does not license that technology in period 2.

Our main interest is in the innovator’s choice of period 1 licensing contracts. We assume that the amount of investment is not observable to outside parties and hence cannot be contracted upon. While it is possible to write a contract that is contingent upon the outcome of the period 2 innovation, it costs $\varphi$ to write such a contract. Since we do not explicitly model the transaction cost $\varphi$ and its impact on the choice of contracts is rather obvious, we assume that $\varphi$ is so large that a contingent contract is never optimal. Therefore, we only consider licenses that specify the payment scheme, the number of licensees and the duration of the contract.

In both periods, the innovator licenses her innovations to $k \leq n$ firms either by a fixed-fee or by a royalty. The duration of a license issued in period 1 can be either one period (short-term) or two periods (long-term). This means that there are four possible types of licensing contracts: short-term fixed fee ($SF$), long-term fixed-fee ($LF$), short-term royalty ($SR$)

---

8This guarantees that the optimal amount of investment will be an interior solution.
9In the case of a drastic innovation, the granting of an exclusive license offers such a large cost advantage that the licensee can effectively monopolize the industry (Arrow, 1962). The case of non-drastic initial innovations is discussed in Section IV.
10There are a variety of reasons why conditional contracts may be even more costly. For example, there may be search costs associated with thinking through the contracts’ implications (Klein 2002) or simply ink costs associated with writing lengthy contracts (Dye 1985).
11To be more specific, it suffices for $\varphi$ to be greater than $\delta \rho (\tau_1 - \tau_2)^2$, where $\tau_1$ and $\tau_2$ are defined in Section III.B.a.
and long-term royalty ($LR$). If the license is a long-term fixed-fee contract, it specifies the payments ($f_1, f_2$) to be made in each of the two periods. Furthermore, a long-term contract may also include an opt-out clause giving the licensees the right to terminate the contract in period 2 without paying the second installment $f_2$. If the license is a royalty contract, then it specifies the royalty rate $r$ for each unit that a licensee sells. All individuals maximize their expected total profits, with a common discount factor of $\delta$. Our solution concept is the subgame perfect equilibrium.

Here is a collection of notations that will be used throughout the paper.

$\Gamma^{LS(k)}$: Gross licensing revenue from a single-period game for licensing scheme $LS \in \{R, FF\}$, where $R$ is royalty, $FF$ is fixed-fee, and $k \in Z^+$ denotes the number of licensees.

$\Pi_t^{LS(k)}$: The innovator’s gross licensing revenue at time $t$ for licensing scheme $LS \in \{SF, LF, SR, LR\}$ and $k \in Z^+$ denotes the number of licensees. It is often convenient to drop the superscript when doing so is unambiguous. Especially in period 2, since there are two innovation outcomes, we use $\Pi_2$ to denote the period 2 gross licensing revenue if the new innovation is unsuccessful and $\Pi_2'$ the revenue if it is successful.

$\pi^M(c)$: Single-period monopoly gross profit for the licensee who has a marginal cost of $c$.

$p^M(c)$: Single-period monopoly price in the downstream market when the marginal cost is $c$.

$q^M(c)$: Single-period monopoly output in the downstream market when the marginal cost is $c$.

II. A Simple Example: The Period 2 Innovation is Drastic

In order to see why royalty licensing may be used when sequential innovations and technology leakage are present, we first must understand why fixed-fee licenses are sub-optimal. The
reason is that they distort an innovator’s incentive to invest in sequential innovations. This is best illustrated by an example, in which the period 2 innovation is also drastic, i.e., $p^M(c_2) < c_1$. Since $c' \geq c_1$, we must have $p^M(c_2) < c'$. Hence a firm who licenses the new innovation will become a monopoly in period 2. This means that the innovator can sell an exclusive license on the new innovation for a fee equal to the period 2 monopoly profit. To put it differently, technology leakage has no consequence for the innovator if she succeeds in the new innovation. It is this feature that makes the example particularly tractable and illustrative.

Clearly, the innovator’s incentive to invest in the period 2 innovation is driven by the payoff difference from the two outcomes of the innovation. We can see that the optimal level of investment is obtained when the innovator were either vertically integrated with a downstream firm or able to commit to an investment level in period 2 at the time when period 1 licenses are issued, under either of which the incentive to invest is driven by the payoff difference $\pi^M(c_2) - \pi^M(c_1)$.

However, if the innovator is neither vertically integrated nor able to commit, then the outcome of a successful innovation may become more attractive. Suppose that the initial license is a standard long-term exclusive contract with an upfront fee, then the innovator receives no income in period 2 unless the new innovation is successful. This means that her incentive to innovate will be driven by $\pi^M(c_2)$. Therefore, the innovator has an incentive to over-invest in period 2, relative to the investment level she would choose if she were able to commit in period 1.

Now suppose that the initial license is a short-term fixed-fee exclusive contract, then the innovator will receive less than the monopoly profit from renewing the license in period 2 due to technology leakage. Let the revenue loss from leakage be $\tau_1$, the innovator’s incentive to innovate will be driven by $\pi^M(c_2) - [\pi^M(c_1) - \tau_1]$. Therefore, the innovator still has an incentive to over-invest in period 2, but the degree of over-investment is smaller.
This example gives us the basic intuition why a short-term fixed-fee contract may be preferred to a long-term fixed fee contract and why neither contract can achieve the benchmark profit. In the standard long-term fixed-fee contract with upfront payments, the innovator faces a classic time-consistency problem (Coasian 1972): Once a license is sold, the innovator is then tempted to invest in new technologies that render the initial license obsolete; expecting this, firms will pay less for the license. On the other hand, a short-term contract entails technology leakage, so the innovator has an incentive to choose an investment level to minimize the cost of technology leakage, but this investment level generally deviates from the optimal.

Of course, the above analysis is far from complete. Clearly, the innovator may want to structure a contract that deals with the time-consistency problem. Since the initial license will be worthless once the period 2 innovation succeeds, a possible solution is to use an installment payment plan, in which the second installment is paid only if a licensee wishes to continue the contract. It is easy to see that the second installment has to be as high as $\pi^M(c_1)$ in order for the innovator to overcome her excessive incentive to invest, but for a payment this high the original licensee will terminate the contract even if the period 2 innovation fails. In other words, a long-term contract that stipulates an opt-out clause with a high continuation fee effectively becomes a short-term contract, which may alleviate the time-consistency problem but not eliminate it. This and other points will be discussed in more detail when we solve the complete model.

**III. Investment and Licensing in Period 2**

We solve the game via backward induction. In this section, we consider the innovator’s period 2 problem. We first find the optimal licensing scheme under a cost asymmetry. It allows us to more precisely define the cost of technology leakage. We then derive the optimal investment level at the beginning of period 2.
A. Licensing Under Cost Asymmetry

In period 2, downstream firms are no longer identical in their pre-licensing costs. Licensees of the initial innovation will have lower marginal costs than non-licensees, either because the former has signed long-term contracts or because of technology leakage. Here we focus on a particular scenario, in which an exclusive license is granted in period 1 so that the period 2 cost asymmetry is between the original licensee and all others. We show that the optimal licensing scheme under such a cost asymmetry is to once again issue a fixed-fee exclusive license to the original licensee.

**Lemma 1** Suppose that firm 1 has a cost of $c_a$ and the other $n - 1$ firms have a cost of $c$, where $c_a \leq c$. If an innovation allows a firm to produce at a cost of $c_b$, where $p^M(c_b) < c$, then it is optimal to offer an exclusive license to firm 1 for a fixed fee.

**Proof.** Suppose that an optimal licensing scheme $S$ exists, in which firm 1’s net profit (profit minus the payment for a license) is $\pi_0$. Since the industry profits are $\pi^M(c_b)$, the innovator’s licensing revenue cannot be greater than $\pi^M(c_b) - \pi_0$ under scheme $S$. Now consider an alternative scheme, in which an exclusive contract is offered for a fixed-fee of $\pi^M(c_b) - \pi_0$ and firm 1 is given the Right of First Offer: if firm 1 accepts, then the game ends; if firm 1 rejects, then the innovator uses scheme $S$ to sell the innovation. Since $p^M(c_b) < c$, firm 1 will be able to earn the monopoly profit if it gets the exclusive license. Therefore, in the subgame perfect equilibrium, firm 1 accepts the offer and the innovator receives $\pi^M(c_b) - \pi_0$ as her revenue. This means that a fixed-fee exclusive contract is at least as profitable as scheme $S$ and is therefore optimal.

