Would Letting People Vote for Several Candidates Yield Policy Moderation?[1]

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We investigate whether letting citizens vote for several candidates would yield policy moderation compared to Plurality Voting. We do so in a setting that takes three key features of elections into account, namely, strategic voting, strategic candidacy and policy motivation on the part of the candidates. We consider two classes of voting rules. One class consists of the voting rules where each voter casts several equally-weighed votes for the different candidates. The other class consists of the voting rules where each voter rank-orders the different candidates. We then identify conditions under which those voting procedures yield policy moderation compared to Plurality Voting. We also show that if any one of those conditions is not satisfied, replacing Plurality Voting with one of those voting procedures may then yield policy extremism! Finally, we find that amongst those voting procedures the extent of policy moderation is maximal under Approval Voting and the Borda Count. Which of these two voting procedures yields the most moderate policy outcomes depends on (1) the symmetry of policy preferences, and (2) the presence of spoiler candidates.

Key Words: Voting rules; Electoral competition; Policy moderation; Citizen-candidate model.
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1. INTRODUCTION

Several countries, such as the U.S. and Canada, elect their policy-makers by means of Plurality Voting (hereafter PV) that is, the voting procedure under which each citizen can vote for exactly one candidate and the candidate who gets the most votes wins the election. Several scholars of electoral systems have claimed that replacing PV with a voting procedure that would let citizens vote for several candidates would yield policy moderation. We define policy moderation as the adoption of policies that lie closer to the median citizen's ideal policy. Their key argument is the so-called wasting-the-vote effect of PV, whereby the fear of wasting one's vote on an underdog induces a citizen to vote for his most-preferred serious contender instead of voting for his most-preferred candidate.[2] Now the wasting-the-vote effect creates a barrier to entry by new candidates. This is because in

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[2] For example, in the 2004 U.S. Presidential election, some of Nader's supporters voted for Kerry or Bush instead of Nader. This is because they anticipated that Nader had no chance of winning the election and, therefore, that a vote for Nader would be wasted. In contrast, there was a (tiny) chance that a vote for Kerry or Bush would be pivotal.

countries where political elections are held under PV, citizens tend to anticipate that a new candidate has no chance of winning the election and, therefore, that a vote for a new candidate would be wasted. The argument then goes that if citizens were given several votes to cast for the different candidates, then they would no longer fear wasting a vote on a new candidate. This would lower the barriers to entry by new candidates, especially by centrists who a majority of citizens would arguably consider as acceptable policy-makers. In other words, this would improve the electoral prospects of the centrists and yield policy moderation compared to PV.

The present paper investigates whether replacing PV with a voting procedure that would let citizens vote for several candidates would yield policy moderation. For this purpose we consider a community that must elect a representative to choose a policy such as a tax rate or the level of public spending. Following the citizen-candidate approach, we model the policy-making process as a three-stage game. At the first stage, policy-motivated citizens decide simultaneously whether to stand for election. At the second stage, citizens decide strategically for whom to vote among the self-declared candidates. At the third stage, the winner of the election assumes office and chooses the policy to implement.

The citizen-candidate approach allows us to take three key features of elections into account. First, the citizen-candidate approach allows us to take strategic voting behavior into account. This is important given that any non-dictatorial voting procedure is open to strategic voting whenever there are at least three alternatives (Gibbard 1973 and Satterthwaite 1975) and that empirical evidence suggests that a sizable fraction of voters vote strategically (e.g., see Forsythe et al. 1996 and Cox 1997). Second, the citizen-candidate approach allows us to endogenize the set of candidates. This is important given that any non-dictatorial and unanimous voting procedure is open to strategic candidacy (Dutta et al. 2001) and that empirical evidence suggests that different voting procedures provide different incentives for candidate entry (e.g., see Persson and Tabellini 2003). Third, the citizen-candidate approach allows for policy motivation on the part of the candidates. This is important for two reasons. A first reason is that policy moderation in PV elections depends critically on whether candidates are policy-motivated and, therefore, whether they can credibly commit to a policy (Alesina 1988). A second reason is that empirical evidence suggests that politicians' own policy preferences matter for their policy decisions (e.g., see Levitt 1996).

There exist countless voting procedures that let people vote for several candidates. In this paper, I consider the scoring rules. This class of voting rules encompasses many of the voting rules that are commonly used and studied. Under a scoring rule, a voter's ballot takes the form of a vector of points that are cast for the different candidates. The candidate who gets the most points wins the election. We shall focus on two broad classes of scoring rules. The first class contains the scoring rules where each vote is weighed equally. We shall call those scoring rules Multiple Voting rules (hereafter MVs). An example of MV is Dual Voting under which each citizen is given two votes to cast for the different candidates. Another example is Approval Voting under which a citizen can cast a vote for as many candidates as he wishes.[3] The second class of scoring rules to be considered contains

[3]Approval Voting is a voting procedure that had been popularized by Brams and Fishburn (1978). It is currently used by several professional and academic associations to elect their office-bearers. Several scholars of electoral systems have recommended that Approval Voting be used for political elections. For more details see, for example, Brams and Fishburn (2005).

the scoring rules where voters rank-order the candidates and where the score that goes to a candidate increases with his position in the ranking. We shall call those scoring rules the Ordinal Voting rules (hereafter OVs). An example of OV is the Borda Count under which the candidate who is ranked last gets no points, the second lowest candidate gets one point, the next-ranked candidate gets two points, and so on.

The present analysis yields a number of interesting results. Our first result concerns MV elections in the context of serious races. An election is said to be serious if in equilibrium, each candidate is likely to win. Our analysis identifies three conditions for a MV to yield policy moderation compared to PV. Those conditions are that (1) voters are allowed to truncate their ballots and cast as many of their votes as they wish, so that the equilibrium set under a MV is a subset of the equilibrium set under PV, (2) each citizen has 'enough' votes to cast, so that the wasting-the-vote effect does not come into play, and (3) policy preferences are symmetric, so that an equal number of candidates stand at each position. However, we also find that replacing PV with another MV may yield policy extremism instead of policy moderation if anyone of those three conditions is not satisfied! Observe that Approval Voting satisfies the first two conditions. Hence, our analysis suggests that the existence of a MV yielding policy moderation compared to PV depends on the symmetry of policy preferences.

Our second result concerns OV elections in the context of serious races. Our analysis shows that replacing PV with an OV always yields policy moderation. We also show that the extent of policy moderation depends on (1) the second-place score in a three-way race and (2) the admissibility of truncated ballots. Keeping everything else unchanged, policy outcomes are more moderate the higher the second-place score in a three-way race is. Allowing citizens to truncate their ballots yields more extreme policy outcomes when the second-place score in a three-way race is relatively high (as under the Borda Count) and more moderate policy outcomes otherwise. This is because when truncated ballots are not admissible, the former OVs cannot deter (and accomodate) multiple similar candidacies while the latter OVs can.

Our third result concerns elections in the context of spoiler races. An election is said to be spoiler if in equilibrium, some citizens (called spoilers) stand for election not to win but because their presence in the race yields an electoral outcome they prefer. The presence of spoilers creates barriers to entry and exit by candidates. As a result, eliminating the wasting-the-vote effect is no longer sufficient for lowering the barriers to entry by centrists and yielding policy moderation.

Our fourth result compares policy moderation under the different MVs and OVs. Which voting procedure yields the most moderate policy outcomes depends on several factors. In the context of serious races, if truncated ballots are admissible, then Approval Voting yields the most moderate outcomes provided that policy preferences are symmetric. If instead, only completely-filled ballots are admissible or policy preferences are asymmetric, then the Borda Count is the voting procedure that yields the most moderate outcomes. Interestingly, the extent of policy moderation is maximal under the Borda Count with only completely-filled ballots admissible. However, this does not extend to spoiler races. The Borda Count with only completely-filled ballots admissible may then yield more extreme policy outcomes than Approval Voting.

The present work is related to a number of papers that study the effect of alternative electoral systems on policy moderation using the Downsian approach to political competition.[4] Under the prototypical Downsian approach, a fixed set of purely office-motivated candidates compete by choosing a platform along a one-dimensional policy space. Cox (1987, 1990) compares policy moderation under alternative voting procedures, assuming that citizens vote sincerely. In this context, he finds that increasing the number of votes a citizen can cast yields policy moderation. Myerson and Weber (1993) relax the sincere voting assumption, adopting instead the pivotal-voter approach whereby a citizen conditions his vote on his anticipation of a close race between the different candidates. They find that while PV imposes few restrictions on the location of the serious contenders, under Approval Voting all serious contenders are standing at the median citizen's ideal policy. The present paper goes further by relaxing the assumption that candidates are purely office-motivated and by endogenizing the set of candidates. In Section 5, we discuss how the differences in assumptions affect policy moderation under the different voting procedures. We do so comparing the present analysis with the Downsian analysis of Cox (1990).

The present paper is also related to the citizen-candidate literature that was initiated by Osborne and Slivinski (1996) and Besley and Coate (1997). In contrast to the Downsian framework, the set of candidates is endogenous and since they are citizens, candidates are policy-motivated. This approach to political competition yields polarization under PV, even in two-candidate races.[5] This is because entry is costly and candidates are policy-motivated, the latter preventing them to credibly commit to the policy they will implement if elected. Previous works in this literature are focused on PV. Notable exceptions are Osborne and Slivinski (1996) who compare policy outcomes under PV and Plurality Runoff, Hamlin and Hjortlund (2000) and Morelli (2004) who study elections under Proportional Representation, and Dellis and Oak (2006) who compare PV and Approval Voting. The present contribution extends this literature by comparing policy moderation under the different MVs and OVs.

The remainder of the paper is organized as follows. Section 2 presents the model. Sections 3 and 4 study policy moderation in the context of serious and spoiler races. A comparison with the Downsian framework is presented in Section 5. Section 6 concludes. All proofs are in the Appendix.

[4] The Downsian framework has also been used to study, for example, the effect of different electoral systems on reducing government corruption (see, for example, Myerson 1993a), on the provision of public goods (see, for example, Lizzeri and Persico 2001 or Milesi-Ferretti et al. 2002) or on the incentives to favor minority interests (see, for example, Myerson 1993b or Lizzeri and Persico 2005).

[5] This contrasts with the Downsian approach where in two-candidate PV elections, both candidates choose to stand at the median citizen's ideal policy. Observe however that we get polarization in the Downsian framework if one of the following assumptions is made: (1) candidates are policy-motivated and uncertain about the distribution of preferences in the electorate (Wittman 1983, Calvert 1985, Roemer 1997), (2) candidates are policy-motivated and unable to credibly commit to a policy if elected (Alesina 1988), (3) the government is divided (Ortuno-Ortin 1997, Alesina and Rosenthal 2000), (4) one of the two candidates is an incumbent (Bernhardt and Ingberman 1985), (5) candidates need to cater to special interests to raise money for their campaign (Baron 1994), (6) voting is costly and voters can abstain (Feddersen 1992), or (7) the two candidates compete against each other under the threat of entry by a third candidate (Palfrey 1984, Weber 1992 and 1998, Callander 2005).

2. THE MODEL

Consider a community that must elect a decision-maker to choose a policy such as a tax rate or the level of public spending. The community is made up of N citizens, indexed by $\ell \in \mathcal{N} \equiv \{1, ..., N\}$. The set of policy alternatives is $X = \mathbb{R}$.[6]

Citizens' preferences over the set of policy alternatives are represented by a utility function $u^{\ell} : X \to \mathbb{R}$, which is assumed to be strictly concave.[7] Let $x_{\ell} \equiv \arg\max_{x \in X} u^{\ell}(x)$ denote citizen ℓ's ideal policy. We assume throughout the analysis that citizens differ only in their ideal policy—that is, $u^{\ell}(x) \equiv u(x_{\ell} - x)$ for all citizen ℓ—with several citizens sharing the same ideal policy.[8] We normalize $u^{\ell}(x_{\ell})$ to 0 and denote by $v_i^{\ell} \equiv u^{\ell}(x_i)$ the utility citizen ℓ derives from x_i, citizen i's ideal policy. Furthermore, let m denote the median citizen's ideal policy and M the number of citizens who share this ideal policy.[9] Finally, let there be $\frac{N-M}{2} \geq 6$ citizens on either side of m and assume $M \leq \frac{N-4}{3}$.[10]

There are three stages to the policy-making process. At the first stage, each citizen decides whether to stand for office. Decisions are made simultaneously. At the second stage, an election is held where each citizen decides for whom to vote among the self-declared candidates. Votes are then added up and the candidate who gets the highest vote total is elected. If several candidates tie for first place, each is elected with an equal probability. At the third stage, the elected decision-maker chooses policy. We assume that a candidate cannot credibly pre-commit to implementing a policy different from his ideal one. Lee et al. (2004) provide empirical justification for this assumption. We now analyze each of these stages in a reverse order.

Policy-making stage. Because this stage is the last one, the elected decision-maker chooses to implement his ideal policy. In case no one runs for election, the status-quo policy $x_0 \in X$ is assumed to be kept.

Election stage. Suppose the election is held under an arbitrary voting rule V. For

[6] The unidimensionality of the policy space facilitates comparisons with the related literature which for the most part, has developed under this assumption. Besides, empirical works have shown that although citizens care about many issues, preferences over those issues tend to be highly correlated and that a unidimensional policy space is relevant (see, for example, Poole and Rosenthal 1991 or Hinich and Munger 1994).

[7] The strict concavity assumption eliminates the knife-edge PV equilibria where three serious contenders are running for election, each standing on a different platform. This is not a very restrictive assumption however in the sense that those equilibria are non-generic—they can be supported only if preferences are Euclidean and citizens' ideal policies are distributed in a very specific way. Besides, those equilibria run counter to empirical as well as experimental evidence (see, for example, Cox 1997 and Forsythe et al. 1996).

