

# Partially-informed Decision Makers in Games of Communication

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## **Abstract**

We incorporate partially informed decision makers into games of communication through cheap talk. We analyze three different extensive-form games in which the expert and the decision maker (DM) privately observe signals about the state of the world. In game 1, the DM reveals her private signal to the expert before the expert reports to her. In game 2, the DM keeping her signal private while the expert reports to her. In game 3, the DM strategically communicates to the expert first before the expert reports to her. We find that the DM's expected equilibrium payoff is not monotonically increasing in the informativeness of her private signal because the expert may reveal less of his information when facing a better-informed DM. Whether the DM extracts more information from the expert in game 1 or in game 2 depends on the parameters. Allowing the DM to communicate strategically to the expert first does not help her extract more information from the expert.

# 1 Introduction

Standard sender-receiver games<sup>1</sup> usually assume that the information asymmetry between the players is one-sided. That is, the sender has relevant information for making decisions and the receiver, while having the decision making power, does not have private information of his own and relies on the sender for useful information. While this assumption simplifies analysis, it also fails to capture an important aspect in real life communication – that the decision maker is usually partially informed as well. For example, the decision maker may have expertise herself. Imagine a CEO trying to decide on the size of an investment project. Although she needs the division manager to inform her how much the project will benefit his division, she has knowledge about her corporation’s overall investments and the general profitability of the project that the division manager does not have. Alternatively, there may exist other sources of information. When a consumer who is trying to decide how much to spend on a product or service consults a salesperson for recommendations, she may already have information about the candidate products or services from the experience of her friends or from the consumers’ reports.

In these situations, the players’s interests are usually not perfectly aligned and the “expert”<sup>2</sup> has an incentive to report dishonestly. For example, the division manager particularly favors investment in projects that help to expand his division. The salesperson wants to promote products that are expensive but not necessarily best matches for the consumer’s needs. When the decision makers are partially informed as well, how does that affect the expert’s communication incentives? Is the expert more likely to lie to someone who knows little or someone who is well informed? Under what circumstances can the decision maker elicit more information from the expert? And how is the welfare of the players affected? To address these questions, we analyze three extensive-form games (which correspond to three different communication environments) with partially-informed decision makers.

In all three games, the players – the expert and the decision maker (DM) – both privately observe signals about the state of the world and communicate (through costless messages<sup>3</sup>) before the DM chooses an action that affects both players’ payoffs. The games differ in the way the communication is structured. In game 1, the DM reveals her information truthfully to the expert before the expert reports to her. In game 2, the DM keeps her information private while the expert communicates to her. In game 3, the DM cannot commit to revealing her information truthfully to the expert, but before the expert reports to her, she has an opportunity to communicate strategically to the expert.

Several interesting results arise. The decision maker’s expected equilibrium payoff is *not*

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<sup>1</sup>For example, Crawford and Sobel’s (1982) classic model of strategic information transmission.

<sup>2</sup>We use the terminology “experts” and “decision makers” instead of “senders” and “receivers” in this paper because in some games that we study, the decision maker also sends messages.

<sup>3</sup>The messages are “cheap talk” except in game 1 in which the DM tells the truth.

monotonically increasing in the informativeness of her own signal. This happens because an enhancement in the quality of the decision maker's information may make it impossible for the expert to reveal his information credibly in equilibrium. The intuition is most clear in the case of quadratic utility functions. With quadratic utilities, whether or not the expert truthfully reveals his information in equilibrium depends on the expected impact of his messages on the DM's choice of action, relative to the divergence of interest between the players. When the impact is sufficiently large/small, the expert can/cannot credibly reveal his information to the DM in equilibrium. For example, suppose the expert has an upward bias so that his ideal point (i.e., the DM's action that maximizes his payoff) is always higher than the DM's ideal point for every state. The expert with the high signal (high type) wants to convince the DM that his signal is indeed high and the question is whether the low type expert can credibly convey his information to the DM. If the DM's responses to different messages are far apart, then the low type would not want to mimic the high type because the action induced would be too high. However, if the DM's responses to different messages are close, both type of the expert prefer the higher action and the low type cannot credibly communicate his information to the DM. If the DM's own signal is highly informative, then the expert's messages do not change her beliefs very much, and the influence on the DM's actions is small. As we explained above, the expert cannot credibly convey his information in equilibrium. Therefore, the DM's expected payoff may fall as a result of an increase in the quality of her own information if the gain does not adequately compensate for the loss of information transmitted by the expert.

