Name Your Own Price at Priceline.com: Strategic Bidding and Lockout Periods

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Abstract

This paper analyzes the Name Your Own Price (NYOP) mechanism adopted by Priceline.com. We characterize a customer's equilibrium bidding path under NYOP and show that the expected payoff of a customer is weakly higher than that in a first-price reverse auction without a reserve price. In addition, we show that Priceline.com's lockout period restriction, a design to protect sellers that seems to hurt customers, can actually benefit a customer in some circumstances.

1 Introduction

Priceline.com, known for its Name Your Own Price (NYOP) system, is a website devoted to helping travelers obtain discount rates for travel-related items such as airline tickets and hotel stays. The NYOP mechanism works as follows. First, a customer enters a bid that specifies the general characteristics of what she wants to buy (travel dates, location, hotel rating, etc.) and the price that she is willing to pay. Next, Priceline.com either communicates the customer's bid to participating sellers or accesses their private database to determine whether Priceline.com can satisfy the customer's specified terms and the bid price. If

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a seller accepts the bid, the offer cannot be cancelled. If no seller accepts the bid, the customer can rebid either by changing the desired specifications or by waiting for a minimum period of time, the *lockout period*, before submitting a new, higher price offer. For a hotel, the lockout period is 24 hours, for rental cars it is three days and for an airline ticket seven days. Priceline says in its seller's guideline that the rule is designed to protect the sellers. Our analysis suggests that the lockout period may often benefit the buyer, because it allows the buyer to commit to fewer rounds of bidding (the bidding must end before the date of travel, of course.)

To represent the Priceline.com auction, we use a dynamic model in which a single buyer suggests prices to N potential sellers for a finite number of rounds. The number of rounds T determines the length of the lockout period. By letting T go to infinity, we can also consider the case of no lockout period. For simplicity, we assume that the buyer's valuation is known. The sellers' costs are privately known and independently drawn from a common distribution.

We first show that without a lockout period and no discounting, there are two kinds of equilibrium bidding paths. As T goes to infinity, either sellers are almost fully discriminated over time or they get pooled into a finite number of cost intervals with bids clustering at the lowest price which is accepted by the seller with the minimum cost. In the latter case, the price pattern is convexly increasing as the buyer keeps her bids close to the price accepted by the minimum-cost seller until the very end. The pattern of bidding will be convex and most of trades (if any) will be realized at the end. This is consistent with empirical evidence and similar to the deadline effect observed in many bargaining processes (see for instance Hart (1989) and Spier (1992) on strikes and pretrial negotiation.)

The buyer's bidding strategy influences the rate at which she learns about the sellers' valuations. Ideally, the buyer would like to commit to a strategy that optimally reveals this information. If she could do that, she would gradually raise the price to price discriminate among the sellers and stop at the optimal reserve price, much like a Dutch auction, but in reverse. But when commitment is impossible, as we assume, the buyer cannot help but respond to the information revealed by rejections. As a consequence, she may want to bid so that her initial bids reveal little information and only at the end will they be more informative. The last minute rush will lead to pooling and inefficient outcomes, because many sellers will accept simultaneously and the winner will be determined by lottery.

The other equilibrium where sellers are discriminated through a gradually increasing bid sequence is, on the other hand, fully efficient, since the maximum bid equals the highest seller cost and the sellers are almost fully discriminated.

We also show that without a lockout period, the expected payoff of a customer is weakly higher than that in a first-price sealed-bid reverse auction (where service providers submit their bids to a customer) without a reserve price, but lower than that in a first-price sealed-bid reverse auction with the optimal reserve price. Moreover, when the expected payoff is strictly higher than that in a first-price reverse auction without a reserve price, the equilibrium bidding path is convexly increasing.

The lockout period, by reducing the number of bidding rounds, affects the process of information revelation. It makes the buyer bid more aggressively early on, because she does not need to be as concerned about the detrimental effects of learning more about the sellers' information while still having many bidding opportunities. This can be especially valuable if the buyer moderately discounts the future, that is, she wants to learn early about bookings. Thus, the lockout period can be advantageous to the buyer, because it permits the buyer to commit to fewer rounds of bidding. However, the welfare effects are ambiguous in general. The finding that the lockout period can be valuable is in line with McAdams and Schwarz (2007)'s view that an intermediary can create value by offering a credible commitment device.

Our analysis also provides insights into the unexplained bidding paths found by Spann and Tellis (2006). They analyze buyers' bidding patterns under NYOP without the lockout period restriction and find that 36% of the patterns are concavely increasing, while 23% are convexly increasing. They argue that the concave patterns can be explained by the positive bidding cost, but the convex ones suggest irrational consumer behavior on the Internet. Our paper shows that a convex pattern where a buyer raises bids more aggressively at the end can occur in a fully rational environment.

The environment studied here is similar to a durable goods monopoly, but with the roles of buyer and seller reversed. In a durable goods monopoly, the seller makes bids. Here the buyer does it. To avoid confusion, call the side that determines the price the principal and the other side the agents. There are two differences between our setting and a durable goods monopoly. First, there is a deadline in our environment, which results in very different equilibrium paths than those of the Coase conjecture.[1] Secondly, there is competition among the agents. With competition, an agent may accept the current price even though the future price path looks attractive, because there is the risk that another agent will accept. Therefore, our model works even when there is no discounting.

The paper is organized as follows. Section 2 describes the model. Section 3 presents an example that motivates our research. Section 4 constructs an equilibrium. Section 5 characterizes the equilibrium bidding behavior. Section 6 analyzes a model with waiting cost to see under what conditions the lockout period rule benefits customers and Section 7 concludes.

2 The Model

There are $N \geq 2$ sellers and 1 buyer in the market. The buyer has one unit of demand for the good provided by the sellers. The buyer's reservation value for the good is v, which is known by everyone. Seller i privately knows his cost θ^i to provide the good. Each θ^i is independently and identically distributed on $[\underline{c}, \overline{c}]$, where $\underline{c} \geq 0$ and $\overline{c} \leq v$, according to a distribution function $\overline{F}$. $\overline{F}$ admits a continuous density f and has full support. And $x + \frac{\overline{F}(x)}{f(x)}$ strictly increases in x. A buyer's payoff is $v - b$, where b is his payment to the seller, if he gets the object, and 0 otherwise. All the players are risk neutral. The setting is common knowledge to everyone in the market.

There is one platform allowing the buyer to submit his bid price to sellers. The buyer is allowed to adjust his bids for T times. In round t, the buyer announces the bid price, and sellers decide whether to accept or not. If n sellers accept the bid, each of them gets the chance to provide the good with probability $\frac{1}{n}$, and the game stops. If no seller accepts and $t < T$, the process proceeds to the next round, and the buyer submits a new price. If $t = T$, then the market closes and no further transaction can happen.

[1] In Stokey's discrete-time model, she also considers the case when there is a deadline and shows that the Coase conjecture still holds when the length of the period shrinks. The conclusion is different from ours because in Stocky's model, the deadline is not just the last day to trade, it is also the last day on which a buyer can enjoy the good and derive utility from it. That is, a buyer derives less utility if he gets the good on a day closer to the deadline. In our model, a buyer derives the same utility no matter when he gets the good.

2.1 Equilibrium concept

The equilibrium concept used in this paper is the perfect Bayesian equilibrium. An equilibrium consists of the buyer's strategy and belief, and the sellers' strategies and beliefs. Only symmetric pure strategy equilibria are considered. Let p_t be the price that the buyer offers the sellers in round t. Denote by $h_t = (p_1, p_2, \cdots, p_t)$ the history of the prices submitted by the buyer in the first t rounds.

Let $b_t(h_{t-1})$ be the price that the buyer would submit in round t given the price history h_{t-1} and the fact that no seller accepts in the first $t-1$ rounds. The buyer's strategy is a set of functions $\{b_t(h_{t-1})\}_{t=1}^{T}$. A seller's strategy can be summarized by functions $\{x_t(h_t)\}_{t=1}^{T}$. In round t, given h_t, a seller accepts the buyer's offer if and only if his cost is less than or equal to $x_t(h_t)$. The buyer's and the sellers' beliefs are summarized by a set of functions $\{y_t(h_{t-1})\}_{t=1}^{T}$, which specifies the greatest lower bound of a seller's cost believed by the buyer and the other sellers given history h_{t-1} and the fact that no seller accepts in the first $t-1$ rounds. Denote by $u_t^0(b, x \mid h_{t-1}, y_t(h_{t-1}))$ the buyer's expected utility given history h_{t-1} and belief $y_t(h_{t-1})$, and $u_t^i\left(b, x^{-i}, x^i \mid h_t, \theta^i, y_t(h_{t-1})\right)$ seller i's expected utility, where x^{-i} is the other sellers' strategy,[2] and x^i is seller i's strategy, given h_t, the realization θ^i of seller i's cost, and belief $y_t(h_{t-1})$.

Definition 1 *A symmetric equilibrium is a (b, y, x) that satisfies*

(a) $x_t(h_t) = y_{t+1}(h_t), \forall t, h_t,$ *and*

(b) $u_t^0(b, x \mid h_{t-1}, y_t(h_{t-1})) \geq u_t^0(b', x \mid h_{t-1}, y_t(h_{t-1}))$ *and*
$u_t^i\left(b, x, x \mid h_t, \theta^i, y_t(h_{t-1})\right) \geq u_t^i\left(b, x, x' \mid h_t, \theta^i, y_t(h_{t-1})\right), \forall b', x', t, h_t, h_{t-1}.$

Condition (a) means that players' belief about seller i's cost is consistent with seller i's strategy, for all i. Condition (b) means that players cannot do better by deviating from the equilibrium strategy.

[2] x^{-i} is a tuple consisting of the other sellers' strategies. But when the other sellers use the same strategies, x^{-i} can be a single function without confusion.

3 An Example

Before proceeding to constructing an equilibrium for the general model, we show calculations for finding the equilibrium path by using the example where $N = 2, v = 1$, and $\overline{F}$ is a uniform distribution on $[0, 1]$, and highlight some interesting points.

In addition to NYOP, a reverse auction is another mechanism commonly used by a buyer to determine allocation. Thus, we are interested in comparing the performances of the two mechanisms. In this example, the reverse auction is analogous to a standard auction with one seller and two buyers whose values are uniformly distributed on $[0, 1]$. In the standard auction, a buyer with value v bids $\frac{v}{2}$ in equilibrium. Therefore, in the reverse auction, a seller with cost x analogously submits ask price $\frac{1}{2} + \frac{1}{2}x$. The buyer buys from the seller with the lowest ask price and gets expected payoff $\frac{1}{3}$.

T=1: Now suppose the buyer and the sellers trade under an NYOP mechanism where $T = 1$. The buyer has one chance to submit his bid b. Seeing the bid, a seller whose cost is below b accepts the offer. Therefore, the buyer maximizes the expected payoff $(1 - b)\left[1 - (1 - b)^2\right]$ by choosing $b = 1 - \frac{1}{\sqrt{3}}$ and gets expected payoff $\frac{2}{3\sqrt{3}}$. From the example, we see that for the buyer, NYOP outperforms a standard auction without a reserve price even when there is only one chance to bid.

T=2: Next consider the case where $T = 2$. Suppose that in round 1, the bid price is b_1 and no one buys. In round 2, the buyer believes that both sellers' costs are above $x_1(b_1)$, and each seller also believes the other one's cost is above $x_1(b_1)$. The updated belief about the distribution of a seller's cost is $U[x_1(b_1), 1]$. Since it is the last round, both sellers will accept if the bid is higher than their costs. Thus $x_2(b_1, b_2) = b_2$. Given the belief, the buyer will bid at $b_2(b_1) = 1 - \frac{1-x_1(b_1)}{\sqrt{3}}$ to maximize his expected revenue.

In round 1, suppose the buyer has submitted a bid at b_1. A seller with cost x decides whether to accept the bid in this round or wait until the next one with the belief that the other seller would accept if his cost is below or equal to $x_1(b_1)$. If the seller accepts in this round, with probability x_1 the other accepts too, and each of them gets to sell with probability $\frac{1}{2}$; and with probability

$1 - x_1$, the seller gets to sell for sure, so the seller's expected payoff is $(b_1 - x)\left[\frac{1}{2}x_1(b_1) + (1 - x_1(b_1))\right]$. If the seller waits, with probability $1 - x_1$, the game moves to the next round. In round 2, the buyer is expected to submit $b_2(b_1)$. With probability $\frac{b_2(b_1)-x_1(b_1)}{1-x_1(b_1)}$, the other seller accepts too and each of them gets to sell with probability $\frac{1}{2}$; and with probability $\frac{1-b_2(b_1)}{1-x_1(b_1)}$, the seller gets to sell for sure, so the seller's expected payoff is $(b_2(b_1) - x)\left[\frac{1}{2}\frac{b_2(b_1)-x_1(b_1)}{1-x_1(b_1)} + \frac{1-b_2(b_1)}{1-x_1(b_1)}\right]$. The seller accepts b_1 in round 1 if

$$(b_1 - x)\left[\frac{1}{2}x_1(b_1) + (1 - x_1(b_1))\right] \geq \max\left\{0, (b_2(b_1) - x)\left[\frac{1}{2}\left[b_2(b_1) - x_1(b_1)\right] + \left[1 - b_2(b_1)\right]\right]\right\}.$$

Note that if a seller with x accepts in round 1, then a seller with $x' < x$ would also accept. In equilibrium, a seller with $x \leq x_1(b_1)$ decides to accept, so we can get $x_1(b_1) = 1 - \frac{-3b_1 + \sqrt{9b_1^2 + 12(1-b_1)}}{2}$ by solving $(b_1 - x_1)\left[\frac{1}{2}x_1 + (1 - x_1)\right] = (b_2(b_1) - x_1)\left[\frac{1}{2}\left[b_2(b_1) - x_1\right] + \left[1 - b_2(b_1)\right]\right]$.