[8] The assumption that citizens differ only in their ideal policy is made to simplify the analysis. All but one of our results can be derived without this assumption. Moreover, for the result that makes use of it—namely, Proposition 2—, it is only as a sufficient condition. The assumption that several citizens share the same ideal policy is not key either. It is only made to avoid corner solutions where more citizens than there are would be willing to stand on a platform. Relaxing this assumption would complicate the statements of the results without adding any new insight.

[9] In the remainder of the paper, we shall abuse notation and call the median citizen m. Also, note that the median citizen's ideal policy is the Condorcet winner—that is, the policy that defeats any other in a pairwise contest.

[10] Assuming there are at least six citizens on either side of the median is made to rule out situations where too few votes are cast. Assuming less than a third of the electorate is at the median is made to rule out situations where the citizens at the median would be able to elect any candidate without the support of others.

a given non-empty set of candidates $\mathcal{C} \subseteq \mathcal{N}$, let $A_V(\mathcal{C})$ denote the set of admissible voting strategies under V and $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ citizen ℓ's voting strategy, where $\alpha_i^\ell(\mathcal{C})$ is the score citizen ℓ gives to candidate i.[11] We denote the profile of voting strategies by $\alpha(\mathcal{C}) = \left(\alpha^1(\mathcal{C}), ..., \alpha^N(\mathcal{C})\right)$. We shall call the *winning set* $W(\mathcal{C}, \alpha)$ the set of candidates with the highest score when $\alpha(\mathcal{C})$ is the voting profile; that is,

$$W(\mathcal{C}, \alpha) \equiv \left\{ i \in \mathcal{C} : \sum_{\ell \in \mathcal{N}} \alpha_i^\ell(\mathcal{C}) \geq \sum_{\ell \in \mathcal{N}} \alpha_j^\ell(\mathcal{C}) \text{ for any } j \in \mathcal{C} \right\}.$$

Given our tie-breaking assumption, citizen i's probability of becoming the decision-maker is then equal to $\frac{1}{\#W(\mathcal{C},\alpha)}$ if $i \in W(\mathcal{C}, \alpha)$ and 0 otherwise, where $\#W(\mathcal{C}, \alpha)$ is the number of winning candidates. Citizen ℓ's expected utility is thus given by $V^\ell(\mathcal{C}, \alpha) \equiv \frac{1}{\#W(\mathcal{C},\alpha)} \sum_{i \in W(\mathcal{C},\alpha)} v_i^\ell$.

We are now ready to define a (pure-strategy) voting equilibrium.

Definition 1 (Voting Equilibrium). Given a non-empty set of candidates $\mathcal{C}$, a strategy profile $\alpha^*(\mathcal{C})$ is a voting equilibrium if for any citizen ℓ, $\alpha^{\ell*}(\mathcal{C}) \in A_V(\mathcal{C})$ is such that
(1) $V^\ell\left(\mathcal{C}; \alpha^{\ell*}, \alpha^{-\ell*}\right) \geq V^\ell\left(\mathcal{C}; \alpha^\ell, \alpha^{-\ell*}\right)$ for all $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$;
(2) $\alpha^{\ell*}(\mathcal{C}) = (0, ..., 0)$ whenever $v_i^\ell = v_j^\ell$ for all two candidates i and j; and
(3) $\alpha^{\ell*}(\mathcal{C})$ is weakly undominated. $\|$

The first condition says that a citizen chooses a voting strategy that maximizes his expected utility, given others' voting strategies and his anticipation of the policy each candidate will implement if elected. The second condition states that a citizen who is indifferent between all candidates abstains from voting. Finally, the third condition is a common refinement of the voting behavior. The following lemma—adapted from Dellis (2006)—characterizes the set of weakly undominated voting strategies under an arbitrary MV or OV.

Lemma 1 (Dellis 2006). *Take a non-empty set of candidates $\mathcal{C}$ and suppose that citizen ℓ is not indifferent between all the candidates in $\mathcal{C}$. A voting strategy $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ is weakly undominated for citizen ℓ if, and only if, there is no other voting strategy $\widetilde{\alpha}^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ such that for all pairs of candidates i and j, $v_i^\ell \geq v_j^\ell$ implies* $\left[\widetilde{\alpha}_i^\ell(\mathcal{C}) - \alpha_i^\ell(\mathcal{C})\right] \geq \left[\widetilde{\alpha}_j^\ell(\mathcal{C}) - \alpha_j^\ell(\mathcal{C})\right]$. $\|$

Thus, for a citizen's voting strategy to be weakly undominated, it must be that there does not exist another admissible voting strategy such that the ordering of score differences between the two voting strategies corresponds to the citizen's preference ordering over the set of candidates.[12]

Candidacy stage. Citizens decide simultaneously whether to stand for election at a utility cost $\delta > 0$. We assume that all citizens anticipate the same voting equilibrium when making their candidacy decision. Denote by $e^\ell \in \{0, 1\}$ citizen ℓ's candidacy decision, with $e^\ell = 1$ if he chooses to stand for election and 0

[11] For example, under PV $\alpha^\ell(\mathcal{C}) \in A_V(\mathcal{C})$ if either $\alpha^\ell(\mathcal{C}) = (0, ..., 0)$—that is, citizen ℓ abstains from voting—, or $\alpha^\ell(\mathcal{C})$ is any permutation of $(1, 0, ..., 0)$.

[12] For example, under PV a citizen's voting strategy is weakly undominated if he does not vote for the candidate(s) he likes the least. Instead, under Approval Voting his voting strategy is weakly undominated if he votes for the candidate(s) he likes the most *and* does not vote for the candidate(s) he likes the least. Finally, under the Borda Count he must rank the candidate(s) he likes the most above the candidate(s) he likes the least.

otherwise. Let $e = \left(e^1, ..., e^N\right)$ be the profile of candidacy decisions and define $\mathcal{C}(e) \equiv \left\{\ell \in \mathcal{N} : e^\ell = 1\right\}$ as the set of candidates. Thus, for a given profile of entry decisions e, citizen ℓ's expected utility is $U^\ell(\mathcal{C}(e), \alpha) \equiv \left[V^\ell(\mathcal{C}(e), \alpha) - \delta e^\ell\right]$ if $\mathcal{C}(e)$ is not empty and $U^\ell(\mathcal{C}(e), \alpha) \equiv u^\ell(x_0)$ otherwise.

We are now ready to define a candidacy equilibrium.

Definition 2 (Candidacy Equilibrium). A profile of candidacy strategies e^* is a candidacy equilibrium if for each citizen ℓ, $e^{\ell *}$ is a best response to $e^{-\ell *}$ given the profile of voting strategies. ‖

Political equilibrium. A political equilibrium (hereafter an equilibrium) is a Subgame Perfect Nash Equilibrium of this game. It consists of a pair $(e^*, \alpha^*(.))$ where: (1) $\alpha^*(.)$ is an equilibrium profile of voting strategies for any non-empty set of candidates $\mathcal{C}$; and (2) e^* is an equilibrium profile of candidacy strategies, given $\alpha^*(.)$.[13] A candidate is said to be serious if he is in the winning set, and spoiler otherwise. We shall call serious an equilibrium where all candidates are serious contenders, and spoiler an equilibrium where some candidates are spoilers.

3. POLICY MODERATION IN SERIOUS RACES

In this section, we study policy moderation in the context of serious races, i.e., elections where in equilibrium all candidates are serious contenders. But before we proceed, two things must be done. First, we need to characterize the equilibrium set under PV. We know from Besley and Coate (1997) that there are only two types of serious equilibria under PV, the one- and two-position equilibria. In a one-position equilibrium, only one citizen enters the race and is elected outright. In a two-position equilibrium, two citizens enter the race, one on either side of the median citizen's ideal policy with the median citizen indifferent between the two candidates, and both candidates tie for first place. The second thing we need to do is to specify how we shall compare equilibria. Given our focus on policy outcomes, we shall say that two equilibria are equivalent if they yield the same lottery over policy outcomes.

3.1. Multiple Voting Rules

Let us first consider elections that are held under a Multiple Voting rule (hereafter MV). A MV is a scoring rule under which each citizen has $q \in \mathbb{N}$ votes to cast—or $(c-1)$ if the number of candidates c does not exceed q—and each vote is weighed equally. Formally, the vector of points a voter can cast $(s_1, s_2, ..., s_c)$ is given by

$$\begin{cases} 1 = s_1 = ... = s_q > s_{q+1} = ... = s_c = 0 & \text{if } c > q \\ 1 = s_1 = ... = s_{c-1} > s_c = 0 & \text{if } c \leq q \end{cases}.$$

Examples of MVs are PV ($q = 1$), Dual Voting ($q = 2$) and Negative Voting ($q = +\infty$).

The following result can be established:

[13] Following previous contributions in the citizen-candidate literature (e.g., Besley and Coate 1997 and Dhillon and Lockwood 2002), we shall focus on equilibria in pure strategies.

PROPOSITION 1. *Let V be a MV with $q \in \mathbb{N}$, the maximum number of votes a citizen is ever allowed to cast. The serious equilibrium set under V is a moderate subset of the serious equilibrium set under PV*[14] *if:*

(1) *truncated ballots are admissible—i.e., a voter can cast as many of his votes as he wishes;*

(2) *each citizen has enough votes to cast—i.e., $q > \overline{q}$ for some $\overline{q} \in \mathbb{Z}_+$ finite; and*

(3) *policy preferences are symmetric—i.e., $u^\ell(x) \equiv u(|x_\ell - x|)$ for all citizen ℓ.*[15] ‖

Thus, replacing PV with another MV yields policy moderation provided that the above three conditions hold. In other words, Proposition 1 provides a (conditional) theoretical underpinning for the claim that letting people vote for several candidates would yield policy moderation. Observe that Approval Voting—i.e., the voting rule under which a citizen can vote for as many candidates as he wishes—satisfies the first two conditions since Approval Voting is equivalent to Negative Voting with truncated ballots admissible. Hence, whether there exists a MV that yields policy moderation compared to PV depends on whether policy preferences are symmetric or not.[16]

On the other hand, if any one of those three conditions does not hold, then replacing PV with another MV may yield policy extremism instead of policy moderation. To see this, let us consider each condition separately.

Admissibility of truncated ballots. Since a citizen is pivotal over the set of serious contenders, he is better off truncating his ballot than casting a vote for a less-preferred candidate. The consequence of admitting truncated ballots is that the serious equilibrium set under any MV is a subset of the serious equilibrium set under PV and, therefore, contains only one- and two-position equilibria. The one-position serious equilibria are equivalent under any MV since only one candidate stands for election and is, therefore, elected outright. On the other hand, the two-position serious equilibrium set under a MV is a subset of the two-position serious equilibrium set under PV. This is because PV is subject to the wasting-the-vote effect and is, therefore, able to deter entry by new candidates. In contrast, other MVs need not be subject to the wasting-the-vote effect and may therefore be unable to deter entry by new candidates. Hence, some of the two-position serious PV equilibria need not have equivalent serious equilibria under some of the other MVs.

On the other hand, if only completely-filled ballots are admissible, then the reverse holds true, i.e., the serious equilibrium set under any MV is a superset of the serious equilibrium set under PV. (For a formal statement and proof of this result, see Proposition 2 in the working paper.) This is illustrated most simply for Dual Voting.

EXAMPLE 1. Consider a community that must elect a representative to choose a tax rate. Suppose there are 39 citizens whose ideal tax rates are distributed as

[14] That is, if a serious equilibrium under V with policy outcome x exists, then for any serious equilibrium under PV that implements $\widetilde{x}$, with $u^m(\widetilde{x}) > u^m(x)$, an equivalent serious equilibrium exists under V.

[15] It is worth mentioning that Proposition 1 is a special case of a more general result where instead of PV, we have a MV $\widetilde{V}$ with $\widetilde{q} < q$.

[16] Observe that if policy preferences are symmetric, then the equilibrium set—and, therefore, the extent of policy moderation—is equivalent under any MV that satisfies the first two conditions. A proof is available from the author upon request.

follows

Ideal tax rates	0	1	2	3	4	5	6	7	8	9	10
Number of citizens	2	2	3	4	5	7	5	4	3	2	2

,i.e., two citizens have 0 as an ideal tax rate, two citizens have 1 as an ideal tax rate, and so on. Assume that preferences can be represented by a quadratic loss utility function, i.e., $u^{\ell}(x) = -(x_{\ell} - x)^2$. (Observe that those preferences satisfy condition 3 of Proposition 1.) Finally, let the utility cost of running for election $\delta = 3$.

If the election is held under PV, then $\{3, 7\}$, $\{2, 8\}$, $\{1, 9\}$ and $\{0, 10\}$ are all two-position serious equilibria, where $\{x, y\}$ denotes an equilibrium with the candidates standing at x and y.[17] In those equilibria, one citizen at each position enters the race, all the citizens at $0, ..., 4$ ($6, ..., 10$, resp.) vote for the left (right, resp.) candidate and the citizens at 5 abstain from voting. Thus, each candidate ties for first place. It is easy to check that neither of the two candidates can be better off not running for election. Moreover, no other citizen is willing to enter the race if he (correctly) anticipates that voters will fear wasting their vote on a new candidate.