Another somewhat surprising result is that allowing an extra round of communication in which the DM strategically communicates her information to the expert does not help her extract more information than if she keeps her signal private. It is obvious that any equilibrium outcome in game 2 is also an equilibrium outcome in game 3 because there always exists an equilibrium in game 3 in which the DM babbles in the first stage and in effect keeps her information private. Why can't the DM do strictly better in game 3? Observe that since the DM is the only player who takes a payoff-relevant action, the sole function of her communication is to extract information from the expert. If a particular message induces more information revelation from the expert, then the DM would strictly prefer to send this message, independent of the realization of her signal. But this undermines the credibility of her communication. Therefore, the DM's equilibrium messages must induce the expert to reveal the same amount of information and the extra round of cheap talk does not help the DM.

What would the DM do if she could choose between the different communication environments, i.e., what game to play? For example, when facing a salesperson, do you want to reveal to him what knowledge you have about the products before he makes his recommendation? The paper shows that whether the DM prefers to reveal her information or to keep

it private depends on the parameters of the model. Fixing the other parameters and varying the expert's bias, we find that the DM is indifferent between playing game 1 and game 2 when the expert's bias is either sufficiently small or sufficiently large, but prefers one to the other when the bias is in the intermediate range. When the bias is very small, the expert reveals his information truthfully in both games and the DM is indifferent. As the bias increases from a small value to a moderate value, the DM may extract more information from the expert by keeping her signal private than by revealing her signal truthfully to the expert. However, as the bias gets even larger, the DM's preference switches – revealing her information to the expert generates a higher expected payoff. When the bias is sufficiently large, the expert babbles in both games and the DM is indifferent again. A similar result is obtained by varying the informativeness of the DM's signal while keeping other parameters fixed.

Only a few papers in the literature have explicitly modeled informed receivers in communication games. Olszewski (2004) analyzes a model in which the receiver, as well as the sender, has private information on the state of the world and the sender wants to be perceived as honest. The paper provides conditions on the information structure with which the unique equilibrium is full information revelation if the sender's reputational concerns are strong enough. Harris and Raviv (2005) compares delegation and communication when both a CEO and a division manager have private information on the profitability of investment projects. They find that if the division manager's information is sufficiently important relative to the CEO's, then it is optimal for the CEO to delegate the investment decision to the division manager instead of making the decision herself with a report from the manager. Seidmann (1990) gives examples which illustrate that when the receiver's type is private information, the sender may communicate effectively in equilibrium even if the sender's types share a common preference ordering over the receiver's actions. The examples show that the uncertainty created by the receiver's private information helps separation in equilibrium because the *distributions* of the receiver's actions may not be stochastically ordered in equilibrium.

Austen-Smith (1993) does not explicitly model informed receivers in communication games. However, since his paper analyzes multiple referrals under open rule in which the receiver gets information from two different and imperfectly-informed sources, some of his results are similar to what our model generates.<sup>4</sup>

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<sup>4</sup>Krishna and Morgan (2001) also look at communication with multiple experts. However, in their model, both experts are perfectly informed. Even though the DM gets information from both sources, one expert knows (in equilibrium) what information the DM gathers from the other expert. So information asymmetry between the expert and the DM is still one-sided.

