Information Asymmetry and Incentives for Active Management

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Abstract

This paper presents a model for delegated portfolio management, given incomplete information about managerial skills and efforts. I show that under information asymmetry, equilibrium outcomes depend on compensation structure and heterogeneity of skills. A performance fee can screen managers of differing ability and lead to a separating equilibrium (high skill managers actively manage funds while low skill managers track indexes), provided that skills for active management are sufficiently superior compared to tracking ability. Otherwise, a pooling equilibrium arises in which managers track indexes. The model suggests that the recent growth in passive management (e.g., closet-indexing) in the mutual fund industry could stem from deterioration in skills, for example, due to a brain drain to the hedge fund industry. Alternatively, investors' better investment options such as hedge funds or investors' weak responses to mutual fund performance could have caused the growth in passive management.

Key words: delegated portfolio management, information asymmetry, mechanism design, compensation, closet-indexing

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Introduction

Although many researchers acknowledge that investors face information asymmetry about fund managers' skills and efforts, few studies actually examine the strategic behavior of fund managers in such a setting. This paper presents a model for delegated portfolio management and examines equilibrium outcomes, given incomplete information about fund managers' skills and efforts. In particular, the main question I seek to answer is: Under what conditions can investors screen managers of differing skills and achieve a separating equilibrium in which high skill managers actively manage funds while low skill managers track indexes (a self-selection mechanism)?

This paper shows that when fund managers withhold information about their skills and management efforts, equilibrium outcomes depend on compensation structures and the heterogeneity of skills. A performance fee for active management can screen managers of differing ability and lead to a separating equilibrium, provided that high skill managers have sufficiently superior ability. Investors provide high skill managers with incentives for active management by paying a performance fee that is enough to compensate them but not enough to entice low skill managers. Yet, when the ability of high skill managers is not sufficiently superior, even high skill managers eschew active management. Intuitively, when skills are virtually homogeneous, separating managers is too costly, and as a result, a pooling equilibrium occurs. Hence, a separating equilibrium is efficient only when active management by high skill managers can lead to sufficiently higher performance compared to passive management (exceeding some threshold of heterogeneity). In the separating equilibrium, the performance fee is positively related to the high skill managers' ability and outside wages, but negatively related to their cost of effort in active management and to investors' reservation utility from other investments.

I show that if compensation for active management is independent of performance, fund managers do not actively manage funds. When investors pay fund managers ex-ante without information about managers' efforts and ability, compensation cannot depend on the fund types (active or passive) in equilibrium. Thus, to avoid costly efforts for active management, fund managers track indexes rather than try to beat them. Therefore, we have a pooling equilibrium with a fixed fee.

Lazear (2000) argues that variable pay, such as performance based compensation, is important because of its sorting role for agents with private information and agents' heterogeneity. In particular, he shows that when agents' ability is heterogeneous, fixed compensation induces efforts that are too low for high ability agents and too high for low ability agents. In my model, with two levels of effort (active or passive management), fixed compensation leads to a distortion: High skill managers' level of effort is low—they track indexes. On the other hand, as Lazear contends, variable pay can sort managers of high ability and induce efficient effort levels. However, my model suggests that when managers' skills are not sufficiently heterogeneous, sorting managers is inefficient. In essence, a distortion in effort may be preferred since inducing different levels of effort is too costly when managers' skills are similar.

A major contribution of the model is its implication that the recent growth in index and closet-index funds[1] could stem from "insufficient" skill levels for active management. The skill levels can become "insufficient" because they deteriorate, for instance, due to a brain drain to the hedge fund industry[2]. Alternatively, even though the mutual fund industry does not experience a decrease in skills, the skill levels for active management can be "insufficient" when separating equilibrium requires higher skills. The model suggests several factors that increase the threshold for heterogeneity in skills that sustains separating equilibrium, for example, investors' better investment alternatives. This implies that growth of other money management industries, such as hedge funds, could have contributed to more passive management in the mutual fund industry.

Moreover, the growth in closet-indexing and index funds can be attributed to weakening performance compensation for mutual funds. With few exceptions, mutual fund fees are not contingent on fund performance and only depend on assets under management. In this case, investors' responses to past performance can be implicit performance compensation when they chase past good performance. Many previous studies document this flow-performance relationship, for example, Ippolito (1992), Patel, Hendricks, and Zeckhauser (1994), Chevalier and Ellison (1995), Goetzmann and Peles (1996), and Siri and Tufano (1998). However, the growth in passive management suggests that the relationship may have weakened in recent years. Indeed, Kim (2009) finds that investors respond to past good performance with weaker sensitivity after 2000 than in the 1990's. Since good performance attracts fewer inflows of investment, managers have fewer incentives for active management.

[1] A closet-index fund is a passively managed, index fund although the manager claims to actively manage the fund and charges a high fee. Cremers and Petajisto (2007) show that closet indexing has steadily grown in the past several years. Moreover, the number of exchange traded funds also increased from 19 in 1997 to 353 in 2006. Their net assets increased around 60 times to $422 billion. Index funds also show comparable growth. See 2007 Investment Company Institute (ICI) Fact Book.

[2] Kostovetsky (2007) provides evidence of the brain drain. He shows that more best-performing young mutual fund managers exited the industry after 1999 when the hedge fund industry became massive and that performance of young mutual fund managers decreased after 1999.

Understanding the cause of closet-indexing helps us seek a solution for discouraging the behavior and increasing investor welfare. Cremers and Petajisto (2007) find that the aggregate percentage of actively managed assets within the nonindex funds with the S&P500 benchmark was just around 30% in 2003 and that the percentage has been steadily declining. Given that roughly 70% of the funds with the S&P500 benchmark are passively managed, investors overpay the managers by around $5 billion a year, assuming that the funds have $700 billion of assets under management and that the annual fee difference between nonindex and index funds is 1%. In addition to the management fees, suboptimal portfolio choices lead to a welfare loss. Investors who planned to allocate their investments in actively managed funds may, in fact, have invested in index funds, thereby distorting their optimal portfolio allocations.

Provided that lower skills have caused closet indexing, the mutual fund industry should focus on attracting and retaining skilled managers. For instance, better compensation for skilled managers may prevent a brain drain to other asset management industries. In addition, creating different pools of managers for active and passive funds can also be an alternative. This can improve heterogeneity of managers' skills and, thus, investors may screen managers of differing ability by chasing good performance of actively managed funds.

Rather, if weakening flow-performance relationships have caused the growth in closet indexing, encouraging explicit performance compensation can discourage closet-indexing. In particular, anecdotal evidence suggests that virtually no incentive fees are used in practice due to the legal restriction for the mutual fund industry: Performance-based compensation must be symmetric, not option-like (The 1970 Amendments to the Investment Company Act). Therefore, removing this restriction may help the industry provide skilled managers with incentives for active management by allowing asymmetric compensation structures.

My model assumes that investors lack information about funds and fund managers. If investors had perfect information about mutual funds and the managers, they could compensate only skilled managers for active management and a separating equilibrium could be achieved. Although in practice mutual fund managers disclose information, such as investment objectives, benchmark indexes and past performance, as required by the Securities and Exchange Commission (SEC), investors still lack important information.[3] For example, past performance is a noisy measure of managerial ability even net of a benchmark since the state of the world also plays a large role. Some studies also

[3] An investment objective of a fund describes whether the fund simply tracks (passive management) or aims to outperform (active management) a benchmark index. A benchmark of a fund is a market index with similar characteristics to those of the fund in question. Benchmarks can help investors obtain information about the funds' investment styles and risks.

argue that performance may be manipulated, for instance, due to product differentiation among funds (Massa (2003)) or favoritism among funds in a fund family (Gaspar, Massa, and Matos (2007)). Moreover, managers' disclosures about their funds can be misleading. The closet-indexing as discussed above is an example of fund managers providing investors with inaccurate information about their funds. Another example is studied by Sensoy (2008), who finds that the majority of stated benchmarks do not match funds' actual investment styles and that some mutual funds could have attracted inflows of investors' money based on questionable benchmarks.

Given private information of fund managers, this paper studies their equilibrium behavior, using the mechanism design theory developed by Hurwicz (1960, 1972), Maskin (1977) and Myerson (1979, 1981). Previous studies in finance have used the approach to suggest optimal contracts. For example, in the corporate finance literature, Darraough and Stoughton (1989) apply the theory to propose an optimal profit sharing rule for joint ventures under information asymmetry about member firms' costs. Harris and Raviv (1998) suggest an optimal capital budgeting rule when division managers have private information about productivity of their projects. Marino and Matsusaka (2005) identify optimal capital allocations and discuss how to implement them through delegation or approval.

Whereas the study of optimal contracts is one important application, the mechanism design theory also helps us better understand observed mechanisms under information asymmetry. For example, Harris and Raviv (1981) use the theory to identify conditions under which sellers—who do not know buyers' value—can maximize their expected revenues in commonly observed auctions. Likewise, given the compensation structures prevalent in the markets, this paper suggests conditions under which the markets can achieve a separating equilibrium (high skill managers actively manage funds while low skill managers track indexes) when investors do not know managers' skills and efforts.

In the mutual fund industry, the market outcome can be viewed as a result of a mechanism. Under SEC rules, fund managers report information about funds in their prospectuses. Fund managers also determine fees in proportion to assets under management, and investors decide investment amounts. Thus, payoffs to fund managers (compensation) and to investors (investment returns) are allocated by both parties together. This enables us to view the outcome as a response-plan equilibrium. A response-plan equilibrium is a collection of response plans that define each agent's strategy for reporting information, given his true type, within which no agent can benefit by unilaterally deviating from his reporting plan. In this equilibrium, agents may lie. In the mutual fund industry, if the accuracy of a prospectus is hard to verify, and a penalty for inaccurate information is unlikely, fund managers may deviate from a strategy of reporting truthfully in their prospectuses. An outcome generated by managers' reporting strategies, whether their reports are truthful or not, can be studied as response-plan equilibrium.