With belief $x_1(b_1)$, the buyer chooses b_1 to maximize his total expected revenue in the two rounds

$$\max_{b_1} \left[1 - b_1\right]\left[1 - (1 - x_1(b_1))^2\right] + \left[1 - b_2(b_1)\right]\left[(1 - x_1(b_1))^2 - (1 - b_2(b_1))^2\right].$$

$\{b_1, x_1(b_1), b_2(b_1), x_2(b_1, b_2)\}$ form a symmetric equilibrium. In equilibrium $b_1 = 0.4214$, $b_2 = 0.5212$, $x_1 = 0.1709$, $x_2 = 0.5212$, and the buyer's payoff is 0.40024.

Numerical results:

In the following table, we show the equilibrium paths of x_t and b_t and the expected buyer's payoffs when $T = 1$, 2, 3, 4, and 5. We assume that the game begins at time 0 and ends at time 1. If the buyer's bid in the tth round is accepted, the transaction occurs at $\frac{(t-1)}{T}$. Column $E(\tau)$ lists the expected transaction time conditional on that transaction occurs.

	Buyer's Payoff	$E(\tau)$	x_{T-4} (b_{T-4})	x_{T-3} (b_{T-3})	x_{T-2} (b_{T-2})	x_{T-1} (b_{T-1})	x_T (b_T)
$T=1$	0.38490	0					0.4225 (0.4225)
$T=2$	0.40024	0.2972				0.1709 (0.4214)	0.5212 (0.5212)
$T=3$	0.40111	0.4563			0.0597 (0.4099)	0.2165 (0.4538)	0.5475 (0.5475)
$T=4$	0.40115	0.5826		0.0154 (0.4007)	0.0597 (0.4127)	0.2165 (0.4538)	0.5475 (0.5475)
$T=5$	0.40115	0.6626	0.0070 (0.3990)	0.0154 (0.4021)	0.0597 (0.4127)	0.2165 (0.4538)	0.5475 (0.5475)

There are several points worth noticing:

1. The buyer's payoff increases in T, the number of rounds,[3] but the increment becomes smaller and smaller. Therefore, the profit of having one more bidding chance shrinks as T increases.

2. The last-round bid increases in T, but the increment also shrinks as T increases. Observe that given T, the bidding path b_t is increasing. But with larger T, the increasing rate is small in the first few rounds and big jumps occur in the last few rounds.

3. In equilibrium the buyer does not get the object only if both sellers' costs are above x_T. Therefore, we know the probability that the buyer gets the object increases in T, but the increment shrinks as T increases. From the table, we see that when T increases from 3 to 4, and to 5, neither the buyer's payoff nor the probability that the buyer gets the object increases much. However, the expected transaction time is much later. This fact suggests that if the buyer has waiting cost and prefers earlier transactions, having fewer rounds might be good for him. The analysis in Section 6 confirms the conjecture.

[3] Note that the numbers in the table are not accurate enough to show small differences.

4 Construction of the Equilibrium

In this section, we construct an equilibrium by solving a series of programs backward and prove the existence of the equilibrium.

To construct the equilibrium, we need to introduce more notations. For convenience, define

$$F(x) = 1 - \overline{F}(x).$$

Note that $F(x)$ strictly decreases in x. Suppose only sellers with costs between x_{t-1} and x_t are willing to provide the good. Let

$$P(x_{t-1}, x_t) = \begin{cases} F(x_{t-1})^N - F(x_t)^N, \text{ if } x_{t-1} \leq x_t \\ 0, \quad \text{if } x_{t-1} > x_t \end{cases}$$

be the probability that the demand is fulfilled. Let

$$\begin{aligned} H(x_{t-1}, x_t) &= \begin{cases} \sum_{n=0}^{N-1} \frac{1}{n+1} \frac{(N-1)!}{n!(N-n-1)!} \left(F(x_{t-1}) - F(x_t)\right)^n \left(F(x_t)\right)^{N-n-1} / F(x_{t-1})^N, \text{if } x_{t-1} \leq x_t \\ 1, \quad \text{if } x_{t-1} > x_t \end{cases} \\ &= \begin{cases} \frac{\sum_{n=0}^{N-1} F(x_t)^{N-1-n} F(x_{t-1})^n}{N F(x_{t-1})^N}, \text{ if } x_{t-1} \leq x_t \\ 1, \quad \text{if } x_{t-1} > x_t \end{cases} \end{aligned}$$

be the conditional probability that a seller gets to provide the good if he accepts the buyer's offer conditional on that the other sellers' costs are above x_{t-1}.[4] Define

$$G(x_{t-1}, x_t) \equiv H(x_{t-1}, x_t) N F(x_{t-1})^N.$$

$\overline{b}_t(x_{t-1})$, $\widehat{x}_t(b_t, x_{t-1})$, and $V_t(x_{t-1})$ defined below are used to characterize equilibrium strategies, beliefs, and the buyer's payoff for the continuation games starting from round t. If $t = T$, let

$$\begin{aligned} V_T(x_{T-1}) &= \max_{\{b_T, x_T\}} (v - b_T) P(x_{T-1}, x_T) \\ &\quad s.t.\ b_T = x_T, \end{aligned}$$

[4] Conditional on that the other sellers' costs are above x_{t-1}, if a seller accepts the buyer's offer, with probability $\frac{(N-1)!}{n!(N-n-1)!} \left(F(x_{t-1}) - F(x_t)\right)^n \left(F(x_t)\right)^{N-n-1} / F(x_{t-1})^N$, there are n other sellers accepting, and each of them gets to sell the good with probability $\frac{1}{n+1}$.

and

$$\left(\overline{b}_T\left(x_{T-1}\right), \overline{x}_T\left(x_{T-1}\right)\right) \in \arg\max_{\{b_T, x_T\}} \left(v - b_T\right) P\left(x_{T-1}, x_T\right) \quad s.t.\ b_T = x_T, \tag{P1}$$

$$\widehat{x}_T\left(b_T, x_{T-1}\right) = \begin{cases} \overline{c} & \text{if } b_T > \overline{c} \\ b_T & \text{if } x_{T-1} \leq b_T \leq \overline{c} \\ x_{T-1} & \text{if } b_T < x_{T-1} \end{cases}. \tag{P2}$$

The constraint $b_T = x_T$ of the program comes from that in the last round, a seller accepts the last-round bid b_T as long as his cost is below b_T, so the cutoff x_T equals b_T. Knowing this and given the belief that all sellers have cost higher than x_{T-1}, the buyer chooses b_T to maximize his payoff—the objective function. Note that there might be multiple solutions to program P1. If there is more than one solution, only those that ensure the existence of equilibrium can be candidates for $\overline{b}_T\left(x_{T-1}\right)$ and $\overline{x}_T\left(x_{T-1}\right)$ (see the proof of Proposition 1 for more details).

If $t < T$, let

$$\begin{aligned} V_t\left(x_{t-1}\right) &= \max_{\{b_t, x_t\}} \left(v - b_t\right) P\left(x_{t-1}, x_t\right) + V_{t+1}\left(x_t\right) \\ s.t.(b_t - x_t)G\left(x_{t-1}, x_t\right) &= C_{t+1}\left(x_t\right) \\ \text{where } C_{t+1}\left(x_t\right) &= (\overline{b}_{t+1}\left(x_t\right) - x_t)G\left(x_t, \overline{x}_{t+1}\left(x_t\right)\right), \\ \text{where } \overline{b}_{t+1}\left(x_t\right), \overline{x}_{t+1}\left(x_t\right) & \text{ are defined as below;} \end{aligned} \tag{P3}$$

and let

$$\left(\overline{b}_t\left(x_{t-1}\right), \overline{x}_t\left(x_{t-1}\right)\right) \in \arg\max_{\{b_t, x_t\}} \left(v - b_t\right) P\left(x_{t-1}, x_t\right) + V_{t+1}\left(x_t\right) \quad s.t.(b_t - x_t)G\left(x_{t-1}, x_t\right) = C_{t+1}\left(x_t\right). \tag{P4}$$

Note that to solve the round-t program, we must solve all the programs for later rounds first, so function $C_{t+1}\left(x\right)$ is determined before solving the program. The right-hand side of the constraint, $C_{t+1}\left(x_t\right)$, is the expected payoff of a seller with cost x_t if he waits and accepts in the next period. The left-hand side is the expected payoff of a seller with cost x_t if he accepts in period t. Given b_t, sellers with costs lower than x_t prefer to accept in period t, and sellers with costs higher than x_t prefer to accept in period $t+1$. So for each b_t, we find the sellers' equilibrium strategy x_t from the constraint. Given the sellers' strategy and the belief that all sellers' cost are above x_{t-1}, the buyer chooses b_t to maximize his payoff—the objective function. The following proposition proves that programs P1 and P4 have a solution

Proposition 1 *There exists a set of solutions* $\left\{\overline{b}_t\left(x_{t-1}\right), \overline{x}_t\left(x_{t-1}\right)\right\}_t$ *that solves program P1 and P4 for all* t.

Proof. The details of the proof are in Appendix A. Here is the sketch. First, by Berge's maximum theorem, $V_T\left(x_{T-1}\right)$ is continuous, and the solution set of x_T for program P1 is upper hemi-contiuous. Therefore, we are able to pick $\overline{x}_T\left(x_{T-1}\right)$ from the solution set such that $C_T\left(x_{T-1}\right)$ is lower semi-continuous. Next, substituting the constraint into the objective function in round $T-1$ in program P4, the objective function is graph-continuous defined in Leininger (1984), and by Leininger's generalized maximum theorem, V_{T-1} is upper semi-continuous, and the solution set of x_{T-1} exists and is upper hemi-continuous. Applying the same procedure backward, we guarantee the existence of a solution to each round-t program. ■

The following assumption is for defining $\widehat{x}_t\left(b_t, x_{t-1}\right)$. We make the assumption to ensure the existence of pure strategy equilibrium.[5] Without the assumption, we are still able to construct an equilibrium in which mixed strategies are applied off the equilibrium path. Therefore, Assumption 1 is not necessary for an equilibrium to exist.

Assumption 1 *Given* x_{t-1}*, assume that there exists* $\underline{b}$ *such that if* $b_t \in [\underline{b}, \overline{c}]$*, there exists* x_t *such that* $(b_t - x_t)G\left(x_{t-1}, x_t\right) = C_{t+1}\left(x_t\right)$*, and if* $b_t < \underline{b}$*,* $(b_t - x_t)G\left(x_{t-1}, x_t\right) < C_{t+1}\left(x_t\right)$ *for all* $x_t \in [x_{t-1}, \overline{c}]$.

[5] Note that $C_{t+1}\left(x_t\right)$ constructed in the proof of Proposition 1 is lower semi-continuous.

Let

$$\widehat{x}_t\left(b_t, x_{t-1}\right) = \begin{cases} \overline{c} & \text{if } b_t \geq \overline{c} \\ x_{t-1} & \text{if } b_t < \underline{b} \text{ (defined in Assumption 1)} \\ \overline{x}_t\left(x_{t-1}\right) & \text{if } b_t = \overline{b}_t\left(x_{t-1}\right) \end{cases}, \tag{P5}$$

otherwise,

$$\widehat{x}_t\left(b_t, x_{t-1}\right) \in \left\{x_t \mid (b_t - x_t)G\left(x_{t-1}, x_t\right) = C_{t+1}\left(x_t\right)\right\}$$

The difference between $\overline{x}_t\left(x_{t-1}\right)$ and $\widehat{x}_t\left(b_t, x_{t-1}\right)$ is that $\overline{x}_t\left(x_{t-1}\right)$ is determined at the same time when the buyer determines b_t, and $\widehat{x}_t\left(b_t, x_{t-1}\right)$ is determined after the buyer submits b_t. When deriving $\widehat{x}_t\left(b_t, x_{t-1}\right)$, we have to take care of the cases when the buyer submits off-equilibrium bids. If an off-equilibrium bid b_t is too high, all the sellers accept and $\widehat{x}_t = \overline{c}$. If b_t is too low, all the sellers reject and $\widehat{x}_t = x_{t-1}$.