## 2 The Model

Suppose there are two players in the game, the expert (player 1) and the decision maker (player 2).<sup>5</sup>

The state of the world,  $\omega$ , is a random variable which takes two values, 0 and 1. The common prior on  $\omega$  is  $\text{prob}\{\omega = 1\} = p$  and  $\text{prob}\{\omega = 0\} = 1 - p$  where  $p \in (0, 1)$ . The expert privately observes a signal  $s_1$  about the state. We assume that  $\text{prob}\{s_1 = x | \omega = x\} = q_1$  and  $\text{prob}\{s_1 = 1 - x | \omega = x\} = 1 - q_1$  for  $x = 0, 1$ . The parameter  $q_1$ , assumed to be in the interval  $(\frac{1}{2}, 1]$ , indicates the informativeness of the expert's signal: when  $q_1 = 1$ , the expert is perfectly informed; when  $q_1 < 1$ , the expert observes the state with noise. In the games that we will analyze, the expert communicates his private information to the DM through a costless message  $m$ . After receiving  $m$ , the DM takes an action  $a$  ( $a \in A = \mathbb{R}$ ) that affects both players' payoffs. Our main departure from the standard models of strategic communication is the assumption that the DM may also have private information on  $\omega$ . Specifically, we assume that she privately observes a signal  $s_2$  with informativeness  $q_2 \in [\frac{1}{2}, 1)$ , i.e.,  $\text{prob}\{s_2 = x | \omega = x\} = q_2$  and  $\text{prob}\{s_2 = 1 - x | \omega = x\} = 1 - q_2$  for  $x = 0, 1$ . Obviously, when  $q_2 = \frac{1}{2}$ , the DM is not informed, as in the standard models.

Both players maximize their expected utilities. Let  $u^{DM}(a, \omega)$  be the DM's von Neumann-Morgenstern utility function and  $u^E(a, \omega, b)$  be the expert's von Neumann-Morgenstern utility function. The parameter  $b$ , sometimes referred to as the expert's "bias," measures the divergence of interests between the two players. We assume that  $u^E(a, \omega, 0) = u^{DM}(a, \omega)$ , i.e., the two parties' interests coincide if the expert's bias equals 0.

To simplify notation, let  $v(a, \omega, b) = u^E(a, \omega, b)$ . So  $v(a, \omega, 0) = u^{DM}(a, \omega)$ . Denoting partial derivatives by subscripts in the usual way, we make the following assumptions: (1)  $v_{11} < 0$ . This implies that the players' preferences are single peaked and for any belief  $\beta$  on the distribution of  $\omega$ ,  $E(v(\cdot) | \beta)$  has a unique maximizer in  $a$ . (2)  $v_{13} \geq 0$ . Without loss of generality, we assume  $b \geq 0$ . This implies that for both  $\omega = 0$  and  $\omega = 1$ , the optimal action for the expert is at least as high as the optimal action for the decision maker. (3)  $v_1(a, 1, b) > v_1(a, 0, b)$  for any  $a \in \mathbb{R}$  and  $b \geq 0$ . This implies that each player's optimal action is higher when  $\omega = 1$  than when  $\omega = 0$ .

The DM's partial information changes the strategic incentives of the players. We will analyze and compare three communication environments (which correspond to three extensive-form games) in which the DM, as well as the expert, has private information.

1. The DM reveals her signal  $s_2$  truthfully to the expert before the expert reports to her. Call this  $\Gamma_1$ .

2. The DM keeps her signal  $s_2$  private and receives the expert's report before she chooses

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<sup>5</sup>We use the pronoun "he" for the expert and the pronoun "she" for the decision maker.

an action. Call this  $\Gamma_2$ .

3. The DM strategically sends a message to the expert about the signal  $s_2$  before the expert reports to her. Call this  $\Gamma_3$ .

Game  $\Gamma_1$  applies to situations in which the DM commits to revealing her signal truthfully to the expert. This can happen, for example, if the DM's information comes from reports of a neutral third party and the DM allows the third party to reveal the same information to the expert. Alternatively, it can happen if the DM's information can be costlessly verified once revealed.

So far we have referred to  $s_2$  as the DM's private signal, but there is an alternative and in some contexts more natural interpretation of  $s_2$  in  $\Gamma_1$  and  $\Gamma_2$ . Think of  $s_2$  as a public signal that both players observe. Then, game  $\Gamma_1$  describes the strategic situation if the expert reports to the DM after the arrival of  $s_2$ . On the other hand, if the expert reports to the DM after observing  $s_1$ , but before the arrival of  $s_2$ , then  $\Gamma_2$  applies. In this interpretation, it is the *timing* of communication that makes the difference.