It should be noted that Lemma 1 in general does not hold if $p^M(c_b) > c$. It is our assumption that the initial innovation is drastic and thus any improvement upon the initial innovation is also drastic against the old technology allows us to dwell on this case, which greatly simplifies our task.
B. The Cost of Technology Leakage

A prominent feature of our model is technology leakage in short-term contracts. Because of technology leakage, the innovator may obtain a smaller licensing revenue in period 2 than she otherwise would. This loss in licensing revenue is the cost of technology leakage.

In order to find the cost of technology leakage, one compares the innovator’s licensing revenues with and without leakage, which in general is not an easy task. However, in our model, this comparison is made simpler by the assumption that the initial innovation is drastic. Due to this assumption and the fact that the period 2 innovation is necessarily an improvement over the initial one, without technology leakage the innovator can always sell an exclusive license in period 2 for a fee equal to the monopoly profits. In other words, the period 2 licensing revenue without leakage is \( M(c_2) \) if the period 2 innovation succeeds, or \( M(c_1) \) if it fails. Therefore, the cost of technology leakage is either \( M(c_2) - \Pi_2 \) or \( M(c_1) - \Pi_2 \).

a. Technology Leakage From an Exclusive License

Moreover, if an exclusive license was issued in period 1, then the cost of technology leakage is exactly equal to the profit that the original licensee can earn in period 2. This is because, according to Lemma 1, it is optimal to offer a second exclusive license to the original licensee so that the innovator and the original licensee split the monopoly profits in period 2. This means that any gain in the bargaining power of the original licensee will directly translate into the innovator’s loss of revenue. It is this linkage that allows us to further quantify the cost of technology leakage based on the latter’s profit in this special but important case of our model.

Like any technology, the cost of the technology leakage not only depends on the amount of cost saving but also the technologies available to firms that it competes with. Suppose that firm 1 has a cost of \( c_a \) and the other \( n - 1 \) firms have a cost of \( c \geq p^M(c_b) \) before a new technology that lowers the cost to \( c_b \) is introduced by the innovator. Denote by \( \tau(c_2, c_b) \) firm
1’s net profits (profits minus the licensing fee) from licensing the new technology \( c_b \). The cost of technology leakage if the period 2 innovation fails can thus be written as \( \tau_1 = \tau (c', c_1) \). Similarly, the cost of technology leakage if the period 2 innovation succeeds is \( \tau_2 = \tau (c', c_2) \). In addition, we find the value of owning an exclusive license to the initial innovation in period 2 in the event of a successful period 2 innovation to be another important variable. Using the notation just introduced, we can write it as \( \tau_l = \tau (c_1, c_2) \).

Throughout the paper, we make the following assumptions on the costs of technology leakage:

Assumption 1 If \( c_a < p^M(c_b) \), then \( \tau (c_a, c_b) > 0 \); if \( c_a \geq p^M(c_b) \), then \( \tau (c_a, c_b) = 0 \).

Assumption 2 \( \tau_l < \pi^M(c_1) \).

Assumption 3 \( \tau_1 - \tau_2 < \pi^M(c_1) - \tau_l \).

Assumption 1 states that the cost of technology leakage is zero if and only if \( c_b \) is drastic against \( c_a \), i.e., the availability of the new technology renders the leaked technology obsolete. Assumption 2 states that the value of having an exclusive access to a production technology of \( c_1 \) cannot exceed the monopoly profit earned with that technology. Assumption 3 further narrows down the range of the costs of technology leakage. In the appendix, we verify that these assumptions are met in homogeneous good, conjectural variation oligopoly models.

One may wonder whether \( \tau_2 \) is always smaller than \( \tau_1 \), since the leakage appears to be less of a concern should the period 2 innovation succeed. The answer is no, due to the integer constraint in the number of licenses that the innovator can sell in period 2. As shown in the proof of Lemma 11, if the original licensee refuses the offer of an exclusive license, then the innovator will auction either 1 or 2 licenses in stage 2 of the period 2 licensing game. For a fixed \( c_a \), when \( c_b \) is close to \( c_a \), 2 is the optimal number of licenses to sell in stage 2; when \( c_b \) decreases, the optimal number of licenses to sell in stage 2 will also decrease and at some point that number will "jump" from 2 to 1, diminishing the threat that can be imposed
on the original licensee. It is this discontinuity in the number of licenses that causes the non-monotonicity in the cost of technology leakage, because of which we cannot rule out the possibility that $\tau_1 < \tau_2$.

C. The Investment Decision and the Value of Investment

Now we solve the innovator’s problem at the investment stage in period 2. Let $\Delta = \Pi'_2 - \Pi_2$, we have

**Lemma 2** The optimal amount of investment is $\rho \Delta^2$, the probability of a successful innovation is $2\rho \Delta$, and the innovator’s expected profit in period 2 is $\Pi_2 + \rho \Delta^2$.

**Proof.** The innovator’s investment decision is $\max_I \Pr(I) \Delta - I = 2\sqrt{\rho I \Delta} - I$. Hence $\sqrt{\rho / I^* \Delta} = 1$, i.e., $I^* = \rho \Delta^2$. So $\Pr(I^*) = 2\rho \Delta$ and the expected profit in period 2 is $\Pi_2 + \Pr(I^*) \Delta - I = \Pi_2 + \rho \Delta^2$.

Lemma 2 shows that the innovator’s incentive to invest in period 2 is entirely determined by $\Delta$, the difference in period 2 licensing revenues from the two outcomes. Hence, $\Delta$ can serve as a convenient indicator of a licensing scheme’s optimality, which we will use repeatedly in this paper. Another result that allows us to easily compare licensing schemes is the following:

**Lemma 3** The innovator’s expected profit in period 2 increases in both $\Pi_2$ and $\Pi'_2$.

**Proof.** The innovator’s expected profit in period 2 is $\max_I \Pi_2 + \Pr(I) (\Pi'_2 - \Pi_2) - I$. Denote it by $\Pi^*_2$. By the envelope theorem, $d\Pi^*_2 / d\Pi_2 = \frac{\partial}{\partial \Pi_2} (\Pr(I) (\Pi'_2 - \Pi_2) - I)|_{I=I^*} = 1 - \Pr(I^*) \geq 0$ and $d\Pi^*_2 / d\Pi'_2 = \Pr(I) \geq 0$.

IV. Licensing in Period 1

In this section, we find the optimal licensing scheme in period 1, which is the central concern of this paper. We start with the first-best scenario for the innovator, whose solution is then

---

12 Because of the earlier assumption that $p < [2\pi^M(c_2)]^{-1}$, this probability will fall between 0 and 1.
used as our benchmark. Then we solve for the payoffs associated with each of the possible licensing schemes. We compare them with the benchmark and discuss each scheme’s advantages/disadvantages. Finally, we carry out some comparative statics exercises by varying the rate of innovation parameter and the cost of technology leakage.

A. The Benchmark

In a first-best scenario, the innovator is vertically integrated with a downstream firm and sells the final output by herself. There is neither a commitment problem nor technology leakage. Since $p^M(c_2) < p^M(c_1) < c_0$, the innovator can monopolize the industry in both periods. Therefore, her incentive to innovate in period 2 is perfectly aligned with the gain in industry profits, which is $\pi^M(c_2) - \pi^M(c_1)$.

**Proposition 1** If the innovator markets the final output by herself, then $\Delta^* = \pi^M(c_2) - \pi^M(c_1)$ and

\[
\Pi^* = (1 + \delta) \pi^M(c_1) + \delta \rho [\pi^M(c_2) - \pi^M(c_1)]^2.
\]

Proposition 1 gives us the upper bound of licensing revenue that the innovator can obtain, which serves as a useful benchmark in comparing different licensing schemes. It also provides a necessary condition for any licensing scheme to generate the benchmark profits: $\Delta = \Pi'_2 - \Pi_2$ must be equal to $\Delta^* = \pi^M(c_2) - \pi^M(c_1)$. This is true because the optimal level of investment is proportional to $\Delta^2$, so the period 2 investment must be inefficient if $\Delta$ deviates from $\Delta^*$.

Vertically integration is not the only way for the innovator to obtain the benchmark profit. If the transaction cost, $\varphi$, is zero, then the benchmark outcome can also be achieved via a license whose payments are contingent upon the innovation outcome. Denote by $f_1$ the period 1 license fee, $f_2$ ($f'_2$) the period 2 license fee if the period 2 innovation fails (succeeds).
Proposition 2  A long-term fixed-fee exclusive contract with \( f_1 = \pi^M(c_1) \), \( f_2 = \pi^M(c_1) \) and \( f'_2 = \pi^M(c_2) \) replicates the benchmark outcome if and only if \( \varphi = 0 \).