Equilibria with the same policy outcomes exist if instead the election is held under Dual Voting and only completely-filled ballots are admissible. The difference with PV is that two candidates are now standing at each position. To see this, let us suppose one candidate stands at 3 and another one at 7, in which case both candidates tie for first place (as under PV). A second citizen at 3 will then want to enter the race since all the citizens at $0, ..., 4$ will then cast their two votes for the two candidates at 3, the citizens at 5 will still abstain from voting, and the citizens at $6, ..., 10$ will cast one vote for the candidate at 7 and will have to cast their second vote for one of the candidates at 3. Consequently, one of the candidates at 3 will be elected the policy-maker and the probability that 3 is implemented increases from 1/2 to 1. A second citizen at 3 is therefore willing to enter the race since his utility gain (equal to 8) exceeds the utility cost of running for election. Likewise, a second citizen at 7 will want to enter the race. Now once two candidates are standing at each position, the wasting-the-vote effect is back. Consequently, no other citizen is willing to enter the race if he (correctly) anticipates that the citizens at $0, ..., 4, 6, ..., 10$ will fear wasting any of their two votes on a new candidate. Thus, $\{3, 7\}$ can be supported as an equilibrium policy outcome under Dual Voting with only completely-filled ballots admissible. The same argument applies to $\{2, 8\}$, $\{1, 9\}$ and $\{0, 10\}$.

On the other hand, $\{3, 7\}$ cannot be supported as a serious equilibrium outcome under Dual Voting if truncated ballots are admissible. Indeed, following the entry of a second candidate at 3, all the citizens at $6, ..., 10$ will now cast a vote only for the candidate at 7. Consequently, all three candidates tie for first place and the probability that 3 is implemented increases from 1/2 to 2/3. A second citizen at 3 is no longer willing to enter the race since his utility gain (now equal to 8/3) is smaller than the cost of running for election. This means that all the citizens at $0, ..., 4, 6, ..., 10$ have one vote left that they can cast for a new candidate. The wasting-the-vote effect is therefore eliminated and the barriers to entry by new candidates are lowered. It is easy to check that a citizen at 5 will then enter the race and win outright, implying that $\{3, 7\}$ cannot be supported as an equilibrium policy outcome under Dual Voting when truncated ballots are admissible. $\square$

[17] We shall ignore the one-position serious equilibria since they are equivalent under all voting rules.

In addition to the one- and two-position equilibria, serious equilibria where candidates are standing at three or more positions are possible under a MV with $q \geq 2$ when only completely-filled ballots are admissible. It is then easy to construct examples where those equilibria are more extreme than any serious equilibrium under PV, in which case we get policy extremism.[18]

Number of votes. This condition requires that each citizen has enough votes to cast so that the wasting-the-vote effect is eliminated. To understand the intuition behind this condition, suppose that policy preferences are symmetric and truncated ballots are admissible. The latter implies that the serious equilibrium set under any MV contains only one- and two-position equilibria. Lemma 2 in the Appendix shows that the one-position serious equilibria are moderate compared to the two-position serious equilibria. This, together with the equivalence of the one-position serious equilibria under all MVs, implies that we need consider only the two-position serious equilibria to compare policy moderation under the different MVs.

The rationale behind this condition is the difference in incentives for multiple entries at the winning positions. Under PV, several candidates standing on the same platform run the risk of splitting their votes, thereby helping the election of rival candidates. The same is not true under the other MVs since voters can then vote for up to q candidates, meaning that up to q candidates can be standing at the same position without entailing a risk of vote splitting. Now, the number of candidates standing at each position increases with the distance between the two positions since the utility gain of implementing one's ideal policy is then bigger. Letting $\overline{q}$ denote the number of citizens who are willing to stand at each of the two most extreme platforms between which the median citizen is indifferent, we get that replacing PV with a MV yields policy moderation if $q > \overline{q}$ but may yield policy extremism otherwise.[19] This is because if $q > \overline{q}$, then the wasting-the-vote effect is eliminated everywhere and the barriers to entry by moderates are lowered. A centrist is then more likely to enter the race the more polarized the two positions are, since his electoral prospects are then better and his utility gain from implementing his ideal policy is then bigger. Thus, the serious PV equilibria that can be supported under those MVs are the most moderate equilibria. In contrast, if $q \leq \overline{q}$, then the wasting-the-vote effect is eliminated only for the moderate equilibria. Only the most extreme PV equilibria may therefore survive, thereby yielding policy extremism. (An example illustrating this discussion can be found in the 'Technical Appendix' of the working paper.)

Symmetry of policy preferences. Suppose that the first two conditions in Proposition 1 are satisfied. The key behind the symmetry of policy preferences condition is the fact that different MVs provide differential incentives for candidate entry at the winning positions. In particular, the vote-splitting effect implies that PV is effective at deterring multiple similar candidacies. This is not true however for the other MVs since a second candidate entering at a winning position does not run

[18] In an earlier version of this paper, we also considered the case where cumulated ballots are admissible, i.e., voters are allowed to cumulate their votes behind fewer candidates. It turns out that the serious equilibrium set is then equivalent under all scoring rules. This is because each voter has then an incentive to stack all his votes on the serious contender he prefers instead of wasting any of them on another candidate. Consequently, the vote totals are scaled up by a common factor while the vote shares are left unchanged.

[19] The proof of Proposition 1 shows that $\overline{q}$ decreases with the cost of running for election δ and increases with the distance between the two platforms.

the risk of splitting votes. If preferences are symmetric, then the utility gain from entering the race is the same for a citizen at the left or the right position, inducing an equal number of citizens at each position to stand for election (as under PV). On the other hand, if preferences are asymmetric, then the utility gain of standing for election may be bigger for, say, a left citizen than for a right citizen, thus inducing more left citizens to enter the race. This reduces the winning probability for a candidate and, therefore, the expected utility gain from entering the race. Since the latter is smaller the less polarized the two positions are, any right citizen may be deterred from entering the race if the two positions are relatively close to each other. Consequently, only the most extreme serious equilibrium policy outcomes under PV may be supported as serious equilibrium policy outcomes under the other MVs, thereby yielding policy extremism. (See Dellis and Oak 2006 for an example under Approval Voting.)[20]

3.2. Ordinal Voting Rules

Let us now consider elections that are held under an Ordinal Voting rule (hereafter OV). An OV is a scoring rule under which a voter must rank-order the candidates and where the candidate who is ranked h^{th} on the ballot gets s_h points, with $1 = s_1 > s_2 \geq ... \geq s_c = 0$. In case truncated ballots are admissible, a candidate who is not ranked gets no points. Examples of OVs are PV ($s_1 = 1$ and $s_2 = ... = s_c = 0$) and the Borda Count ($s_h = \frac{c-h}{c-1}$ for $h = 1, ..., c$).

The following result can be established.

Proposition 2. *Let V and $\widetilde{V}$ be any two OVs with s and $\widetilde{s}$ their respective second-place scores in three-way races. Suppose $s > \widetilde{s}$. Then, the serious equilibrium set under V is a moderate subset of the serious equilibrium set under $\widetilde{V}$.* ||

To appreciate this result, take $\widetilde{V}$ to be PV (i.e., $\widetilde{s} = 0$). Thus, Proposition 2 establishes that replacing PV with an OV yields policy moderation. Observe that contrary to MVs, no further condition is required for an OV to yield policy moderation. The key factor driving this difference between MVs and OVs is that a voter can cast a one-point vote for up to q candidates under a MV but for only one candidate under an OV. This implies that under an OV two (or more) candidates standing on the same platform run the risk of splitting their top-score votes, thereby helping the election of rival candidates. On the other hand, there is no such vote-splitting effect under a MV with $q \geq 2$. In other words, MVs can accomodate multiple similar candidacies, but OVs cannot. As a result, getting policy moderation under an OV does not require any of the three conditions stated in Proposition 1 since the purpose of those conditions is to deal with the issue of multiple similar candidacies.

However, the extent of policy moderation depends on (1) the second-place score in a three-way race s and (2) the admissibility of truncated ballots. Let us first study the effect of s on policy moderation. Given that a voter can top-rank only one candidate, the serious equilibrium set under an OV contains only one- and two-position equilibria with only one candidate standing at each position. (For a formal statement and proof of this result, see Proposition 1 in Dellis 2006.) Lemma 2 in

[20] Interestingly, both this condition and the previous one deal with the issue of multiple similar candidacies. However, the former condition is concerned with its consequences on the barriers to entry by moderates while the latter condition is concerned with its consequences on the barriers to entry at the winning positions.

the Appendix shows that the one-position serious equilibria are moderate compared to the two-position serious equilibria. This, together with the equivalence of the one-position serious equilibria under the different OVs, implies that we can restrict our attention to the two-position serious equilibria. Now suppose that a moderate enters the race, and assume for simplicity that truncated ballots are admissible. Then, in the worst-case scenario all the citizens at the median rank the moderate first and truncate their ballot. At the same time, all the leftists (rightists, resp.) rank the left (right, resp.) candidate first and the moderate second if he is a most-preferred candidate and truncate their ballot otherwise. Consequently, a larger s implies a larger vote share for the moderate. In other words, keeping everything else unchanged, the larger s is, the more effective at lowering the barriers to entry by moderates an OV is. The following example sheds some light on this result.

Example 2. Consider the community described in Example 1 and assume for simplicity that truncated ballots are admissible. Recall from Example 1 that if the election is held under PV (i.e., $s = 0$), $\{3, 7\}$, $\{2, 8\}$, $\{1, 9\}$ and $\{0, 10\}$ are all two-position serious equilibria. The same holds true if the election is held under the OV with $s = 1/3$. To see this, consider the situation where one citizen at 0 and one citizen at 10 stand for election. It is easy to check that neither of them can be better off staying out of the race and that the risk of vote-splitting deters any other citizen at 0 and 10 from entering the race. Moreover, no moderate is willing to enter the race if he (correctly) anticipates that following his entry in the race the citizens at $0, ..., 4$ ($6, ..., 10$, resp.) will continue top-ranking the candidate at 0 (10, resp.), the citizens at 5 will rank him first and truncate their ballot, and all the other citizens for whom he is a most-preferred candidate will rank him second. A moderate would then get at most 14 points while the candidates at 0 and 10 would get 16 points each. Hence, the barriers to entry by moderates are kept unmoved. (The same argument applies to $\{3, 7\}$, $\{2, 8\}$ and $\{1, 9\}$.)

This is not true however if the election is held under the Borda Count (i.e., $s = 1/2$). Indeed, a citizen at 4 will then enter the race, anticipating he will get at least 17.5 points and win outright. Consequently, $\{0, 10\}$ cannot be supported as an equilibrium policy outcome under the Borda Count. The same argument applies to $\{1, 9\}$. On the other hand, it is easy to check that $\{2, 8\}$ and $\{3, 7\}$ can be supported as serious equilibrium policy outcomes. □

Let us now study the effect of admitting truncated ballots. For that purpose, we shall distinguish those OVs that are worst-punishing—i.e., those with $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$—from those that are best-rewarding—i.e., those with $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$.[21] (Observe that the Borda Count is worst-punishing.)

We then have:

Proposition 3. *Suppose that the election is held under an OV. If it is worst-punishing, then the serious equilibrium set when only completely-filled ballots are admissible is a moderate subset of the serious equilibrium set when truncated ballots are admissible. The reverse holds true if it is best-rewarding.* ||

Thus, the effect of allowing for vote truncation depends on whether the OV is best-rewarding or worst-punishing. This is because the worst-punishing OVs are

[21] This corresponds to the classification proposed by Cox (1987), except for the term $\left(\frac{N-M-2}{N-M-1}\right)$ which is due to strategic voting.

unable to deter and accomodate multiple entries at the winning positions when only completely-filled ballots are admissible. Consequently, the only serious equilibria are the one-position equilibria, which we know to be the most moderate equilibria.[22] This is illustrated most simply in the case of the Borda Count.

EXAMPLE 3. Consider the community described in Example 1 and suppose that the election is held under the Borda Count. We know from Example 2 that $\{3,7\}$ and $\{2,8\}$ are all two-position serious equilibria when truncated ballots are admissible. This is no longer true however if only completely-filled ballots are admissible. To see this, suppose one citizen at 3 and one citizen at 7 stand for election, and let a second citizen at 3 enters the race. Then all the citizens at $0,...,4$ rank first and second the two candidates at 3, the citizens at 5 abstain from voting since they are indifferent between all three candidates, and all the citizens at $6,...,10$ rank the candidate at 7 first and one of the candidates at 3 second. Consequently, the candidate at 7 gets 16 points while the two candidates at 3 share 32 points. It is then easy to see that in any voting equilibrium, one of the candidates at 3 wins outright. Thus, the entry in the race of a second citizen at 3 increases the probability that 3 is implemented from 1/2 to 1. A second citizen at 3 is therefore willing to enter the race since his utility gain (equal to 8) exceeds the cost of running for election. Hence, $\{3,7\}$ cannot be supported as an equilibrium policy outcome. The same argument applies to $\{2,8\}$. $\square$

On the other hand, a best-rewarding OV is effective at deterring multiple entries at the winning positions since the second-place score in a three-way race is relatively small. But what about entry by moderates? To look at this question, assume that a moderate enters the race. If truncated ballots are admissible, then in the worst-case scenario, all the left (right, resp.) citizens rank the left (right, resp.) candidate first and the moderate second if the latter is a most-preferred candidate and truncate their ballot otherwise. At the same time, the median citizens rank the moderate first and truncate their ballot. If instead only completely-filled ballots are admissible, then the median citizens and all the citizens for whom the moderate is not a most-preferred candidate will now rank second the left or right candidate. Hence, the vote total of the moderate is here the same whether truncated ballots are admissible or not, but the vote totals of the left and right candidates are bigger when only completely-filled ballots are admissible. Consequently, if a moderate can be deterred from entering the race when truncated ballots are admissible, so can he when only completely-filled ballots are admissible. However, the reverse is not true.