Throughout the analysis of these three games, we will use  $m$  to denote the message that the expert sends to the DM. In  $\Gamma_3$ , we will use  $l$  to denote the message that the DM sends to the expert. Since we have a binary state space, we assume, for simplicity, that the message spaces are also binary, i.e.,  $L = M = \{0, 1\}$ .

Let  $\sigma_i$  denote the expert's mixed reporting strategy and  $s_i$  denote his pure reporting strategy in game  $\Gamma_i$ . Due to the strict concavity of the DM's payoff function, she never plays a mixed strategy in equilibrium and we use  $a_i$  to denote the DM's pure action strategy in game  $\Gamma_i$ .

Since the expert observes both  $s_1$  and  $s_2$  when sending his report in  $\Gamma_1$ , we have  $\sigma_1 : S_1 \times S_2 \times M \rightarrow [0, 1]$ , where  $\sigma_1(s_1, s_2, m)$  is the probability that the expert with observation  $s_1$  sends  $m$  if the DM reveals her signal to be  $s_2$ . The expert's pure strategy is  $m_1 : S_1 \times S_2 \rightarrow M$ . In  $\Gamma_1$ , the DM's choice of action depends on the signal  $s_2$  as well as the message sent by the expert. Her strategy is  $a_1 : M \times S_2 \rightarrow A$ .

In  $\Gamma_2$ , the DM keeps  $s_2$  private and the expert does not observe  $s_2$  when sending a message to the DM. Therefore, the expert's mixed strategy is  $\sigma_2 : S_1 \times M \rightarrow [0, 1]$  where  $\sigma_2(s_1, m)$  is the probability that the expert with observation  $s_1$  sends message  $m$ . His pure strategy is  $m_2 : S_2 \rightarrow M$ . The DM's action strategy is  $a_2 : M \times S_2 \rightarrow A$ .

In  $\Gamma_3$ , the DM sends a message to the expert in the first round of communication. Let  $\rho : S_2 \times L \rightarrow [0, 1]$  denote her mixed strategy where  $\rho(s_2, l)$  is the probability that the DM with observation  $s_2$  sends a message  $l$  to the expert. In the second round of communication, the expert's choice of message depends on  $s_1$  as well as the DM's cheap-talk message  $l$ . Therefore, the expert's mixed strategy is  $\sigma_3 : S_1 \times L \times M \rightarrow [0, 1]$  where  $\sigma_3(s_1, l, m)$  is the probability that the expert with observation  $s_1$  sends message  $m$  if the message sent by the DM is  $l$ . His pure strategy is  $m_3 : S_1 \times L \rightarrow M$ . The DM's action strategy is

$a_3 : L \times M \times S_2 \rightarrow A$ .

We use Perfect Bayesian Equilibrium (PBE) as our solution concept. As is typical in cheap-talk models, the problem of multiple equilibria arises in all three games. We will address this problem later. Define a truth-telling equilibrium in  $\Gamma_i$  as a PBE in which  $m_i(s_1, \cdot) = s_1$  for  $s_1 = 0, 1$ . For now, we will focus on finding the conditions under which a truth-telling equilibrium exists.

### 3 Baseline case : $q_2 = \frac{1}{2}$

When  $q_2 = \frac{1}{2}$ , the DM is not informed. Since the DM has no useful information to reveal or communicate, we will analyze  $\Gamma_2$  in which there is only one round of communication from the expert to the DM. The intuition we develop in analyzing this baseline case will help us understand the results in more complicated settings. So we will discuss the derivation of our results in detail.

Fix  $p$  and  $q_1$ . Let  $a^*(0) = \arg \max_{\{a\}} E(u^{DM}(a, \omega) | s_1 = 0)$  and  $a^*(1) = \arg \max_{\{a\}} E(u^{DM}(a, \omega) | s_1 = 1)$ . In a truth-telling equilibrium, the DM's strategy  $a(m)$  satisfies  $a(0) = a^*(0)$  and  $a(1) = a^*(1)$ .