Given a response-plan equilibrium, I focus on incentive-compatible direct mechanisms to examine the equilibrium and its efficiency according to the revelation principle.[4] I describe managers' types in terms of their skills (high or low) and fund types (active or passive) to incorporate both incomplete information about managerial ability (adverse selection) and management effort (moral hazard). This is a generalized version of the principal-agent problem in the spirit of Myerson (1982). To determine efficiency, I adopt a bargaining approach in a Bayesian setting as proposed by Myerson (1979) and assume that the fund managers know their own types (*interim-welfare criterion*). I find optimal compensation by maximizing surpluses of investors and fund managers. The optimal compensation is also renegotiation-proof (durable) since only fund managers have private information and the model results hold for any number of managers (see Holmstrom and Myerson (1983)).

The rest of the paper is organized as follows. Section 1 discusses related literature. Section 2 states the problem and Section 3 proposes a solution to the problem. A conclusion follows and the appendix provides some proof.

1 Related Literature

This paper is related to the theoretical literature on optimal compensation for delegated portfolio management. Heckerman (1975) shows that an optimal compensation contract can induce managers with private information about stocks to align their interests with those of investors. The literature expanded greatly following Bhattacharya and Pfleiderer (1985). The main strand of the literature designs optimal compensation contracts for actively managed funds, when investors cannot observe fund managers' *effort* for collecting superior information or their risk-taking behavior. Stoughton (1993) and Li and Tiwari (2008) suggest that an option-like incentive fee is optimal for the moral hazard on effort.[5] Heinkel and Stoughton (1994), in their two-period model, propose a dynamic contract that has an option-like performance compensation in the

[4] The revelation principle states that for an equilibrium outcome of any arbitrary mechanism, a mechanism that ensures agents' *truthful* reports of their *types (*an incentive-compatible direct mechanism) can generate the same outcome.

[5] An option-like incentive fee is asymmetric. Under a symmetric fee contract , a manager also receives a penalty for a negative return in the same way that she receives a bonus for a positive return.

second period.[6] However, Grinblatt and Titman (1989) show that an asymmetric performance fee may encourage fund managers to take excessive risk. Their results support Starks's (1987) model, wherein a symmetric performance fee is optimal and preferred to an asymmetric performance fee, given the moral hazard on risk-taking.[7] Carpenter (2000), Elton, Gruber and Blake (2003), and Ross (2004) challenge these results and argue that an option-like fee does not necessarily distort fund managers' risk-taking incentives. In Panageas and Westerfield's (2007) model, risk-seeking behavior arises from an option-like incentive fee, only in finite horizons. Furthermore, Palomino and Prat (2003) show that an option-like performance compensation is superior for inducing fund managers to take optimal risk in their investment decisions. Das and Sundaram (2002) incorporate risk sharing and adverse selection and conclude that an option-like contract is optimal for maximizing investors' welfare.

Another strand of the literature studies optimal compensation in the presence of information asymmetry about managerial *skills*. Bhattacharya and Pfleiderer (1985) present a model with information asymmetry about managers' forecasting ability and propose an optimal compensation contract that screens out managers with inferior ability. But their model only considers actively managed portfolios and, thus, no information asymmetry about managers' efforts. Ippolito (1992) suggests a simple model for actively managed funds, in which managers are either good (skilled and diligent) or bad (unskilled or fraudulent) by an exogenously given probability. The model shows that markets can maintain good funds (actively managed funds by good managers) by investors' chasing good performers. However, the model does not discuss the equilibrium choices of effort by the managers. On the other hand, Dybvig, Farnsworth and Carpenter (2004) propose the optimal compensation contract that can also affect managers' investment strategies in the presence of private information about stock prices. Yet, in their model, managerial skills are common knowledge and only actively managed funds are considered.

The model presented in this paper is also comparable to Berk and Green's (2004) model, which explains why investors chase performance even when it

[6] Most studies assume that a nonzero optimal level of effort exists for collecting information. An exception is Dow and Gorton's (1997) model, in which talented managers do not always find superior information. In this case, managers may do noise trading although "actively" doing nothing is optimal because investors cannot distinguish it from "simply" doing nothing.

[7] Admati and Pfleiderer (1997) show that a choice of benchmark used to evaluate performance is critical for risk-taking incentives. Also see Ou-Yang (2003), Agarwal, Gomez and Priestley (2007), Basak, Pavlova and Shapiro (2008) for discussions about benchmarking. In Brennan's (1993) model, a choice of benchmark portfolios for evaluating relative performance of fund managers affects asset returns. Murphy (2000) examines the role and the choice of performance standards for corporate executives' compensation.

is not persistent. They do so because past performance is the evidence of superior managerial skills. Nevertheless, investors cannot earn excess profits by investing in the funds of managers with superior ability because managers' skills have decreasing returns to scale and the investors compete to invest their money in those funds until the excess returns they earn decrease to zero. The model, therefore, shows that the fund flow-performance relationship is a result of investors' rational responses, and that the lack of performance persistence is due to the competition among investors. However, Berk and Green's model does not consider information asymmetry because managers actively manage funds (no moral hazard) and they do not know their own skills.

This paper departs from earlier studies in three important ways. First, my model considers *both* adverse selection and moral hazard problems while many studies focus on the latter. Allen (2001) argues that adverse selection may be more salient than moral hazard in the agency context for delegated portfolio management. Considering incomplete information about both ability and effort is critical for studying a self-selection mechanism in the markets. To this end, I view moral hazard on effort as managers' misreporting of fund types. For example, closet-indexing can be viewed as misreporting index funds as actively managed funds. Then I define managers' types by their skills and fund types. This approach simplifies the analysis through the revelation principle.

Second, I identify optimal compensation for managers as a solution to a bargaining problem between investors and managers. This is comparable with standard principal-agent problems in which a principal designs compensation schemes for agents. Since fund managers (agents) typically offer their fee schemes to investors, solving a problem from the perspective of investors (principal) may not represent the industry. Moreover, competition among fund managers to attract flows and among investors to direct their money to profitable funds suggests that neither parties have full bargaining power. Rather, equilibrium can be better described as a result of a bargaining between two parties. Therefore, I find equilibrium by maximizing a social welfare function, a weighted sum of surpluses of investors and fund managers.

Finally, my model proposes conditions for separating equilibrium for any given performance fees, rather than proposing specific performance fee structure that can provide fund managers with incentives for active management. Many studies examine which performance fees—for instance, an option-like or a symmetric fee—are better for inducing efforts. Those studies impose the constraint that requires managers to exert a positive level of effort. This excludes pooling equilibrium in which managers exert no effort. I show that such equilibrium can be efficient under some conditions, namely, skills for active management are not sufficiently superior, compared to tracking ability. My model also suggests how growth of other asset management industries can influence mutual fund managers' incentives for active management.

2 A statement of the problem

I consider an investor who makes contracts with a fund manager. [8] To focus on the compensation contract, I assume that the investor's utility is additively separable between investment income and transaction cost. This separability implies that the investor's optimal investment problem is independent of compensation for the fund manager. More specifically, I assume the following.

Assumption 1 *The investor's utility is given by*

$$U_0 = u(\cdot) - d$$

where $u(\cdot)$ is an increasing and strictly concave function of utility from investment in funds and d is compensation for the fund manager. I assume the disutility from compensating the manager is linear, in particular, equal to the compensation, without loss of generality.

On the other hand, the fund manager's utility is additively separable for money and effort. It is increasing and concave in compensation and decreasing in management effort. For simplicity, I assume that disutility from effort is equal to effort, but one can assume any form of disutility from effort (e.g., convex).

Assumption 2 *Fund manager's utility is given by*

$$U = v(\cdot) - e$$

for the manager where $v(\cdot)$ is a continuous, differentiable, increasing and strictly concave function of utility from compensation and e is management effort. I assume the disutility from management effort is equal to the effort without loss of generality. I normalize so that $v(0) = 0$.

As a result of these separability assumptions, I can restrict my attention to the contract about fees between the investor and the fund manager. In other words, the problem is to find an equilibrium compensation contract that minimizes the investor's transaction cost and maximizes the fund manager's utility.

I assume that the investor is of only one type but that the fund manager can have high (H) skill with probability λ or low (L) skill with probability $1 - \lambda$. After the manager realizes her skill type, she can choose to manage an active (A) or a passive (P) fund. Choosing a fund type corresponds to choosing an effort level between high effort (active management) and low effort (passive management). The probability of choosing a fund is endogenous and depends

[8] The previous version of the paper considers two fund managers. All results hold for any number of managers. For notation simplicity, I assume one manager in this paper.

on her skill type. I denote the probability that the high skill manager and the low skill manager choose an active fund by μ_H and μ_L respectively. Thus, the probability that a passive fund is chosen is equal to $1 - \mu_H$ and $1 - \mu_L$ respectively. I denote the probability distribution over the four types by Λ, a four by one vector.