In reality, however, both vertical integration and writing complete contracts may be impractical: a research university may want to keep arms’ length from the product market in order to avoid conflicts of interest; certain “transaction costs” may prevent future contingencies from being contracted ex ante. Therefore, we must also examine the optimal licensing scheme when the above two options are unavailable.

B. Fixed-fee Licenses

We first consider fixed-fee licenses. Since we assume that the initial innovation is drastic, a fixed-fee exclusive license is optimal in an one-innovation model (Katz and Shapiro 1986, Kamien and Tauman 1986), but we show in this section that it is generally not true when there are sequential innovations and technology leakage. In so doing, we also solve for the optimal fixed-fee contracts. To streamline our exposition, we restrict our attention to exclusive contracts in period 1. We will verify in Section V. that this restriction is inconsequential.

a. Short-term Fixed-Fee Exclusive License

In a short-term fixed-fee exclusive contract, a licensee has the right to use the cost-reducing technology for just one period, during which he earns the monopoly profit \( \pi^M(c_1) \). After the contract expires at the end of period 1, the original licensee enjoys a cost of \( c' < c_0 \) because of technology leakage, while the other \( n - 1 \) firms only have the old technology of \( c_0 \). In period 2, a new license is issued regardless of whether the innovator is successful in her R&D efforts.
**Lemma 4** If a short-term fixed-fee exclusive license is offered in period 1, then $\Pi^{SF} = \Pi^* - \delta \rho (\tau_1 - \tau_2)^2$; if $c' > p^M(c_1)$, then a short-term fixed-fee exclusive license replicates the benchmark outcome.

**Proof.** According to Lemma 1, the period 2 license will be granted to the original licensee. Thus it is easy to see that

$$\Pi_2 = \pi^M(c_1) - \tau_1$$

and

$$\Pi'_2 = \pi^M(c_2) - \tau_2.$$ 

Hence,

$$I = \rho[\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2]^2,$$

$$\Pi_2^{SF} = \pi^M(c_1) - \tau_1 + \rho (\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2)^2$$

and

$$\Pi_1^{SF} = \pi^M(c_1) + \delta (1 - Pr) \tau_1 + \delta Pr \tau_2,$$

where

$$Pr = 2\rho (\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2).$$

Therefore,

$$\Pi^{SF} = (1 + \delta) \pi^M(c_1) - 2 (\tau_1 - \tau_2) \delta \rho (\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2) + \delta \rho (\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2)^2$$

(3)

$$= \Pi^* - \delta \rho (\tau_1 - \tau_2)^2.$$ 

if $c' > p^M(c_1)$, then $\tau_1 = \tau_2 = 0$ by Assumption 1 hence $\Pi^{SF} = \Pi^*$. It is also easy to verify that the innovator will choose the optimal investment level $\rho[\pi^M(c_2) - \pi^M(c_1)]^2$ if she has the ability to commit in period 1.

Lemma 4 tells us that the benchmark outcome can potentially be replicated by a series of short-term contracts if there is no technology leakage, otherwise technology leakage costs the innovator $\delta \rho (\tau_1 - \tau_2)^2$, where $\tau_1 - \tau_2$ represents the difference in the costs of technology leakage between the two outcomes of the period 2 innovation. From this result, we can see that the innovator’s period 2 revenue loss from technology leakage does not directly translate into the loss in her total licensing revenue: after all, expecting a leakage, firms will pay more for the initial license. Rather, it is the innovator’s attempt to minimize the revenue loss from leakage that causes distortion in her incentive to invest in sequential innovations and this lowers licensees’ willingness to pay for the initial innovation. It can be seen most clearly by examining $\Delta = \Pi_2 - \Pi_2$, which equals $\pi^M(c_2) - \pi^M(c_1) + (\tau_1 - \tau_2)$ under a short-term contract. Thus, as long as the costs of technology leakage are not identical under different
innovation outcomes, the innovator’s incentive to invest will deviate from the optimal level. It is this deviation that results in the innovator’s loss in total revenue. In other words, the presences of both technology leakage and sequential innovation are essential for short-term fixed-fee contracts not to be able to replicate the benchmark outcome.

b. **Long-term Fixed-fee Exclusive License**

Now we examine in detail long-term fixed-fee contracts and their optimality. In our simple example, long-term fixed-fee contracts entail a time-consistency problem, but only contracts with an upfront payment are considered. To deal with the problem, the innovator may choose to add an opt-out clause, which allows a licensee to terminate a long-term contract after the innovation outcome realizes in period 2. More specifically, the period 1 contract specifies the fees to be paid in each of the two periods and we denote them by $f_1$ and $f_2$; if a licensee opts out the contract in period 2, then the contract terminates and $f_2$ will not be paid.\(^{13}\)

Clearly, a long-term fixed-fee contract $(f_1, f_2)$ without the opt-out clause is equivalent to a long-term fixed-fee contract $(f_1 + \delta f_2, 0)$ with the opt-out clause. This means that any long-term fixed-fee contracts without the opt-out clause are just special cases of a long-term fixed-fee contract with an opt-out clause. Therefore, it suffices to find the optimal long-term fixed-fee contract with an opt-out clause.

**Lemma 5** *If $\tau_1 - \tau_2 < 0$, then there exists a long-term fixed-fee exclusive contract that replicates the benchmark outcome.*

**Proof.** Consider a long-term fixed-fee contract $(f_1, f_2)$ with $f_1 = \pi^M(c_1) + \delta \tau_2$ and $f_2 = \pi^M(c_1) - \tau_2$.

\(^{13}\)Here we implicitly assume a zero termination fee, but this assumption is without loss of generality, since only the difference in the payments affects a licensee’s decision whether to continue or to terminate the contract and the innovator’s incentive to invest in a new innovation. If the contract instead specifies a non-zero termination fee of $f_2'$, then such a contract is equivalent to $(f_1 + \delta f_2', f_2 - f_2')$. 

18
If the period 2 innovation is not successful, then the period 2 surplus that the original licensee can obtain is \( M(c_1) - f_2 = \tau_2 \) by continuing the contract and \( \tau_1 \) by opting out. Since \( \tau_1 < \tau_2 \), the contract will be continued after period 1 and thus the original licensee is willing to pay \( f_1 = \pi^M(c_1) + \delta \tau_2 \) in period 1. Also, we obtain that \( \Pi_2 = f_2 = \pi^M(c_1) - \tau_2 \).

If the period 2 innovation is successful, then the period 2 surplus that the original licensee can obtain is \( \tau_1 - f_2 \) by continuing the initial contract and \( \tau_2 \) by opting out. Since \( \tau_1 - \pi^M(c_1) + \tau_2 < \tau_2 \), the initial contract will be terminated after period 1 and the original licensee’s period 2 surplus is \( \tau_2 \). This again means that the original licensee is willing to pay \( \pi^M(c_1) + \delta \tau_2 \) in period 1, so we obtain that \( \Pi'_2 = \pi^M(c_2) - \tau_2 \).

Since \( \Delta = \Pi'_2 - \Pi_2 = \pi^M(c_2) - \pi^M(c_1) \), the innovator’s incentive to invest in period 2 is optimal. Therefore, the given contract replicates the benchmark outcome.

**Lemma 6**  
In a long-term fixed-fee exclusive contract \((f_1, f_2)\) with an opt-out clause, if \( \tau_1 - \tau_2 > 0 \) and \( f_2 \leq \pi^M(c_1) - \tau_1 \), then \( \Pi^{LF} \) increases with \( f_2 \).

**Proof.** In period 2, there are two states of nature: (i) innovation is not successful; (ii) innovation is successful. In case (i), since \( \pi^M(c_1) - f_2 > \tau_1 \), the original licensee will continue the contract and get \( \pi^M(c_1) - f_2 \). Hence \( \Pi_2 = f_2 \).