The following lemma implies that  $a^*(1) > a^*(0)$ .

Define  $\hat{a}(\beta) = \arg \max_{\{a\}} (\beta u^{DM}(a, 1) + (1 - \beta) u^{DM}(a, 0))$ , i.e.,  $\hat{a}(\beta)$  is the DM's optimal action if she believes that  $\omega = 1$  with probability  $\beta$  and  $\omega = 0$  with probability  $(1 - \beta)$ .

**Lemma 1**  $\hat{a}(\beta)$  is increasing in  $\beta$ .

**Proof.** Since  $\hat{a}(\beta) = \arg \max_{\{a\}} (\beta u^{DM}(a, 1) + (1 - \beta) u^{DM}(a, 0))$  and  $u_{11}^{DM} < 0$ , we have

$$\beta u_1^{DM}(\hat{a}(\beta), 1) + (1 - \beta) u_1^{DM}(\hat{a}(\beta), 0) = 0 \Rightarrow \frac{u_1^{DM}(\hat{a}(\beta), 1)}{u_1^{DM}(\hat{a}(\beta), 0)} = -\frac{1 - \beta}{\beta}$$

Taking derivatives with respect to  $\beta$  on both sides of the equation, we have

$$\frac{u_{11}^{DM}(\hat{a}(\beta), 1) u_1^{DM}(\hat{a}(\beta), 0) - u_{11}^{DM}(\hat{a}(\beta), 0) u_1^{DM}(\hat{a}(\beta), 1)}{(u_1^{DM}(\hat{a}(\beta), 0))^2} \frac{d(\hat{a}(\beta))}{d\beta} = \frac{1}{\beta^2}.$$

Since  $u_1^{DM}(\hat{a}(\beta), 1) > 0$ ,  $u_1^{DM}(\hat{a}(\beta), 0) < 0$  and  $u_{11}^{DM} < 0$ , we have  $\frac{d(\hat{a}(\beta))}{d\beta} > 0$ . ■

Since  $\text{prob}(\omega = 1 | s_1 = 1) > \text{prob}(\omega = 1 | s_1 = 0)$ , Lemma 1 implies that  $a^*(1) > a^*(0)$ . So if the DM believes that the expert is telling the truth, she responds to message 1 with a higher action than to message 0. Since  $v_{13} \geq 0$  and we assume that the expert has an upward bias, i.e.,  $b \geq 0$ , we have  $E(v(a^*(1), \omega, b) | s_1 = 1) \geq E(v(a^*(0), \omega, b) | s_1 = 1)$ . So the expert with the observation  $s_1 = 1$  (call him the "type 1" expert) would like to convince the DM that his signal is indeed 1 instead of 0. Hence the IC constraint for truth telling is satisfied for the type-1 expert and we need only look at type 0's IC constraint, which requires that he prefers sending message 0 to message 1.

Fix  $p$  and  $q_1$ . Define  $\bar{b}$  to be the value such that  $E(u^E(a^*(0), \omega, \bar{b}) | s_1 = 0) = E(u^E(a^*(1), \omega, \bar{b}) | s_1 = 0)$ .

**Proposition 1** *A truth-telling equilibrium exists if and only if  $b \in [0, \bar{b}]$ .*

**Proof.** Since  $v_{13} \geq 0$ ,  $\beta v_{13}(a, 0, b) + (1 - \beta)v_{13}(a, 1, b) \geq 0$  for any  $\beta \in [0, 1]$ . Since  $a^*(1) > a^*(0)$  and  $E(v(a^*(0), \omega, 0) | s_1 = 0) > E(v(a^*(1), \omega, 0) | s_1 = 0)$ ,  $E(u^E(a^*(0), \omega, \bar{b}) | s_1 = 0) = E(u^E(a^*(1), \omega, \bar{b}) | s_1 = 0)$  implies  $\bar{b} > 0$ . Moreover, we have  $E(u^E(a^*(0), \omega, b) | s_1 = 0) \geq E(u^E(a^*(1), \omega, b) | s_1 = 0)$  if  $b \leq \bar{b}$  and  $E(u^E(a^*(0), \omega, b) | s_1 = 0) < E(u^E(a^*(1), \omega, b) | s_1 = 0)$  if  $b > \bar{b}$ . Therefore the IC constraint for type 0 expert is satisfied if and only if  $b \leq \bar{b}$ . ■

Proposition 1 says that for a fixed prior and fixed informativeness of the expert's signal, a truth-telling equilibrium exists if the divergence of interest between the two players is sufficiently small ( $b \leq \bar{b}$ ). Below is some discussion on comparative statics.