Definition 3 *The type of fund manager describes her skill and the fund type that she manages. We denote the type of manager by t and the set of four possible types by $T = \{(H, A), (H, P), (L, A), (L, P)\}$.*

Assumption 4 *Managing the A fund takes less management effort for the H skill manager than it does for the L skill manager, which I denote by $\overline{e}_H < \overline{e}_L$. On the other hand, tracking an index (managing the P fund) takes same effort for both skills, which I normalize to zero without loss of generality. Thus, $e_{H,A} = \overline{e}_H$, $e_{L,A} = \overline{e}_L$, and $e_{H,P} = e_{L,P} = 0$.*

Assumption 5 *The return distribution of the P fund does not depend on the manager's skill but that of the A fund does. The return distribution of the A fund managed by the H skill manager first-order stochastically dominates the return distribution when managed by the L skill manager. Similarly, the return distribution of the P fund first-order stochastically dominates the return distribution of the A fund managed by the L skill manager. I denote the stochastic dominance by $F_P(\widetilde{r}) \succ F_P(\widetilde{r})$ and $F_P(\widetilde{r}) \succ F_P(\widetilde{r})$ where $F(\cdot)$ is a cumulative distribution function that depends on skills and fund types as represented in the subscript, and $\widetilde{r}$ is the return on the funds.*

It is worth discussing an equilibrium implication of the stochastic dominance assumption. When the manager incurs costs of effort for active management but the return distribution of active management by the L skill type are stochastically dominated by that of passive management, the L skill manager will not choose the A fund in equilibrium (see Appendix for proof). By the revelation principle, the type (L, P) manager should earn as much as she would earn by pretending to actively manage the fund (type (L, A)). Since return distribution of passive management is superior than that of active management by the L skill manager, the payoffs when the passive manager pretends to be of the type (L, A) must exceed the payoffs when the manager is actually of the type (L, A). As a result, the type (L, P) manager must obtain larger payoffs than the type (L, A) manager in equilibrium. This leads the L skill manager to always choose the P fund in equilibrium. Therefore, only two equilibria are plausible: pooling equilibrium with passive management and separating equilibrium in which the H skill manager chooses the A fund and the L skill manager chooses the P fund.

Skills can be measured in many ways. In particular, I define skills as "alpha," the performance premium for active management by the H skill manager compared to passive management:

$$\alpha = \frac{E_{H,A}[\overline{v}(\widetilde{r})] - E_P[\overline{v}(\widetilde{r})]}{E_P[\overline{v}(\widetilde{r})]}, \tag{1}$$

where $E_{H,A}[\cdot]$ and $E_P[\cdot]$ are expectations evaluated under the distribution $F_P(\widetilde{r})$ and $F_P(\widetilde{r})$ respectively, and $\overline{v}(\widetilde{r})$ is the manager's payoffs from some compensation that is determined only by $F_P(\widetilde{r})$ and $F_P(\widetilde{r})$. In other words, once we have $F_P(\widetilde{r})$ and $F_P(\widetilde{r})$, $\overline{v}(\widetilde{r})$ is exogenously given. The expected value of $\overline{v}(\widetilde{r})$ is larger under $F_P(\widetilde{r})$ than under $F_P(\widetilde{r})$, which suggests that α is positive. I discuss $\overline{v}(\widetilde{r})$ in detail in Appendix.

When only the fund manager (as opposed to investors) knows the critical information for the investment contracts—management skill and fund type—, an equilibrium with transactions between them may not be guaranteed. Suppose, therefore, that the investor can ask an arbitrator to mediate the contract with the fund manager. The arbitrator receives confidential information about the fund manager's skill and the fund type. Then the arbitrator suggests compensation for each manager type (skill and fund type). If both the investor and the fund manager agree on the suggestions, the investor makes the investment and pays the fees according to the advice. Otherwise, there is no transaction between them: no investment and no fund management. The arbitrator, thus, must select a compensation rule (*mechanism*) for the fund manager, given the manager's report of her own skill and fund type.

There are many kinds of compensation rules. I restrict my attention to some specific fee structures that mimic the industry practice: fixed fees and performance fees. [9] The investor pays management fees that are independent of performance to any type of the manager and additional performance fees to the H skill manager who manages the A fund. In essence, the (H, A) type manager earns management fees plus performance fees that depend on some "performance" returns. [10] One can also consider performance fees for the (L, A) type manager, but this does not change any equilibrium results since there is no (L, A) type in equilibrium as discussed earlier. For simplicity, thus, I assume performance fees for the (H, A) type manager only.

[9] The purpose of the model is not discussing whether those fee structures are optimal or not. Rather it aims to find the conditions for separating equilibrium given those fee structures, to understand the recent growth in passive management in the mutual fund industry. Thus, the paper propses a partial equilibrium model. The model also does not consider the whole money management industry including other funds such as hedge funds.

[10] Performance returns are the returns that a performance fee depends on. For example, an option-like fee uses max [0, return]. The model in this paper does not require a specific incentive fee structure and, thus, the results apply to any given incentive fee structure.

Definition 6 *The set of alternatives* Γ *for the investor and the fund manager consists of the elements* (m, β, π) *where* m *is the dollar amount of fixed compensation for the manager with* $m \in [0, \overline{m}]$*, and* $\beta \in [0, 1]$ *is the performance fee rate for the type (H,A) manager, which is multiplied by some (dollar) performance returns on funds. Finally,* π *is the probability of compensating the manager with* $\pi \in [0, 1]$.

Definition 7 *The Bayesian collective choice problem is summarized by (*Γ*,* T*,* U_0*,* U*,* Λ*) where* Γ *is the set of alternatives,* T *the set of the manager's types,* U_0 *the investor's utility,* U *the manager's utility, and* Λ *the probability distribution over* T.

Definition 8 *A response set* S *is the set of all possible responses of the fund manager to the arbitrator, given her true type* $t \in T$*. A standard response set is* $S = T$*, when responses are restricted to reports of types.*

I assume that each response of the managers to the arbitrator is confidential and noncooperative. The fund managers are expected to be truthful as long as they have no incentive to lie. For a standard response set, a truthful response is the identity map.

Definition 9 *A choice mechanism* Π *by the arbitrator is choosing an element in* Γ *given the information reported to the arbitrator by the manager,* $s \in S$ *or* $t \in T$.

Choosing the compensation probability π corresponds to a mechanism that chooses a probability distribution over the manager types that receive compensation in the case of more than one managers. When there are multiple managers, a mechanism fully specifies the probabilities that each manager type obtains compensation, for every report of types (see Myerson (1979)). Yet, for one manager, specifying only the compensation probability for her reported type is sufficient and simplifies the model.

A response plan equilibrium is a response plan of the fund manager such that she cannot be better off by changing to another response plan, given a mechanism. The revelation principle states that we can generate any response plan equilibrium by an incentive-compatible direct mechanism. Hence, I restrict my analysis to a mechanism by which managers report their *true types.*

Definition 10 *A Bayesian incentive compatible mechanism* Π *is the one that does not give any type of the manager an incentive to lie:*

$$Z(\Pi, t|t) \geq Z(\Pi, s|t)$$

for all $s \neq t \in T$ *for every* $t \in T$ *where*

$$Z(\Pi, s|t) = \pi(s)v(m(s), \beta(s)) - e_t. \tag{2}$$

In words, $Z(\Pi, s|t)$ is the expected payoff when the fund manager of type t reports her type as s, given a choice mechanism Π. A Bayesian incentive compatible mechanism leads to a response plan equilibrium in which responses are standard and the response plans are the identity map.

An allocation of conditionally-expected payoffs when the manager is honest is denoted by $H(\Pi) \equiv \{H(\Pi|t)\}_{t\in T}$ where $H(\Pi|t) = Z(\Pi, t|t)$ given Π. In essence, this allocation of payoffs is what we can achieve with *truthful* responses when the response set is *standard* (report types). Equivalently, we can achieve this allocation by a Bayesian incentive compatible mechanism. This leads to the following definition of an incentive-feasible set.

Definition 11 *An incentive-feasible set of allocations of conditionally-expected payoffs is* $F^* = \{H(\Pi)\colon \Pi$ *is is a Bayesian incentive compatible mechanism*$\}$.

The allocation of conditionally-expected payoffs when all managers are honest is possible only through a mechanism which ensures that all fund managers reveal their types, that is, a Bayesian incentive compatible mechanism. Therefore, an incentive-feasible set is restricted to the set of allocations that can be achieved by a Bayesian incentive compatible mechanism.

Definition 12 *The conflict outcome occurs when the investor and the fund manager fail to agree. It is no-transaction between them. They receive their reservation utilities in the conflict outcome.*

Definition 13 *A reference point is the payoffs from the conflict outcome: reservation utilities.*

For simplicity, I assume that the fund manager's reservation utility is zero for every type (relaxing the assumption does not change the results). The reservation utility of the investor is expected payoffs from other investment alternatives such as hedge funds. I denote the investor's reservation utility by w_0 and will discuss how equilibrium outcomes change as w_0 changes, e.g., as other money management industries such as hedge funds grow.

The conflict outcome can always be achieved by a Bayesian-incentive compatible mechanism since no fund manager has an incentive to lie when the mechanism chooses the conflict outcome no matter which types are reported. The mechanism is choosing $m(t) = \beta(t) = \pi(t) = 0$ for every $t \in T$. Thus, the payoffs from the conflict outcome are in the incentive-feasible set F^*.

3 Efficient bargaining solution

To achieve efficiency, the arbitrator must find a Bayesian incentive compatible mechanism that is Pareto optimal. No other incentive compatible mechanism can make some better off without making others worse off. I use the *interim welfare criterion* for efficiency, which considers the expected payoff for the fund manager conditional on her type. [11] In other words, the fund manager's private information about her true type is considered in her expected payoffs when we determine whether the manager would be better or worse off by an alternative allocation. The interim-efficient allocation is durable in a sense that the investor and the fund manager will not unanimously approve a change to any other allocations because only the fund manager has private information (Holmstrom and Myerson (1983)).