Lastly,

$$\widehat{x}_0 = \underline{c}.$$

Theorem 1 *Assume Assumption 1. Let $\overline{b}_t$ be as defined in (P1) and (P4), and $\widehat{x}_t$ be as defined in (P2) and (P5). The following (b, y, x) is an equilibrium of the game.*

$$\begin{aligned} b_t\left(h_{t-1}\right) &= \overline{b}_t\left(\widehat{x}_{t-1}\left(p_{t-1}, \widehat{x}_{t-2}\left(p_{t-2}, \cdots \widehat{x}_1\left(p_1, \widehat{x}_0\right)\ldots\right)\right)\right), \\ x_t\left(h_t\right) &= \widehat{x}_t\left(p_t, \widehat{x}_{t-1}\left(p_{t-1}, \cdots \widehat{x}_1\left(p_1, \widehat{x}_0\right)\ldots\right)\right), \\ y_{t+1}\left(h_t\right) &= x_t\left(h_t\right). \end{aligned}$$

Proof. See Appendix A. ■

Corollary 1 *The equilibrium path $\left\{\left(b_1, \cdots, b_T\right), \left(x_1, \cdots, x_T\right)\right\}$ can be found by solving the recursive program*

$$V_1\left(\underline{c}\right) = \max_{\{b_1, x_1\}} \left(v - b_1\right) P\left(\underline{c}, x_1\right) + V_2\left(x_1\right) \tag{P7}$$

$$s.t.(b_1 - x_1)G\left(\underline{c}, x_1\right) = C_2\left(x_1\right).$$

The value of the program is the buyer's payoff in equilibrium.

The program shows that the equilibrium path $\left\{\left(b_1, \cdots, b_T\right), \left(x_1, \cdots, x_T\right)\right\}$

maximizes the buyer's payoff but is subject to two constraints. The first one is the sellers' IC constraint, which exists in every mechanism and is shown in the constraint part of the program. The second constraint comes from the recursive form of the program. In each round, the buyer makes his bidding decision based on his current information and is not able to commit to a bidding path at the beginning. The second constraint keeps the buyer from achieving the outcome derived from the optimal mechanism stated by Myerson (1981).

5 Equilibrium Bidding Behavior

With T chances to submit prices, the buyer is able to segment the sellers in up to T groups according to their costs. However, the buyer cannot commit to a bidding path in advance, and in each round, he will choose a price that maximizes his expected payoff based on his belief. Thus, the buyer would suffer from the inability to commit and get lower payoff than when commitment is possible. In this section, we focus on the case when there is no lockout period restriction so that the buyer can submit as many bids as he wants. We first show that when committing to a bidding path is impossible, the optimal outcome for the buyer stated by Myerson (1981) is not attainable if the optimal auction design involves setting a reserve price. Next, we characterize the equilibrium bidding behavior.

5.1 Commitment and optimality

Note that in our setting, a first-price or second-price reverse auction works as follows — sellers submit their asks and the buyer chooses to buy an object from the seller with the lowest ask price. The buyer can announce a reserve price before the auction starts so that the buyer buys the object only if there is at least one ask price below the reserve price. A first-price or second-price reverse auction with a reserve price r such that $r + \frac{\overline{F}_i(r)}{f_i(r)} = v$ is an optimal mechanism prescribed by Myerson (1981). Under NYOP, it is the buyer who submits bids. When there are a large number of bidding chances, if the buyer commits to raise bids gradually and stop at r, then to the sellers, the game, like a reverse Dutch auction with a reserve price, is almost strategically equivalent to a first-price reverse auction with reserve price r, and the optimal outcome for the buyer can be approximately achieved. The following proposition elucidates this point.

Proposition 2 *Let $\pi(T)$ be the buyer's maximum payoff when there are T rounds and commitment to a path is possible. Let π^* be the buyer's payoff in Myerson's optimal mechanism. Given any $\epsilon > 0$, there exists T' such that for all $T > T'$, $\pi^* - \pi(T) < \epsilon$.*

Proof. See Appendix B. The proof shows that by committing to a path $(b_1, b_2, \cdots, b_T)$ such that in round t, sellers with cost below $x_t = \underline{c} + t\frac{r-\underline{c}}{T}$ (where r is the optimal reserve price) accept, the buyer's payoff can be arbitrarily close to π^* when T goes to infinity. ■

However, when commitment is not possible, even though the buyer is allowed to adjust the price as many times as he wants, the maximum payoff resulting from the optimal mechanism is not approximately achievable. By corollary 1 and (P1), we know that on the equilibrium path, the last-round b_T and x_T can be found by solving

$$x_T = b_T = \arg\max_b (v - b) \left[F(x_{T-1})^N - F(b)^N \right].$$

A necessary condition for b_T is

$$F(x_{T-1})^N = F(b_T)^N + (v - b_T) NF(b_T)^{N-1} f(b_T). \tag{1}$$

Suppose the optimal auction involves setting a reserve price $r < \bar{c}$. If the optimal auction can be approximately implemented when T goes to infinity, then it must be that $\lim_{T\to\infty} b_T = \lim_{T\to\infty} x_T = r$ and $\lim_{T\to\infty} x_{T-1} = r$. But by equation (1), if $\lim_{T\to\infty} b_T = r$, $\lim_{T\to\infty} x_{T-1} < r$, so the optimal auction cannot be approximately implemented.

Proposition 3 *When commitment to a path is impossible, the buyer's payoff under NYOP is bounded away from the payoff in Myerson's optimal auction if the optimal auction involves setting a reserve price.*

5.2 Equilibrium bidding path with no lockout period restriction

In this section, we characterize the pattern of the equilibrium bidding path when $T \to \infty$ (i.e. when there is no lockout period restriction). The question is how the buyer designs a bidding path to discriminate sellers. When commitment is possible, it is optimal for the buyer to induce sellers to reveal information about their costs gradually in every round. But when commitment is impossible, acquiring new information will change the buyer's pricing strategy later on, and it is not clear whether doing so is beneficial for the buyer. Theorem 2 shows that the equilibrium bidding path might have the feature that the speed of information elicited from the bidding path is very slow at the beginning, and most of the information is revealed at the end.

First we characterize how the buyer's payoff changes when the number of rounds increases.

Proposition 4 *The buyer's payoff increases with T, and the payoff converges when $T \to \infty$.*

Proof. When the number of rounds increases from M to $M+1$, the buyer can submit price $\underline{c}$ in the first round and then in the remaining rounds, do the same thing as when there are M rounds. Following this strategy, the buyer's payoff is the same as when $T = M$, and he might be able to do better by using other strategies. Therefore, the buyer's payoff is weakly increasing with T. Moreover, the buyer's payoff is bounded by the payoff in Myerson's optimal mechanism, so the payoff converges when $T \to \infty$. ■

Therefore, when the buyer does not have time preference, having more rounds is weakly better for him. We need the following condition for subsequent discussion.

Condition 1 *Assume that F is such that $\overline{x}_t^T(x_{t-1})$ defined in (P1) and (P4) is continuous on $[\underline{c}, \overline{c}]$ for all t and T.*

It can be proved that if the distribution F is uniform on $[\underline{c}, \overline{c}]$, Condition 1 holds.[6] Condition 1 ensures that the objective functions and constraints of the programs in Section 4 are continuous, so the generalized envelope theorem by Milgrom and Segal (2002) can be applied.

[6] If

$$\begin{aligned}\phi_t^T(x_{t-1}, x_t) \equiv\ & (v - x_t)\left[F(x_{t-1})^N - F(x_t)^N\right] \\ & - C_{t+1}^T(x_t, \delta)\left[F(x_{t-1}) - F(x_t)\right] + V_{t+1}^T(x_t, \delta)\end{aligned}$$

is concave in x_t for any t and T, then Condition 1 holds.

For convenience, we denote x_t and b_t on the equilibrium path when there are T rounds by x_t^T and b_t^T. The following proposition shows a convergence property of x_{T-t}^T when T goes to infinity.

Proposition 5 *Assume Condition 1.* $\lim_{T\to\infty} x_{T-t}^T$ *exists for all* $t \in \{0, 1, \cdots\}$.
Proof. Note that given any t and T, $\overline{x}_t^T(\cdot) = \overline{x}_{t+1}^{T+1}(\cdot)$ (defined in program P4 on page 10). When we increase the number of rounds from T to $T+1$, $x_1^{T+1} \geq x_0^{T+1} = x_0^T$. By Lemma 1 in Appendix B, $x_1^{T+1} \geq x_0^T$ implies $x_{T+1-t}^{T+1} \geq x_{T-t}^T$ for all t. Hence, x_{T-t}^T increases in T. Furthermore, x_{T-t}^T has an upper bound $\overline{c}$, so we conclude that $\lim_{T\to\infty} x_{T-t}^T$ exists. ■

Let $X^T = \left\{x_t^T\right\}_{t=1}^T$. The following defines a cluster point of the cutoff set X^T when $T \to \infty$.

Definition 2 $z \in [\underline{c}, \overline{c}]$ *is a cluster point if for any* $\epsilon > 0$*, there exists* y *such that (i)* $0 < |y - z| < \epsilon$*, and (ii) for any* $\delta > 0$*, there exists* T' *such that for all* $T > T'$*, there exists* $x \in X^T$ *such that* $|y - x| < \delta$*.*

Let B be the set of cluster points, and $[\underline{c}, \overline{c}] \setminus B$ be the complement of B.

Theorem 2 *Assume Condition 1.*

1. *The cluster point set* B *is either the whole interval* $[\underline{c}, \overline{c}]$ *or a single point* $\{\underline{c}\}$*, i.e.* $B = [\underline{c}, \overline{c}]$ *or* $\{\underline{c}\}$*.*

2. *The cluster point set* B *is a single point* $\{\underline{c}\}$ *if and only if the last period cutoff* x_T^T *is bounded away from* $\overline{c}$ *when* $T \to \infty$*, i.e.* $B = \{\underline{c}\}$ *if and only if* $\lim_{T\to\infty} x_T^T < \overline{c}$*.*

3. *If* $B = [\underline{c}, \overline{c}]$*, the buyer's payoff is approximately the same as that in a first-price auction without a reserve price.*

Proof. The details of the proof are in Appendix B. Here is the sketch. Lemma 3 shows that if the number of rounds left in a continuation game starting with belief x_{t-1} goes to infinity, then the difference between x_t and x_{t-1} goes to 0. So, if $a \in [\underline{c}, \overline{c}]$ is a cluster point, any point $x < a$ must be a cluster point too. However, Lemma 6 shows that it cannot be the case that $a \in (\underline{c}, \overline{c})$, $[\underline{c}, a]$ belongs to the cluster point set B, and $(a, \overline{c}]$ belongs to the complement of B because it violates the necessary condition under which the buyer chooses the optimal strategy for himself in every round. Therefore, the cluster point set is either $[\underline{c}, \overline{c}]$ or $\{\underline{c}\}$. The third statement comes from the revenue equivalence principle. ■

The first statement of the theorem implies that there are only two possible equilibrium paths: one with the cluster point set B equal to the whole interval $[\underline{c}, \overline{c}]$ and one with the cluster point set equal to a single point $\{\underline{c}\}$. When the number of rounds is large, if the cluster point set is $[\underline{c}, \overline{c}]$, sellers are approximately fully discriminated; on the other hand, if the cluster point set is $\{\underline{c}\}$, sellers are segmented into groups, and types in the same group are pooled, and moreover, the segmentation becomes extremely fine when approaching to $\{\underline{c}\}$. When the later situation happens, information is barely revealed in the first many rounds and transactions mostly occur in the last few rounds. The second statement says that the occurrence that $\{\underline{c}\}$ is the only cluster point occurs if and only if $\lim_{T\to\infty} x_T^T$ is strictly lower than $\overline{c}$. In other words, late transaction and information revelation coincide with the possibility that the buyer's demand is not fulfilled.

By Theorem 2, we can also characterize the buyer's bidding pattern when the number of bidding chances is large. When the cluster point set is $[\underline{c}, \overline{c}]$, the buyer raises bids constantly, and information about sellers' costs is revealed gradually over time. When the cluster point set is $\{\underline{c}\}$, the buyer only raises the bids largely at the very end and does not try to induce much information revelation most of the time.

The result could explain the puzzle proposed by Spann and Tellis. Spann and Tellis (2006) employ the data of a NYOP retailer in Germany that sells airline tickets for various airlines and allows multiple bidding to analyze buyers' bidding patterns. They argue that with positive bidding cost, the pattern should be concavely increasing because at the beginning consumers try to increase the probability of successful bidding by bidding higher, but when the bids are closer to their reservation value, the increasing rate slows down; and with zero bidding cost, the pattern should reflect linearly increasing bids. However, the result shows that only 36% of the data fit the first pattern and 5% fit the second pattern. 23% of the data fit the pattern which is convexly increasing, so they conclude that consumer behavior on the internet is not so rational. Nevertheless, a convexly increasing pattern corresponds to the case $B = \{\underline{c}\}$ in Theorem 2. Thus, a convex path can actually occur in a fully rational environment.[7] In addition to the convex bidding path, the case $B = \{\underline{c}\}$ also implies that most transactions happen near the end. This is related to the deadline effect that has been observed in many negotiation processes such as bargaining during strikes and pretrial negotiation. Our model thus provides insight into this phenomenon.