In case (ii), we separate \( f_2 \) further into two regions: a) \( f_2 \leq \tau_1 - \tau_2 \) and b) \( \tau_1 - \tau_2 < f_2 < \pi^M(c_1) - \tau_1 \).

\[ (ii.a) \] \( f_2 \leq \tau_1 - \tau_2 \). If the original licensee continues the initial contract and produces at a cost of \( c_1 \), then he gets \( \tau_1 - f_2 \); if he opts out, then he gets \( \tau_2 \). Since \( \tau_1 - f_2 \geq \tau_2 \), it is optimal for the original licensee to continue the original license. This means that \( \Pi'_2 = \pi^M(c_2) - \tau_1 + f_2 \) and \( \Delta = \pi^M(c_2) - \tau_1 \). So the innovator’s incentive to invest is independent of \( f_2 \). Therefore, her licensing revenue is a constant if \( f_2 \leq \tau_1 - \tau_2 \).

\[ (ii.b) \] \( f_2 > \tau_1 - \tau_2 \). If the original licensee continues the initial contract and produces at a cost of \( c_1 \), then he gets \( \tau_1 - f_2 \); if he opts out, then he gets \( \tau_2 \). Since \( \tau_1 - f_2 < \tau_2 \), the original licensee’s right to use the old innovation has no value and he will opt out the initial contract.
This means that $\Pi'_2 = \pi^M(c_2) - \tau_2$ and $\Delta = \pi^M(c_2) - \tau_2 - f_2$. In period 1, a firm is willing to pay $\Pi^L_1 = \pi^M(c_1) + \delta(1 - 2\rho (\pi^M(c_2) - \tau_2 - f_2)) (\pi^M(c_1) - f_2) + 2\delta \rho (\pi^M(c_2) - \tau_2 - f_2) \tau_2$ for an exclusive license. At the same time, $\Pi^L_2 = f_2 + \rho (\pi^M(c_2) - \tau_2 - f_2)^2$. Hence, the total licensing revenue is $\Pi^{LF} = \pi^M(c_1) + \delta(1 - 2\rho (\pi^M(c_2) - \tau_2 - f_2)) (\pi^M(c_1) - f_2) + 2\delta \rho (\pi^M(c_2) - \tau_2 - f_2) \tau_2 + \delta (f_2 + \rho (\pi^M(c_2) - \tau_2 - f_2)^2)$, so $\frac{\partial}{\partial f_2} \Pi^{LF} = 2\rho \delta (\pi^M(c_1) - \tau_2 - f_2) \geq 2\rho \delta (\tau_1 - \tau_2) > 0$. Last, it is also easy to verify that $\Pi^{LF}$ is continuous at $f_2 = \tau_1 - \tau_2$.

**Lemma 7** If $\tau_1 - \tau_2 > 0$, then any equilibrium long-term fixed-fee exclusive contract ($f_1, f_2$) with an opt-out clause and $f_2 \geq \pi^M(c_1) - \tau_1$ is equivalent to a short-term exclusive contract with a fixed fee of $f_1$.

**Proof.** If the period 2 innovation is not successful, then the period 2 surplus that the original licensee can obtain is $\pi^M(c_1) - f_2$ by continuing the contract and $\tau_1$ by opting out. Since $\tau_1 \geq \pi^M(c_1) - f_2$, the contract will be terminated after period 1.

If the period 2 innovation is successful, then the period 2 surplus that the original licensee can obtain is $\tau_1 - f_2$ by continuing the initial contract and $\tau_2$ by opting out. Since $\tau_1 - f_2 < \tau_1 + \tau_1 - \pi^M(c_1) < \tau_2$, the initial contract will also be terminated after period 1.

Using the above lemmas, we obtain the following result for fixed-fee contracts.

**Proposition 3** For homogeneous good, conjectural variation oligopoly models, (i) If $\tau_1 - \tau_2 \leq 0$, then there exists a long-term fixed-fee contract that replicates the benchmark outcome; (ii) if $\tau_1 - \tau_2 > 0$, then a short-term contract is optimal among fixed-fee licenses.

The intuition for the above result is easy to understand. As shown in the simple example, in a long-term fixed-fee contract, the innovator has an incentive to over-invest in order to make the initial license obsolete. To mitigate this incentive, the innovator can increase $f_2$, the continuation fee on the initial license. But too high a continuation fee will lead the original licensee to terminate the initial contract regardless of the innovation outcome, replicating a short-term contract. Hence $f_2$ can not exceed $\pi^M(c_1) - \tau_1$. On the other
hand, the continuation fee that allows the innovator to replicate the benchmark outcome is \( \pi^M(c_1) - \tau_2 \). The two conditions can both be met only if \( \pi^M(c_1) - \tau_2 < \pi^M(c_1) - \tau_1 \), i.e., \( \tau_1 < \tau_2 \),\(^{14}\) otherwise a long-term fixed-fee contract is at best as profitable as a short-term one.

As shown in the proof, the optimal fixed-fee contract depends on comparing the costs of technology leakage, especially \( \tau_1 - \tau_2 \) and \( \pi^M(c_1) - \tau_1 \). In homogenous good, conjectural variation models, we have Assumption 2 and 3, which significantly reduce the number of cases to consider. For more general models, the result and the proof are somewhat tedious, so we leave them in the appendix.

C. Royalty Licenses

Next we consider the optimality of short-term and long-term royalty licensing schemes. We will use a result attributed to Kamien and Tauman (1986): in a one-innovation licensing game, under Cournot competition with a linear demand, the licensing revenue from royalty \( \Gamma^R(k)(r) \) on a drastic innovation that reduces the production cost from \( c_0 \) to \( c_1 \) is maximized at \( r^* = (a - c_1) / 2 \) and \( k^* = n \) for a maximum of \( \Gamma^R(r^*) = \frac{n}{n+1} \pi^M(c_1) \); under Bertrand competition, \( \Gamma^R(r^*) = \pi^M(c_1) \).

a. Short-term Royalty

Like a short-term fixed-fee license, short-term royalty contracts last only one period, but they generally admit more licensees in period 1. Hence, in period 2, more than one firms may have access to the part of cost saving that is irreversible. This makes an explicit solution to the period 2 licensing game difficult to obtain. Therefore, we simply compare the two licensing schemes and rule out short-term royalty as a possible optimal scheme.

**Lemma 8** Short-term royalty is less profitable than short-term fixed-fee exclusive licensing.

\(^{14}\)It should also be noted that the continuation fee does not have to be positive. In fact, if \( \pi^M(c_1) < \tau_2 \), then the continuation fee will be negative; or to put it differently, the continuation fee will be greater than the termination fee.
**Proof.** Recall that the innovator’s total licensing revenue net of investment is \( \Pi = \Pi_1 + \Pi_2 + \delta \rho (\Pi'_2 - \Pi_2)^2 \). Our plan of the proof is to show that all three terms, \( \Pi_1, \Pi_2 \) and \( \Pi'_2 \), are lower under short-term royalty \((SR)\) than under short-term fixed-fee exclusive licensing \((SF)\) and therefore the same must be true for \( \Pi \) according to Lemma 3.

First, it is easy to see that \( \Pi^{SR}_1 \leq \pi^M(c_1) \leq \Pi^{SF}_1 \). Next we consider period 2 licensing if the innovation fails so that the best technology available remains \( c_1 \). Because of technology leakage, \( k_1 \geq 1 \) firms have a cost of \( c' \) at the beginning of period 2 under \( SR \) but only 1 firm has \( c' \) under \( SF \). All other firms have a cost of \( c_0 > c' \).

Let the period 2 optimal licensing scheme under \( SR \) be \( O \). We want to show that under \( SF \) a scheme based on \( O \) can give the innovator a period 2 licensing revenue at least as much as \( \Pi^{SR}_2 \). Consider scheme \( O^+ \), under which scheme \( O \) is used along with a royalty contract offered to \( k_1 - 1 \) firms that allows them to use the cost-reducing technology \( c_1 \) for a rate of \( c' - c_1 \). For the \( k_1 - 1 \) firms offered a royalty, their cost effectively becomes \( c_1 + r = c' \). Thus, in total, \( k_1 \) of firms will have a cost of \( c' \) when they participate in scheme \( O \). This means that scheme \( O^+ \) allows period 2 licensing under \( SF \) to replicate the licensing game played under scheme \( O \) and therefore \( \Gamma^{O^+} \geq \Pi^{SR}_2 \), where the inequality holds if the royalty offer is taken by a positive number of firms. Since \( O^+ \) is not necessarily the optimal scheme under \( SF \), we must have \( \Pi^{SF}_2 \geq \Gamma^{O^+} \) and therefore \( \Pi^{SF}_2 \geq \Pi^{SR}_2 \). The same argument, except that the royalty rate should be set at \( c' - c_2 \), can be applied to the case of a successful period 2 innovation to show that \( \Pi'_2 \) is lower under \( SR \) than under \( SF \).