Thus, among the OVs, the Borda Count with only completely-filled ballots admissible yields the most moderate policy outcomes. This is because this voting procedure is unable to deter and accomodate multiple entries at the winning positions. Interestingly, this contrasts with MVs such as Approval Voting which cannot deter multiple similar candidacies either but can accomodate them. Consequently, while policy moderation is robust to asymmetry in policy preferences under the Borda Count, it is not under Approval Voting.

[22] Observe that for a cost of running δ sufficiently small, the median citizen's ideal policy is the unique serious equilibrium outcome. This provides a theoretical underpinning for the claim sometimes made that the Borda Count elects the Condorcet winner more often than the other commonly-discussed voting rules.

3.3. Comparing MVs and OVs

Let us now compare policy moderation under the different MVs and OVs. We know from Proposition 1 that when policy preferences are symmetric, the extent of policy moderation is identical under any MV that gives each citizen enough votes to cast and that allows voters to truncate their ballots.[23] On the other hand, if any of those two conditions is not satisfied and/or policy preferences are asymmetric, then we may get policy extremism instead. At the same time, we know that OVs always yield policy moderation and that the extent of policy moderation is more substantial if (1) the second-place score in a three-way race s is bigger, (2) truncated ballots are admissible when the OV is best-rewarding, and (3) only completely-filled ballots are admissible when the OV is worst-punishing. Moreover, the serious equilibrium set under a worst-punishing OV with only completely-filled ballots admissible contains only one-position equilibria which are the most moderate equilibria.

We then have:

PROPOSITION 4. *Let V be a MV with $q > \overline{q}$, where $\overline{q}$ is defined as in Proposition 1. Suppose that policy preferences are symmetric and that V admits truncated ballots. Then,*

$$WPOV_C \prec V \prec WPOV_T \prec BROV_T \prec BROV_C$$

where $Y \prec Z$ reads as "the equilibrium set under Y is a moderate subset of the equilibrium set under Z", WPOV (BROV, resp.) stands for worst-punishing OV (best-rewarding OV, resp.) and the subscript T (C, resp.) for truncated (only completely-filled, resp.) ballots admissible. ‖

To understand why the serious equilibrium set under V is a moderate subset of the serious equilibrium set under an OV with truncated ballots admissible, recall that when truncated ballots are admissible the extent of policy moderation depends on whether moderates can be deterred from entering the race or not. Now a moderate does not want to enter the race if it is relatively too costly to do so and/or his electoral prospects are bleak. While the former does not depend on the voting procedure, the latter does. Indeed, all the citizens for whom the moderate would be a most-preferred candidate must cast a one-point vote for him if the election is held under V, but only a s-point vote if the election is held under an OV. Consequently, if a moderate can be deterred from entering the race under V, so can he be under an OV. However, the reverse does not hold true.

To sum up, in the context of serious races, the extent of policy moderation is maximal under the Borda Count with only completely-filled ballots admissible. This is because this voting procedure is unable to deter and accomodate multiple entries at the winning positions. Observe that this conclusion is robust to asymmetry in policy preferences.

4. POLICY MODERATION IN SPOILER RACES

In this section, we study policy moderation in the context of spoiler races, i.e., elections where in equilibrium some candidates are spoilers. A spoiler is a citizen who enters the race (correctly) anticipating that he has no chance of winning the

[23] One may interpret those two conditions as ensuring that enough information about voters' preferences is captured. I thank Michel Truchon for having suggested this interpretation.

election but that his presence in the race will yield an electoral outcome that he prefers to the outcome that would result otherwise.

The presence of spoilers in the race creates barriers to entry and exit by candidates, thus preventing voting procedures from lowering the barriers to entry by moderates and yielding policy moderation. Intuitively, candidates are deterred from exiting the race and potential candidates are deterred from entering the race if they anticipate a reaction on the part of the voters that will result in the election of less-preferred candidates. By increasing the number of candidates/platforms, the presence of spoilers makes possible such beliefs even when they cannot be supported in the context of serious races. The following example illustrates this for the case of Approval Voting.

Example 4. Consider the community described in Example 1 and suppose that the election is held under Approval Voting. It is easy to check that the serious equilibrium set contains only one-position equilibria. This is no longer true however in the context of spoiler races. For example, $\{1, 9\}$ can be supported by a spoiler equilibrium. To see this, let there be four candidates whose platforms are 0, 1, 9 and 10, respectively. Moreover, let all the citizens at $1, ..., 5$ $(5, ..., 9$, resp.) vote for the candidate at 1 (9, resp.) and all the citizens at 0 (10, resp.) vote for the candidates at 0 and 1 (9 and 10, resp.). Thus, the candidates at 1 and 9 tie for first place and no citizen has an incentive to deviate from his voting strategy. Moreover, neither of the candidates at 0 and 1 (9 and 10, resp.) is willing to exit the race and no other citizen at $0, ..., 5$ $(6, ..., 10$, resp.) is willing to enter the race if they anticipate that all citizens would then vote for the candidate at 9 (1, resp.). □

That the presence of spoilers creates barriers to entry and exit implies that the results derived in the context of serious races do not carry through in the context of spoiler races. To see this in a simple way, let us consider three key voting procedures, namely, PV, the Borda Count with only completely-filled ballots admissible (hereafter BC) and Approval Voting. We have that:

Proposition 5. *Suppose that* (1) *a two-position serious equilibrium exists under PV that implements the two most extreme positions, and* (2) *the utility cost of running for election is not excessively high so that the median citizen would stand for election if a citizen at one of those positions was running unopposed.*[24] *Then, BC yields as extreme equilibrium policy outcomes as PV, and more extreme equilibrium policy outcomes than Approval Voting.* ‖

Proposition 5 contrasts with our previous results on two counts. First, it reverses the result from Proposition 4 that BC yields more moderate equilibrium policy outcomes than Approval Voting. Second, the result in Proposition 5 holds true whether policy preferences are symmetric or not. Consequently, the result that Approval Voting may yield policy extremism compared to PV when policy preferences are asymmetric is not robust to the presence of spoilers in the race.

The key factor driving this result is the difference in restrictions that weak undominance imposes on voting behavior. Under Approval Voting, a voter is required to vote for all his most-preferred candidate(s). No such restriction is imposed under PV and the only restriction under BC is that a voter ranks any most-preferred candidate above any least-preferred candidate. Consequently, multiple entries at the

[24] It is worth mentioning that those conditions are only sufficient and made to simplify the analysis. The community described in Example 1 satisfies those conditions.

winning positions are sufficient to deter a moderate from entering the race under PV and BC, but need not under Approval Voting. (An example can be found in the 'Technical Appendix' of the working paper.)

5. A COMPARISON WITH THE DOWNSIAN FRAMEWORK

The present analysis differs from the standard Downsian analysis of Cox (1987, 1990). In this section, we highlight the differences between the two approaches and discuss their implications for our results. Our assumptions differ from those of Cox on three counts:

(1) Strategic candidacy: We endogenize the set of candidates, while Cox assumes an exogenous set of at least three candidates.

(2) Strategic voting: We assume citizens cast a ballot that maximizes their expected utility anticipating the probability each candidate wins the election, while Cox assumes that citizens vote sincerely.

(3) Policy commitment: We assume that candidates are policy-motivated and, therefore, cannot credibly commit to implementing a policy different from their ideal one, while Cox assumes that candidates are purely office-motivated and can therefore credibly commit to implementing any policy if elected.

These differences in assumptions yield important differences in conclusions. For example, Cox (1990) finds that in the context of MV elections, giving a citizen more votes to cast strengthens the centripetal forces of political competition when only completely-filled ballots are admissible, while allowing citizens to truncate their ballot dampens those forces. On the other hand, our model predicts that in the context of serious races, (1) MVs may yield policy extremism when only completely-filled ballots are admissible and (2) admitting truncated ballots yields policy moderation provided that each citizen has enough votes to cast and policy preferences are symmetric. We also find that the centripetal forces of political competition are dampened by the presence of spoilers in the race. The key factor driving this reversal of Cox's results is the differential incentives for candidate entry under alternative voting procedures. To see this, we shall focus on serious MV elections where only completely-filled ballots are admissible.

Replacing PV with another MV would yield policy extremism if candidates were able to credibly commit to the policy they will implement if elected (keeping unchanged our other assumptions). This is because candidates converge towards the median citizen's ideal policy under PV while they need not under the other MVs. To see this, note that under PV, only one candidate stands on a platform since the presence of a second candidate would run the risk of splitting votes, thereby helping the election of rival candidates. But with only one candidate running on each of at most two platforms, the forces of political competition induce each candidate to move closer to the median citizen's ideal policy since by doing so a candidate can capture some of his rival's votes without losing any of his extreme votes. (For a formal proof, see Dellis and Oak 2007.) On the other hand, there is no such risk of vote splitting if the election is held under another MV. This implies that in equilibrium, several candidates can be running on the same platform. Candidates can therefore be deterred from moving closer to the median citizen's ideal policy if they anticipate that by doing so they will lose some of their extreme votes. In other words, the possibility of multiple similar candidacies neutralizes the centripetal forces of political competition, thus yielding policy extremism compared to PV. (An example illustrating this discussion can be found in the 'Technical Appendix' of the working paper.)[25]

Replacing PV with another MV would yield policy extremism as well if voting was sincere (keeping unchanged our other assumptions). To understand this result, note that when voting is assumed to be sincere, the key argument behind the claim that letting people vote for several candidates yields policy moderation is the so-called squeezing effect of PV, whereby a centrist candidate is 'squeezed' between a left and a right candidates whose presence in the race creates barriers that absorb the extreme votes. The argument then goes that if we keep the set of candidates fixed and increase the number of votes a citizen casts, then some of the extreme votes will eventually overflow those barriers and reach a centrist candidate. Consequently, increasing the number of votes a citizen casts would improve the electoral prospects of the centrists and yield policy moderation. This argument is no longer true however with an endogenous set of candidates. This is because if voting is sincere and the election is held under a q-MV, then in any two-position serious equilibrium, q candidates are standing at each position. Indeed, more than q candidates running on the same platform would split their votes while less than q candidates would let some extreme votes go to the candidates at the other position, thereby helping the election of rival candidates. Now following the entry by a moderate, the loss of centrist votes is more diluted for the left and right candidates the more such candidates there are. Consequently, increasing the number of votes expands the equilibrium set towards the extreme since the loss of centrist votes is bigger when the two platforms are more polarized. (A formal proof can be found in the 'Technical Appendix' of the working paper.)

6. CONCLUSION

This paper studies whether letting citizens vote for several candidates would yield policy moderation compared to Plurality Voting. In contrast to previous contributions, we do so in a setting that takes into account three key features of elections, namely, strategic voting, strategic candidacy and policy-motivation on the part of the candidates. We investigate two ways to let people express their preferences for the different candidates. One way is to give each citizen several equally-weighed votes to cast for the different candidates. The other way is to ask citizens to rank-order the candidates. We call the former voting procedures Multiple Voting rules and the latter Ordinal Voting rules.

Focusing on serious races (i.e., elections where all candidates are serious contenders), we found that Ordinal Voting rules yield policy moderation compared to Plurality Voting. However, the extent of policy moderation depends on the second-place score in a three-way race and the admissibility of truncated ballots. On the other hand, we found that replacing Plurality Voting with a Multiple Voting rule yields policy moderation provided that (1) truncated ballots are admissible, (2) each citizen has enough votes to cast, and (3) policy preferences are symmetric. However, we can get policy extremism instead of policy moderation if any one of those three conditions is not satisfied! These conditions are required for the Multiple Voting rules but not for the Ordinal Voting rules because the former can accomodate multiple similar candidacies while the latter cannot. Observe that Approval Voting is a Multiple Voting rule that satisfies the first two conditions. Consequently, the

[25] Dellis and Oak (2007) present a similar argument for Approval Voting, suggesting that the argument is not specific to the case where only completely-filled ballots are admissible.

existence of a Multiple Voting rule yielding policy moderation compared to Plurality Voting depends on the symmetry of policy preferences. Finally, we found that the presence of spoilers in the race (i.e., citizens who stand for election not to win but because their presence in the race yields an electoral outcome they prefer) creates barriers to entry and exit by candidates, in which case policy moderation need no longer be substantial.

Comparing policy moderation under the different Multiple and Ordinal Voting rules, we found that when all candidates are serious contenders, the extent of policy moderation is maximal under the Borda Count with only completely-filled ballots admissible. However, this result is not robust to the presence of spoilers in the race. Indeed, in the context of spoiler races, the Borda Count may yield more extreme policy outcomes than Approval Voting.

The present analysis has several limitations that deserve further research. First of all, we have assumed that candidates are only policy-motivated. Allowing for a mix of policy- and office-motivation is of interest. However, this extension is not trivial. The complication lies in the discontinuity that comes from the fact that a candidate will no longer be indifferent if another candidate running on the same platform wins the election instead of him. A second shortcoming is the assumption that information is perfect and complete. Relaxing this assumption in the citizen-candidate framework has yet to be done. Moreover, it would be interesting to extend the analysis to multi-dimensional policy spaces and to other electoral systems like, for example, multi-seat or runoff elections.