The arbitrator's problem is a *bargaining* problem since the feasible set of expected allocation of payoffs F^* includes a reference point when the investor and fund managers receive their reservation utilities, as discussed in Section 2. Moreover, in practice, both investment amount by investors and fee ratios charged by fund managers determine compensation for fund managers. Thus, a mechanism that gives all surpluses to either the investor or the fund manager may not represent the industry well. Rather, I consider mechanisms that provide both parties with positive surpluses. In particular, Harsanyi and Selton (1972), extending Nash (1950), identify a feasible solution to a bargaining problem for N agents as a vector $\{x_{t_i}\}_{i=0}^{N}$ that maximizes a generalized Nash Product. The generalized Nash Product is given by

$$\prod_{i=1}^{N} \left\{ \prod_{t_i \in T_i} (x_{t_i} - w_{t_i})^{p(t_i)} \right\},$$

where x_{t_i} is the (expected) utility and w_{t_i} the reference point of the agent i of type $t_i \in T_i$, and $p(t_i)$ is the probability that the agent i is of type t_i. In my model with one investor and one manager, the generalized Nash Product is simplified to $(x_0 - w_0) \prod_{t \in T} x_t^{p(t)}$ where x_0 and w_0 are the investor's expected utility and reservation utility and x_t and w_t are the type t manager's expected utility and reservation utility respectively.

Consequently, the arbitrator maximizes the generalized Nash Product over the set of feasible allocations of conditionally-expected payoffs denoted by F^* (Definition 4). However, no one will participate in the contract if the expected payoff by a mechanism suggested by the arbitrator is less than the reference point (Definition 5). Therefore, at a minimum, an implementable mechanism

[11] See Holmstrom and Myerson (1983) for detailed discussions and other welfare criteria.

design should provide reservation utility to the investor and the fund managers. Thus, an efficient solution for N agents is the one that maximizes the generalized Nash Product over the set F^*_+ given by

$$F^*_+ = F^* \cap \{x : x_{t_i} \geq w_{t_i} \text{ for all } i \text{ and all } t_i \in T_i\}.$$

Assumption 14 *The allocation of conflict outcome is strictly dominated.*

Since the choice set is compact (m, β and π are bounded), there exists a unique incentive-feasible bargaining solution under Assumption 5 (Myerson (1979)). In addition, Assumption 5 implies that the solution that maximizes the generalized Nash Product should also strictly dominate the conflict outcome. Thus, I can maximize the log of the generalized Nash Product,

$$\log(x_0 - w_0) + \sum_{t\in T} p(t) \log x_t.$$

Therefore, the arbitrator must find an efficient Bayesian incentive compatible mechanism, $\{\pi(t),\, m(t),\, \beta(t)\}_{t\in T}$, which solves the following problem:

$$\max_{\{\pi(t),m(t),\beta(t)\}_{t\in T}} \log(x_0 - w_0) + \sum_{t\in T} p(t) \log x_t \tag{3}$$

subject to

$$Z(\Pi, t|t) \geq Z(\Pi, s|t) \text{ for all } s \neq t \text{ for every } t \in T \tag{4}$$
$$Z(\Pi, t|t) \geq 0 \text{ for every } t \in T \tag{5}$$
$$\pi(t) \in [0,1],\ m(t) \in [0, \overline{m}],\ \beta(t) \in [0,1] \text{ for every } t \in T, \tag{6}$$

where x_0 is the expected utility of the investor and $x_t = Z(\Pi, t|t)$ is the expected utility of the manager of type t when she truthfully reports her type. According to Assumption 1, the expected utility of the investor is given by

$$\begin{aligned} x_0 &= \sum_{t\in T} p(t)\{u(t) - \pi(t)d(t)\} \\ &= u^* - \sum_{t\in T} p(t)\pi(t)d(t), \end{aligned} \tag{7}$$

where $u^* \equiv \sum_{t\in T} p(t)u(t)$ is the expected utility from investment. By Assumption 2, the expected utility of the manager of type t, $Z(\Pi, t|t)$, is given by

$$x_t \equiv Z(\Pi, t|t) = \pi(t) E_t[v(m(t) + \beta(t)\tilde{r} 1_{\{t=(H,A)\}})] - e_t, \tag{8}$$

where $1_{\{t=(H,A)\}}$ is the indicator function for the type (H, A) since the performance fees are only given to that type. $E_t[\cdot]$ is the expectation under the distribution of returns that depends on the type t.

Now we solve the arbitrator's problem given by (3) to (6) for two cases: no performance fee, and performance fee for the type (H, A) manager. Considering that few mutual funds actually charge performance fees, the case without performance fees might represent the industry. On the other hand, without explicit performance fees, investors' fund flows can compensate performance by changing investment balance depending on performance when compensation is proportional to assets under management. Thus, provided that fund flows are responsive to recent performance, the second case can represent the industry.

3.1 Special case (no performance fee)

Suppose that the manager receives only a management fee. This is a special case in which $\beta(t) = 0$ for any report $t \in T$.

Then the equation (8), the expected utility of the manager of type t, becomes

$$x_t = \pi(t)v(m(t)) - e_t. \tag{9}$$

Proposition 1 The incentive compatibility constraints imply that the manager receives the same expected utility from the management fee irrespective of her skill and fund type. In essence, the set of feasible allocations F^* consists of those allocations that provide every type of the manager with the same expected utility.

PROOF. When the fund manager of type t truthfully reports her type, she receives the expected utility given by (9). However, if she reports a type $s \neq t$, she receives

$$Z(\Pi, s|t) = \pi(s)v(m(s)) - e_t.$$

She will not lie if and only if

$$\pi(t)v(m(t)) - e_t \geq \pi(s)v(m(s)) - e_t \text{ for all } s \neq t. \tag{10}$$

The inequality (10) should hold for every $t \in T$, which implies

$$\pi(t)v(m(t)) = \pi(s)v(m(s)) \equiv \overline{y} \text{ for all } t, s \in T. \tag{11}$$

In essence, the expected utility from the management fee does not depend on fund types and managerial skills. I denote this expected utility for all types by $\overline{y}$.

Corollary 1 All managers choose to manage the P fund rather than the A fund (a pooling equilibrium).

PROOF. By Proposition 1, the expected utility from fee income is the same for both fund types. While the P fund requires no cost of effort, managing the A fund decreases expected utility by the disutility from managing the A fund. Therefore, the manager is better off by choosing the P fund. ■

Proposition 2 In the efficient pooling equilibrium, the expected utility from passive management $\overline{y}$ decreases as (1) the investor's utility from investment u^* decreases, or (2) the investor's reservation utility from other investments w_0 increases.

PROOF. Let me define $v_t = v(m(t))$ and $Q(v_t) = m(t)$ where $Q = v^{-1}$. That is, v_t is the utility of the fund manager of type t when she receives fee income $m(t)$. Since v is strictly concave, Q is strictly convex. In equilibrium, as Appendix shows, the expected utility from passive management $\overline{y}$ is given by

$$\overline{y} = \frac{u^* - w_0}{2Q'(\overline{v})}, \tag{12}$$

where $\overline{v}$ satisfies $Q'(\overline{v}) = \frac{Q(\overline{v})}{\overline{v}}$. Therefore, Proposition 2 follows.■

Propositions 1 and 2 highlight the main results of the model when the manager's compensation does not depend on performance. The investor does not know who is skilled and for which funds managerial skills matter. Yet, the investor must compensate the fund manager before he observes the fund performance. As a result, the investor ends up paying the same management fee on average to every type of the manager, and all types of the manager receive the same expected utility from fee income. This, in turn, leads every type of the manager to passively manage a fund. In this case, it is socially optimal that the investor pays fewer management fees when he has better outside investment opportunities like hedge funds.

Table 1. Comparative statics in an efficient pooling equilibrium.

	u^*	w_0
$\overline{y}$	+	−

Note: $\overline{y}$ (a manager's expected utility from the management fee), u^* (investors' utility from investment in mutual funds), w_0 (investors' reservation utility from other investments).

I solve a more general problem without restricting $\beta(t)$ to zero for all $t \in T$, i.e., the type (H, A) manager receives a performance fee plus a management fee. The expected utility of the manager of type t is given by the equation (8). With a performance fee, we can achieve a separating equilibrium provided that the skill for active management is sufficiently superior compared to the skill for passive management as suggested by Proposition 3.

Proposition 3 A separating equilibrium is possible when skills for active management are sufficiently superior as given by

$$\alpha > \alpha^*,$$

where α measures as given by the equation (1) and α^* is some threshold. More specifically, α measures an H skill manager's performance for the A fund compared to that for the P fund. When α exceeds some threshold α^*, a performance fee can lead to an equilibrium, in which the H skill manager chooses the A fund and the L skill manager chooses the P fund. Otherwise, we have a pooling equilibrium in which the manager always passively manages funds even though performance compensation exists.

PROOF. A sketch of the proof is as follows (Appendix provides a complete proof). The H skill manager chooses the A fund if and only if the expected utility premium (difference of expected utility between managing the A fund and the P fund) is positive, which I denote by [12]

$$\Delta > 0.$$

As shown in Appendix, Δ is given by

$$\Delta = \alpha \overline{y} - \overline{e}_H,$$

where

$$\alpha = \frac{E_{H,A}[v_{H,A}(\tilde{r})] - E_P[v_{H,A}(\tilde{r})]}{E_P[v_{H,A}(\tilde{r})]}$$

measures skills and

$$\overline{y} = \frac{(u^* - w_0)c + \overline{e}_H}{g(\lambda)c + \alpha + 1}$$

is the expected utility from passive management.

[12] I exclude the case where the manager is indifferent between active management and passive management. In this case, we can consider a mixed strategy by which the manager chooses the A fund with some positive probability less than 1.

The skill measure α is explained in the equation (1). In the equation for $\overline{y}$, $g(\cdot)$ is a positive, increasing function and c is some positive constant.