The intuition behind the proof goes as follows: when compared with short-term fixed-fee licensing, short-term royalty generates a smaller period 1 licensing revenue and leads to a greater degree of technology leakage, which lowers the licensing revenue in period 2 regardless of the innovation outcome. In particular, under a fixed-fee contract the innovator is able to capture a licensee’s gain from technology leakage, whereas under a pure royalty the innovator is unable to. This, however, suggests that a two-part tariff can potentially be a more profitable licensing scheme, an observation that we will discuss in Section V..
b. Long-term Royalty

Under a long-term royalty scheme, a licensee is entitled to use the period 1 innovation \( c_1 \) for both periods and pay \( r \) for every unit of output.\(^{15} \) We will show that the innovator’s incentive to invest under the long-term royalty scheme is time-consistent and therefore, the optimal royalty rate that maximizes revenue in a one-innovation licensing game is also optimal in a long-term contract.

**Lemma 9** Under Cournot competition with linear demand, the optimal royalty rate is \( r^* = \left( \frac{a - c_1}{2} \right) \) in a long-term royalty contract and the licensing revenue is \( \Pi^{LR} = (1 + \delta) \Gamma^R(r^*) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2 \); under Bertrand competition, the optimal royalty rate is \( r^* = \left( \frac{a - c_1}{2} \right) \) in a long-term royalty contract and the licensing revenue is \( \Pi^{LR} = \Pi^* \).

**Proof.** If the period 2 innovation is not successful, then \( \Pi_2 = \Gamma^R(r) \). Hence \( \Pi^{LR} = (1 + \delta) \Gamma^R(r) + \delta \rho (\Pi_2' - \Gamma^R(r))^2 \). In order to find the optimal royalty rate \( r \), we separate the possible choice of \( r \) into two regions: (i) \( r \geq r^* \) or (ii) \( r < r^* \), where \( r^* = \left( \frac{a - c_1}{2} \right) \) is the optimal royalty rate in a one-innovation licensing game.

(i) If \( r \geq r^* \), then the period 2 innovation \( c_2 \) is drastic against an original licensee’s total cost \( c_1 + r \), since \( p^M(c_2) < p^M(c_1) \leq c_1 + r \). Therefore, if the period 2 innovation is successful, then the licensing revenue from it is maximized via a fixed-fee exclusive license. This means that \( \Pi_2 = \pi^M(c_2) \) and \( \Pi^{LR} = (1 + \delta) \Gamma^R(r) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r) \right)^2 \). We can see that the innovator’s total profits \( \Pi^{LR} \) increases with \( \Gamma^R(r) \), since

\[
\frac{\partial}{\partial \Gamma^R(r)} \Pi^{LR} = (1 + \delta) - 2 \delta \rho \left( \pi^M(c_2) - \Gamma^R(r) \right) \geq (1 + \delta) - \delta > 0.
\]

Therefore, \( r \) should be chosen so as to maximize \( \Gamma^R(r) \), i.e., \( r = r^* \). Thus,

\[
\Pi^{LR} = (1 + \delta) \Gamma^R(r^*) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2.
\]

\( ^{15} \)Allowing a more elaborate royalty scheme that possibly includes opt-out clauses or time-dependent royalty rates, etc., can only make royalty more attractive and strengthen our results.
(ii) In order for \( r < r^* \) to be a profitable deviation from \( r^* \), we must have \((1 + \delta) \Gamma^R(r) + \delta \rho (\Pi'_2 - \Gamma^R(r))^2 > (1 + \delta) \Gamma^R(r^*) + \delta \rho (\pi^M(c_2) - \Gamma^R(r^*))^2 \) and \( \Pi'_2 > \Gamma^R(r) \). But \((1 + \delta) \Gamma^R(r^*) + \delta \rho (\pi^M(c_2) - \Gamma^R(r^*))^2 > (1 + \delta) \Gamma^R(r) + \delta \rho (\Pi'_2 - \Gamma^R(r))^2 \), a contradiction. The first inequality is due to Eq. (4) and the second is due to \( \Pi'_2 \leq \pi^M(c_2) \).

Under Bertrand competition, \( \Gamma^R(r^*) = \pi^M(c_1) \) hence \( \Pi^{LR} = \Pi^* \).

Royalty contracts generally cannot replicate the benchmark outcome under Cournot competition. In standard one-innovation model, they are inferior to fixed-fee contracts. But in a model with sequential innovations, a long-term royalty contract can avoid both the technology leakage problem in a short-term contract and the time-consistency problem in a fixed-fee contract: Eq. (4) shows that maximizing period 1 licensing revenue is consistent with maximizing the total profits. Essentially, the use of royalty to collect payments on an ongoing basis eliminates the innovator’s commitment problem. It is this advantage that makes royalty a potentially optimal licensing scheme.

It should be noted that the licensing revenue obtained in Eq. (5) is likely to be the lower bound for a long-term royalty contract. If renegotiations are allowed, the innovator can potentially increase her revenue. For example, in the above discussion, we have implicitly assumed that the innovator cannot change the licensing scheme for the period 1 innovation in period 2 if the new innovation is not successful. Now suppose that the innovator can modify licensing contracts with individual licensees, then it is optimal to move from the royalty scheme to a fixed-fee exclusive licensing in period 2. This change will increase the period 2 licensing revenue without affecting the period 1 royalty rate and thus may increase the innovator’s total profits. Under Cournot competition, it can be shown that the innovator’s total revenue will then become \( \Gamma^R(r^*) + \delta \left( 1 - \frac{1}{(n+1)^2} \right) \pi^M(c_1) + \delta \rho \left( \pi^M(c_2) - \left( 1 - \frac{1}{(n+1)^2} \right) \pi^M(c_1) \right)^2 \), which is greater than \( \Pi^{LR} \) obtained in Eq.(5), since \( \Gamma^R(r^*) = \frac{n}{n+1} \pi^M(c_1) < \left( 1 - \frac{1}{(n+1)^2} \right) \pi^M(c_1) \).
D. Summary of results and Comparative Statics

Now we summarize the comparison of different licensing schemes and discuss how the choice of licensing schemes varies with the model parameters. In so doing, we also provide some potential testable hypotheses, which can serve as guidance for future empirical work.

**Proposition 4** Under Cournot competition with linear demand, (i) If $\tau_1 - \tau_2 < 0$, then the optimal licensing contract is a long-term fixed-fee contract with an opt-out clause, where $\Pi^{LF} = \Pi^*$; (ii) if $\tau_1 - \tau_2 > 0$, then the optimal licensing contract is either $SF$ or $LR$, where $\Pi^{SF} = \Pi^* - \rho \delta (\tau_1 - \tau_2)^2$ and $\Pi^{LR} = (1 + \delta) \Gamma^R(r^*) + \delta \rho (\pi^M(c_2) - \Gamma^R(r^*))^2$, which approaches $\Pi^*$ when $n \to \infty$. Under Bertrand competition, the optimal licensing contract is a long-term royalty with $\Pi^{LR} = \Pi^*$.

**Corollary 1** Denote by $\tilde{n}$ the number of firms such that $\Pi^{LR} = \Pi^{SF}$ and $A = \pi^M(c_2)/\pi^M(c_1)$.

Under Cournot competition with linear demand, $\partial \tilde{n}/\partial \rho < 0$.

**Proof.** From Proposition 4, we get

$$\frac{\partial}{\partial \rho} \tilde{n} = -\delta (n + 1) (1 + 2(n + 1)(A - 1) + (\tau_1 - \tau_2)^2(n + 1)^2) / ((n + 1)(1 + \delta - 2A\rho \delta) + 2\delta \rho n) < 0.$$  

When we vary the parameter governing the probability of innovation, we find that non-exclusive royalty contracts are optimal for higher levels of innovation. This occurs because increasing the rate of innovation magnifies the incentives to engage in R&D. This makes the degree to which there is over-investment under fixed-fee contracts larger. This suggests that industries where sequential innovations are common are more likely to use non-exclusive royalty contracts. Although there is no direct evidence to support this prediction, Anand and Khanna (2000) do find that the incidence of exclusivity varies considerably across industries.\footnote{They have not been able to gather reliable information on the form of payment (royalties versus fixed fees) agreed to in the licensing contracts.} Exclusive transfers are much less common in Computers (18%) and Electronics (16%), two industries that are well known for sequential innovations (Bessen 2004, Bessen and Maskin 2000), than in the other industries (38%).
Our model also suggests that short-term exclusive contracts may be more likely used in industries that have strong intellectual property protection so that technology leakage is of little concern. This hypothesis is also compatible with the above pattern of licensing observed by Anand and Khanna: the chemical industries have high invent-around costs, patents deliver strong appropriability (Levin et al. 1987, Cohen et al. 2000), and these industries also have higher incidences of exclusive licensing than computers and electronics industries, which have low invent-around costs, patents deliver low appropriability. Further research, however, is needed to identify which of the two factors is more responsible for the observed pattern.