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APPENDIX

Let us first introduce some extra notation. Define $\mathcal{N}_L \equiv \{\ell \in \mathcal{N} : x_\ell < m\}$ the set of leftists, and $\mathcal{N}_R \equiv \{\ell \in \mathcal{N} : x_\ell > m\}$ the set of rightists. Denote by $\mathcal{M} \equiv \{\ell \in \mathcal{N} : x_\ell = m\}$ the set of citizens who share the same ideal policy as the median citizen. Also, for any non-empty set of candidates $\mathcal{C}$ and every citizen ℓ, let $G^\ell(\mathcal{C}) \equiv \left\{i \in \mathcal{C} : v_i^\ell \geq v_j^\ell \text{ for all } j \in \mathcal{C}\right\}$ be the set of citizen ℓ's most-preferred candidates. Similarly, let $L^\ell(\mathcal{C}) \equiv \left\{i \in \mathcal{C} : v_i^\ell \leq v_j^\ell \text{ for all } j \in \mathcal{C}\right\}$ be the set of citizen ℓ's least-preferred candidates. Finally, let $n_h \equiv \#\{\ell \in \mathcal{N} : x_\ell = x_h\}$ and $c_h \equiv \#\{i \in \mathcal{C} : x_i = x_h\}$ be the numbers of citizens and candidates at x_h, respectively. Also, for simplicity I shall abuse notation and designate an equilibrium as $(\mathcal{C}, \alpha)$ instead of (e, α).

We now state and prove an additional lemma. It shows that when the election is held under an OV or vote truncation is allowed, the one-position serious equilibrium set is moderate compared to the two-position serious equilibrium set.

LEMMA 2. *Let the election be held under V, and suppose that V is either an arbitrary MV with truncated ballots admissible, or an arbitrary OV. Take $(\mathcal{C}_1, \alpha)$ a one-position serious equilibrium, and let x_1 denote the policy outcome. Take $(\mathcal{C}_2, \alpha)$ a two-position serious equilibrium, and let $\{x_L, x_R\}$ be the policy outcome. Then $u^m(x_1) > u^m(x_h)$ for $h = L, R$.* $\|$

Proof of Lemma 2. W.l.o.g. suppose $x_1 \leq m$ and $x_L < x_R$. Observe that in any two-position serious equilibrium we must have $v_L^m = v_R^m$ and $x_L < m < x_R$. Assume by way of contradiction that $u^m(x_h) \geq u^m(x_1)$ for $h = L, R$. Hence $x_1 \leq x_L$. Since $(\mathcal{C}_2, \alpha)$ is an equilibrium, it must be that a candidate standing at x_R is better off running for election. It must then be that $-u^R(x_L) \geq \delta$. At the same time $(\mathcal{C}_1, \alpha)$ an equilibrium implies that this citizen would be worse off entering the race against x_1. This cannot be the case if $x_1 = x_L$ since $(\mathcal{C}_2, \alpha)$ is an equilibrium. Hence, it must be that $x_1 < x_L$. But then $u^m(x_R) > u^m(x_1)$, and our citizen at x_R would win outright if he enters the race. It must then be that $-u^R(x_1) < \delta$, which contradicts $-u^R(x_L) \geq \delta$ and $x_1 < x_L$. **Q.E.D.**

We now turn to proving the propositions stated in the text.[26]

Proof of Proposition 1. Suppose that the election is held under a MV V with $q \in \mathbb{N}$ and that truncated ballots are admissible.

I first show that the serious equilibrium set under V is a subset of the serious equilibrium set under PV. To do so, observe first that Proposition 3 in Dellis (2006) implies that in any serious equilibrium under V either $\mathcal{C} = \{i\}$ with $-v_i^m < \delta$, or $\mathcal{C} = \{i, j\}$ with $x_i < m < x_j$ and $v_i^m = v_j^m$. Moreover, the one-position serious equilibrium set is equivalent under any MV and is moderate compared to the two-position serious equilibrium set (the latter from Lemma 2). Consequently, we only need to show that for any two-position serious equilibrium under V, an equivalent serious equilibrium exists under PV. Pick $(\mathcal{C}, \alpha)$ a two-position serious equilibrium under V with policy outcome $\{x_L, x_R\}$. I am now going to construct $\left(\overline{\mathcal{C}}, \overline{\alpha}\right)$ a two-position serious equilibrium under PV. Suppose that the election is held under PV and construct $\overline{\mathcal{C}} = \{i, j\}$ for some $i, j \in \mathcal{C}$ with $x_i = x_L$ and $x_j = x_R$. Weak undominance, together with $\overline{c} = 2$, implies that in equilibrium, voting is sincere. Moreover, $(\mathcal{C}, \alpha)$ an equilibrium under V implies that $-\frac{c_R}{c\,(c-1)} v_j^i \geq \delta$ and $-\frac{c_L}{c\,(c-1)} v_i^j \geq \delta$; that is, neither candidate i, nor candidate j would be better off not running for election. Now, $\frac{1}{2} \geq \frac{c_h}{c\,(c-1)}$ (for $h = L, R$) and $\overline{c} = 2$ imply that the same holds true under PV when $\overline{\mathcal{C}}$ is the set of candidates. It remains to show that no other citizen wants to run for election. Pick $k \in \left(\mathcal{N} \backslash \overline{\mathcal{C}}\right)$ arbitrarily, and define $\widetilde{\mathcal{C}} \equiv \left(\overline{\mathcal{C}} \cup \{k\}\right)$. Construct $\overline{\alpha}\left(\widetilde{\mathcal{C}}\right)$ such that: (1) $\overline{\alpha}_i^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{N}_L$, (2) $\overline{\alpha}_j^\ell\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{N}_R$, and

[26] For the sake of brevity, some of the details are omitted (they are available from the author). Moreover, we shall maintain two assumptions. One assumption is that the number of citizens on either side of the median citizens' ideal policy—i.e., $\#\mathcal{N}_h$, $h = L, R$—is assumed to be even. The other assumption is that a citizen decides to stand for election when he is indifferent between entering the race or not. Both assumptions are w.l.o.g., and are made only to simplify notation.

(3)

$$\begin{cases} \overline{\alpha}_k^{\ell}\left(\widetilde{\mathcal{C}}\right)=1 & \text{if } x_k \in (x_L, x_R) \\ \overline{\alpha}_h^{\ell}\left(\widetilde{\mathcal{C}}\right)=1 & \text{if } x_k \notin [x_L, x_R]\text{, with } h \in L^k\left(\overline{\mathcal{C}}\right) \\ \overline{\alpha}^{\ell}\left(\widetilde{\mathcal{C}}\right)=(0,0,0) & \text{if } x_k \in \{x_L, x_R\} \end{cases}$$

for all $\ell \in \mathcal{M}$. Hence, $W\left(\widetilde{\mathcal{C}},\overline{\alpha}\right)=\overline{\mathcal{C}}$ if $x_k \in [x_L, x_R]$ and $W\left(\widetilde{\mathcal{C}},\overline{\alpha}\right)=\{h\}$ otherwise. This, together with $\delta > 0$, implies $U^k\left(\widetilde{\mathcal{C}},\overline{\alpha}\right) < U^k\left(\overline{\mathcal{C}},\overline{\alpha}\right)$. Thus, $\left(\overline{\mathcal{C}},\overline{\alpha}\right)$ is a two-position serious equilibrium under PV with policy outcome $\{x_L, x_R\}$. Since we took $(\mathcal{C},\alpha)$ arbitrarily, this holds true for any two-position serious equilibrium under V. Thus, the serious equilibrium set under V is a subset of the serious equilibrium set under PV.

For any x_L,$x_R \in X$, with $v_L^m = v_R^m$ and $n_L, n_R \geq 1$, define $q\left(x_L, x_R\right)$ as the integer that satisfies

$$\frac{1}{2}\left(-1-\frac{v_R^L}{2\delta}\right) < q\left(x_L, x_R\right) \leq \frac{1}{2}\left(1-\frac{v_R^L}{2\delta}\right)$$

and let $\overline{q}$ be the largest $q\left(x_L, x_R\right)$. Hence, $\overline{q}$ is the largest number of candidates who would be willing to run on the same platform in a two-position serious equilibrium.

Suppose that $q > \overline{q}$ and that $u^{\ell}(x) = u\left(|x_{\ell} - x|\right)$ for all citizen ℓ. I am now going to show that the serious equilibrium set under V is a moderate subset of the serious equilibrium set under PV. That the serious equilibrium set under V is a subset of the serious equilibrium set under PV, together with Lemma 2, implies it is sufficient to show that the two-position serious equilibrium set under V is moderate compared to the two-position serious equilibrium set under PV. To show this, pick $(\mathcal{C},\alpha)$ a two-position serious equilibrium under V with policy outcome $\{x_L, x_R\}$. Suppose that a two-position serious equilibrium $\left(\overline{\mathcal{C}},\overline{\alpha}\right)$ with policy outcome $\{\widetilde{x}_L, \widetilde{x}_R\}$ such that $[\widetilde{x}_L, \widetilde{x}_R] \subsetneq [x_L, x_R]$ exists under PV. (Observe that $\overline{c}_L = \overline{c}_R = 1$ and that $\widetilde{x}_L < m < \widetilde{x}_R$ with $\widetilde{v}_L^m = \widetilde{v}_R^m$, where $\widetilde{v}_h^m \equiv u^m\left(\widetilde{x}_h\right)$ for $h = L, R$. We must show that an equivalent serious equilibrium $\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right)$ exists under V. Construct $\widetilde{\mathcal{C}}$ such that (1) $x_i \in \{\widetilde{x}_L, \widetilde{x}_R\}$ for all $i \in \widetilde{\mathcal{C}}$, (2) $\overline{\mathcal{C}} \subseteq \widetilde{\mathcal{C}}$, (3) $-\frac{\widetilde{c}_R}{\widetilde{c}\,(\widetilde{c}-1)}\widetilde{v} \geq \delta$ for all $i \in \widetilde{\mathcal{C}}_L$, where $\widetilde{v} \equiv \widetilde{v}_R^L = \widetilde{v}_L^R$, and a similar condition holds for any $j \in \widetilde{\mathcal{C}}_R$, and (4) there does not exist $i \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ with $x_i = \widetilde{x}_L$ and $-\frac{\widetilde{c}_R}{\widetilde{c}\,(\widetilde{c}+1)}\widetilde{v} \geq \delta$, and a similar condition holds for $j \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ with $x_j = \widetilde{x}_R$. Note that $\widetilde{c}_L = \widetilde{c}_R < q$, the equality by symmetry of preferences and the inequality by $q > \overline{q}$. Moreover, note that for every citizen ℓ either $G^{\ell}\left(\widetilde{\mathcal{C}}\right) = L^{\ell}\left(\widetilde{\mathcal{C}}\right)$, or $L^{\ell}\left(\widetilde{\mathcal{C}}\right) \neq G^{\ell}\left(\widetilde{\mathcal{C}}\right)$ and $\widetilde{\mathcal{C}} = \left(L^{\ell}\left(\widetilde{\mathcal{C}}\right) \vee G^{\ell}\left(\widetilde{\mathcal{C}}\right)\right)$. The same is true for every set of candidates $\left(\widetilde{\mathcal{C}}\backslash\{k\}\right)$, $k \in \widetilde{\mathcal{C}}$. This, together with weak undominance, implies that voting on $\widetilde{\mathcal{C}}$ is sincere. Consequently, $W\left(\widetilde{\mathcal{C}},\alpha\right) = \widetilde{\mathcal{C}}$ since $\widetilde{v}_L^m = \widetilde{v}_R^m$. This, together with condition (3), implies that neither candidate in $\widetilde{\mathcal{C}}$ would be better off not running for election under V. It remains to show that there exists a profile of voting strategies such that no other citizen wants to run for election. Pick $k \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ arbitrarily. Condition (4) implies that citizen k does not want to run for election if $x_k \in \{\widetilde{x}_L, \widetilde{x}_R\}$. Suppose instead that $x_k < \widetilde{x}_L$. Construct

$\alpha\left(\widetilde{\mathcal{C}}\cup\{k\}\right)$ such that

$$\begin{cases} \alpha_i^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)=1 & \text{for all } \ell\in\mathcal{N}_L \text{ and } i\in\widetilde{\mathcal{C}}_L \\ \alpha_k^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)=1 & \text{for all } \ell\in\mathcal{N} \text{ with } k\in G^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right) \\ \alpha_j^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)=1 & \text{for all } \ell\in(\mathcal{M}\cup\mathcal{N}_R) \text{ and } j\in\widetilde{\mathcal{C}}_R \end{cases}$$