What would happen on the equilibrium path depends on the distribution of sellers' cost F. Under NYOP, the buyer is allowed to set up a price path so that $\lim_{T\to\infty} b_T^T = \lim_{T\to\infty} x_T^T < \overline{c}$, which functions as a reserve price. But since there is no commitment, to sustain $\lim_{T\to\infty} b_T^T < \overline{c}$, the buyer must have $\lim_{T\to\infty} x_T^T > \lim_{T\to\infty} x_{T-1}^T > \lim_{T\to\infty} x_{T-2}^T > \cdots > \lim_{T\to\infty} x_1^T$ (by Lemma 2 in Appendix B), and this requirement incurs some costs. First, the buyer has to charge the same price for sellers between x_t and x_{t-1}, and hence sellers receive more information rent than when fully discriminated. Furthermore, sellers in $[x_t, x_{t-1})$ get to sell the good with the same probability. Hence, the allocation is not efficient under NYOP. If the benefit dominates the loss of having $\lim_{T\to\infty} b_T^T < \overline{c}$, the equilibrium path will lead to $\lim_{T\to\infty} b_T^T < \overline{c}$.

Figure 1 shows the path of x_t when a seller's cost is uniformly distributed on $[0, 1]$ and $T = 20, N = 2$. When $v = 1, v = 1.2$ and $v = 1.4$, the optimal reserve prices are $0.5, 0.6$ and 0.7 respectively. So when $v = 1$, the buyer is more inclined to have x_{20} much lower than $\overline{c} = 1$, and in equilibrium, a seller with cost higher than 0.1 would not sell the good until the last two periods, which implies transactions are much more likely to occur in the last two periods. On the other hand, when $v = 1.4$, the loss of having x_{20} much lower than $\overline{c} = 1$ dominates the benefit, so in equilibrium, the buyer raises bids gradually to a price close to 1, and transactions happen constantly in every period.

One thing that deserves mention is that when the benefit of having a reserve price is large enough, in order to attain $\lim_{T\to\infty} b_T^T < \overline{c}$, the buyer has to restrict himself from getting too much information about sellers' costs. Supposing he raises bids early so that sellers with higher cost also accept, once the bid is rejected, he believes that sellers' costs are over a higher threshold and will raise bids further in the next rounds. In the end, $\lim_{T\to\infty} b_T^T = \overline{c}$. Therefore, he has to keep the bids low most of the time and his belief about sellers' costs does not

[7] After the buyer's waiting cost is incorporated in the next section, all the three patterns can occur in our model with different parameters.

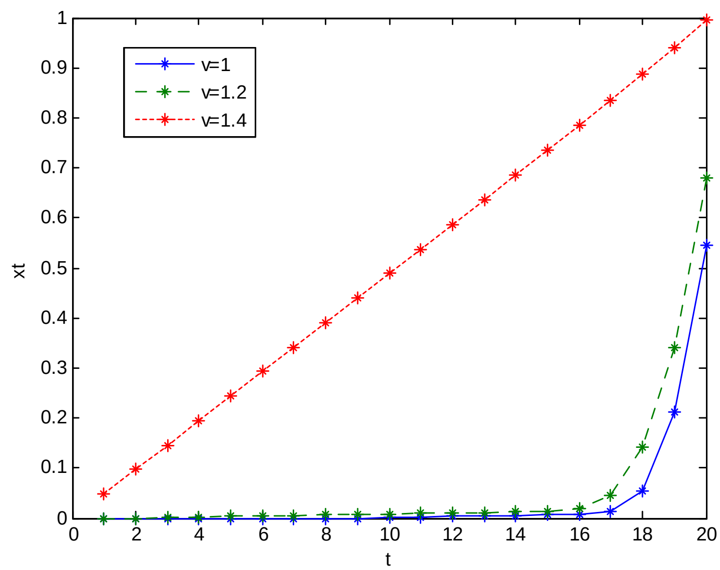

Figure 1: Path of x_t

change much until the last few rounds; and since he only has a few chances left, he cannot raise bids to $\bar{c}$, so $\lim_{T\to\infty} b_T^T < \bar{c}$. The following proposition provides some means to check whether $\lim_{T\to\infty} b_T^T < \bar{c}$ or $\lim_{T\to\infty} b_T^T = \bar{c}$.

Proposition 6 *If $\lim_{T\to\infty} x_T^T = \bar{c}$, there does not exist a finite number M such that the buyer's expected payoff when there are M rounds is higher than that in a first-price reverse auction without a reserve price.*

Proof. If the buyer's payoff when $T = M$ is higher than that in a first-price reverse auction without a reserve price, by Proposition 4, the buyer's payoff when $T \to \infty$ is weakly higher than when $T = M$. Hence, by the third statement of Theorem 2, $\lim_{T\to\infty} b_T^T = \bar{c}$ would not happen. ■

For example, when $N = 2$, $\overline{F}(x) = x$ on $[0, 1]$ and $v = 1$, the expected payoff of the buyer is $\frac{1}{3}$ in a first-price reverse auction. But if the buyer is allowed to submit the price once, and he chooses $b = 0.4225$, then the expected payoff is 0.3849. Thus, we know that x_T is bounded away from $\bar{c}$ when $T \to \infty$.

The proposition and the theorem give insights about why Priceline.com has to limit bidding chances within a period of time. Suppose travelers realize their demand for a hotel room M days in advance. If allowed to submit bids many times a day, under some circumstances, travelers would not submit serious bids until the last day, and so successful transactions only occur on the day just before the trip. This would somewhat inconvenience the hotels and travelers. If only one bid is allowed a day, then transactions will occur much earlier, but the negative impact on travelers' payoff is infinitesimal. This intuition is formalized and analyzed in the next section.

Based on the analysis above, we can also characterize the buyer's payoff with different equilibrium paths and obtain an upper bound and a lower bound for the buyer's expected payoff under NYOP.

Theorem 3 *When $T \to \infty$, if on the equilibrium path, $\lim_{T\to\infty} b_T^T = \lim_{T\to\infty} x_T^T < \bar{c}$, the buyer's expected payoff is strictly greater than that in a reverse auction without a reserve price. Thus, when $T \to \infty$, the buyer's expected payoff is between the payoff in a reverse auction without a reserve price and the payoff in a reverse auction with the optimal reserve price.*

Proof. Note that when $T \to \infty$, a path that almost fully discriminates sellers and satisfies sellers' IC constraint is a feasible solution candidate to program P7 (it is the stationary solution to program P7 when $T = \infty$, see Appendix B, Proposition 9) and it brings the buyer almost the same expected payoff as in a reverse auction with no reserve price. Therefore, if the solution to program P7 is the path with $\lim_{T\to\infty} b_T^T = \lim_{T\to\infty} x_T^T < \bar{c}$, it must yield a higher value for the program than in a reverse auction with no reserve price. This proves the first statement. The second statement follows from Theorem 2, Proposition 3, and the first statement. ■

We can consider the mechanism used in Hotwire.com as a first-price reverse auction without a reserve price. Hotels submit their prices to Hotwire.com, and Hotwire.com picks the lowest one and announces it on the website. Customers see the price and decide whether to buy or not. Therefore, we should expect that customers get higher expected savings under NYOP.

6 Model with Buyer's Waiting Cost

At Priceline, when a bid is rejected, a customer has to wait for a period of time to submit another bid, but some other NYOP websites in Europe allow customers to rebid immediately once their bids are rejected. In this section, we examine the conditions under which having the lockout period restriction benefits customers.

6.1 The model and an example

We modify the model in Section 2 to fit the real environment better. In reality, buyers would like to pin down their travel plans as early as possible, so late transactions actually incur some waiting costs. Therefore, we incorporate buyers' waiting cost and show that setting an appropriate lockout period rule may benefit the buyer. However, we assume that sellers have no preference for early or late transactions.

The model is modified as follows. The buyer realizes his demand for the good at time 0 and tries to fulfill the demand in time period $[0, M]$. After time M, the buyer no longer needs the good. If the buyer gets the good at price B at time t, his utility is $\delta^{\frac{t}{M}}(v - B)$, where $\delta \in (0, 1)$. The platform sets a lockout period rule which regulates how frequently the buyer can submit a bid. If the lockout period is s, the buyer can submit bids for $\lfloor \frac{M}{s} \rfloor$ times, that is, $T = \lfloor \frac{M}{s} \rfloor$.

After incorporating waiting cost, let us revisit the example in Section 3 and confirm our conjecture about how the lockout period improves the buyer's payoff.

The following table is for the case when $\delta = 0.9$.

	Buyer's Payoff	x_{T-4} (b_{T-4})	x_{T-3} (b_{T-3})	x_{T-2} (b_{T-2})	x_{T-1} (b_{T-1})	x_T (b_T)
$T = 1$	0.3849					0.4225 (0.4225)
$T = 2$	0.3897				0.2066 (0.4405)	0.5418 (0.5418)
$T = 3$	0.3844			0.1204 (0.4422)	0.2885 (0.5006)	0.5891 (0.5891)
$T = 4$	0.3802		0.0959 (0.4479)	0.1840 (0.4855)	0.3356 (0.5343)	0.6163 (0.6163)
$T = 5$	0.3773	0.0579 (0.4391)	0.1086 (0.4617)	0.1875 (0.4885)	0.3356 (0.5348)	0.6163 (0.6163)

Compared to the result when $\delta = 1$, we see that a buyer with waiting cost trades more eagerly. However, he would still like to have x_5 much lower than $\bar{c}$ to serve as a reserve price, so he has to suppress his intention to induce early transaction and cannot raise bids too fast. With the conflict, the table shows that allowing two bidding chances yields the highest payoff for the buyer. Having more rounds causes delay, which is costly to the buyer. The example illustrates that the lockout period rule which puts restriction on the buyer's bidding chances might actually help the buyer.

6.2 Equilibrium bidding path with no lockout period and $\delta < 1$

When $\delta = 1$, we show in Section 5.2 that when there is no lockout period, there are two possible equilibrium paths – either sellers are almost fully discriminated over time or they get pooled into some cost intervals. In the latter case, the price pattern is convexly increasing, and most of trades will be realized at the end. In this section, we show that with $\delta < 1$, there is one more possible path along which sellers with costs below some level are almost fully discriminated and sellers with costs above the level are pooled in intervals.

First, let

$$\left(\overline{b}_T^T(x_{T-1},\delta),\overline{x}_T^T(x_{T-1},\delta)\right) \in \arg\max_{\{b_T,x_T\}} (v-b_T)P(x_{T-1},x_T) \quad \text{(P1)}$$
$$s.t.\ b_T = x_T,$$

and

$$\left(\overline{b}_t^T(x_{t-1},\delta),\overline{x}_t^T(x_{t-1},\delta)\right) \in \arg\max_{\{b_t,x_t\}} (v-b_t)P(x_{t-1},x_t) + \sqrt[T]{\delta}V_{t+1}(x_t,\delta) \quad \text{(P4)}$$
$$s.t.(b_t-x_t)G(x_{t-1},x_t) = C_{t+1}(x_t,\delta).$$

for $t<T$. We need Condition 2 and Condition 3 for subsequent discussion.

Condition 2 *Assume that F is such that $\overline{x}_t^T(x_{t-1},\delta)$ is continuous in x_{t-1} on $[\underline{c},\overline{c}]$ for all t and T.*

Condition 3 *Assume that F is such that for any T, k, and $x\in[\underline{c},\overline{c}]$, $\overline{x}_{T-k}^T(x,\delta)$ converges to $\overline{x}_{T-k}^T(x,1)$ when T goes to infinity.*

Note that $\overline{x}_{T-k}^T(\cdot,1)$ is independent of T. If the distribution F is uniform on $[\underline{c},\overline{c}]$, it can be proved that Condition 2 and Condition 3 hold.[8]

Proposition 7 *Assume Conditions 2 and 3. Given δ, $\lim_{T\to\infty} x_{T-k}^T$ exists for all $k\in\{0,1,\cdots\}$.*
Proof. See Appendix B ■

The following theorem is a companion of Theorem 2, which characterizes the equilibrium path given $\delta\in(0,1]$ when there is no lockout period. The cluster point set B is defined on page 16.

[8] Given δ, if

$$\phi_t^T(x_{t-1},x_t) \equiv (v-x_t)\left[F(x_{t-1})^N - F(x_t)^N\right] - C_{t+1}^T(x_t,\delta)\left[F(x_{t-1})-F(x_t)\right] + \sqrt[T]{\delta}V_{t+1}^T(x_t,\delta)$$

is concave in x_t for any t and T, then Condition 2 holds. If

$$\phi_t^T(x_{t-1},x_t) \equiv (v-x_t)\left[F(x_{t-1})^N - F(x_t)^N\right] - C_{t+1}^T(x_t,\delta)\left[F(x_{t-1})-F(x_t)\right] + V_{t+1}^T(x_t,\delta)$$

is concave in x_t for any t and T, then Condition 3 holds.