V. Discussion

In this section, we explore the effects of relaxing some of our assumptions made in the basic model. Our results appear robust to these extensions.

A. Multiple Fixed-Fee Licenses

For ease of exposition, we only allowed the innovator to sell an exclusive fixed-fee license on the initial innovation in the main results. Here we consider the possibility that the innovator sells multiple fixed-fee licenses in period 1. Issuing multiple licenses decreases the industry profit in period 1, which in turn decreases the potential licensing revenue, but it might allow the innovator to better resolve the time-consistency problem by being able to more closely approximate the optimal investment level, which is $\pi^M(c_2) - \pi^M(c_1)$. However, this is never optimal under homogenous good, conjectural variation oligopoly models. We prove this result in the appendix, but the intuition is as follows: if $\tau_1 < \tau_2$, then a long-term exclusive fixed-fee license is able to replicate the benchmark case and is optimal; if $\tau_1 > \tau_2$, then the decrease in industry profit resulting from multiple licensees in period 1 is larger than $\Pi^* - \Pi^{SF}$, that is to say, the total decrease in profits as a result of technology leakage is always
smaller than the one-period decrease in industry profits from moving from a monopoly to a
duopoly.

B. Non-drastic Innovation

Our current analysis has been limited to drastic innovations in period 1. If the innovation
in period 1 is not drastic, then it may no longer be optimal to offer an exclusive license in
period 1.\textsuperscript{17} Instead, a fixed-fee license may be offered to multiple firms, either as a long-term
contract or a short-term contract. A detailed analysis will be complicated and is beyond
the scope of this paper.\textsuperscript{18} However, the basic trade-off between the value of the original
innovation and the incentive to engage in future innovations remains the same. Therefore,
we expect our main result concerning the optimality of royalty and short-term contracts to
continue to hold.

C. Delayed Licensing

Another possibility to consider is whether the innovator can choose not to offer any license
in period 1 and only offer licenses after the outcome of period 2 innovation is realized.
Such a delay leads to a revenue loss of $\pi^M(c_1)$. Since $\delta \rho (\pi^M(c_1) - \pi_1)^2 \leq \delta \rho (\pi^M(c_1))^2 \leq \delta (\pi^M(c_1))^2 / 2\pi^M(c_2) < \pi^M(c_1)$, delayed licensing is less profitable than a long-term fixed-fee contract with an upfront payment.

D. Two-part Tariff

In this model, the licensing policies are confined to either pure fixed fees or pure royalty. An
important extension of the model would be to allow the licensor to include two-part tariffs as
a possible payment scheme. Here we discuss what happens if the innovator can use two-part

\textsuperscript{17}In Kamien and Tauman (1986), it has been shown that a fixed-fee license sold to multiple firms is optimal
in a one-period game if the innovation is not drastic.

\textsuperscript{18}The major complication involves solving the period 2 competition outcome with four different types of
firms, whose marginal costs can be any of $c_0, c_1, c_2$ and $c'$. 27
tariffs, i.e., a combination of fixed fees and royalties. Since both fixed fee and royalty are special cases of the more general licensing scheme, the innovator cannot do worse by having the ability to use the combination of the two. The question is therefore about which existing licensing scheme can be improved by its combination with the other. First, it is clear that any pure fixed-fee contracts cannot be improved by adding a positive royalty, for otherwise the fee currently set could not have been optimal; second, royalty contracts can potentially become more profitable since the innovator can use the fixed-fee part of a two-part tariff to extract licensees’ profits. In particular, in short-term contracts, a two-part tariff can be used to extract some of the technology leakage that will occur in period 2.\footnote{This is consistent with Vishwasrao (2007)’s empirical finding that contracts of longer duration are generally associated with a combination of fees and royalties rather than royalties alone.} Thus, allowing two-part tariffs increases the circumstances under which contracts with positive royalty rates are used.

**VI. Conclusion**

In this paper, we have extended the literature on technology licensing by adding to the literature on the duration of contracts, sequential innovations and a model of technology leakage. We show in this framework that it may be optimal for the innovator to limit the length of fixed-fee license to less than the length of underlying patent. We find that long-term, but not short-term, royalty contracts can be optimal, even under complete information and risk neutrality, because they allow the innovator to resolve a time-consistency problem caused by sequential innovation and technology leakage. This implies that royalty contracts are on average of longer duration than fixed-fee contracts, a result generally consistent with empirical findings.

It has long been recognized that the market of technology licensing is imperfect (e.g., Caves, Crookell and Killing 1983). While other papers in the literature of technology licensing have dwelt on incomplete information, moral hazard, risk and uncertainty, our paper
focuses on the irreversibility of technology transfer and the incentive to engage in sequential innovations. In particular, we introduce the notion of technology leakage, which is shown to be an important determinant in an innovator’s choice of licensing contracts. Nonetheless, it remains an under-explored topic, which we believe will lead to fruitful researches.

The model presented here has made strong assumptions that can potentially relaxed. First, one would like to relax the assumptions of the drastic and exogenous nature of the initial innovation. Second, one may examine whether our results extend beyond the artificial two-period model. Third, in our model, technology leakage is studied in detail only when one firm has a cost advantage over other firms hence an exclusive license is the best form of contract. It can be challenging yet worthwhile to quantify technology leakage in more general cases. On a related point, the optimal contract when there is cost asymmetry among potential licensees deserves more attention in the literature. Last, one can extend the analysis by allowing more general licensing schemes, including two-part tariffs. These topics are left for future research.
A Proofs

Lemma 10 If the period 1 innovation is drastic, then any fixed-fee contract with \( k \geq 2 \) licensees in period 1 cannot be optimal.

Proof. Under Cournot competition with linear demand, if \( k \geq 2 \) licenses are sold in period 1 for a fixed fee, then the period 1 industry profits will be at most \( 8\pi^M(c_1)/9 \), since \( \pi^D(c_1) = 4\pi^M(c_1)/9 \). This means that the total licensing revenue will be at least \( \pi^M(c_1)/9 \) less than the benchmark profit. Now consider two cases. First, if \( \tau_1 \leq \tau_2 \) then according to Lemma 5, a long-term fixed-fee exclusive contract can replicate the benchmark outcome. Therefore, issuing multiple fixed-fee licenses cannot be optimal. Second, if \( \tau_1 > \tau_2 \), then \( \delta \rho (\tau_1 - \tau_2)^2 \leq \delta \rho \tau_1^2 / (2\pi^M(c_2)) < \delta (\frac{1}{4}\pi^M(c_1))^2 / (2\pi^M(c_2)) = \delta \pi^M(c_1)/32 < \pi^M(c_1)/9 \), where the third inequality follows from Lemma 11 that \( \tau_1 \leq \pi^M(c_1)/4 \). Since \( \Pi^* - \Pi^{SF} = \delta \rho (\tau_1 - \tau_2)^2 \), we must have \( \Pi^{LF}(k \geq 2) - \Pi^{SF} \leq \delta \rho (\tau_1 - \tau_2)^2 - \pi^M(c_1)/9 < 0 \).

Under Bertrand competition, if \( k \geq 2 \) licenses are sold in period 1, then the industry profits will be zero in period 1. This means that the total licensing revenue will be at least \( \pi^M(c_1) \) less than the benchmark profit. According to Lemma 11, \( 0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_1 \), we have \( \delta \rho (\tau_1 - \tau_2)^2 < \delta \rho (\pi^M(c_1) - \tau_1)^2 \leq \delta (\pi^M(c_1))^2 / (2\pi^M(c_2)) < \pi^M(c_1) \). Hence \( \Pi^{LF}(k \geq 2) < \Pi^{SF} \).

B The cost of technology leakage in Homogenous Good, Conjectural Variation Oligopoly Models

In the main text, we make assumptions on the value of leakages. Here we verify that these assumptions are met in homogenous good, conjectural variation oligopoly models including Cournot competition with linear demand and Bertrand competition.

Proof of Lemma 3. Suppose that a licensing scheme \( S \) exists such that the innovator’s licensing revenue equals \( \pi^M(c_b) \). Since post-licensing competition takes place among firms
who sell homogenous good at a uniform price, we must have \( p = p^M(c_b) > c_a \) and \( \pi_i = 0 \) for all \( i = 1, 2, ..., n \). Now if firm 1 chooses not to license the innovation, then \( \pi_1 = q_1(p^M(c_b))(p^M(c_b) - c_a) > 0 \). This means that \( S \) cannot be an equilibrium. Contradiction.