and $\alpha_h^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)=0$ otherwise. Hence $W\left(\widetilde{\mathcal{C}}\cup\{k\},\alpha\right)=\widetilde{\mathcal{C}}_R$, and $U^k\left(\widetilde{\mathcal{C}}\cup\{k\},\alpha\right)<U^k\left(\widetilde{\mathcal{C}},\alpha\right)$. Construct a similar profile of voting strategies for $x_k>\widetilde{x}_R$. Finally, suppose $x_k\in(\widetilde{x}_L,\widetilde{x}_R)$. Given that $[\widetilde{x}_L,\widetilde{x}_R]\subsetneq[x_L,x_R]$ and $(\mathcal{C},\alpha)$ is a two-position serious equilibrium under V, it must be that $-\left(\frac{v_L^k+v_R^k}{2}\right)<\delta$ and/or there exists α such that $\pi_k(\mathcal{C}\cup\{k\},\alpha)=\gamma^k<\left(\frac{N-M}{2}-1\right)$, where $\gamma^k\equiv\#\left\{\ell\in\mathcal{N}:k\in G^\ell(\mathcal{C}\cup\{k\})\right\}$ and $\pi_k(.)$ denotes candidate k's vote total. Since $x_k>\widetilde{x}_L>x_L$ and $u^k(.)$ single-peaked, we have that $\widetilde{v}_L^k>v_L^k$. Similarly, $\widetilde{v}_R^k>v_R^k$. Hence $-\left(\frac{\widetilde{v}_L^k+\widetilde{v}_R^k}{2}\right)<-\left(\frac{v_L^k+v_R^k}{2}\right)$. Moreover, $\widetilde{\gamma}^k\leq\gamma^k$, where $\widetilde{\gamma}^k\equiv\#\left\{\ell\in\mathcal{N}:k\in G^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)\right\}$. To see this, take a citizen ℓ such that $k\in G^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)$. Then $x_\ell\in(\widetilde{x}_L,\widetilde{x}_R)$, and $\widetilde{v}_h^\ell>v_h^\ell$ (for $h=L,R$). Consequently, $\max\left\{\widetilde{v}_L^\ell,\widetilde{v}_R^\ell\right\}>\max\left\{v_L^\ell,v_R^\ell\right\}$. But $k\in G^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)$ implies that $v_k^\ell\geq\max\left\{\widetilde{v}_L^\ell,\widetilde{v}_R^\ell\right\}$, and thus $v_k^\ell>\max\left\{v_L^\ell,v_R^\ell\right\}$. Hence $G^\ell(\mathcal{C}\cup\{k\})=\{k\}$. Since we took ℓ arbitrarily, this holds true for any citizen ℓ for whom $k\in G^\ell\left(\widetilde{\mathcal{C}}\cup\{k\}\right)$, and thus $\widetilde{\gamma}^k\leq\gamma^k$. To sum up, $-\left(\frac{\widetilde{v}_L^k+\widetilde{v}_R^k}{2}\right)<\delta$ and/or $\widetilde{\gamma}^k<\left(\frac{N-M}{2}-1\right)$, implying that there exists α such that $U^k\left(\widetilde{\mathcal{C}}\cup\{k\},\alpha\right)<U^k(\mathcal{C}\cup\{k\},\alpha)$. Since $U^k(\mathcal{C}\cup\{k\},\alpha)\leq U^k(\mathcal{C},\alpha)$—$(\mathcal{C},\alpha)$ an equilibrium implies that citizen k does not want to enter the race when the set of candidates is $\mathcal{C}$—and $U^k(\mathcal{C},\alpha)<U^k\left(\widetilde{\mathcal{C}},\alpha\right)$—since $\widetilde{v}_h^k>v_h^k$ (for $h=L,R$) while $c_L=c_R$ and $\widetilde{c}_L=\widetilde{c}_R$, the latter by symmetry of preferences—, we have $U^k\left(\widetilde{\mathcal{C}}\cup\{k\},\alpha\right)<U^k\left(\widetilde{\mathcal{C}},\alpha\right)$.

Finally, pick α an equilibrium profile of voting strategies for any other set of candidates. Hence, $\left(\widetilde{\mathcal{C}},\alpha\right)$ is a two-position serious equilibrium under V with policy outcome $\{\widetilde{x}_L,\widetilde{x}_R\}$. This, together with $\widetilde{c}_L=\widetilde{c}_R$, implies $\left(\widetilde{\mathcal{C}},\alpha\right)$ is equivalent to $(\overline{\mathcal{C}},\overline{\alpha})$. **Q.E.D.**

Proof of Proposition 2. Let V and $\widetilde{V}$ be two OVs and denote by s and $\widetilde{s}$ their respective second-place scores in a three-way race. W.l.o.g. suppose that $s>\widetilde{s}$. We shall start by making a couple of observations. First, under an OV a citizen has a unique top-score vote to cast (i.e., $s_1\neq s_2$). This, together with Proposition 1 in Dellis (2006), implies that the equilibrium set contains only one- and two-position equilibria. Moreover, the one-position serious equilibrium set is equivalent under any OV. Finally, Lemma 2 shows that under an OV, the one-position serious equilibrium set is moderate compared to the two-position serious equilibrium set. To prove the result, it is therefore sufficient to show that the two-position serious equilibrium set under V is a subset of the two-position serious equilibrium set under $\widetilde{V}$ and is moderate in the sense that if a serious equilibrium with policy outcome $\{x_L,x_R\}$ exists under V, then for every serious equilibrium under $\widetilde{V}$ with policy outcome $\{\widetilde{x}_L, \widetilde{x}_R\}$, with $[\widetilde{x}_L, \widetilde{x}_R] \subsetneq [x_L, x_R]$, an equivalent serious equilibrium exists under V. To do so, we shall distinguish two cases.

Case 1: Only completely-filled ballots are admissible. Before proving the result, we need to characterize the two-position serious equilibrium sets under the different OVs. First, it must be that $\mathcal{C} = \{L, R\}$ with $x_L < m < x_R$ and $v_L^m = v_R^m$. Moreover, equilibrium voting strategies must be sincere. Also, it must be that: (1) neither of the two candidates would be better off not running for election; and (2) no other citizen would be better off entering the race. Necessary and sufficient conditions for the former to hold are $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$. The necessary and sufficient conditions for the latter to hold are given in the next three claims. The first one considers the citizens whose ideal policies are x_L and x_R.

Claim 2.1. *Let the election be held under an OV. There exists an equilibrium profile of voting strategies such that* $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$ for any $k \in (\mathcal{N} \backslash \mathcal{C})$ *with* $x_k \in \{x_L, x_R\}$ *if, and only if,* $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$. ∥

Proof of Claim 2.1. W.l.o.g. take $k \in (\mathcal{N} \backslash \mathcal{C})$ with $x_k = x_L$, and let $\widetilde{\mathcal{C}} \equiv \{k, L, R\}$. Observe that Lemma 3 in Dellis (2006) implies that in any voting equilibrium, $W\left(\widetilde{\mathcal{C}}, \alpha\right) \subsetneq \widetilde{\mathcal{C}}$.

(Necessity) Assume by way of contradiction that $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$ and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$ for some equilibrium profile of voting strategies. First note that $-\frac{v_R^L}{2} \geq \delta$ (i.e., citizen L is willing to stand for election) and $v_R^k = v_R^L$ (since $x_k = x_L$) imply $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$ only if $R \in W\left(\widetilde{\mathcal{C}}, \alpha\right)$, i.e., $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ or $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{h, R\}$ for some $h \in \{k, L\}$. Note also that $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{k, L\}$ ($\{R\}$, resp.) and $L^\ell\left(\widetilde{\mathcal{C}}\right) = \{R\}$ ($\{k, L\}$, resp.) for all $\ell \in \mathcal{N}_L$ ($\mathcal{N}_R$, resp.), and $L^\ell\left(\widetilde{\mathcal{C}}\right) = G^\ell\left(\widetilde{\mathcal{C}}\right)$ for all $\ell \in \mathcal{M}$. Hence, in any voting equilibrium the citizens at the median abstain, while the others vote sincerely. This means that $\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) = \frac{N-M}{2}$ and $\pi_i\left(\widetilde{\mathcal{C}}, \alpha\right) \geq \left(\frac{N-M}{2}\right)\left(\frac{1}{2} + s\right)$ for some $i \in \{k, L\}$. It follows that $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ only if $\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) > \left[\pi_i\left(\widetilde{\mathcal{C}}, \alpha\right) + (1 - s)\right]$ for all $i \in \{k, L\}$, which cannot be true if $s \geq \frac{1}{2}\left(\frac{N-M-4}{N-M-2}\right)$. And for $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{h, R\}$ it must be that for $i \in \{k, L\}$, $i \neq h$, $\alpha_i^\ell\left(\widetilde{\mathcal{C}}\right) = s$ for all $\ell \in (\mathcal{N} \backslash \mathcal{M})$—otherwise some voters would be better off permuting their ranking of the two left candidates—, and $\pi_h\left(\widetilde{\mathcal{C}}, \alpha\right) = \pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) = \frac{N-M}{2}$ and $\pi_i\left(\widetilde{\mathcal{C}}, \alpha\right) = (N - M)\,s$. This, together with $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$, implies that $\pi_i\left(\widetilde{\mathcal{C}}, \alpha\right) + (1 - s) > \pi_R\left(\widetilde{\mathcal{C}}, \alpha\right)$. A leftist would then be better off permuting his ranking of the two left candidates. Hence, in any voting equilibrium, $R \notin W\left(\widetilde{\mathcal{C}}, \alpha\right)$, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) \geq U^k(\mathcal{C}, \alpha)$ a contradiction.

(Sufficiency) Suppose that $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ such that

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (s, 1, 0) & \text{for all } \ell \in \mathcal{N}_L \\ (s, 0, 1) & \text{for all } \ell \in \mathcal{N}_R \\ (0, 0, 0) & \text{for all } \ell \in \mathcal{M} \end{cases}$$

where $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \equiv \left(\alpha_k^\ell, \alpha_L^\ell, \alpha_R^\ell\right)$. Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = W(\mathcal{C}, \alpha)$ and neither citizen is willing to deviate from his voting strategy. This, together with $\delta > 0$, implies $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$. ■

The next claim considers the entry by extremists.

Claim 2.2. *Let the election be held under an OV with* $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$. *Then for any citizen* k *with* $x_k \notin [x_L, x_R]$, *there exists an equilibrium profile of voting strategies such that* $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$. $\|$

Proof of Claim 2.2. Pick a citizen k with $x_k \notin [x_L, x_R]$, and let $\widetilde{\mathcal{C}} \equiv \{k, L, R\}$. W.l.o.g. suppose $x_k < x_L$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (s,1,0) & \text{for all } \ell \in \mathcal{N}_L,\ \ell \neq k \\ (1,s,0) & \text{for } \ell = k \\ (0,s,1) & \text{for all } \ell \in \mathcal{M} \\ (s,0,1) & \text{for all } \ell \in \mathcal{N}_R \end{cases}$$

where $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \equiv \left(\alpha_k^\ell, \alpha_L^\ell, \alpha_R^\ell\right)$. It is easy to check that $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ and that $\alpha\left(\widetilde{\mathcal{C}}\right)$ is a voting equilibrium. It follows that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$. ■

It remains to consider the entry by moderates. Before doing so, let us introduce some extra notation. For a candidate k define $\Gamma^k(\mathcal{C}) \equiv \left\{\ell \in \mathcal{N} : k \in G^\ell(\mathcal{C})\right\}$ the set of citizens for whom k is a most-preferred candidate, and $\gamma^k(\mathcal{C}) \equiv \#\Gamma^k(\mathcal{C})$ the number of such citizens. Similarly, define $\Gamma_h^k(\mathcal{C}) \equiv \left\{\ell \in \mathcal{N}_h : k \in G^\ell(\mathcal{C})\right\}$ and $\gamma_h^k(\mathcal{C}) \equiv \#\Gamma_h^k(\mathcal{C})$ for $h = L, R$.[27] The next claim provides necessary and sufficient conditions for the existence of an equilibrium profile of voting strategies such that no moderate is willing to enter the race.[28]

Claim 2.3. *Let* $\mathcal{C} = \{L, R\}$ *with* $x_L < m < x_R$ *and* $v_L^m = v_R^m$. *Suppose that the election is held under* an OV *with* $s \in \left(0, \frac{1}{2}\frac{N-M-2}{N-M-1}\right]$. *Let* k *be a citizen with* $x_k \in (x_L, m]$. *There exists an equilibrium profile of voting strategies* $\alpha(.)$ *such that* $U^k(\mathcal{C} \cup \{k\}, \alpha) < U^k(\mathcal{C}, \alpha)$:
(1) *if* M *is even and* $s \leq \frac{N-3M-2}{2N-3M-2}$, *with the latter inequality strict if there exists a citizen* ℓ *with* $x_\ell \neq m$ *and* $G^\ell(\mathcal{C} \cup \{k\}) = \{k\}$;
(2) *if* $\left(\gamma_L^k - \gamma_R^k\right) \geq (M-1)$, where $\gamma_h^k \equiv \gamma_h^k(\mathcal{C} \cup \{k\})$ for $h = L, R$;
(3) *and otherwise only if at least one of the following conditions hold: (i)* $-\left(\frac{v_L^k + v_R^k}{2}\right) < \delta$; *(ii)* $\frac{N-3M-2}{2} > s_R$ *and* $\left(\frac{v_L^k - v_R^k}{2}\right) < \delta$; *or (iii)* $\frac{N-3M-2}{2} > s_L$, *where* $s_h \equiv \left(2\gamma_h^k + \gamma_i^k - \frac{N+M-2}{2}\right) s$ *for* $h, i \in \{L, R\}$, $h \neq i$. *Moreover condition (i) is sufficient. Condition (iii) is sufficient if in addition one of the following hold: (a)* $\Gamma_R^k = \emptyset$; *(b)* $\frac{N-3M-4}{2} > \left(2\gamma_L^k + \gamma_R^k - \frac{N+M+2}{2}\right) s$; *or (c)* $\frac{1-s}{s} \geq \left(M + \gamma_R^k - \gamma_L^k\right)$ *and* $\frac{N-3M-2}{2} > \left(\gamma_L^k + 2\gamma_R^k - \frac{N-M+2}{2}\right) s$. *Similar conditions can be formulated for condition (ii) if* $(M-1) > \left(\gamma_R^k - \gamma_L^k\right)$.[29]

[27] In order to simplify notation I shall omit $\mathcal{C}$ whenever it is possible to do so without causing a confusion.

[28] While Claim 2.3 does not consider PV (i.e., $s = 0$) it is worth mentioning that $M \leq \frac{N-4}{3}$ is a sufficient condition in that case.

[29] Note that the set of sufficient conditions stated here is not tight. However, if neither of those conditions holds, then in any other set of sufficient conditions either condition (b) must hold with

Similar conditions can be formulated for the case where $x_k \in (m, x_R)$. $\|$

Proof of Claim 2.3. Pick a citizen k with $x_k \in (x_L, m]$. Define $\widetilde{\mathcal{C}} \equiv (\mathcal{C} \cup \{k\})$. Let $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \equiv \left(\alpha_L^\ell, \alpha_k^\ell, \alpha_R^\ell\right)$ denote citizen ℓ's voting strategy.