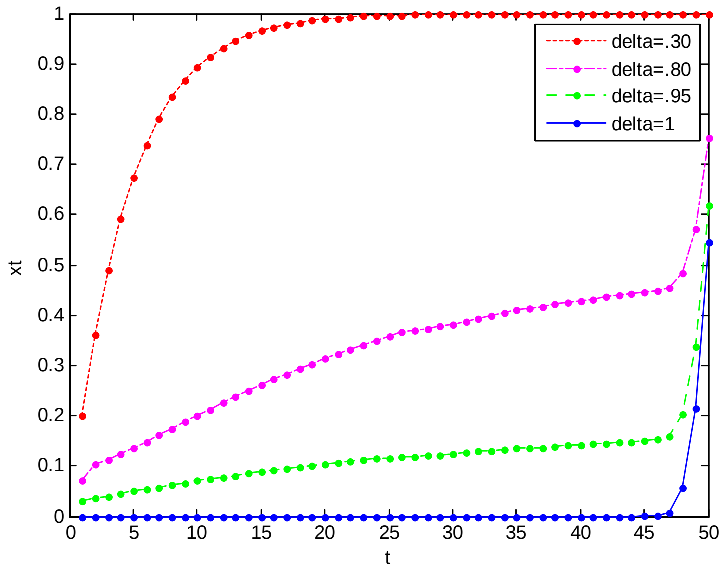

Figure 2: Path of x_t with different values of δ

Theorem 4 *Assume Conditions 2 and 3. Given* $\delta \in (0,1]$,

1. *The cluster point set* B *is* $[\underline{c}, a]$, *where* $a \in [\underline{c}, \overline{c}]$.
2. *The cluster point set is not the whole interval* $[\underline{c}, \overline{c}]$ *if and only if the last period cutoff* x_T^T *is bounded away from* $\overline{c}$ *when* $T \to \infty$, *i.e.* $a < \overline{c}$ *if and only if* $\lim_{T\to\infty} x_T^T < \overline{c}$.

Proof. See Appendix B. ■

Figure 2 illustrates the points made in Theorem 4. It shows the paths of x_t for different values of δ when $v = 1$, $T = 50$, $N = 2$, and a seller's cost is uniformly distributed on $[0,1]$. The paths with $\delta = 1$, 0.95, and 0.8 are consistent with the case of $a < \overline{c}$, and the path with $\delta = 0.3$ is consistent with the case of $a = \overline{c}$. When $\delta = 0.3$, the differences between adjacent x_t's in the first few rounds are relatively large. However, they shrink as the number of rounds increases, as shown in Figure 3. Figure 3 depicts the paths of x_t in the first 30 rounds given $\delta = 0.3$ with $T = 50, 100$, and 150. When $T \to \infty$, the differences between adjacent x_t's

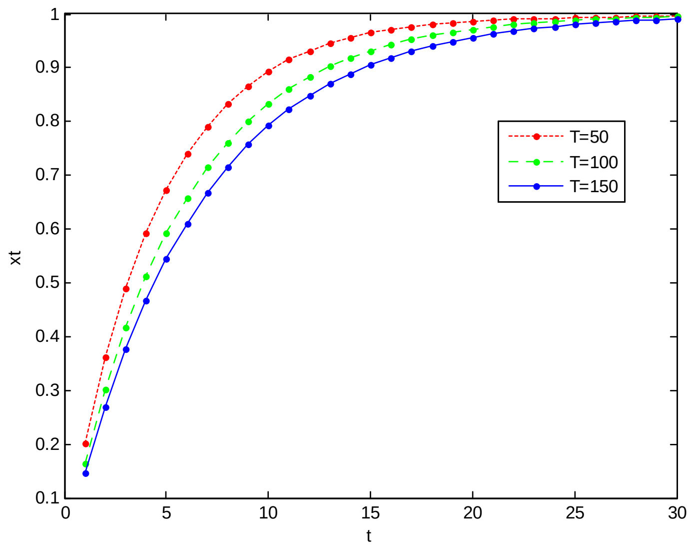

Figure 3: Path of x_t in the first 30 rounds with different numbers of total rounds, $\delta = 0.3$.

go to 0. When $\delta = 1$, 0.95, and 0.8, the differences between adjacent x_t's in the last three rounds are large. However, they do not shrink when the number of rounds increases, as shown in Figure 4. Figure 4 depicts the paths of x_t given $\delta = 0.95$ with $T = 50, 100$, and 150.

Theorem 2 is a special case of Theorem 4. When $\delta = 1$, $a = \underline{c}$ or $\overline{c}$; and when $\delta \in (0, 1)$, a can be anything in $[\underline{c}, \overline{c}]$, and Figure 2 shows that a decreases in δ. The difference comes from the fact that with $\delta < 1$, after several rounds, some waiting cost has been sunk and the remaining time left before the deadline is shorter. It is as if the buyer now has a higher discount factor, so the buyer's bidding behavior changes accordingly. We can see from Figure 2 that with lower δ, the path is more concave at the beginning since the buyer is more eager to get the good. As time passes by and less time is left, the path turns convex.

6.3 Optimal lockout period

In this section, we use some numerical examples to study the pros and cons of the lockout period rule and characterize the circumstances under which setting

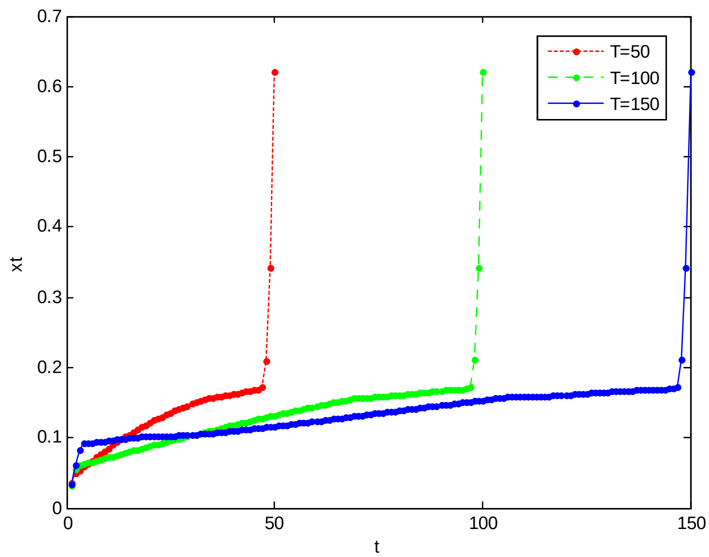

Figure 4: Path of x_t with different numbers of total rounds, $\delta = 0.95$.

an appropriate lockout period increases the buyer's payoff.

With a discount factor δ lower than 1, the example in Section 6.1 shows that the buyer's payoff does not monotonically increase with the number of rounds, which contrasts to the result in Proposition 4.

Proposition 8 *With $\delta \in (0, 1)$, the buyer's payoff might not monotonically increase with the number of rounds T.*

The following discusses how a lockout period rule affects the buyer's payoff given different values of δ. We focus on the settings in which Myerson's optimal mechanism involves setting a reserve price. If setting a reserve price is unnecessary, having more rounds always benefits the buyer because it helps the buyer discriminate the sellers better and be able to close the transaction earlier.

With high discount factor When the discount factor is high but lower than 1, if there is no lockout period, in equilibrium, the path of x_t is convex (see Figure 2), the last-round price is lower than $\bar{c}$, and most transactions occur late. If there is a lockout period, the buyer has fewer bidding chances and will bid seriously from the beginning, so transactions occur earlier. However, the buyer also loses chances to discriminate sellers with cost around $\underline{c}$.

With low discount factor When the discount factor is low, if there is no lockout period, the buyer raises the bid aggressively, and the bidding path is concave. With a lockout period, the buyer cannot raise the bid all the way up to $\bar{c}$, so there is a reserve-price-like effect. But the lockout period limits the buyer's bidding chances so that the buyer cannot discriminate the sellers well, and it also prevents the buyer from bidding aggressively and getting the good early.

We consider the example where $N = 2$, $v = 1.2$, and a seller's cost is uniformly distributed on $[0, 1]$. The following table summarizes the number of rounds T^* that maximizes the buyer's payoff and the corresponding buyer's payoff $\pi(T^*)$, given different values of δ.

δ	$\lim_{T\to\infty} x_T$	T^*	$\pi(T^*)$
1.00	0.668	∞	0.5559
0.95	0.774	2	0.5461
0.90	1	2	0.5399
0.85	1	∞	0.5333

The result shows that setting a lockout period so that the buyer has two bidding chances maximizes the buyer's payoff when $\delta = 0.95$ and 0.9. With $\delta = 0.95$, when there is no lockout period, $\lim_{T\to\infty} x_T < 1$, so the equilibrium path of x_t is mostly convex, and transaction is very likely to occur late. By setting a lockout period, the buyer benefits from having early transactions but suffers from not being able to discriminate sellers with costs around $\underline{c}$. With $\delta = 0.9$, when there is no lockout period, $\lim_{T\to\infty} x_T = 1$, so the equilibrium path of x_t is concave, and transactions occur early. By setting a lockout period, the buyer benefits from having a last-round price lower than $\bar{c}$, which functions like a reserve price, but suffers from not being able to close transaction early and discriminate sellers finely. In these two cases, the benefit of having a lockout period dominates the loss. However, with δ very close to 1 and δ lower than 0.85, the loss dominates the benefit, so setting a lockout period hurts the buyer.

In addition, setting a lockout period can be valuable for the buyer when having a reserve price benefits the buyer a lot. Consider another example where

$v = 1$ and the other parameters are the same as before. The optimal reserve price is 0.5. In this case, if δ is lower than 0.62, $\lim_{T\to\infty} x_T = 1$, so the buyer's payoff when there is no lockout period is at most $\frac{1}{3}$, the payoff in a reverse auction with no reserve price. On the other hand, the buyer's payoff when only one bidding chance is allowed is 0.3849 for all δ. Therefore, setting a lockout period benefits the buyer if $\delta < 0.62$ (it also benefits the buyer for higher values of δ.)

From the discussion above, we see that NYOP websites with different designs of rebidding rules are preferred by different kinds of customers. Priceline's lockout period rule seems to hurt customers by restricting their rebidding opportunities, but in fact, a customer with waiting cost might find it beneficial.

7 Conclusion

This paper analyzes the Name Your Own Price (NYOP) mechanism adopted by Priceline.com. We characterize the buyer's and the sellers' equilibrium strategies and show that Priceline.com's lockout period restriction, a design to protect sellers that seems to hurt customers, can actually benefit a customer with moderate discount factor.

We show that when there is no lockout period and no waiting cost, the equilibrium paths can be categorized into two classes. In the first class, the cluster point set of the sellers' cost cutoffs in all rounds is the whole cost interval $[\underline{c}, \overline{c}]$, which implies that sellers with different costs are almost fully discriminated and information about sellers' cost is revealed gradually over time. In this case, the buyer raises bids constantly, the ending price is the highest possible cost $\overline{c}$, and the buyer's payoff is approximately the same as the payoff in a reverse auction without a reserve price. In the second class, the cluster point set is a single point $\{\underline{c}\}$, which implies that sellers with different costs are pooled in intervals except the one with the lowest possible cost, and information about the sellers' cost is barely revealed in the first many rounds. In this case, the buyer does not raise the bid much until the very end, the ending price is lower than $\overline{c}$, and the buyer's payoff is greater than the payoff in a reverse auction without a reserve price. In the second type of equilibrium paths, most transactions occur just before the deadline. The delay of transactions incurs waiting cost if the buyer has time preference. Therefore, setting a lockout period might actually benefit a buyer by moving transactions forward.

This paper also indicates some interesting extensions for future research. Based on our analysis, one might be curious about whether Priceline can do better by adopting other measures, such as restricting the number of bidding chances instead of the frequency of bidding. Moreover, one can extend the model to consider the case when there are multiple buyers with private information about their own valuations, which better characterizes the situation of high travel season.

A Appendix

Proof of Proposition 1. There exists a set of solutions $\left\{\bar{b}_t\left(x_{t-1}\right), \overline{x}_t\left(x_{t-1}\right)\right\}_t$ that solves program P1 and P4 for all t. In the last period, recall that

$$\begin{aligned} V_T\left(x_{T-1}\right) &= \max_{x_T\in\left[x_{T-1},\overline{c}\right]}\left(v-x_T\right)P\left(x_{T-1},x_T\right), \\ \overline{x}_T\left(x_{T-1}\right) &\in X_T\left(x_{T-1}\right)=\arg\max_{x_T\in\left[x_{T-1},\overline{c}\right]}\left(v-x_T\right)P\left(x_{T-1},x_T\right), \\ \text{and } C_T\left(x_{T-1}\right) &= \left(\overline{x}_T\left(x_{T-1}\right)-x_{T-1}\right)G\left(x_{T-1},\overline{x}_T\left(x_{T-1}\right)\right). \end{aligned}$$

By Berge's maximum theorem, we know $V_T\left(x_{T-1}\right)$ is continuous and $X_T\left(x_{T-1}\right)$ is upper hemi-continuous. In period $t, t<T$, let

$$\begin{aligned} \phi_t\left(x_{t-1},x_t\right) &= \left(v-x_t\right)\left[F\left(x_{t-1}\right)^N-F\left(x_t\right)^N\right] \\ &\quad -C_{t+1}\left(x_t\right)\left[F\left(x_{t-1}\right)-F\left(x_t\right)\right]+V_{t+1}\left(x_t\right), \\ \alpha\left(x_{t-1}\right) &= \left[x_{t-1},\overline{c}\right]. \end{aligned}$$

Then

$$\begin{aligned} V_t\left(x_{t-1}\right) &= \max_{x_t\in\alpha\left(x_{t-1}\right)}\phi_t\left(x_{t-1},x_t\right) \\ \overline{x}_t\left(x_{t-1}\right)\in X_t\left(x_{t-1}\right) &= \arg\max_{x_t\in\alpha\left(x_{t-1}\right)}\phi_t\left(x_{t-1},x_t\right). \end{aligned}$$

We show that by picking a proper $\overline{x}_t\left(x_{t-1}\right)$ from $X_t\left(x_{t-1}\right), t\leq T$, each round-$t$ program has a solution.