Therefore, $\Delta > 0$ holds if

$$\alpha\overline{y} > \overline{e}_H,$$

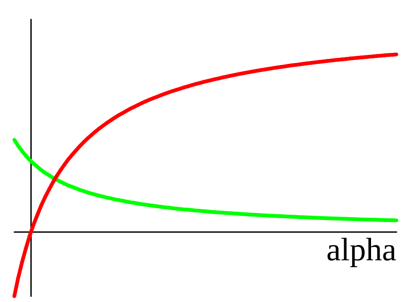
which gives us

$$\alpha > \frac{g(\lambda)c + 1}{(u^* - w_0)c}\overline{e}_H \equiv \alpha^*. \blacksquare$$

A performance fee can screen managers of differing ability when it is enough to compensate the H skill manager but not enough to entice the L skill manager. Yet, performance compensation cannot be large on average when skills for active management are not sufficiently superior (there is not much difference between active management and passive management). The intuition is that when the two skill levels, H and L, are similar, the low skill manager can easily mimic the high skill manager. Thus, separating them requires more bribes to the low skill manager and less performance compensation for active management. When the decreased incentive fee cannot cover the high skill manager's cost of effort for active management, the manager is better off by passively managing a fund. This leads to a pooling equilibrium in which both high and low skill managers passively manage funds even in the presence of performance compensation.

As a special case, consider $\overline{e}_H = 0$. When the H skill manager does not take management effort for active management, the threshold becomes zero and the condition for separating equilibrium always holds since α is positive. Thus, separating the high and low skill managers is always efficient when the high skill manager does not take any effort for active management.

Figure 1.



The green line is the expected utility from passive management $\overline{y}(\alpha)$ and the red line is the expected utility from active management $(\overline{y}+\Delta)(\alpha)$. The two functions cross at $\alpha^*> 0$ if a high skill manager's cost of effort for active management is positive ($\overline{e}_H> 0$). A separating equilibrium arises for $\alpha > \alpha^*$.

Figure 1 shows the expected utility from passive management $\overline{y}$ (green line) and from active management $\overline{y} + \Delta$ (red line) as a function of skills α with other parameters fixed (especially, $\overline{e}_H > 0$). The expected utility premium Δ is positive when $\overline{y} + \Delta$ is above $\overline{y}$. In other words, Δ is positive if and only if alpha is larger than the value at which two functions cross (α^*). In the region right to the crossing value, we have a separating equilibrium while a pooling equilibrium arises in the region left to the crossing value.

Suppose Proposition 3 holds and we have a separating equilibrium. Corollaries 2 and 3 show how payoffs change as some parameters, such as managerial skills and cost of effort for active management, change.

Corollary 2 In the separating equilibrium, the expected utility from passive management decreases as (1) managerial skills for active management α increase, (2) skilled managers are more efficient for active management (their cost of effort for active management $\overline{e}_H$ decreases), (3) the fraction of skilled managers λ increases, (4) the investor's utility from investment in the funds u^* decreases, or (5) the investor's reservation utility from other investments w_0 increases.

PROOF. See Appendix.■

Corollary 3 In the separating equilibrium, the expected utility premium from active management increases as (1) managerial skills for active management α increase, (2) skilled managers are more efficient for active management (their cost of effort for active management $\overline{e}_H$ decreases), (3) the fraction of skilled managers λ decreases, (4) the investor's utility from investment in the funds u^* increases, or (5) the investor's reservation utility from other investments w_0 decreases.

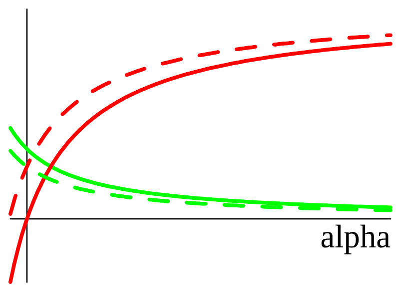
PROOF. See Appendix.■

The main results in Corollaries 2 and 3 are summarized in Table 2. Intuitively, as active management adds more value compared to passive management (as α increases), it is socially optimal that the investor pays a larger performance fee but a smaller (fixed) management fee.

Likewise, if skilled managers are more efficient in active management (less cost of effort $\overline{e}_H$), the performance fee is larger while the management fee is smaller. Figure 2 shows the case where $\overline{e}_H$ decreases to zero. Then the expected utility for passive managers decreases (green dash line) while the expected utility for active managers increases (red dash line) so that they meet at the value zero

($\alpha^* = 0$). In this case, a separating equilibrium always arises irrespective of α as discussed above.

Figure 2.



The green line is the expected utility from passive management $\overline{y}(\alpha)$ and the red line is the expected utility from active management $(\overline{y}+\Delta)(\alpha)$. The dashed lines show when a high skill manager's cost of effort for active management $\overline{e}_H$ decreases to zero. When $\overline{e}_H = 0$, the two functions cross at $\alpha^* = 0$. As a result, the region for a separating equilibrium expands to all positive values of α.

On the other hand, in a separating equilibrium, the investor pays fewer performance and management fees when the skilled managers account for a larger fraction of managers in the industry. Compensation for active management is higher than that for passive management. The investor's cost of fee would increase as the manager is more likely to have the high skills. Therefore, in an efficient outcome, the average performance and management fees must decrease as there are more skilled managers.

Finally, investors pay more performance and management fees if their utility from investment in mutual funds increases, while they pay fewer if they have better outside investment opportunities.

Table 2. Comparative statics in an efficient separating equilibrium

	α	$\overline{e}_H$	λ	u^*	w_0
$\overline{y}$	$-$	$+$	$-$	$+$	$-$
Δ	$+$	$-$	$-$	$+$	$-$
$\overline{y}+\Delta$	$+$	$-$	$-$	$+$	$-$

Note: $\overline{y}$ (a manager's expected utility from the management fee), Δ (expected utility from the performance fee), α (skills), $\overline{e}_H$ (cost of effort for active management),

λ (fraction of skilled managers), u^* (investors' utility from investment in mutual funds), w_0 (investors' reservation utility from other investments).

Corollary 4 A separating equilibrium is not likely if (1) skilled managers are inefficient for active management (their cost of effort for active management $\overline{e}_H$ is high), (2) the fraction of skilled managers λ is large, (3) skilled managers' outside wages are low (4) the investor's utility from investment in the funds u^* is low, or (5) the investor's reservation utility from other investments w_0 is high.

PROOF. See Appendix.■

Proposition 3 suggests that a separating equilibrium requires a sufficient level of skills for active management. In particular, a separating equilibrium is unlikely when managers' skills are not higher than the threshold. This can happen due to a lower skills or a higher threshold. Corollary 4 suggests the conditions that increase the threshold (Table 3 shows how the threshold changes). For instance, as managers are less efficient for active management, the required level of skills increases. Moreover, as there are more skilled managers in the industry, the benchmark level of skills becomes higher. On the other hand, if investors receive fewer payoffs from investment in mutual funds or more from other investments like hedge funds, the standard also increases. When the manager's skills are inferior compared to the higher standard, we have a pooling equilibrium in which managers do not actively manage funds.

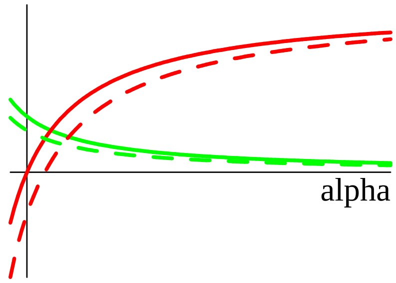
Table 3. Comparative statics for the skill threshold α^*

	$\overline{e}_H$	λ	u^*	w_0
α^*	+	+	−	+

Note: α^* (threshold of skills), $\overline{e}_H$ (cost of effort for active management), λ (fraction of skilled managers), u^* (investors' utility from investment in mutual funds), w_0 (investors' reservation utility from other investments).

As an example, Figure 3 illustrates the case in which investors have better other investments (w_0 increases). Then both the expected utility for passive managers and active managers decrease but the latter is more sensitive. As a result, two expected utility functions cross at a larger value and the region for separating equilibrium shrinks.

Figure 3.



The green line is the expected utility from passive management $\overline{y}(\alpha)$ and the red line is the expected utility from active management $(\overline{y}+\Delta)(\alpha)$. The dashed lines show when investors' reservation utility from other investments w_0 increases. When w_0 becomes larger, the two functions cross at a larger value. As a result, the region for a separating equilibrium shrinks.

4 Conclusion

When investors do not have information about managers' skills and fund types (active or passive), the fund managers who have the information can behave strategically. Yet, even under information asymmetry, if managers' compensation for active management depends on performance and managerial skills are sufficiently heterogeneous, the market can screen managers of differing ability and achieve a separating equilibrium—in which high skill managers actively manage funds while low skill managers track indexes. Otherwise, the market fails to reward skills and active management. This, in turn, makes it optimal for fund managers to passively manage funds regardless of their skills and creates a pooling equilibrium.

The recent growth in passively managed mutual funds (e.g., closet-indexing) suggests that skilled managers have few incentives for active management. I show that a lack of incentives for active management can result from lower skill in the mutual fund industry or from investors' better other investments (e.g., hedge fund growth). Alternatively, more closet-indexing can be consistent with a separating equilibrium in which only fewer skilled managers can actively manage funds as managers with superior ability migrate to the hedge fund industry.

Appendix

Proof for Proposition 2

As given by (9), the expected utility of the manager of type t is

$$x_t = Z(\Pi, t|t) = \pi(t)v(m(t)) - e_t. \tag{1-1}$$

Proposition 1 and Corollary 1 reduce the arbitrator's problem to

$$\max_{\overline{y},\{m(t),\pi(t)\}_{t\in T}} \quad \log(x_0 - w_0) + \sum_{t\in T} p(t)\log(\overline{y} - e_t) \tag{1-2}$$

$$\text{subject to } \pi(t)v(m(t)) = \overline{y} \text{ for every } t \in T, \tag{1-3}$$

where $x_0 = u^* - \sum_{t\in T} p(t)\pi(t)m(t)$.

Note that I drop the participation constraints because an optimal allocation gives the manager of type t more than her reservation utility w_t (Assumption 5) and, therefore, the constraints are not binding.

Let me define $v_t = v(m(t))$ and $Q(v_t) = m(t)$ where $Q \equiv v^{-1}$. That is, v_t is utility of the manager of type t when she receives fee income $m(t)$. Since V is strictly concave, Q is strictly convex.