(1) Suppose that M is even and $s \leq \frac{N-3M-2}{2N-3M-2}$, with a strict inequality if there exists a citizen ℓ with $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{k\}$ and $x_\ell \neq m$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (1, s, 0) & \text{for all } \ell \in \mathcal{N}_L \\ (0, s, 1) & \text{for all } \ell \in \mathcal{N}_R \end{cases}$$

and $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) \in \{(s, 1, 0), (0, 1, s)\}$ for all $\ell \in \mathcal{M}$ with $\sum_{\ell \in \mathcal{M}} \alpha_h^\ell\left(\widetilde{\mathcal{C}}\right) = \frac{M}{2}s$ (for $h = L, R$). Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \mathcal{C}$ with $\alpha\left(\widetilde{\mathcal{C}}\right)$ an equilibrium profile of voting strategies given $\widetilde{\mathcal{C}}$, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$.

Suppose from now on that at least one of the conditions in (1) does not hold. It follows that in any voting equilibrium $\#W\left(\widetilde{\mathcal{C}}, \alpha\right) = 1$ (the proof is available from the author).

(2) Suppose $\left(\gamma_L^k - \gamma_R^k\right) \geq (M-1)$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (1, 0, s) & \text{for all } \ell \in \mathcal{N} \text{ with } G^\ell\left(\widetilde{\mathcal{C}}\right) = \{L\} \\ (1, s, 0) & \text{for all } \ell \in \Gamma_L^k \\ (0, 1, s) & \text{for all } \ell \in \mathcal{M} \\ (0, s, 1) & \text{for all } \ell \in \Gamma_R^k \end{cases}$$

and let $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = (0, s, 1)$ for $\left(\gamma_L^k - \gamma_R^k - M + 2\right)$ of the citizens for whom $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{R\}$ and $(s, 0, 1)$ for the other ones. Note that $\left[\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) - \pi_L\left(\widetilde{\mathcal{C}}, \alpha\right)\right] = 2s$ and $\left[\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) - \pi_k\left(\widetilde{\mathcal{C}}, \alpha\right)\right] > (1+s)$, the latter following from $\left(3\gamma_L^k + 3 - \frac{N+3M}{2}\right) < \frac{N-3M-2}{2}$ (the proof is available from the author). Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ with $\alpha\left(\widetilde{\mathcal{C}}\right)$ an equilibrium profile of voting strategies, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k(\mathcal{C}, \alpha)$ since $R \in L^k(\mathcal{C})$ and $\delta > 0$.

(3) Suppose $\left(\gamma_L^k - \gamma_R^k\right) < (M-1)$. I first prove the necessity part. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as above but with $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = (s, 0, 1)$ for all citizen ℓ for whom $G^\ell\left(\widetilde{\mathcal{C}}\right) = \{R\}$. Hence, $\alpha\left(\widetilde{\mathcal{C}}\right)$ is the profile of weakly undominated voting strategies that maximizes $\left[\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) - \pi_k\left(\widetilde{\mathcal{C}}, \alpha\right)\right]$. If condition (iii) is not satisfied this difference is lower than $(1+s)$, and either k wins outright or citizen L wants to deviate and cast a ballot $(s, 1, 0)$; that is, $W\left(\widetilde{\mathcal{C}}, \alpha\right) \in \{\{k\}, \{L\}\}$ in any voting equilibrium. Now, suppose that condition (ii) does not hold either. Either $\left(\frac{v_L^k - v_R^k}{2}\right) \geq \delta$, in which

an equality, or condition (c) must hold with the second inequality replaced with an equality (the proof is available from the author). In any case, the proof of Proposition 2 below would still hold if we include those conditions.

case condition (i) does not hold and $U^k\left(\widetilde{\mathcal{C}},\alpha\right) \geq U^k(\mathcal{C},\alpha)$. Or $s_R \geq \frac{N-3M-2}{2}$, in which case it cannot be that candidate L wins outright. To see why, construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as above, replacing $\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = (0,1,s)$ with $(s,1,0)$ for all $\ell \in \mathcal{M}$. Hence, $\alpha\left(\widetilde{\mathcal{C}}\right)$ is the profile of weakly undominated voting strategies that maximizes $\left[\pi_L\left(\widetilde{\mathcal{C}},\alpha\right) - \pi_k\left(\widetilde{\mathcal{C}},\alpha\right)\right]$. But this difference is smaller than $(1+s)$ (given the assumption on s_R), and either k wins outright or citizen R wants to deviate and cast a ballot $(0,1,s)$ since then candidate k would either tie with or defeat candidate L. Hence, if neither condition (ii), nor condition (iii) holds, then in any voting equilibrium $W\left(\widetilde{\mathcal{C}},\alpha\right) = \{k\}$. And if condition (i) does not hold either, then $U^k\left(\widetilde{\mathcal{C}},\alpha\right) \geq U^k(\mathcal{C},\alpha)$.

Condition (i) implies that in any voting equilibrium $U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k(\mathcal{C},\alpha)$, and is thus sufficient. Condition (iii) is sufficient as well if the additional conditions stated hold since no $\ell \in \mathcal{N}_R$ for whom $G^\ell(\mathcal{C}) = \{k\}$ wants to deviate from his voting strategy, either because there is no such citizen—(a)—, or he is not pivotal for candidate k—(b)—, or candidate L would be winning—(c). Similarly for condition (ii) if $(M-1) > \left(\gamma_R^k - \gamma_L^k\right)$, the latter condition for candidate L to defeat candidate R.[30] ■

We are now ready to prove Proposition 2 for the case where only completely-filled ballots are admissible. From Claim 2.1 we know that if $s > \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$ the set of two-position serious equilibria under V is empty, and the result holds trivially. Suppose from now on that $s \leq \frac{1}{2}\left(\frac{N-M-2}{N-M-1}\right)$.

Let us first show that the two-position serious equilibrium set under V is a subset of the two-position serious equilibrium set under $\widetilde{V}$. Take $(\mathcal{C},\alpha)$ a two-position serious equilibrium under V, and call L and R the two candidates. I am going to construct $\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right)$ an equivalent serious equilibrium under $\widetilde{V}$. Let $\widetilde{\mathcal{C}} \equiv \mathcal{C}$ and $\widetilde{\alpha}\left(\widetilde{\mathcal{C}}\right) \equiv \alpha(\mathcal{C})$. Since $(\mathcal{C},\alpha)$ is a two-position serious equilibrium under V, it must be that neither candidate would be better off not running; that is, $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$. Pick $k \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ arbitrarily, and let $\overline{\mathcal{C}} \equiv \{L,R,k\}$. We need to construct $\widetilde{\alpha}\left(\overline{\mathcal{C}}\right)$ an equilibrium profile of voting strategies such that $U^k\left(\overline{\mathcal{C}},\widetilde{\alpha}\right) < U^k\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right)$. Claims 2.1 and 2.2, together with $s > \widetilde{s}$, imply that such an equilibrium profile of voting strategies exists if $x_k \notin (x_L, x_R)$. Suppose instead that $x_k \in (x_L, x_R)$. Since $(\mathcal{C},\alpha)$ is a serious equilibrium under V, there must exist an equilibrium profile of voting strategies $\alpha\left(\overline{\mathcal{C}}\right)$ such that $U^k\left(\overline{\mathcal{C}},\alpha\right) < U^k\left(\widetilde{\mathcal{C}},\alpha\right)$. And given that $\widetilde{s} < s$ and $M \leq \frac{N-4}{3}$ the same holds true under $\widetilde{V}$. Hence, for any $k \in \left(\mathcal{N}\backslash\widetilde{\mathcal{C}}\right)$ there exists $\widetilde{\alpha}\left(\overline{\mathcal{C}}\right)$ such that $U^k\left(\overline{\mathcal{C}},\widetilde{\alpha}\right) < U^k\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right)$. Finally, for any other set of candidates take any equilibrium profile of voting strategies. Hence, $\left(\widetilde{\mathcal{C}},\widetilde{\alpha}\right)$ is a two-position serious equilibrium under $\widetilde{V}$ and is equivalent to $(\mathcal{C},\alpha)$.

Let us now show that the two-position serious equilibrium set under V is mod-

[30] Note that this condition is necessarily satisfied if condition (iii) does not hold.

erate compared to the two-position serious equilibrium set under $\widetilde{V}$. To do so, it is sufficient to show that the two-position serious equilibrium set under any OV is moderate compared to the two-position serious equilibrium set under PV. Let V be an OV and suppose $(\mathcal{C},\alpha)$ a two-position serious equilibrium under V with policy outcome $\{x_L, x_R\}$. Assume that a two-position serious equilibrium $\left(\widehat{\mathcal{C}},\widehat{\alpha}\right)$ exists under PV, with policy outcome $\{\widehat{x}_L, \widehat{x}_R\}$ such that $[\widehat{x}_L, \widehat{x}_R] \subsetneq [x_L, x_R]$. I am going to construct $\left(\widehat{\mathcal{C}},\alpha\right)$ an equivalent serious equilibrium under V. Let $\alpha\left(\widehat{\mathcal{C}}\right) \equiv \widehat{\alpha}\left(\widehat{\mathcal{C}}\right)$. Since $\left(\widehat{\mathcal{C}},\widehat{\alpha}\right)$ is an equilibrium, it must be that neither candidate would be better off not running that is, $-\frac{\widehat{v}_R^L}{2} \geq \delta$ and $-\frac{\widehat{v}_L^R}{2} \geq \delta$ where $\widehat{v}_R^L = u(\widehat{x}_L - \widehat{x}_R)$ and $\widehat{v}_L^R = u(\widehat{x}_R - \widehat{x}_L)$.

Pick $k \in \left(\mathcal{N}\backslash\widehat{\mathcal{C}}\right)$ arbitrarily, and define $\widetilde{\mathcal{C}} \equiv \left(\widehat{\mathcal{C}} \cup \{k\}\right)$. Claims 2.1 and 2.2 imply that if $x_k \notin (\widehat{x}_L, \widehat{x}_R)$, then a voting equilibrium $\alpha\left(\widetilde{\mathcal{C}}\right)$ exists such that $U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k\left(\widehat{\mathcal{C}},\alpha\right)$. Suppose instead that $x_k \in (\widehat{x}_L, \widehat{x}_R)$. W.l.o.g. let $x_k \in (\widehat{x}_L, m]$. Since $(\mathcal{C},\alpha)$ is a two-position serious equilibrium under V, there must exist an equilibrium profile of voting strategies $\alpha(\mathcal{C}\cup\{k\})$ such that $U^k(\mathcal{C}\cup\{k\},\alpha) < U^k(\mathcal{C},\alpha)$. It can then be shown that the same holds true under $\widehat{\mathcal{C}}$ (the proof is available from the author). Hence, for any $k \in \left(\mathcal{N}\backslash\widehat{\mathcal{C}}\right)$ an equilibrium profile of voting strategies exists such that $U^k\left(\widetilde{\mathcal{C}},\alpha\right) < U^k\left(\widehat{\mathcal{C}},\alpha\right)$.

Finally, for any other set of candidates take any equilibrium profile of voting strategies.

Hence, $\left(\widehat{\mathcal{C}},\alpha\right)$ is a two-position serious equilibrium under V which is equivalent to $\left(\widehat{\mathcal{C}},\widehat{\alpha}\right)$.

Case 2: Truncated ballots are admissible. The characterization of the two-position serious equilibrium set is identical to the one where only completely-filled ballots are admissible, except that Claims 2.1-2.3 are replaced with the following claim.

Claim 2.4. *Let the election be held under an OV, and suppose that citizens are allowed to truncate their ballot. Then, an equilibrium profile of voting strategies exists such that* $U^k(\mathcal{C}\cup\{k\},\alpha) < U^k(\mathcal{C},\alpha)$ for any $k \in (\mathcal{N}\backslash\mathcal{C})$ *with* $x_k \notin (x_L, x_R)$. *The same holds true for any citizen* k *with* $x_k \in (x_L, x_R)$ *if, and only if, at least one of the following two conditions holds:* (1) $-\left(\frac{v_L^k + v_R^k}{2}\right) < \delta$; *or* (2) $\left(\gamma_L^k + \gamma_R^k\right)s \leq \frac{N-3M-2}{2}$. $\|$

Proof of Claim 2.4. Let $\mathcal{C} = \{L, R\}$ be a set of candidates with $x_L < m < x_R$ and $v_L^m = v_R^m$. Pick $k \in (\mathcal{N}\backslash\mathcal{C})$ arbitrarily, and define $\widetilde{\mathcal{C}} \equiv \{k, L, R\}$.

First consider the case where $x_k \notin [x_L, x_R]$. W.l.o.g. suppose $x_k < x_L$. Construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows:

$$\alpha^\ell\left(\widetilde{\mathcal{C}}\right) = \begin{cases} (s,1,0) & \text{for all } \ell \in \mathcal{N}_L \text{ with } k \in G^\ell\left(\widetilde{\mathcal{C}}\right) \\ (0,1,0) & \text{for all } \ell \in \mathcal{N}_L \text{ with } k \notin G^\ell\left(\widetilde{\mathcal{C}}\right) \\ (0,s,1) & \text{for all } \ell \in \mathcal{M} \\ (0,0,1) & \text{for all } \ell \in \mathcal{N}_R \end{cases}$$

where $\alpha^{\ell}\left(\widetilde{\mathcal{C}}\right) = \left(\alpha_k^{\ell}, \alpha_L^{\ell}, \alpha_R^{\ell}\right)$. Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$ with $\alpha\left(\widetilde{\mathcal{C}}\right)$ an equilibrium profile of voting strategies. This means that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\mathcal{C}, \alpha\right)$.