First observe that for upper hemi-continuous correspondence X_T, we are able to find n_T closed intervals $[a_k, a_{k+1}], k = 1, \cdots, n_T$, such that $\cup_k [a_k, a_{k+1}] = [\underline{c}, \overline{c}]$, and n_T continuous functions $\overline{x}_{T,k} : [a_k, a_{k+1}] \rightarrow [a_k, \overline{c}]$ such that $\overline{x}_{T,k}(x) \in X_T(x), \forall x \in [a_k, a_{k+1}]$. Let

$$\begin{aligned} C_T(x_{T-1}) &= \left\{ \begin{array}{c} (\overline{x}_{T,k}(x_{T-1}) - x_{T-1})G(x_{T-1}, \overline{x}_{T,k}(x_{T-1})), \text{ if } x_{T-1} \in (a_k, a_{k+1}) \\ \min \left\{ \begin{array}{c} (\overline{x}_{T,k}(x_{T-1}) - x_{T-1})G(x_{T-1}, \overline{x}_{T,k}(x_{T-1})), \\ (\overline{x}_{T,k+1}(x_{T-1}) - x_{T-1})G(x_{T-1}, \overline{x}_{T,k+1}(x_{T-1})) \end{array} \right\}, \text{ if } x_{T-1} = a_{k+1}, k < n_T \end{array} \right., \\ \overline{x}_T(x_{T-1}) &= \left\{ \begin{array}{c} \overline{x}_{T,k}(x_{T-1}), \text{ if } x_{T-1} \in (a_k, a_{k+1}) \\ \arg\min_{x \in \{\overline{x}_{T,k}(x_{T-1}), \overline{x}_{T,k+1}(x_{T-1})\}} (x - x_{T-1})G(x_{T-1}, x), \text{ if } x_{T-1} = a_{k+1}, k < n_T \end{array} \right., \\ \overline{b}_T(x_{T-1}) &= \overline{x}_T(x_{T-1}). \end{aligned}$$

C_T is lower semi-continuous and V_T is continuous, so ϕ_{T-1} is upper semi-continuous. Note that ϕ_{T-1} is graph-continuous with respect to α, which is defined in Leininger (1984). So by Leininger's generalized maximum theorem, V_{T-1} is upper semi-continuous, and X_{T-1} is upper hemi-continuous.

Similarly, since X_{T-1} is upper hemi-continuous, we are able to find n_{T-1} closed intervals $[a'_k, a'_{k+1}], k = 1, \cdots, n_{T-1}$, such that $\cup_k [a'_k, a'_{k+1}] = [\underline{c}, \overline{c}]$, and n_{T-1} continuous functions $\overline{x}_{T-1,k} : [a'_k, a'_{k+1}] \rightarrow [a'_k, \overline{c}]$ such that $\overline{x}_{T-1,k}(x) \in X_{T-1}(x), \forall x \in [a'_k, a'_{k+1}]$. Let

$$C_{T-1}(x_{T-2}) = \\ \left\{ \begin{array}{c} (\overline{x}_{T-1,k}(x_{T-2}) - x_{T-2})G(x_{T-2}, \overline{x}_{T-1,k}(x_{T-2})) + C_T(x_{T-1}), \text{ if } x_{T-2} \in (a'_k, a'_{k+1}) \\ \min \left\{ \begin{array}{c} (\overline{x}_{T-1,k}(x_{T-2}) - x_{T-2})G(x_{T-2}, \overline{x}_{T-1,k}(x_{T-2})) + C_T(x_{T-1}), \\ (\overline{x}_{T-1,k+1}(x_{T-2}) - x_{T-2})G(x_{T-2}, \overline{x}_{T-1,k+1}(x_{T-2})) + C_T(x_{T-1}) \end{array} \right\}, \text{ if } x_{T-2} = a'_{k+1} \end{array} \right.,$$

$$\overline{x}_{T-1}(x_{T-2}) = \\ \left\{ \begin{array}{c} \overline{x}_{T-1,k}(x_{T-2}), \text{ if } x_{T-2} \in (a'_k, a'_{k+1}) \\ \arg\min_{x \in \{\overline{x}_{T-1,k}(x_{T-2}), \overline{x}_{T-1,k+1}(x_{T-2})\}} (x - x_{T-2})G(x_{T-2}, x) + C_T(x), \text{ if } x_{T-2} = a'_{k+1} \end{array} \right.,$$

$$\overline{b}_{T-1}(x_{T-2}) = \overline{x}_{T-1}(x_{T-2}) + \frac{C_T(\overline{x}_{T-1}(x_{T-2}))}{G(\overline{x}_{T-2}, \overline{x}_{T-1}(x_{T-2}))}.$$

C_{T-1} is lower semi-continuous and V_{T-1} is upper semi-continuous, so ϕ_{T-2} is upper semi-continuous. Check that ϕ_{T-2} is graph-continuous with respect to α. Applying the same procedure, we conclude that there exists a set of solutions

$\left\{\overline{b}_t\left(x_{t-1}\right), \overline{x}_t\left(x_{t-1}\right)\right\}_t$ that solves program P1 and P4 for all t. ■

Proof of Theorem 1. First we show that $u_t^i\left(b, x, x \mid h_t, \theta^i, y_t\left(h_{t-1}\right)\right) \geq u_t^i\left(b, x, x' \mid h_t, \theta^i, y_t\left(h_{t-1}\right)\right)$. If $t = T$, $x_T\left(h_T\right) = p_T$. Seller i with cost $\theta^i \leq p_T$ gets positive expected payoff if accepting and 0 if not, so he should accept. Seller i with cost $\theta^i > p_T$ gets negative expected payoff if accepting and 0 if not, so he would not accept. Therefore, he should follow x. For $t < T$, let $x_{t-1} = x_{t-1}\left(h_{t-1}\right)$. In the continuation game, the price path $\left(b_{t+1}, b_{t+2}, \cdots, b_T\right)$ and the belief path $\left(y_{t+1}, y_{t+2}, \cdots, y_T\right) = \left(x_t, x_{t+1}, \cdots, x_{T-1}\right)$ can be found by solving the recursive program

$$\begin{aligned} &\max_{x_t} \left(v - b_t\right) P\left(x_{t-1}, x_t\right) + V_{t+1}\left(x_t\right) \\ s.t.\text{if } &\left(b_t - x_{t-1}\right) < C_{t+1}\left(x_{t-1}\right), x_t = x_{t-1}, \\ &\text{otherwise, } (b_t - x_t) G\left(x_{t-1}, x_t\right) = C_{t+1}\left(x_t\right). \end{aligned}$$

Seller i's deviation does not affect $\left(b_{t+1}, \cdots, b_T\right)$ and $\left(y_{t+1}, \cdots, y_T\right)$. Suppose seller i's cost θ^i is in $(x_{s-1}, x_s]$, $s \geq t$, so he should buy in round s. If he accepts in round $s' \neq s$, $u_t^i\left(b, x, x' \mid h_t, \theta^i, y_t\left(h_{t-1}\right)\right) = (b_{s'} - \theta^i)\frac{G\left(x_{s'-1}, x_{s'}\right)}{NF(x_{t-1})^{N-1}}$. If he sticks to x (accepts in round s), $u_t^i\left(b, x, x \mid h_t, \theta^i, y_t\left(h_{t-1}\right)\right) = (b_s - \theta^i)\frac{G\left(x_{s-1}, x_s\right)}{NF(x_{t-1})^{N-1}}$. If $s' > s$, we know that

$$\begin{aligned} (b_s - x_s) G\left(x_{s-1}, x_s\right) &= (b_{s+1} - x_s) G\left(x_s, x_{s+1}\right) \\ &\vdots \\ (b_{s'-1} - x_{s'-1}) G\left(x_{s'-2}, x_{s'-1}\right) &= (b_{s'} - x_{s'-1}) G\left(x_{s'-1}, x_{s'}\right) \end{aligned}$$

Since $G\left(x_{s'-1}, x_{s'}\right) < NF\left(x_{s'-1}\right)^{N-1} < G\left(x_{s'-2}, x_{s'-1}\right)$, for any $x < x_{s'-1}$, $(b_{s'-1} - x) G\left(x_{s'-2}, x_{s'-1}\right) > (b_{s'} - x) G\left(x_{s'-1}, x_{s'}\right)$. Applying the same argument, since $\theta^i < x_s \leq \cdots \leq x_{s'-1}$, $(b_s - \theta^i) G\left(x_{s-1}, x_s\right) > (b_{s+1} - \theta^i) G\left(x_s, x_{s+1}\right) \geq \cdots \geq (b_{s'} - \theta^i) G\left(x_{s'-1}, x_{s'}\right)$. On the other hand, if $t \leq s' < s$, applying similar arguments, since $\theta^i > x_{s-1} \geq \cdots \geq x_{s'}$, $(b_s - \theta^i) G\left(x_{s-1}, x_s\right) > (b_{s-1} - \theta^i) G\left(x_{s-2}, x_{s-1}\right) \geq \cdots \geq (\theta^i - b_{s'}) G\left(x_{s'-1}, x_{s'}\right)$. Therefore, $(b_s - \theta^i)\frac{G(x_{s-1}, x_s)}{NF(x_{t-1})^{N-1}} > (b_{s'} - \theta^i)\frac{G\left(x_{s'-1}, x_{s'}\right)}{NF(x_{t-1})^{N-1}}$.

Next, we show that $u_t^0\left(b, x \mid h_{t-1}, y_t\left(h_{t-1}\right)\right) \geq u_t^0\left(b', x \mid h_{t-1}, y_t\left(h_{t-1}\right)\right)$. For any t and any h_{t-1}, given x, the buyer's optimal strategy must generate the path that maximizes his conditional utility

$$\max_{p_t} \frac{(v-p_t)\, P\left(x_{t-1}\left(h_{t-}\right), x_t\left(\left(h_{t-}, p_t\right)\right)\right) + V_{t+1}\left(x_t\left(\left(h_{t-}, p_t\right)\right)\right)}{F\left(x_{t-1}\left(h_{t-}\right)\right)^N}. \tag{P6}$$

That is, the strategy b is consistent with the solution $(p_t, \cdots, p_T)$ derived from (P6) in the sense that $b_t\left(h_{t-1}\right) = p_t, b_{t+1}\left(h_{t-1}, p_t\right) = p_{t+1}, \cdots$. Under our construction of $x_t\left(h_t\right)$, the solution to (P6) is the same as $(b_t, \cdots, b_T)$ derived from (P3). Hence the strategy b constructed from (P3) is consistent with $(p_t, \cdots, p_T)$ and is optimal. ■

B Appendix

Proof of Proposition 2. Let r be the optimal reserve price. Submitting a path $(b_1, b_2, \cdots, b_T)$ so that in round t, sellers with cost below $x_t = \underline{c} + t\frac{r-\underline{c}}{T}$ accept, is a feasible choice. We show that the buyer's payoff with the path can be arbitrarily close to π^* when T goes to infinity.

Given the path, in the last round, sellers with cost below r accept, so $b_T = x_T = r$. In round t, $t < T$, a seller with cost x_t feels indifferent between accepting now or accepting in the next round, so the following constraint holds:

$$\begin{aligned}
&(b_t - x_t)G\left(x_{t-1}, x_t\right) = (b_{t+1} - x_t)G\left(x_t, x_{t+1}\right) \\
\Rightarrow &(b_t - x_t)\left(F(x_t)^{N-1} + F(x_t)^{N-2}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-1}\right) \\
&= (b_{t+1} - x_t)\left(F(x_{t+1})^{N-1} + F(x_{t+1})^{N-2}F(x_t) + \cdots + F(x_t)^{N-1}\right).
\end{aligned}$$

If $\delta \equiv x_{t+1} - x_t$ and $\Delta \equiv b_{t+1} - b_t$ are small, an approximation of the equation is

$$\begin{aligned}
&(b_t - x_t)\left(NF(x_t)^{N-1} + \frac{(N-1)\,N}{2}F(x_t)^{N-2}f\left(x_t\right)\delta\right) \\
&\approx (b_t + \Delta - x_t)\left(NF(x_t)^{N-1} - \frac{(N-1)\,N}{2}F(x_t)^{N-2}f\left(x_t\right)\delta\right) \\
\Rightarrow &\frac{\Delta}{\delta} \approx \frac{(b_t - x_t)\,(N-1)\,F(x_t)^{N-2}f\left(x_t\right)}{F(x_t)^{N-1}} = \frac{(N-1)\,f\left(x_t\right)(b_t - x_t)}{F\left(x_t\right)}.
\end{aligned}$$

In a reverse Dutch auction with reserve price r, a seller with cost x accepts at price

$$b(x) = r\frac{F(r)^{N-1}}{F(x)^{N-1}} + \frac{1}{F(x)^{N-1}}\int_x^r y(N-1)F(y)^{N-2}f(y)\,dy,$$

which is also the price submitted by a seller with cost x in a first-price reverse auction with reserve price r.