Then, the above problem can be rewritten as

$$\max_{\overline{y},\{v_t,\pi(t)\}_{t\in T}} \quad \log(u^* - \sum_{t\in T} p(t)\pi(t)Q(v_t) - w_0) + \sum_{t\in T} p(t)\log(\overline{y} - e_t) \tag{1-4}$$

$$\text{subject to } \pi(t)v_t = \overline{y} \text{ for every } t \in T. \tag{1-5}$$

The constraint (1-5) is the incentive compatibility constraint by Proposition 1.

Assuming $\pi(t) \in (0, 1)$, I define a Lagrangian,

$$L(\overline{y}, v_t, \pi(t), \mu_t) = \log(u^* - \sum_{t\in T} p(t)\pi(t)Q(v_t) - w_0)$$
$$+ \sum_{t\in T} p(t)\log(\overline{y} - e_t) + \sum_{t\in T} \mu_t\{\pi(t)v_t - \overline{y}\}. \tag{1-6}$$

Since my objective function is now strictly concave and the constraints are linear, the Kuhn-Tucker theorem applies. Thus, the first-order conditions are also sufficient.

FOC for (1-6) with respect to $\pi(t)$ gives us

$$\frac{p(t)}{x_0 - w_0}\frac{Q(v_t)}{v_t} = \mu_t. \tag{1-7}$$

Since $p(1) = p(3) = 0$ by Corollary 1, we should have $\mu_{H,A} = \mu_{L,A} = 0$.

On the other hand, FOC for (1-6) with respect to v_t is given by

$$\frac{p(t)}{x_0 - w_0}Q^{'}(v_t) = \mu_t. \tag{1-8}$$

(1-7) and (1-8) imply

$$Q^{'}(v_t) = \frac{Q(v_t)}{v_t} \text{ for } t = (H,P),(L,P). \tag{1-9}$$

When Q is increasing and convex, there is a unique $\overline{v}$ that satisfies (1-9). Moreover, $v_{H,A}$ and $v_{L,A}$ that satisfy the equation (1-9) also satisfy the FOCs. Therefore, we have

$$v_t = \overline{v} \text{ for every } t \in T. \tag{1-10}$$

Then the probability of receiving payment, $\pi(t)$, must be the same by Proposition 1:

$$\pi(t) = \frac{\overline{y}}{\overline{v}} \equiv \overline{\pi} \text{ for every } t \in T. \tag{1-11}$$

Finally, FOC for (1-5) with respect to $\overline{y}$ yields,

$$\sum_{t\in T}\frac{p(t)}{\overline{y}} = \sum_{t\in T}\mu_t. \tag{1-12}$$

By the equation (1-7) and

$$x_0 = u^* - \overline{\pi}Q(\overline{v}),$$

I can rewrite (1-12) as

$$\frac{1}{\overline{y}} = \frac{Q^{'}(\overline{v})}{u^* - \overline{\pi}Q(\overline{v}) - w_0}.$$

By (1-9) and (1-11), the above equation becomes

$$\frac{1}{\overline{y}} = \frac{Q^{'}(\overline{v})}{u^* - \overline{y}Q^{'}(\overline{v}) - w_0}. \tag{1-13}$$

Therefore,

$$\overline{y} = \frac{u^* - w_0}{2Q'(\overline{v})}, \tag{1-13}$$

and

$$\frac{\partial \overline{y}}{\partial u^*} > 0, \frac{\partial \overline{y}}{\partial w_0} < 0. \blacksquare$$

Proof for Proposition 4

The H skill manager who manages the A type fund receives a performance fee plus a fixed fee. If the type (H, A) manager receives compensation, she receives utility,

$$v_{H,A}(\tilde{r}) \equiv v(m(H, A) + \beta(H, A)\tilde{r})$$

where $\tilde{r}$ is a random variable, which is some "performance" return on which performance compensation depends.

On the other hand, the other manager of type $t \neq (H, A)$ only receives the management fee as given by

$$v_t \equiv v(m(t)) \; for \; t \neq (H, A).$$

Similar to the case without performance fee, I use the inverse function of a manger's utility from income, $Q = v^{-1}$, and write

$$\begin{aligned} Q(v_t) &= m(t) \\ Q(v_{H,A}(\tilde{r})) &= m(H, A) + \beta(H, A)\tilde{r} \; for \; t \neq (H, A). \end{aligned} \tag{2-1}$$

The expected utility for the type t manager with the truthful report is

$$x_{H,A} = \pi(H, A)E_{H,A}[v_{H,A}(\tilde{r})] - \overline{e}_H \tag{2-2}$$
$$x_t = \pi(t)v_t \; for \; t = (H, P) \text{ and } (L, P) \tag{2-3}$$
$$x_{L,A} = \pi(L, P)v_{L,A} - \overline{e}_L, \tag{2-4}$$

where $E_{H,A}[\cdot]$ is the expectation with respect to $F_{H,A}(\tilde{r})$, the distribution of "performance" returns of the A fund when managed by an H skill manager.

Now, I first derive the incentive compatibility (IC) constraints for each type of the manager. The ICs for $t = (H, P)$ (or (L, P)) not to report a type (L, P) (or (H, P)) or (L, A) yield

$$\pi(t)v_t = \overline{y} \; for \; t = (H,P) \text{ and } (L,P) \tag{2-5}$$
$$\overline{y} \geq \pi(L,A)v_{L,A}. \tag{2-6}$$

If the manager of type (H,P) or (L,P) pretends to be of type (H,A), she receives

$$\pi(H,A)E_P[v_{H,A}(\tilde{r})],$$

where $E_P[\cdot]$ is the expectation with respect to $F_P(\tilde{r})$, the distribution of "performance" returns of the P fund. Therefore, I need another IC constraint,

$$\overline{y} \geq \pi(H,A)E_P[v_{H,A}(\tilde{r})]. \tag{2-7}$$

A manager of type (L,A) will not pretend to be of type (H,P) or (L,P) if

$$\pi(L,P)v_{L,A} - \overline{e}_L \geq \overline{y} - \overline{e}_L.$$

Then, by (2-6), we should have

$$\pi(L,P)v_{L,A} = \overline{y}, \tag{2-8}$$

and therefore

$$\overline{y} - \overline{e}_L$$

is the expected utility of the type (L,A) manager from the truthful report.

Notice that a type 4 manager receives more than a type (L,A) manager since

$$x_{L,P} = \overline{y} > \overline{y} - \overline{e}_L = x_{L,A}.$$

Therefore, it is optimal for the low skill manager to choose the P fund (become type (L,P)) rather than the A fund (become type (L,A)). So in equilibrium, the low skill manager always passively manages a fund.

On the other hand, if the manager of type (L,A) lies to be the type (H,A), she receives

$$\pi(H,A)E_{L,A}[v_{H,A}(\tilde{r})] - \overline{e}_L,$$

where $E_{L,A}[\cdot]$ is the expectation with respect to $F_{L,A}(\tilde{r})$, the distribution of "performance" returns of the A fund when managed by an L skill manager.

Then the IC that the type (L,A) manager does not lie to be the type (H,A) manager is

$$\overline{y} \geq \pi(H,A)E_{L,A}[v_{H,A}(\tilde{r})]. \tag{2-9}$$

For the type (H,A) manager, she will not lie to be of any other type if

$$\pi(H,A)E_{H,A}[v_{H,A}(\tilde{r})] \geq \overline{y}. \tag{2-10}$$

Let me define the difference of the H skill manager's expected utility between active management and passive management as

$$\Delta = (\pi(H, A)E_{H,A}[v_{H,A}(\tilde{r})] - \bar{e}_H) - \bar{y},$$

which allows me to rewrite (2-10) as

$$\Delta \geq -\bar{e}_H. \tag{2-11}$$

For future reference, let me derive the condition for the separating equilibrium. The H skill manger will choose the A type fund if

$$\pi(H, A)E_{H,A}[v_{H,A}(\tilde{r})] - \bar{e}_H > \bar{y}, \tag{2-12}$$

which is equivalent to

$$\Delta > 0. \tag{2-13}$$

The arbitrator's problem (3)-(6) in Section 3 becomes

$$\max_{\bar{y},\Delta,v_{H,A}(\widetilde{r}),\pi(H,A)\{v_t,\pi(t)\}_{t\in T,t\neq(H,A)}} \log(x_0 - w_0) + \sum_{t\in T} p(t)\log(x_t)$$

subject to

$$\begin{aligned}
\pi(H, A)E_{H,A}[v_{H,A}(\tilde{r})] - \bar{e}_H &= \bar{y} + \Delta \ for \ t = (H, A) \\
\Delta &\geq -e_H && \text{(IC 1)} \\
\pi(t)v_t &= \bar{y} \ for \ t = (H, P) \text{ and } (L, P) && \text{(IC 2)} \\
\pi(L, P)v_{L,A} &= \bar{y} && \text{(IC 3)} \\
\pi(H, A)E_{L,A}[v_{H,A}(\tilde{r})] &\leq \bar{y} && \text{(IC 4)} \\
\pi(H, A)E_P[v_{H,A}(\tilde{r})] &\leq \bar{y}, && \text{(IC 5)}
\end{aligned}$$

where

$$x_0 = u^* - \{p(H, A)\pi(H, A)E[Q(v_{H,A}(\tilde{r}))] + \sum_{j\neq 1} p(t)\pi(t)Q(v_t)\}$$

and x_t's are given by (2-2) to (2-4).

(IC 1) states that the manager of type (H, A) does not misreport her type as derived in (2-11). (IC 2) is the condition that the L (H) skill fund manager who manages the P fund will not lie to have the H (L) skill, and this is derived in (2-5). (IC 3) is that the type (L, A) manager will not report to be of the type (H, P) or (L, P) as I derive in (2-8). (IC 4) ensures that the L skill manager who actively manages a fund (type (L, A)) will not pretend to have the H skill (type (H, A)), as derived in the condition (2-9). The condition that an index fund manager (type (H, P) or (L, P)) will not lie to be a type manager to earn the performance fee is provided by (IC 5), which is derived in (2-7).