Now consider the case where $x_k \in \{x_L, x_R\}$. W.l.o.g. suppose $x_k = x_L$. First note that in any voting equilibrium, $\alpha^{\ell}\left(\widetilde{\mathcal{C}}\right) = (0,0,0)$ for all $\ell \in \mathcal{M}$ since $G^{\ell}\left(\widetilde{\mathcal{C}}\right) = L^{\ell}\left(\widetilde{\mathcal{C}}\right)$. Moreover, $\alpha_R^{\ell}\left(\widetilde{\mathcal{C}}\right) = 1$ and $\alpha_k^{\ell}\left(\widetilde{\mathcal{C}}\right) = \alpha_L^{\ell}\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in \mathcal{N}_R$ since $G^{\ell}\left(\widetilde{\mathcal{C}}\right) = \{R\}$ and $L^{\ell}\left(\widetilde{\mathcal{C}}\right) = \{k, L\}$. Similarly, $\alpha_h^{\ell}\left(\widetilde{\mathcal{C}}\right) \in \{s, 1\}$ (for $h = k, L$) and $\alpha_R^{\ell}\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in \mathcal{N}_L$. This means that in any voting equilibrium, $\pi_R\left(\widetilde{\mathcal{C}}, \alpha\right) = \frac{N-M}{2}$, while $\pi_h\left(\widetilde{\mathcal{C}}, \alpha\right) \leq \frac{N-M}{2}$ for $h = k, L$, with the inequality strict for at least one of those two candidates. Hence, either $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{R\}$, or $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{h, R\}$ for some $h \in \{k, L\}$, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\mathcal{C}, \alpha\right)$.

It remains to consider the case where $x_k \in (x_L, x_R)$. Let us first prove the necessity part. Assume by way of contradiction that there exists an equilibrium profile of voting strategies $\alpha\left(\widetilde{\mathcal{C}}\right)$ such that $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\mathcal{C}, \alpha\right)$, but that neither of the two conditions hold. Weak undominance implies that $\alpha_L^{\ell}\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in (\mathcal{M} \cup \mathcal{N}_R)$, $\alpha_R^{\ell}\left(\widetilde{\mathcal{C}}\right) = 0$ for all $\ell \in (\mathcal{M} \cup \mathcal{N}_L)$, and $\alpha_k^{\ell}\left(\widetilde{\mathcal{C}}\right) \in \{s, 1\}$ for all citizen ℓ for whom $k \in G^{\ell}\left(\widetilde{\mathcal{C}}\right)$ with $\alpha_h^{\ell}\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{M}$. Hence, in any voting equilibrium $\pi_k\left(\widetilde{\mathcal{C}}, \alpha\right) \geq \left[M + \left(\gamma_L^k + \gamma_R^k\right) s\right]$ and $\pi_h\left(\widetilde{\mathcal{C}}, \alpha\right) \leq \frac{N-M}{2}$, for $h = L, R$. Now, since condition (2) does not hold, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{k\}$ in any voting equilibrium. And since condition (1) does not hold, $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) \geq U^k\left(\mathcal{C}, \alpha\right)$, a contradiction.

Let us now prove the sufficiency part. Suppose condition (1) holds. Then, $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\mathcal{C}, \alpha\right)$ even if $W\left(\widetilde{\mathcal{C}}, \alpha\right) = \{k\}$. Suppose condition (2) holds, and construct $\alpha\left(\widetilde{\mathcal{C}}\right)$ as follows: (i) $\alpha_h^{\ell}\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{N}_h$, for $h = L, R$; (ii) $\alpha_k^{\ell}\left(\widetilde{\mathcal{C}}\right) = 1$ for all $\ell \in \mathcal{M}$; (iii) $\alpha_k^{\ell}\left(\widetilde{\mathcal{C}}\right) = s$ for all $\ell \in (\mathcal{N} \backslash \mathcal{M})$ for whom $k \in G^{\ell}\left(\widetilde{\mathcal{C}}\right)$; and (iv) $\alpha_j^{\ell}\left(\widetilde{\mathcal{C}}\right) = 0$ otherwise. Hence, $W\left(\widetilde{\mathcal{C}}, \alpha\right) = W\left(\mathcal{C}, \alpha\right)$, and $U^k\left(\widetilde{\mathcal{C}}, \alpha\right) < U^k\left(\mathcal{C}, \alpha\right)$. ■

We are now ready to prove Proposition 2 for the case where truncated ballots are admissible. Let V and $\widetilde{V}$ be any two OVs with $s > \widetilde{s}$. Take $(\mathcal{C}, \alpha)$ a two-position serious equilibrium under V, and call L and R the two candidates. We are now going to construct $(\mathcal{C}, \widetilde{\alpha})$ an equivalent serious equilibrium under $\widetilde{V}$. Let $\widetilde{\alpha}(\mathcal{C}) \equiv \alpha(\mathcal{C})$. Given that $\mathcal{C}$ is an equilibrium set of candidates under V, we have $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$—i.e., neither candidate is better off not entering the race. Obviously, those two conditions hold as well under $\widetilde{V}$. Take $k \in (\mathcal{N} \backslash \mathcal{C})$, and let $\overline{\mathcal{C}} \equiv \{k, L, R\}$. By Claim 2.4 we know that if $x_k \notin (x_L, x_R)$, then an equilibrium profile of voting strategies exists such that $U^k\left(\overline{\mathcal{C}}, \widetilde{\alpha}\right) < U^k\left(\mathcal{C}, \widetilde{\alpha}\right)$. Suppose instead that $x_k \in (x_L, x_R)$. Since $(\mathcal{C}, \alpha)$ is an equilibrium under V, condition (1) and/or condition (2) in Claim 2.4 must hold. If condition (1) holds, then we are done since this condition does not depend on s. If condition (2) holds, then so does it under $\widetilde{V}$ since $\widetilde{s} < s$. It follows that for any $k \in (\mathcal{N} \backslash \mathcal{C})$ an equilibrium profile of voting strategies $\widetilde{\alpha}\left(\overline{\mathcal{C}}\right)$ exists such that $U^k\left(\overline{\mathcal{C}}, \widetilde{\alpha}\right) < U^k\left(\mathcal{C}, \widetilde{\alpha}\right)$. For any other set of candidates, take any equilibrium profile of voting strategies. Hence, $(\mathcal{C}, \widetilde{\alpha})$ is a serious equilibrium under $\widetilde{V}$ and is equivalent to $(\mathcal{C}, \alpha)$.

Given that condition (2) in Claim 2.4 holding under $\widetilde{V}$ does not imply that it holds under V as well, we have proved the subset part of the result. The moderation part follows directly from the fact that it is only for moderates that there may not exist an equilibrium profile of voting strategies that deters them from entering the race. **Q.E.D.**

Proof of Proposition 3. Suppose that the election is held under an OV with second-place score in a three-way race s. W.l.o.g. suppose that $s > 0$. By the same argument as in the proof of Proposition 2, we need consider only the two-position serious equilibrium set.

The result is trivial if V is worst-punishing since no two-position serious equilibrium exists when only completely-filled ballots are admissible.

Suppose instead that V is best-rewarding. Take $(\mathcal{C}, \alpha)$ a two-position serious equilibrium when truncated ballots are admissible. We need to show that an equivalent serious equilibrium exists when only completely-filled ballots are admissible. Let $\mathcal{C} = \{L, R\}$. Since $(\mathcal{C}, \alpha)$ is an equilibrium, it must be that $-\frac{v_R^L}{2} \geq \delta$ and $-\frac{v_L^R}{2} \geq \delta$; that is, neither candidate would be better off not running for election. It remains to show that for any $k \in (\mathcal{N} \backslash \mathcal{C})$, an equilibrium profile of voting strategies $\alpha\left(\mathcal{C} \cup \{k\}\right)$ exists such that $U^k\left(\mathcal{C} \cup \{k\}, \alpha\right) < U^k\left(\mathcal{C}, \alpha\right)$. Pick $k \in (\mathcal{N} \backslash \mathcal{C})$ arbitrarily. If $x_k \notin (x_L, x_R)$, then we know from Claims 2.1 and 2.2 that such a profile of voting strategies exists. If $x_k \in (x_L, x_R)$, we know from Claim 2.4 and $(\mathcal{C}, \alpha)$ a serious equilibrium that either $-\left(\frac{v_L^k + v_R^k}{2}\right) < \delta$, or $\left(\gamma_L^k + \gamma_R^k\right) s \leq \frac{N - 3M - 2}{2}$ (or both). If the former, then we are done. Instead, suppose that only the latter holds, and assume w.l.o.g. that $x_k \in (x_L, m]$. First, note that condition (iii) of Claim 2.3 holds. To see this, assume the contrary. Then,

$$\left(2\gamma_L^k + \gamma_R^k - \frac{N + M - 2}{2}\right) s \geq \frac{N - 3M - 2}{2} \geq \left(\gamma_L^k + \gamma_R^k\right) s$$

, and thus $\left[\left(\gamma_L^k - \frac{N - M}{2}\right) + (1 - M)\right] \geq 0$, a contradiction. Moreover, either condition (b) of Claim 2.3 holds, or condition (c) (or both). To see this, suppose the contrary. First note that the second inequality in (c) necessarily holds since otherwise we would have

$$\left(\gamma_L^k + 2\gamma_R^k - \frac{N - M + 2}{2}\right) s \geq \frac{N - 3M - 2}{2} \geq \left(\gamma_L^k + \gamma_R^k\right) s$$

, and thus $\left[\left(\gamma_R^k - \frac{N - M}{2}\right) - 1\right] \geq 0$, a contradiction.

It must then be that $\frac{1 - s}{s} < \left(M + \gamma_R^k - \gamma_L^k\right)$, or $\left(M + \gamma_R^k + 1\right) s > \left(1 + \gamma_L^k s\right)$. Together with condition (b) not holding, we have

$$\left(\gamma_L^k + 2\gamma_R^k - \frac{N - M}{2}\right) s > \frac{N - 3M - 2}{2} \geq \left(\gamma_L^k + \gamma_R^k\right) s$$

, and thus $\left(\gamma_R^k - \frac{N - M}{2}\right) > 0$, a contradiction.

Hence, a serious equilibrium equivalent to $(\mathcal{C}, \alpha)$ exists when only completely-filled ballots are admissible. Now, given that $(\mathcal{C}, \alpha)$ was taken arbitrarily this holds true for any such equilibrium. Thus the subset part of the result. The subset is moderate since the only difference lies in whether the entry by moderates can be deterred. **Q.E.D.**

Proof of Proposition 4. Given Propositions 1-3 and the discussion in the text, we only need to show that the serious equilibrium set under V is a moderate subset of the serious equilibrium set under an OV where truncated ballots are admissible. Take $(\mathcal{C}, \alpha)$ a two-position serious equilibrium under V. Let $\widetilde{V}$ be an OV and suppose that truncated ballots are admissible. We are now going to construct $\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$ a serious equilibrium under $\widetilde{V}$ which is equivalent to $(\mathcal{C}, \alpha)$. Pick $i, j \in \mathcal{C}$ with $x_i = x_L$ and $x_j = x_R$, and construct $\widetilde{\mathcal{C}} = \{i, j\}$. Given $\widetilde{\mathcal{C}}$ there is a unique voting equilibrium under $\widetilde{V}$, with $W\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right) = \widetilde{\mathcal{C}}$. Since $(\mathcal{C}, \alpha)$ is an equilibrium under V and preferences are symmetric, then $c_L = c_R$ and $\frac{-v_j^i}{2(c-1)} \geq \delta$ and $\frac{-v_i^j}{2(c-1)} \geq \delta$—i.e., neither candidate in $\mathcal{C}$ is better off not running for election under V. Given that $c \geq 2$, the same holds true for candidates i and j under $\widetilde{V}$. It remains to show that for any $k \in \left(\mathcal{N} \backslash \widetilde{\mathcal{C}}\right)$ there exists an equilibrium profile of voting strategies $\widetilde{\alpha}\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$ such that $U^k\left(\widetilde{\mathcal{C}} \cup \{k\}, \widetilde{\alpha}\right) < U^k\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$. Pick $k \in \left(\mathcal{N} \backslash \widetilde{\mathcal{C}}\right)$ arbitrarily. If $x_k \notin (x_L, x_R)$, then we are done (by Claim 2.4). Suppose instead that $x_k \in (x_L, x_R)$. Since $(\mathcal{C}, \alpha)$ is a serious equilibrium and that $c_L = c_R < q$, either $-\left(\frac{v_i^k + v_j^k}{2}\right) < \delta$, or $\left(\gamma_L^k + \gamma_R^k\right) < \frac{N-3M-2}{2}$ (or both). But then one of the two conditions of Claim 2.4 holds, and we are done. Now for any $\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$ pick one such $\widetilde{\alpha}\left(\widetilde{\mathcal{C}} \cup \{k\}\right)$, and for any other set of candidates $\widehat{\mathcal{C}}$ let $\widetilde{\alpha}\left(\widehat{\mathcal{C}}\right)$ be any profile of voting strategies. It then follows that $\left(\widetilde{\mathcal{C}}, \widetilde{\alpha}\right)$ is an equilibrium under $\widetilde{V}$, which is equivalent to $(\mathcal{C}, \alpha)$. Hence the subset part of the result.

It is only when there exists a citizen k with $x_k \in (x_L, x_R)$ for whom $\left(\gamma_L^k + \gamma_R^k\right) \geq \frac{N-3M-2}{2} \geq \left(\gamma_L^k + \gamma_R^k\right) s$ that there may not exist a serious equilibrium under V which is equivalent to a two-position serious equilibrium under $\widetilde{V}$. Hence the moderation result since $\left(\gamma_L^k + \gamma_R^k\right)$ increases with the distance between x_L and x_R. **Q.E.D.**

Proof of Proposition 5. For the sake of brevity, the proof is omitted here. It can be found in the 'Technical Appendix' of the working paper.