$$\begin{aligned} b'(x) &= (N-1)f(x)\left[\frac{xF(x)^{N-1} + \int_x^r F(y)^{N-1}dy}{F(x)^N} - \frac{xF(x)^{N-2}}{F(x)^{N-1}}\right] \\ &= \frac{(N-1)f(x)(b(x)-x)}{F(x)}. \end{aligned}$$

Since $b(x_T) = b_T = r$ and $b'(x) = \frac{\Delta}{\delta}$, $b(x_t) - b_t = O(\delta^2) \cdot T(\delta) = O(\delta)$, where $T(\delta) = \frac{r-\underline{c}}{\delta}$. The optimal payoff is $\pi^* = \int_{b=\underline{c}}^r (v - b(x))\,dF(x)^N$. By the Riemann-Stieltjes integral, for all $\epsilon > 0$, there exists $\delta' > 0$ such that for all $\delta < \delta'$,

$$\left|\sum_{t=1}^{T(\delta)}(v - b(x_t))\left(F(x_t)^N - F(x_{t-1})^N\right) - \pi^*\right| < \frac{\epsilon}{2}.$$

Since $b(x_t) - b_t = O(\delta)$ and $\sum_{t=1}^{T(\delta)}\left(F(x_t)^N - F(x_{t-1})^N\right) = 1$, there exists $\delta'' > 0$ such that for all $\delta < \delta''$,

$$\left|\sum_{t=1}^{T(\delta)}[(v - b_t) - (v - b(x_t))]\left(F(x_t)^N - F(x_{t-1})^N\right)\right| < \frac{\epsilon}{2}.$$

Therefore, for any $\delta \leq \min\left\{\delta', \delta''\right\}$, i.e. for any $T \geq \frac{r-\underline{c}}{\min\{\delta',\delta''\}}$,

$$\left|\pi^* - \sum_{t=1}^{T(\delta)}(v - b_t)\left(F(x_t)^N - F(x_{t-1})^N\right)\right| < \epsilon.$$

The buyer can do weakly better by choosing a better path, so $\pi^* - \pi(T(\delta)) < \epsilon$. ■

Lemma 1 *Assume Condition 1.* $\overline{x}_t^T(x_{t-1})$ *(defined in program P4) increases in* x_{t-1}.

Proof. It is easy to check that $\overline{x}_T^T(x_{T-1})$ defined in program P1 increases in x_{T-1}. With $t < T$, $\overline{x}_t^T(x_{t-1})$ is derived from program P4. Let

$$\begin{aligned}\varphi(x_t, x_{t-1}) &\equiv (v - b_t(x_t; x_{t-1}))\left[F(x_{t-1})^N - F(x_t)^N\right] + V_{t+1}^T(x_t),\\ \text{where } b_t(x_t; x_{t-1}) &= \frac{C_{t+1}^T(x_t)}{F(x_t)^{N-1} + F(x_t)^{N-2}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-1}} + x_t.\end{aligned}$$

$$\begin{aligned}\frac{\partial\varphi(x_t, x_{t-1})}{\partial x_t} &= -\left[F(x_{t-1}) - F(x_t)\right]\left[\left(F(x_{t-1})^{N-1} + \cdots + F(x_t)^{N-1}\right) + \frac{dC_{t+1}^T(x_t)}{dx_t}\right]\\ &\quad + f(x_t)\left(\overline{x}_{t+1}(x_t) - x_t\right)\left[NF(x_t)^{N-1} - \left(F(\overline{x}_{t+1}(x_t))^{N-1} + \cdots + F(x_t)^{N-1}\right)\right]\end{aligned}$$

and

$$\frac{\partial^2\varphi(x_t, x_{t-1})}{\partial x_t \partial x_{t-1}} = \frac{1}{\overline{c} - \underline{c}}\left[NF(x_{t-1})^{N-1} + \frac{dC_{t+1}^T(x_t)}{dx_t}\right].$$

For any x_t, x_{t-1} and $x'_{t-1} \in [x_{t-1}, x_t]$, if $\left(F(x_{t-1})^{N-1} + \cdots + F(x_t)^{N-1}\right) + \frac{dC_{t+1}^T(x_t)}{dx_t} < 0$, $\frac{\partial\varphi(x_t, x_{t-1})}{\partial x_t} > 0$ and $\frac{\partial\varphi\left(x_t, x'_{t-1}\right)}{\partial x_t} > 0$. If $\left(F(x_{t-1})^{N-1} + \cdots + F(x_t)^{N-1}\right) + \frac{dC_{t+1}^T(x_t)}{dx_t} > 0$, $\frac{\partial^2\varphi(x_t, x_{t-1})}{\partial x_t \partial x_{t-1}} > 0$, so $\frac{\partial\varphi}{\partial x_t}(x_t, x_{t-1}) > 0$ implies $\frac{\partial\varphi}{\partial x_t}\left(x_t, x'_{t-1}\right) > 0$. Therefore, $\varphi(x_t, x_{t-1})$ satisfies single crossing property of marginal returns. By Milgrom-Shannon theorem, $\overline{x}_t^T(x_{t-1})$ increases in x_{t-1}. ■

We use Lemmas 2, 3, and 4 to prove Lemma 5 and Lemma 6, and use Lemmas 5, 6, and 4 to prove Theorem 2. We sometimes add superscript T to $V_t(x)$ and $C_t(x)$ (defined in (P3)) for clarification. Note that for two sets (t, T) and (t', T'), if $T - t = T' - t'$, then $V_t^T(x) = V_{t'}^{T'}(x)$ and $C_t^T(x) = C_{t'}^{T'}(x)$. So, we let $c_k(x) = C_{T-k}^T(x)$.

Proof of Theorem 2. By Lemma 6, if $B \neq \{\underline{c}\}$, there does not exist $a \in (\underline{c}, \overline{c})$ such that $(a, \overline{c}] \subset [\underline{c}, \overline{c}] \setminus B$. Then by Lemma 5, $B = [\underline{c}, \overline{c}]$. So the first statement is proved. The third statement follows from the revenue equivalence principle. For the second statement, if $\lim_{T\to\infty} x_T^T < \overline{c}$, it must be that $B = \{\underline{c}\}$. On the other hand, if $B = \{\underline{c}\}$, there exists $t < \infty$ such that $\overline{c} - \lim_{T\to\infty} x_{T-t}^T > 0$. By Lemma 4, $\lim_{T\to\infty} x_T^T < \overline{c}$. ■

Lemma 2 *Assume Condition 1. Given $k > 0$, there exists $\delta_k(\epsilon, x) > 0$ such that for any t and T where $T - t = k$, if $x_t^T = x_{t+1}^T - \epsilon$, $\epsilon > 0$, then $x_{t-1}^T \leq x_t^T - \delta_k\left(\epsilon, x_t^T\right)$. $\delta_k(\epsilon, x)$ does not depend on T.*

Proof. Given any t, T such that where $T - t = k$ and given belief x_{t-1}, the continuation equilibrium x_t^* and b_t^* are derived from

$$V_t^T(x_{t-1}) = \max_{\{b_t, x_t\}} (v - b_t)\left[F(x_{t-1})^N - F(x_t)^N\right] + V_{t+1}^T(x_t) \tag{P8}$$

$$s.t.(b_t - x_t)(F(x_t)^{N-1} + F(x_t)^{N-2}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-1}) = C_{t+1}^T(x_t). \tag{2}$$

From (2),

$$b_t = \frac{C_{t+1}^T(x_t)}{F(x_t)^{N-1} + F(x_t)^{N-2}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-1}} + x_t,$$

$$\begin{aligned}\frac{db_t}{dx_t} &= \frac{C_{t+1}^{T\prime}(x_t)}{F(x_t)^{N-1} + F(x_t)^{N-2}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-1}} + 1 \\ &+ \frac{(b_t - x_t)\left[(N-1)F(x_t)^{N-2} + (N-2)F(x_t)^{N-3}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-2}\right]f(x_t)}{F(x_t)^{N-1} + F(x_t)^{N-2}F(x_{t-1}) + \cdots + F(x_{t-1})^{N-1}}\end{aligned}$$

The solution $\{x_t^*, b_t^*\}$ must satisfy the first order condition

$$\begin{aligned} & 0 = -\frac{db_t(x_t^*)}{dx_t}\left[F(x_{t-1})^N - F(x_t^*)^N\right] + (v - b_t(x_t^*))\left(NF(x_t^*)^{N-1}f(x_t^*)\right) + V_{t+1}^{T\prime}(x_t^*) \\ \Rightarrow\ & 0 = \left[F(x_{t-1}) - F(x_t^*)\right]\left[-\left(F(x_{t-1})^{N-1} + \cdots + F(x_t^*)^{N-1}\right) - C_{t+1}^{T\prime}(x_t^*)\right] \\ & -C_{t+1}^T(x_t^*)f(x_t^*) + N(v - x_t^*)F(x_t^*)^{N-1}f(x_t^*) + V_{t+1}^{T\prime}(x_t^*). \end{aligned} \tag{3}$$

Note that

$$\begin{aligned} V_{t+1}^T(x_t) &= \max_{\{b_{t+1}, x_{t+1}\}} (v - b_{t+1}(x_{t+1}; x_t))\left[F(x_t)^N - F(x_{t+1})^N\right] + V_{t+2}^T(x_{t+1}), \qquad (4) \\ \text{where } b_{t+1}(x_{t+1}; x_t) &= \frac{C_{t+2}^T(x_{t+1})}{F(x_{t+1})^{N-1} + F(x_{t+1})^{N-2}F(x_t) + \cdots + F(x_t)^{N-1}} + x_{t+1}. \end{aligned}$$

Let $\left\{x_{t+1}^*(x_t), b_{t+1}^*(x_t)\right\}$ be the solution to program (4). By (2), $C_{t+2}^T\left(x_{t+1}^*\right) = (b_{t+1}^* - x_{t+1}^*)(F(x_{t+1}^*)^{N-1} + F(x_{t+1}^*)^{N-2}F(x_t) + \cdots + F(x_t)^{N-1})$. By the envelope theorem,

$$V_{t+1}^{T\prime}(x_t) = -NF(x_t)^{N-1} f(x_t)\left(v - x_{t+1}^*\right) + f(x_t)\, C_{t+2}^{T}\left(x_{t+1}^*\right). \tag{5}$$

Plugging into (3), we get

$$\begin{aligned} 0 &= [F(x_{t-1}) - F(x_t^*)]\left[-C_{t+1}^{T\prime}(x_t^*) - \left(F(x_{t-1})^{N-1} + \cdots + F(x_t^*)^{N-1}\right)\right] \\ &\quad - \left[C_{t+1}^{T}(x_t^*) - C_{t+2}^{T}\left(x_{t+1}^*\right)\right] f(x_t^*) - NF(x_t^*)^{N-1} f(x_t^*)\left[x_t^* - x_{t+1}^*\right] \\ &= -[F(x_{t-1}) - F(x_t^*)]\left[\left(F(x_{t-1})^{N-1} + \cdots + F(x_t^*)^{N-1}\right) + c_k'(x_t^*)\right] \\ &\quad + f(x_t^*)\left(x_{t+1}^* - x_t^*\right)\left[NF(x_t^*)^{N-1} - \left(F(x_{t+1}^*)^{N-1} + \cdots + F(x_t^*)^{N-1}\right)\right] \end{aligned} \tag{6}$$

If $x_t^* = x_{t+1}^* - \epsilon$,
$\left(x_{t+1}^* - x_t^*\right)\left[NF(x_t^*)^{N-1} - \left(F(x_{t+1}^*)^{N-1} + \cdots + F(x_t^*)^{N-1}\right)\right]$ is strictly positive, and so by (6),
$[F(x_{t-1}) - F(x_t^*)]\left[\left(F(x_{t-1})^{N-1} + \cdots + F(x_t^*)^{N-1}\right) + c_k'(x_t^*)\right]$ is strictly positive. Condition 1 implies that $c_k(x)$ is continuous on $[\underline{c}, \overline{c}]$, and $c_k'(x)$ exists almost everywhere and is bounded. Therefore, by (6), if $x_{t+1}^* - x_t^* > 0$, $x_t^* - x_{t-1} > 0$. Moreover, the difference between x_{t-1} and x_t^* only depends on x_t^*, ϵ ($\epsilon = x_{t+1}^* - x_t^*$), and k. ■

Lemma 3 *Assume Condition 1. In a continuation game starting from round t ($t < T - 1$) with the belief that the greatest lower bound of a seller's cost is x_{t-1}, when the number of rounds left in the continuation game goes to infinity, $x_t \to x_{t-1}$ on the continuation equilibrium path but $x_t \neq x_{t-1}$.*

Proof. In the continuation game, the equilibrium path $\{x_\tau\}_{t\leq\tau\leq T}$ and $\{b_\tau\}_{t\leq\tau\leq T}$ are derived from program P8. As $T-t\to\infty$, the value of the program converges, so the additional payoff a buyer can get by adding one more round goes to 0. In the following proof, we show that given any $T-t$, when one more round is added to the continuation game, the additional payoff the buyer can get is strictly positive if $x_t > x_{t-1} + \epsilon, \epsilon > 0$. However, the buyer's payoff is bounded by the payoff in Myerson's optimal mechanism, so when $T-t\to\infty$, $x_t \to x_{t-1}$.