Note that when the DM has low "confidence" in his prior, (i.e., when  $p$  is close to  $\frac{1}{2}$ ), the expert's information has a large impact on the DM's posterior. Accordingly, the difference between the DM's responses to difference messages (when he believes that they are truthful) decreases in the confidence of the prior, i.e.,  $(a^*(1) - a^*(0))$  decreases in  $|p - \frac{1}{2}|$ .

As we vary  $q_1$ , the informativeness of the expert's signal, the DM's optimal responses  $a^*(0)$  and  $a^*(1)$  change. It is straightforward to show that  $\frac{da^*(0)}{dq_1} < 0$  and  $\frac{da^*(1)}{dq_1} > 0$ . Intuitively, the more informative the expert's signal, the more sensitive the DM's posterior beliefs are to the messages she receives. By Lemma 1, her optimal actions in response to the messages diverge as  $q_1$  increases.

How does  $q_1$  affect the expert's truth-telling incentives? In particular, how does the threshold value  $\bar{b}$  vary with  $q_1$ ? Recall that type 0's IC constraint is  $E(u^E(a^*(0), \omega, b) | s_1 = 0) \geq E(u^E(a^*(1), \omega, b) | s_1 = 0)$ . The change in  $q_1$  affects the expert's incentives by changing both his posterior on  $\omega$  and the DM's action choices. A more informative signal makes the type-0 expert believe with higher probability that  $\omega = 0$  and this strengthens type-0 expert's incentive to tell the truth. An increase in  $q_1$  also drives down  $a^*(0)$  and drives up  $a^*(1)$ . Whether this strengthens or weakens the type-0 expert's truth-telling incentive depends on the curvatures of both the expert's and the DM's utility functions. Without making further assumptions, it is not clear whether an increase in  $q_1$  raises or lowers  $\bar{b}$ . But if we make the common assumption that the players have quadratic utility functions, then we can provide stronger comparative static results, with intuitive interpretations.<sup>6</sup>

<sup>6</sup>Green and Stokey (1982) also study how changing the information structure of the sender's signal affects equilibrium outcomes and the welfare of the players. They find that in the class of "partition equilibrium," the sender benefits from a Blackwell improvement of his signal and when the change in the information structure is a "success enhancing" change – a particular kind of Blackwell improvement – the receiver also benefits.



**Example 1 Quadratic Utilities.**

Suppose  $u^{DM} = -(a - \omega)^2$  and  $u^E = -(a - \omega - b)^2$ . Note that the quadratic utility functions satisfy all the assumptions we made on the derivatives.

In a truth-telling equilibrium, the DM's best responses  $a(0)$  and  $a(1)$  are conditional expectations of  $\omega$ . That is,  $a(0) = a^*(0) = E(\omega|s_1 = 0) = \frac{p(1-q_1)}{p(1-q_1)+(1-p)q_1}$  and  $a(1) = a^*(1) = E(\omega|s_1 = 1) = \frac{pq_1}{pq_1+(1-p)(1-q_1)}$ .

The expert's expected utility is  $Eu^E = E(-(a - \omega - b)^2) = -(a - E(\omega) - b)^2 - \text{Var}(\omega)$ . Since the variance terms do not depend on the messages the expert sends, they usually cancel out in IC constraints.