**Lemma 11** Under Cournot competition with linear demand, if \( c_a \geq c_b \), then \( \tau(c_a, c_b) \leq \frac{1}{4} \pi^M(c_b) < \frac{9}{16} \pi^M(c_a) \).

**Proof.** Following Lemma 1, we consider a licensing scheme that involves the Right of First Offer to the firm who has a cost of \( c_b \) (firm 1) with the threat of an auction of fixed-fee licenses: in stage 1, the innovator offers firm 1 a fixed-fee exclusive license; if firm 1 accepts, then the game ends; but if firm 1 rejects, then the innovator sells licenses in stage 2 via a sealed-bid first-price auction. In stage 2, the number of new licenses that will be issued and whether the original licensee places a winning bid depend upon the parameter values. Divide the range of possible values for \( c_a \) and \( c_b \) into three regions: (i) \( c_b < c_a < \frac{a + 2c_b}{3} \), (ii) \( \frac{a + 2c_b}{3} \leq c_a < \frac{a + c_b}{2} \) or (iii) \( c_a \geq \frac{a + c_b}{2} \).

i) Within this region we will separately consider two possibilities for \( k \), either \( k \geq 2 \) licenses are auctioned or \( k = 1 \) licenses are auctioned. When \( k \geq 2 \) licenses are auctioned, the original licensee might or might not win one of the licenses depending on the parameter values. Here we will show that in either case, the cost of technology leakage is bounded from above and the licensing revenue from issuing \( k \geq 2 \) licenses is bounded from below. Then we will show that the lower bound on licensing revenue for \( k \geq 2 \) licenses is greater than the licensing revenue from \( k = 1 \) new licenses.

First, suppose that \( k \geq 2 \) licenses are issued in period 2. The range of costs considered in (i) are chosen such that the original licensee with a cost of \( c_a \) will still earn positive profit in competition with two licensees each with cost \( c_b \). Therefore, if \( k \geq 2 \) licenses are auctioned, then the amount that an outside firm is willing to pay for one of these licenses depends upon whether or not the initial licensee also wins the license or not.

If the original licensee never wins one of the new licenses, then her profit is equal to \( \frac{1}{(k + 2)^2} (a - c_a - kc_a + kc_b)^2 \). This is decreasing in \( k \) and \( c_a \). Therefore the case where \( k = 2 \)
and \( c_b = c_a \) gives us an upper bound for the period 2 profit for the original licensee if she
does not win a new license. The period 2 profit to the original licensee is equal to the cost
of technology leakage, thus \( \tau(c_a, c_b) \leq \frac{1}{16}(a - c_b)^2 = \frac{1}{4} \pi^M(c_b) \). The licensing revenue if \( k = 2 \)
and the original licensee does not win one of the new licenses is equal to \( \frac{1}{8}(a + c_a - 2c_b)^2 \) and
is a lower bound on possible licensing revenue.

In the case above, an outside firm was only willing to bid as much as she could earn in
profit competing with \( k \) firms with the cost \( c_b \) and one firm with a cost of \( c_a \). If, however,
the original licensee sometimes wins one of the licenses then the problem is complicated by
the fact that new licensees will be willing to place a higher bid. A unique pure strategy
equilibrium may not exist. For our purposes, it will suffice to show that even if the original
licensee receives a period 2 license, the cost of technology leakage is bounded from above
and the value of licensing revenue is bounded from below.

We know that the minimum winning bid placed by a firm that was not an original licensee
is at least \( \frac{1}{(k+2)^2}(a + c_a - 2c_b)^2 \); the amount they would bid if the original firm never wins the
license. Using this as a lower bound for the winning bid we know that if, \( k \geq 2 \) the minimum
licensing revenue is equal to \( \frac{k-1}{(k+2)^2}(a + c_a - 2c_b)^2 \geq \frac{1}{8}(a + c_a - 2c_b)^2 \) and \( \tau \leq \pi^k(c_b) - \frac{1}{(k+2)^2}(a + c_a - 2c_b)^2 \leq \frac{1}{9}(a - c_b)^2 - \frac{1}{16}(a - c_b)^2 = \frac{7}{36} \pi^M(c_b) < \frac{1}{4} \pi^M(c_b) \).

Combining the two possibilities we see that if \( k \geq 2 \) licenses are issued, then regardless
of whether the original licensee receives a new license, \( \tau \leq \frac{1}{4} \pi^M(c_b) \) and licensing revenue is
at least \( \frac{1}{8}(a + c_a - 2c_b)^2 \).

If \( k = 1 \), then the original licensee with a cost of \( c_a \) will outbid non-licensees with a cost of
c_0, since \( \pi^M(c_b) > \pi^D(c_a, c_b) + \pi^D(c_b, c_a) \). In this case, the winning bid will be the profits that a
new licensee would earn in competition with the original licensee, \( \pi^D(c_b, c_a) = \frac{1}{9}(a + c_a - 2c_b)^2 \).
This is less than the minimum licensing revenue from \( k > 2 \) licenses. Therefore, if \( c_a < \)
\( \frac{a+2c_a}{3} \), more than one license will be issued and \( \tau \leq \frac{1}{4} \pi^M(c_b) \).

Notice that \( \frac{1}{4} \pi^M(c_b) \) is increasing in \( c_b \) and that within (i) \( c_b \geq \frac{3c_a-a}{2} \). Therefore,
\( \frac{1}{4} \pi^M(c_b) \leq \frac{1}{4} \pi^M \left( \frac{3c_a-a}{2} \right) = \frac{a}{16} \pi^M(c_a) \).

32
ii) Since $c_a \geq \frac{a+2c_b}{3}$, the original licensee with cost $c_a$ will produce nothing in competition with two firms with cost $c_b$, but since $c_a < \frac{a+c_b}{2}$ it is not driven from the industry if there is only one firm with cost $c_b$. Therefore, if two licenses are issued, regardless of whether the original licensee wins one of the two licenses, only two firms will compete in the market. Thus, outside firms will bid duopoly profits with technology $c_b$, the innovator earns $\frac{8}{9} \pi^M(c_b)$ in licensing revenue, and $\tau = 0$. If, on the other hand, only one license is issued, then the original licensee will win the auction with a bid equal to what a firm with cost $c_b$ would earn in a duopoly with the original licensee, where the original licensee has a cost of $c_a$. The innovator earns $\pi^D(c_b,c_a) = \frac{1}{9}(a+c_a-2c_b)^2$ in licensing revenue and $\tau = \pi^M(c_b) - \frac{1}{9}(a+c_a-2c_b)^2$. The innovator will find issuing one license to be optimal if $\pi^D(c_b,c_a) = \frac{1}{9}(a+c_a-2c_b)^2 \geq \frac{8}{9} \pi^M(c_b)$. Thus, if it is optimal to issue only one license, then $\tau \leq \pi^M(c_b) - \frac{8}{9} \pi^M(c_b) = \frac{1}{9} \pi^M(c_b)$. To summarize, within this region, if at least two licenses are issued $\tau = 0$, while if one license is issued $\tau \leq \frac{1}{9} \pi^M(c_b)$. Again, $\pi^M(c_b)$ is decreasing in $c_b$ and within $(ii)$ $c_b \geq (2c_a - a)$, therefore, $\frac{1}{9} \pi^M(c_b) \leq \frac{1}{9} \pi^M(2c_a - a) = \frac{1}{9} \frac{4}{9} \pi^M(c_a) = \frac{4}{9} \pi^M(c_a)$.

iii) The innovator will issue one license and earn monopoly profit as her licensing revenue, there will be no leakage. Combining the three possibilities we conclude that $\tau(c_a,c_b) \leq \frac{1}{4} \pi^M(c_b) < \frac{9}{16} \pi^M(c_a)$.

**Lemma 12** Under Cournot competition with linear demand, $\tau_1 - \tau_2 < \pi^M(c_1) - \tau_1$.

**Proof.** From lemma 11 we know that $\tau_1 < \frac{1}{4} \pi^M(c_1)$ and $\tau_1 < \frac{9}{16} \pi^M(c_1)$, therefore $\tau_1 - \tau_2 + \tau_1 < \frac{13}{16} \pi^M(c_1)$.