Let $\{x_\tau^*, b_\tau^*\}_{t\leq\tau\leq T}$ be the equilibrium path when there are $T-t$ rounds left, which can be derived from program P8. If we add a constraint $x_t = x_{t-1}$ to P8 and let $\{x_\tau', b_\tau'\}_{t\leq\tau\leq T}$ be the solution to the program, then the buyer's payoff and $\{x'_\tau, b'_\tau\}_{t+1\leq\tau\leq T}$ would be the same as those in the continuation game with $T-t-1$ rounds. The value of the program is

$$V_t(x_{t-1}) = (v - b'_t)\left[F(x_{t-1})^N - F(x'_t)^N\right] + V_{t+1}(x'_t), \text{ where } x'_t = x_{t-1}.$$

Without the constraint, x'_t can be increased by ε, and $V_t(x_{t-1})$ increases approximately by

$$\begin{aligned} & \left[(v-b'_t)\, NF(x'_t)^{N-1} f(x'_t) + V'_{t+1}(x'_t)\right]\varepsilon \\ = & \ [(v-b'_t)\, NF(x'_t)^{N-1} f(x'_t) - NF(x'_t)^{N-1}(v - x'_{t+1})\, f(x'_t) \\ & +(b'_{t+1} - x'_{t+1})(F(x'_{t+1})^{N-1} + F(x'_{t+1})^{N-2}F(x'_t) + \cdots + F(x'_t)^{N-1}) f(x'_t)]\varepsilon \\ = & \ (x'_{t+1} - x'_t)\left[NF(x'_t)^{N-1} - \left(F(x'_{t+1})^{N-1} + F(x'_{t+1})^{N-2}F(x'_t) + \cdots + F(x'_t)^{N-1}\right)\right] f(x'_t)\,\varepsilon. \end{aligned}$$

The second equation comes from $(b'_t - x'_t)\, NF(x'_t)^{N-1} = (b'_{t+1} - x'_t)\left(F(x'_t)^{N-1} + \cdots + F(x'_{t+1})^{N-1}\right)$. Therefore, if $x'_{t+1} > x'_t + \epsilon$, the value is positive and increasing the number of rounds from $T-t-1$ to $T-t$ strictly increases the buyer's payoff. ■

Lemma 4 *Given any T and $t < \infty$, if $x^T_{T-t} < \bar{c}$, then $x^T_T < \bar{c}$ and $x^T_T - x^T_{T-1} > 0$.*
Proof. When $t = 1$, by (1), $x^T_T < \bar{c}$ and $x^T_T - x^T_{T-1} > 0$. When $t = 2$, if $x^T_{T-1} = \bar{c}$, then $x^T_T = \bar{c}$ and the buyer pays for the good at a price higher than or equal to $\bar{c}$, which cannot happen in equilibrium. Therefore, $x^T_{T-1} < \bar{c}$, and we can apply the result we get in the case when $t = 1$.

Applying the same argument to the case where $t = 3, 4, \cdots$, we can conclude that, for any t, if $x^T_{T-t} < \bar{c}$, then $x^T_T < \bar{c}$ and $x^T_T - x^T_{T-1} > 0$. ■

Recall that B is the set of cluster points, and $[\underline{c}, \bar{c}] \setminus B$ is the complement of B.

Lemma 5 *Assume Condition 1. If $a \in B$, $[\underline{c}, a] \subset B$.*
Proof. By Lemma 3, $\underline{c} \in B$. If not the whole interval $[\underline{c}, a]$ belongs to B, there must exist $[b, c] \subset [\underline{c}, a]$ such that $(b, c) \subset [\underline{c}, \bar{c}] \setminus B$ and $b, c \in B$. Since $(b, c) \subset [\underline{c}, \bar{c}] \setminus B$, there exist functions $t(T)$ and $t'(T)$, $t(T) < t'(T)$, such that $\left\{x^T_{t(T)}\right\}_T$ and $\left\{x^T_{t'(T)}\right\}_T$ converge, $b \leq \lim_{T\to\infty} x^T_{t(T)} < \lim_{T\to\infty} x^T_{t'(T)} \leq c$, and no other sequences $\{x^T_k\}_T$, $x^T_k \in \{x^T_t\}_{t(T)<t<t'(T)}$, converge. However, since $a \in B$, i.e. $\lim_{T\to\infty} T - t(T) = \infty$, Lemma 3 implies $x^T_{t(T)+1}$ is arbitrarily close to $x^T_{t(T)}$

when T is large. Therefore, $\lim_{T\to\infty} x^T_{t(T)+1} - \lim_{T\to\infty} x^T_{t(T)} = 0$, a contradiction.
■

Lemma 6 *Assume Condition 1. If there exists $a \in (\underline{c}, \overline{c})$ such that $(a, \overline{c}] \subset [\underline{c}, \overline{c}] \setminus B$, then $[\underline{c}, \overline{c}] \setminus B = (\underline{c}, \overline{c}]$.*

Proof. By Lemma 5, we only need to show that it cannot be the case that $(a, \overline{c}] \subset [\underline{c}, \overline{c}] \setminus B$ and $[\underline{c}, a] \subset B$. Suppose $[\underline{c}, a) \subset B$. First, we show that when $T \to \infty$, there exists $x \in X^T$, $x > a$, that is arbitrarily close to a. Since $(a, \overline{c}] \subset [\underline{c}, \overline{c}] \setminus B$, there exists $t < \infty$ such that $\overline{c} - \lim_{T\to\infty} x^T_{T-t} > 0$. By Lemma 4, $\lim_{T\to\infty} x^T_T < \overline{c}$ and $\lim_{T\to\infty} x^T_T - \lim_{T\to\infty} x^T_{T-1} > 0$. By Lemma 2, $\lim_{T\to\infty} x^T_{T-s+1} - \lim_{T\to\infty} x^T_{T-s} > 0$, for all $s < \infty$. Since $(a, \overline{c}] \subset [\underline{c}, \overline{c}] \setminus B$, it must be the case that $\lim_{t\to\infty} \lim_{T\to\infty} x^T_{T-t} = a$, which also implies $a \in B$.

Since $[\underline{c}, a] \subset B$ and $\lim_{t\to\infty} \lim_{T\to\infty} x^T_{T-t} = a$, we can rewrite the necessary condition (6) for the optimality problem as

$$0 = \left[F\left(x - dx^-\right) - F(x)\right]\left[-F\left(x - dx^-\right)^{N-1} - \cdots - F(x)^{N-1} - C'(x)\right] \tag{7}$$
$$-f(x)\, dx^+ \left[F\left(x + dx^+\right)^{N-1} + \cdots + F(x)^{N-1} - NF(x)^{N-1}\right]$$

where $C(x) = \lim_{k\to\infty} c_k(x)$, $x \in [\underline{c}, a]$, and dx^- and dx^+ are two positive numbers which can be arbitrarily small. For $x \in [\underline{c}, a)$, $dx^- \in O(dx^+)$ but $dx^- \notin o(dx^+)$, and an approximation of equation (7) is

$$\Rightarrow \quad 0 = f(x)\, dx^- \left[-NF(x)^{N-1} - \frac{(N-1)N}{2} F(x)^{N-2} f(x)\, dx^- - C'(x)\right]$$
$$-f(x)\, dx^+ \left[NF(x)^{N-1} - \frac{(N-1)N}{2} F(x)^{N-2} f(x)\, dx^+ - NF(x)^{N-1}\right]$$

Since dx^- and dx^+ are arbitrarily small, the equation implies $C'(x) = -NF(x)^{N-1}$ for $x \in [\underline{c}, a)$.

However, $\lim_{x\to a+} C'(x) \neq -NF(x)^{N-1}$. If $\lim_{x\to a+} C'(x) = -NF(x)^{N-1}$, in order to satisfy equation (7), there exists $\epsilon > 0$ such that for $x \in (a, a+\epsilon)$, $dx^- \in O(dx^+)$ but $dx^- \notin o(dx^+)$.[9] So $(a, a+\epsilon) \subset B$, a contradiction. Since $C'(x)$ is not continuous at a, and dx^- and dx^+ can be arbitrarily small, the

[9] Suppose $dx^- \in o\left(dx^+\right)$. Since dx^- is derived from equation (7), $dx^- \in o\left(dx^+\right)$ implies $dx^- \in O\left(\left(dx^+\right)^2\right)$ and $C'(x) = -NF(x)^{N-1} + \frac{(N-1)N}{2} F(x)^{N-2} f(x) \neq -NF(x)^{N-1}$.

necessary condition (7) does not hold around a. Therefore, a path that $[\underline{c}, a] \subset B$ and $(a, \overline{c}] \subset [\underline{c}, \overline{c}] \setminus B$ cannot occur in equilibrium. Note that there must be at least one cluster point in $[\underline{c}, \overline{c}]$. Since only $\underline{c}$ can be in B, $[\underline{c}, \overline{c}] \setminus B = (\underline{c}, \overline{c}]$. ■

Proposition 9 *A path that fully discriminates sellers is a stationary solution to program P7 when $T = \infty$.*

Proof. If the buyer fully discriminates sellers, we can rewrite the necessary condition (6) for a stationary solution as

$$\begin{aligned} & 0 = [F(x - dx) - F(x)] \left[-F(x - dx)^{N-1} - \cdots - F(x)^{N-1} - C'(x) \right] \\ & -f(x)\, dx \left[F(x + dx)^{N-1} + \cdots + F(x)^{N-1} - NF(x)^{N-1} \right] \\ \Rightarrow \quad & 0 = f(x)\, dx \left[-NF(x)^{N-1} - \frac{(N-1)N}{2} F(x)^{N-2} f(x)\, dx - C'(x) \right] \\ & -f(x)\, dx \left[NF(x)^{N-1} - \frac{(N-1)N}{2} F(x)^{N-2} f(x)\, dx - NF(x)^{N-1} \right], \end{aligned}$$

where dx is a positive number which can be arbitrarily small. Note that $\frac{C(x)}{N}$ can be considered as the information rent given to a seller with cost x. In our setting, in an incentive compatible mechanism that fully discriminates sellers with different costs, the information rent $R(x)$ has the property that $R'(x) = -F(x)^{N-1}$, so $\frac{C'(x)}{N} = -F(x)^{N-1}$. Therefore, the necessary condition holds. Given x_{t-1}, supposing $x_{t+1} - x_t$ is arbitrarily small, one can check that the objective function of (P8) is concave in x_t. Therefore, a path that fully discriminates sellers is a stationary solution. ■

Proof of Proposition 7. Since $\overline{x}^T_{T-k}(x_{T-k-1}, \delta)$ is continuous in x_{T-k-1} and $\lim_{T\to\infty} \overline{x}^T_{T-k}(x_{T-k-1}, \delta) = \overline{x}^T_{T-k}(x_{T-k-1}, 1)$, given any x,
$\lim_{T\to\infty} \overline{x}^T_{T-k}\left(\overline{x}^T_{T-k-1}\left(\cdots \overline{x}^T_{T-k-t}(x, \delta) \cdots\right), \delta\right) = \overline{x}^T_{T-k}\left(\overline{x}^T_{T-k-1}\left(\cdots \overline{x}^T_{T-k-t}(x, 1) \cdots\right), 1\right)$
for $t \in \{0, 1, \cdots\}$. Since the sequence $\{\lim_{T\to\infty} \overline{x}^T_{T-k}\left(\overline{x}^T_{T-k-1}\left(\cdots x^T_{T-k-t}(x, \delta) \cdots\right), \delta\right)$, $\lim_{T\to\infty} \overline{x}^T_{T-k-1}\left(\cdots x^T_{T-k-t}(x, \delta) \cdots\right), \cdots, \lim_{T\to\infty} x^T_{T-k-t}(x, \delta)\}$ decreases and is bounded below by $\underline{c}$, there exists a limit of the sequence
$\{\lim_{T\to\infty} \overline{x}^T_{T-k}\left(\overline{x}^T_{T-k-1}\left(\cdots x^T_{T-k-t}(x, \delta) \cdots\right), \delta\right), \lim_{T\to\infty} \overline{x}^T_{T-k-1}\left(\cdots x^T_{T-k-t}(x, \delta) \cdots\right),$
$\cdots, \lim_{T\to\infty} x^T_{T-k-t}(x, \delta)\}$ when $t \to \infty$. Let χ be the supremum of the cluster

point set B given δ. Then $\lim_{T\to\infty} x_{T-k}^T = \lim_{t\to\infty} \overline{x}_{T-k}^T \left(\overline{x}_{T-k-1}^T \left(\cdots x_{T-k-t}^T (\chi, 1) \cdots\right), 1\right)$.
■

Proof of Theorem 4. The model in Section 2 is a special case when $\delta = 1$. Given $\delta < 1$, since $\lim_{T\to\infty} \sqrt[T]{\delta} = 1$, Lemmas 3 and 4 hold, so Lemma 5 holds for $\delta < 1$. However, Lemma 6 might not hold with $\delta < 1$. By Lemma 5, the first statement of Proposition 4 is proved. For the second statement, if $\lim_{T\to\infty} x_T^T < \overline{c}$, it must be that $a < \overline{c}$. On the other hand, if $a < \overline{c}$, there exists $t < \infty$ such that $\overline{c} - \lim_{T\to\infty} x_{T-t}^T > 0$. By Lemma 4, $\lim_{T\to\infty} x_T^T < \overline{c}$. ■

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