Under the assumption that return on the P fund stochastically dominates the return on the A fund managed by the L skill manager, only (IC 5) can hold with equality and (IC 4) holds with strict inequality. Thus, I can ignore (IC 4).

To solve the problem, I consider a separating equilibrium, which requires the condition (2-13). Then the inequality constraint (IC 1) is not binding. Also, we have $p(H, A) = \lambda$, $p(L, P) = 1 - \lambda$, and $p(H, P) = p(L, A) = 0$. As a result, we can also ignore (IC 2) since the Lagrangian multiplier must be zero.

Define a Lagrangian,

$$\begin{aligned}
&L(\overline{y}, \Delta, v_{H,A}(\widetilde{r}), \pi(H, A), v_t, \pi(t), \mu_{H,A}, \mu_{L,P}, \theta_{L,P}) \\
&= \log(x_0 - w_0) + \{\lambda \log(\overline{y} + \Delta - w_{H,A}) + (1 - \lambda) \log(\overline{y} - w_{L,P})\} \\
&+\mu_{H,A}\{\pi(H, A)E_{H,A}[v_{H,A}(\widetilde{r})] - \overline{e}_H - \overline{y} - \Delta\} + \mu_{L,P}\{\pi(L, P)v_{L,P} - \overline{y}\} \\
&+\theta_{L,P}\{\overline{y} - \pi(H, A)E_P[v_{H,A}(\widetilde{r})]\}. \qquad (2\text{-}14)
\end{aligned}$$

where $x_0 = u^* - \lambda\pi(H, A)E[Q(v_{H,A}(\widetilde{r}))] - (1 - \lambda)\pi(L, P)Q(v_{L,P})$.

Lemma 1 In the separating equilibrium, a manager's utility from the management fee is only determined by managers' preference for fee income (e.g., risk aversion).

PROOF. We need to solve v_t for every $t \neq (H, A)$ and show they are equal to some exogenous value (given the manager's utility function). FOC for (2-14) with respect to $\overline{y}$ gives us

$$\frac{\lambda}{\overline{y} + \Delta} + \frac{1 - \lambda}{\overline{y}} = \mu_{H,A} + \mu_{L,P} - \theta_{L,P}. \qquad (2\text{-}15)$$

FOC for (2-14) with respect to Δ is

$$\frac{\lambda}{\overline{y} + \Delta} = \mu_{H,A}. \qquad (2\text{-}16)$$

FOC for (2-14) with respect to $\pi(L, P)$ gives

$$\frac{1}{x_0 - w_0} \frac{Q(v_{L,P})}{v_{L,P}} = \frac{\mu_{L,P}}{1 - \lambda}. \qquad (2\text{-}17)$$

On the other hand, FOC for (2-14) with respect to $v_{L,P}$ yields

$$\frac{1}{x_0 - w_0} Q'(v_{44}) = \frac{\mu_{L,P}}{1-\lambda}. \tag{2-18}$$

(2-17) and (2-18) imply

$$\frac{Q(v_{L,P})}{v_{L,P}} = Q'(v_{L,P}).$$

Since Q is increasing and strictly convex, we have a unique solution

$$v_{L,P} = \overline{v}.$$

By (IC 2),

$$\pi(L,P) = \frac{\overline{y}}{\overline{v}}.$$

In addition,

$$\begin{aligned} v_{H,P} = v_{L,A} &= \overline{v} \text{ and} \\ \pi(H,P) = \pi(L,A) &= \frac{\overline{y}}{\overline{v}} \end{aligned}$$

satisfy the IC constraints (with zero Lagrangian multipliers). Therefore, we have

$$v_t = \overline{v} \; for \; t \neq (H,A) \tag{2-19}$$

and

$$\pi(t) = \frac{\overline{y}}{\overline{v}} \; for \; t \neq (H,A). \tag{2-20}$$

■

Lemma 2 In the efficient outcome, a manager's utility from management plus performance fees is only determined by managers' preference for fee income and the density functions of "performance" returns of the A fund and of the P fund.

PROOF. We need to solve $v_{H,A}(\widetilde{r})$ and show that it depend only on $F_{H,A}(\widetilde{r})$ and $F_P(\widetilde{r})$ (given the manager's utility function). FOC for (2-14) with respect to $\pi(H,A)$ is

$$\frac{1}{x_0 - w_0} \frac{E_{H,A}[Q(v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]} = \frac{\mu_1}{\lambda} - \frac{\theta_4}{\lambda} \frac{E_P[v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]}$$

I can replace $\frac{1}{x_0 - w_0}$ in the above equation using (2-17) and (2-19) as

$$\frac{\mu_4}{(1-\lambda)} \frac{\overline{v}}{Q(\overline{v})} \frac{E_{H,A}[Q(v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]} = \frac{\mu_1}{\lambda} - \frac{\theta_4}{\lambda} \frac{E_P[v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]}.$$

For simplicity, if I define

$$a_1 \equiv \frac{\mu_1}{\lambda}, a_4 \equiv \frac{\mu_4}{(1-\lambda)} \frac{\overline{v}}{Q(\overline{v})}, \text{ and } b_4 \equiv \frac{\theta_4}{\lambda}, \tag{2-21}$$

the above equation becomes

$$a_4 \frac{E_{H,A}[Q(v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]} = a_1 - b_4 \frac{E_P[v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]}. \tag{2-22}$$

If I differentiate (2-14) with respect to $v_{H,A}(\widetilde{r})$, the pointwise optimization yields

$$\frac{\lambda \pi_1(1) Q'(v_{H,A}(\widetilde{r})) F_{H,A}(\widetilde{r})}{x_0 - w_0} = \mu_1 \pi(H,A) F_{H,A}(\widetilde{r}) - \theta_4 \pi(H,A) F_P(\widetilde{r})$$

and by rearranging it, I obtain

$$\frac{Q'(v_{H,A}(\widetilde{r}))}{x_0 - w_0} = \frac{\mu_1}{\lambda} - \frac{\theta_4}{\lambda} \frac{F_P(\widetilde{r})}{F_{H,A}(\widetilde{r})}. \tag{2-23}$$

Using (2-17) and (2-19), I replace $\frac{1}{x_0 - w_0}$ in (2-23) to get

$$\frac{\mu_4}{(1-\lambda)} \frac{\overline{v}}{Q(\overline{v})} Q'(v_{H,A}(\widetilde{r})) = \frac{\mu_1}{\lambda} - \frac{\theta_4}{\lambda} \frac{F_P(\widetilde{r})}{F_{H,A}(\widetilde{r})}. \tag{2-24}$$

If I multiply both sides of (2-24) by $F_{H,A}(\widetilde{r})$ and then integrate, I get

$$\frac{\mu_4}{(1-\lambda)} \frac{\overline{v}}{Q(\overline{v})} E_{H,A}[Q'(v_{H,A}(\widetilde{r})] = \frac{\mu_1}{\lambda} - \frac{\theta_4}{\lambda}. \tag{2-25}$$

Using (2-21), I can rewrite (2-25) as

$$a_1 = b_4 + a_4 E_{H,A}[Q'(v_{H,A}(\widetilde{r})]. \tag{2-26}$$

If I use (2-21) and (2-26), the equation (2-24) becomes

$$a_4 \{Q'(v_{H,A}(\widetilde{r})) - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]\} = b_4 \{\frac{f_{H,A}(\widetilde{r}) - f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})}\}.$$

Then

$$a_4 = \frac{b_4\{\frac{f_{H,A}(\widetilde{r})-f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})}\}}{Q'(v_{H,A}(\widetilde{r})) - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]}, \tag{2-27}$$

and by (2-26),

$$a_1 = b_4\{1 + \frac{E_{H,A}[Q'(v_{H,A}(\widetilde{r})]}{Q'(v_{H,A}(\widetilde{r})) - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]} \frac{f_{H,A}(\widetilde{r}) - f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})}\}. \tag{2-28}$$

Since we can express (2-22) as

$$a_4 \frac{E_{H,A}[Q(v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]} = a_1 - b_4 \frac{E_P[v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]},$$

we have

$$\begin{aligned} &\frac{b_4\{\frac{f_{H,A}(\widetilde{r})-f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})}\}}{Q'(v_{H,A}(\widetilde{r})) - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]} \frac{E_{H,A}[Q(v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]} \\ &= b_4\{1 + \frac{E_{H,A}[Q'(v_{H,A}(\widetilde{r})]}{Q'(v_{H,A}(\widetilde{r})) - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]} \frac{f_{H,A}(\widetilde{r}) - f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})} - \frac{E_P[v_{H,A}(\widetilde{r})]}{E_{H,A}[v_{H,A}(\widetilde{r})]}\}. \end{aligned} \tag{2-29}$$

If I cancel b_4 in (2-29) and rearrange it, I obtain

$$\frac{f_{H,A}(\widetilde{r}) - f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})} = \frac{\{Q'(v_{H,A}(\widetilde{r})) - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]\}\{E_{H,A}[v_{H,A}(\widetilde{r})] - E_P[v_{H,A}(\widetilde{r})]\}}{E_{H,A}[Q(v_{H,A}(\widetilde{r})] - E_{H,A}[Q'(v_{H,A}(\widetilde{r})]E_{H,A}[v_{H,A}(\widetilde{r})]} \tag{2-30}$$

which gives us a solution for $v_{H,A}(\widetilde{r})$. Notice that $v_{H,A}(\widetilde{r})$ does not depend on the parameters such as λ and other manager's type but only on $f_{H,A}(\widetilde{r})$, $f_P(\widetilde{r})$ and the function $Q = v^{-1}$. I denote the solution by $\overline{v}(\widetilde{r})$. ■