**Lemma 13** Under Bertrand Competition, $0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_1$.

**Proof.** Under Bertrand competition, licenses can only be profitably sold to a single firm in each period. This is because Bertrand competition yields zero profit to each firm, unless there is only one firm with a superior technology. Thus, we can obtain an explicit solution
for technology leakage:

\[
\tau(c_a, c_b) = \begin{cases} 
\pi^M(c_b) - (c_a - c_b)D(c_a) & \text{if } c_a < p^M(c_b) \\
0 & \text{otherwise.}
\end{cases}
\]

First, suppose that \(c_2\) is not drastic relative to \(c'\), then \(\tau_1 = \pi^M(c_1) - (c' - c_1)D(c')\) and \(\tau_2 = \pi^M(c_2) - (c' - c_2)D(c')\). Because \(q^M(c_1)\) maximizes profits when cost is \(c_1\), we know \(\tau_1 - \tau_2 = \pi^M(c_1) - \pi^M(c_2) + (c_1 - c_2)D(c') > (p^M(c_2) - c_1)q^M(c_2) - \pi^M(c_2) + (c_1 - c_2)D(c').\)

Note that \(\pi^M(c_2) = (p^M(c_2) - c_1)q^M(c_2) + (c_1 - c_2)q^M(c_2).\) Therefore, \(\tau_1 - \tau_2 > (p^M(c_2) - c_1)q^M(c_2) - (p^M(c_2) - c_1)q^M(c_2) - (c_1 - c_2)q^M(c_2) + (c_1 - c_2)D(c') = (c_1 - c_2)(D(c') - q^M(c_2)) = (c_1 - c_2)(D(c') - D(p^M(c_2))).\) Since \(c_2\) is not drastic relative to \(c'\) this must be positive.

Second, suppose that \(c_2\) is drastic relative to \(c'\). In this case \(\tau_1 = \pi^M(c_1) - (c' - c_1)D(c')\) and \(\tau_2 = 0\), thus \(\tau_1 - \tau_2 > 0\).

Last, \(\tau_1 - \tau_2 + \tau_l = \pi^M(c_1) - (c_1 - c_2)[D(c_1) - D(c')] < \pi^M(c_1)\).

\section*{C More General Classes of Downstream Competition}

In the main text, we have focused on the case of homogenous good, conjectural variation oligopoly models, which allows us to impose restrictions on the costs of technology leakage. For completeness, in this appendix, we solve for the optimal fixed-fee licensing contracts without restricting the nature of downstream competition. In addition to generalizing our main result, the following results further illustrate the important role played by technology leakage in our model.

\begin{lemma}
If either (i) \(\tau_1 - \tau_2 < 0\) and \(\pi^M(c_1) - \tau_l > 0\), or (ii) \(\pi^M(c_1) - \tau_l < 0\) and \(\tau_1 - \tau_2 > 0\), then there exists a long-term fixed-fee contract that replicates the benchmark outcome.
\end{lemma}

\begin{proof}
Case (i) is proved in the proof of Lemma 5. An analogous proof can be constructed for case (ii).
\end{proof}
Lemma 15 If either (i) \( 0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_i \) and \( f_2 \geq \pi^M(c_1) - \tau_i \), or (ii) \( \pi^M(c_1) - \tau_i < \tau_1 - \tau_2 < 0 \) and \( f_2 \geq \tau_i - \tau_2 \), or (iii) \( \tau_1 - \tau_2 < \pi^M(c_1) - \tau_i < 0 \) and \( f_2 \geq \pi^M(c_1) - \tau_i \), or (iv) if \( \tau_1 - \tau_2 > \pi^M(c_1) - \tau_i > 0 \) and \( f_2 \geq \tau_i - \tau_2 \), then any equilibrium long-term fixed-fee contract \((f_1, f_2)\) with the opt-out clause is equivalent to a short-term fixed-fee contract with \( f = f_1 \).

Proof. Case (i) is proved in the proof of Lemma 7. Analogous proofs can be constructed for the other cases.

Lemma 16 In a long-term fixed-fee contract \((f_1, f_2)\) with the opt-out clause, if either (i) \( \tau_1 - \tau_2 < \pi^M(c_1) - \tau_i < 0 \) and \( f_2 \leq \pi^M(c_1) - \tau_i \), or (ii) \( \tau_1 - \tau_2 > \pi^M(c_1) - \tau_i > 0 \) and \( f_2 \leq \tau_i - \tau_2 \), then \( \Pi^{LF} \) decreases with \( f_2 \).

Proof. The following proof applies to case (i). An analogous proof can be constructed for case (ii).

In period 2, there are two states of nature: 1. innovation is not successful; 2. innovation is successful. In case 1, the original licensee will continue the contract and get \( \pi^M(c_1) - f_2 \), since \( \pi^M(c_1) - f_2 > \tau_i \). Hence \( \Pi_2 = f_2 \).

In case 2, we separate \( f_2 \) further into two regions: a) \( f_2 \leq \tau_i - \tau_2 \) and b) \( \tau_i - \tau_2 < f_2 < \pi^M(c_1) - \tau_i \).

(2.a) \( f_2 \leq \tau_i - \tau_2 \). If the original licensee continues the initial contract and produces at a cost of \( c_1 \), he gets \( \tau_i - f_2 \); if he opts out, he gets \( \tau_2 \). Since \( \tau_i - f_2 \geq \tau_2 \), it is optimal for the original licensee to continue the original license. This means that \( \Pi'_2 = \pi^M(c_2) - \tau_i + f_2 \) and \( \Delta = \pi^M(c_2) - \tau_i \). So the innovator’s incentive to invest is independent of \( f_2 \). Therefore, her licensing revenue is a constant if \( f_2 \leq \tau_i - \tau_2 \).

(2.b) \( f_2 > \tau_i - \tau_2 \). If the original licensee continues the initial contract and produces at a cost of \( c_1 \), then he gets \( \tau_i - f_2 \); if he opts out, then he gets \( \tau_2 \). Since \( \tau_i - f_2 < \tau_2 \), the original licensee’s right to use the old innovation has no value and he will opt out the initial contract. This means that \( \Pi'_2 = \pi^M(c_2) - \tau_2 \) and \( \Delta = \pi^M(c_2) - \tau_2 - f_2 \). In period 1, a firm is willing to
pay \( \Pi_1^{LF} = \pi^M(c_1) + \delta(1 - 2\rho (\pi^M(c_2) - \tau_2 - f_2)) (\pi^M(c_1) - f_2) + 2\delta \rho (\pi^M(c_2) - \tau_2 - f_2) \tau_2 \) for an exclusive license. At the same time, \( \Pi_2^{LF} = f_2 + \rho (\pi^M(c_2) - \tau_2 - f_2)^2 \). Hence, the total licensing revenue is \( \Pi^{LF} = \pi^M(c_1) + \delta(1 - 2\rho (\pi^M(c_2) - \tau_2 - f_2)) (\pi^M(c_1) - f_2) + 2\delta \rho (\pi^M(c_2) - \tau_2 - f_2) \tau_2 + \delta \left( f_2 + \rho (\pi^M(c_2) - \tau_2 - f_2)^2 \right) \), so \( \frac{\partial}{\partial f_2} \Pi^{LF} = 2 \rho \delta (\pi^M(c_1) - \tau_2 - f_2) < 2 \rho \delta (\pi^M(c_1) - \tau_1) < 0 \). Last, it is easy to verify that \( \Pi^{LF} \) is continuous at \( f_2 = \tau_1 - \tau_2 \).

**Lemma 17** In a long-term fixed-fee contract \((f_1, f_2)\) with the opt-out clause, if either (i) \( 0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_1 \) and \( f_2 \leq \pi^M(c_1) - \tau_1 \), or (ii) \( \pi^M(c_1) - \tau_1 < \tau_1 - \tau_2 < 0 \) and \( f_2 \leq \tau_1 - \tau_2 \), then \( \Pi^{LF} \) increases with \( f_2 \).

**Proof.** Case (i) is proved in the proof of Lemma 6. An analogous proof can be constructed for case (ii).

From the above lemmas, we can conclude the following for fixed-fee contracts.

**Proposition 5** (i) If \( \tau_1 - \tau_2 \) and \( \pi^M(c_1) - \tau_1 \) have different signs, then there exists a long-term fixed-fee contract that replicates the benchmark outcome; (ii) if they have the same sign, then the optimal fixed-fee license depends on the comparison of their absolute values: if \( |\tau_1 - \tau_2| > |\pi^M(c_1) - \tau_1| \), then a long-term contract with an upfront payment is optimal and generates a licensing revenue that is \( \delta \rho \left[ \pi^M(c_1) - \tau_1 \right]^2 \) below the benchmark profit, otherwise a short-term contract is optimal and generates a licensing revenue that is \( \delta \rho (\tau_1 - \tau_2)^2 \) below the benchmark profit.
References