Now I turn to the proof for Proposition 3. FOCs with respect to the Lagrangian multipliers are

$$\begin{aligned} \pi(H,A)E_{H,A}[v_{H,A}(\widetilde{r})] - \overline{e}_H &= \overline{y} + \Delta \\ \pi(L,P)v_{L,P} &= \overline{y} \\ \pi(H,A)E_P[v_{H,A}(\widetilde{r})] &= \overline{y}. \end{aligned}$$

By Lemma 1 and Lemma 2, the above conditions can be simplified to

$$\pi(H,A)E_{H,A}[\overline{v}(\widetilde{r})] = \overline{y} + \Delta + \overline{e}_H \tag{2-31}$$

$$\pi(L,P)\overline{v} = \overline{y} \tag{2-32}$$

$$\overline{\pi}E_P[\overline{v}(\widetilde{r})] = \overline{y}. \tag{2-33}$$

Using (2-32) and (2-33) respectively, we have

$$\pi(L,P) = \frac{\overline{y}}{\overline{v}} \quad (2\text{-}34)$$

$$\pi(H,A) = \frac{\overline{y}}{E_P[\overline{v}(\widetilde{r})]}. \quad (2\text{-}35)$$

Using (2-34) and (2-35), I can rewrite

$$x_0 = u^* - \{\lambda \frac{E_{H,A}[Q(\overline{v}(\widetilde{r}))]}{E_P[\overline{v}(\widetilde{r})]} + (1-\lambda)\frac{Q(\overline{v})}{\overline{v}}\}\overline{y}.$$

If I define the inside the bracket as

$$g(\lambda) = (1-\lambda)\frac{Q(\overline{v})}{\overline{v}} + \lambda \frac{E_{H,A}[Q(\overline{v}(\widetilde{r}))]}{E_P[\overline{v}(\widetilde{r})]}, \quad (2\text{-}36)$$

I can write

$$x_0 = u^* - g(\lambda)\overline{y}. \quad (2\text{-}37)$$

In the equations (2-28) and (2-29), let me define

$$c_4^* \equiv \frac{\frac{f_{H,A}(\widetilde{r}) - f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})}}{Q'(\overline{v}(\widetilde{r})) - E_{H,A}[Q(\overline{v}(\widetilde{r})]}$$

and

$$c_1^* \equiv 1 + \frac{E_{H,A}[Q(\overline{v}(\widetilde{r})]}{Q'(\overline{v}(\widetilde{r})) - E_{H,A}[Q(\overline{v}(\widetilde{r})]} \frac{f_{H,A}(\widetilde{r}) - f_P(\widetilde{r})}{f_{H,A}(\widetilde{r})}.$$

Note that c_4^* and c_1^* are constants. Using the constants, I can also express a_1 and a_4 as

$$a_1(\equiv \frac{\mu_1}{\lambda}) = c_1^* b_4 \text{ and } a_4(\equiv \frac{\mu_4}{(1-\lambda)}\frac{\overline{v}}{Q(\overline{v})}) = c_4^* b_4$$

where

$$b_4 \equiv \frac{\theta_4}{\lambda}.$$

By (2-17) (or (2-17)) and Lemma 1,

$$\frac{1}{x_0 - w_0} = a_4,$$

which gives us

$$x_0 = w_0 + \frac{1}{a_4}. \quad (2\text{-}38)$$

(2-37) and (2-38) imply

$$\frac{1}{a_4} = u^* - g(\lambda)\overline{y} - w_0. \quad (2\text{-}39)$$

In addition, by (2-16) and definitions $a_1 \equiv \frac{\mu_1}{\lambda} = c_1^* b_4$ and $a_4 = c_4^* b_4$, we have

$$\frac{2}{\overline{y} + \Delta} = c_1^* b_4 = \frac{c_1^*}{c_4^*} a_4,$$

which gives us

$$\overline{y} + \Delta = \frac{1}{a_4} \frac{c_4^*}{c_1^*}. \tag{2-40}$$

If we divide (2-31) by (2-33), we get

$$\overline{y} + \Delta = \frac{E_{H,A}[\overline{v}(\tilde{r})]}{E_P[\overline{v}(\tilde{r})]} \overline{y} - \overline{e}_H. \tag{2-41}$$

Then (2-40) along with (2-39) and (2-41) becomes

$$\frac{E_{H,A}[\overline{v}(\tilde{r})]}{E_P[\overline{v}(\tilde{r})]} \overline{y} - \overline{e}_H = \{u^* - g(\lambda)\overline{y} - w_0\} \frac{c_4^*}{c_1^*},$$

by which I solve $\overline{y}$,

$$\overline{y} = \frac{(u^* - w_0)\frac{c_4^*}{c_1^*} + \overline{e}_H}{g(\lambda)\frac{c_4^*}{c_1^*} + \underbrace{\left(\frac{E_{H,A}[\overline{v}(\tilde{r})] - E_P[\overline{v}(\tilde{r})]}{E_P[\overline{v}(\tilde{r})]} \right)}_{\equiv \alpha \text{ (skill)}} + 1}. \tag{2-42}$$

Here I define the performance premium earned by active management by high skill managers compared to passive management as *skill* and denote by α,

$$\alpha = \frac{E_{H,A}[\overline{v}(\tilde{r})] - E_P[\overline{v}(\tilde{r})]}{E_P[\overline{v}(\tilde{r})]} \tag{2-43}$$

By (2-41), I obtain

$$\Delta = \alpha \overline{y} - \overline{e}_H. \tag{2-44}$$

(2-45) allows us to rewrite the condition for a separating equilibrium (2-8) as

$$\alpha \overline{y} > \overline{e}_H.$$

Since $\overline{y}$ is given by (2-42), I can rewrite the above condition as

$$\frac{\left\{(u^* - w_0)\frac{c_4^*}{c_1^*} + \overline{e}_H\right\} \alpha}{g(\lambda)\frac{c_4^*}{c_1^*} + \alpha + 1} > \overline{e}_H,$$

which gives us the threshold α^* of skills for active management,

$$\alpha > \frac{g(\lambda)\overline{e}_H \frac{c_4^*}{c_1^*} + \overline{e}_H}{(u^* - w_0)\frac{c_4^*}{c_1^*}} \equiv \alpha^*. \blacksquare \tag{2-45}$$

Proof for Corollary 2

First, I show $\frac{Q(\overline{v})}{\overline{v}} < \frac{E_{H,A}[Q(\overline{v}(\widetilde{r}))]}{E_P[\overline{v}(\widetilde{r})]}$. Since the expected fee to the type (L, P) manager is less than the expected fee to the type (H, A) manager by $\Delta > 0$, we have

$$\pi(L, P)Q(\overline{v}) < \pi(H, A)E_{H,A}[Q(\overline{v}(\widetilde{r}))],$$

where

$$Q(\overline{v}) = v^{-1}(\overline{v}) = \overline{m}$$
$$E_{H,A}[Q(\overline{v}(\widetilde{r}))] = E_{H,A}[v^{-1}(\overline{v}(\widetilde{r}))] = \overline{m} + \overline{\beta}E_{H,A}[\widetilde{r}]$$

as derived in (2-1). Then, by (2-36) and (2-37), the above inequality becomes

$$\frac{Q(\overline{v})}{\overline{v}} < \frac{E_{H,A}[Q(\overline{v}(\widetilde{r}))]}{E_P[\overline{v}(\widetilde{r})]},$$

as desired. Thus, by (2-36), we have

$$g'(\lambda) = -\frac{Q(\overline{v})}{\overline{v}} + \frac{E_{H,A}[Q(\overline{v}(\widetilde{r}))]}{E_P[\overline{v}(\widetilde{r})]} > 0.$$

and, therefore,

$$\frac{\partial \overline{y}}{\partial \lambda} < 0,$$

since $\overline{y}$ is given by (2-42). In words, a type 4 manager's expected utility from the management fee decreases as the fraction of skilled managers increases in an efficient outcome.

On the other hand, the relationships with skills α and inefficiency $\overline{e}_H$ are

$$\frac{\partial \overline{y}}{\partial \alpha} = -\frac{\overline{y}}{g(\lambda)\frac{c_4^*}{c_1^*} + \alpha + 1} < 0$$

and

$$\frac{\partial \overline{y}}{\partial \overline{e}_H} = \frac{1}{g(\lambda)\frac{c_4^*}{c_1^*} + \alpha + 1} > 0$$

respectively. In essence, as skills or the efficiency increases, fixed management fee decreases. Other relationships are straightforward.■

Proof for Corollary 3

As (2-44) shows, the expected utility premium from active management Δ is positively related to the expected utility from passive management $\overline{y}$ but negatively related to inefficiency for active management $\overline{e}_H$. Note that skills

α do not depend on the parameters such as $\lambda, \overline{e}_H$, and w_0 while $\overline{y}$ does. Therefore, we have

$$\frac{\partial \Delta}{\partial \alpha} = \overline{y} + \alpha \frac{\partial \overline{y}}{\partial \alpha} = \overline{y} \left\{ 1 - \frac{1}{g(\lambda)\frac{c_4^*}{c_1^*} + \alpha + 1} \right\} > 0$$

and

$$\frac{\partial \Delta}{\partial \overline{e}_H} = \alpha \frac{\partial \overline{y}}{\partial \overline{e}_H} - 1 = \frac{\alpha}{g(\lambda)\frac{c_4^*}{c_1^*} + \alpha + 1} - 1 < 0.$$

Finally, other relationships are trivial since they have the same signs as the relationships with $\overline{y}$.■

Proof for Corollary 4

(2-45) provides the threshold of skills for active management α^*. As already discussed in Proof for Corollary 2, $g'(\lambda) > 0$. The comparative statics are straightforward.■

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