The threshold value  $\bar{b}$  satisfies  $E(u^E(a^*(0), \omega, \bar{b})|s_1 = 0) = E(u^E(a^*(1), \omega, \bar{b})|s_1 = 0)$ , which implies that  $-(a^*(0) - E(\omega|s_1 = 0) - \bar{b})^2 - \text{Var}(\omega|s_1 = 0) = -(a^*(1) - E(\omega|s_1 = 0) - \bar{b})^2 - \text{Var}(\omega|s_1 = 0)$ . Therefore  $\bar{b}^2 = (a^*(1) - a^*(0) - \bar{b})^2$ . The solution  $\bar{b} = \frac{a^*(1) - a^*(0)}{2} = \frac{1}{2} \left( \frac{p(1-p)(2q_1-1)}{pq_1+(1-p)(1-q_1)(p(1-q_1)+(1-p)q_1)} \right)$ .

With quadratic utilities, whether a truth-telling equilibrium exists depends on the difference between  $a^*(1)$  and  $a^*(0)$ , relative to the size of  $b$ . When  $a^*(1)$  and  $a^*(0)$  are far apart, type 0 expert has no incentive to deviate from telling the truth: although  $a^*(0)$  is below his expected ideal point,  $a^*(1)$  is too high to be beneficial. On the other hand, when the distance between  $a^*(1)$  and  $a^*(0)$  is small relative to  $b$ , type 0 expert has an incentive to lie and induce the DM to choose  $a^*(1)$  which is higher than  $a^*(0)$ , but not too high for the expert. In this case, type 0's IC constraint is violated and truth telling cannot happen in equilibrium.

The range of  $b$  for which a truth-telling equilibrium exists increases as the diffusion of the prior increases because the DM's optimal actions are more sensitive to the expert's messages when the prior is more diffuse.

Since  $\frac{da^*(0)}{dq_1} < 0$  and  $\frac{da^*(1)}{dq_1} > 0$ ,  $\frac{d\bar{b}}{dq_1} > 0$ . So an increase in the informativeness of the expert's signal increases the range of  $b$  for which truth telling can happen in equilibrium. Intuitively, when  $s_1$  is highly noisy, even if the DM believes the expert's messages, the messages have a small impact on the DM's choice of actions, i.e.,  $(a^*(1) - a^*(0))$  is small. Truth telling cannot happen in equilibrium because type 0's IC constraint is violated. In contrast, if  $s_1$  is highly informative, then the expert expects the DM's choice of action to be sensitive to the messages that he sends. In this case, type 0 would like to be separated from type 1 because  $a^*(1)$  is too high to be beneficial and truth telling is sustainable in equilibrium.

So far we have focused on truth-telling equilibria. We see that when the expert's bias is higher than  $\bar{b}$ , truth-telling fails to exist. But does there exist an equilibrium in which some information is transmitted, that is, an equilibrium other than babbling, when  $b > \bar{b}$ ? The discussion below shows that the answer is no.

Fix the parameters  $p$ ,  $q_1$  and  $b > \bar{b}$ . Suppose there exists a non-babbling equilibrium  $\hat{E}$  (we also refer to it as a semi-separating equilibrium). Then the DM responds to the messages 0 and 1 with different actions in  $\hat{E}$ , i.e.,  $a(m=1) \neq a(m=0)$ . This immediately implies that the type 1 expert does not play a mixed strategy in  $\hat{E}$  since he strictly prefers the higher action. Without loss of generality, assume that the type 1 expert sends  $m=1$  with probability 1, i.e.,  $\sigma(1,1) = 1$ . Since  $\hat{E}$  is neither a truth-telling equilibrium nor a babbling equilibrium, the type 0 expert sends both messages 0 and 1 with positive probability, i.e.,  $\sigma(0,0), \sigma(0,1) \in (0,1)$ . Given these reporting strategies, if the DM receives  $m=0$ , she infers that it was sent by type 0 with probability 1 and responds with  $a(0) = a^*(0)$ ; if the DM receives  $m=1$ , she infers that it could be sent by either types with positive probability and responds with  $a(1) \in (a^*(0), a^*(1))$ . In  $\hat{E}$ , type 0 is indifferent between inducing  $a(0) (= a^*(0))$  and inducing  $a(1) (< a^*(1))$ . Since  $u^E$  is single peaked and  $u_{11}^E < 0$ , type 0 strictly prefers  $a^*(0)$  to  $a^*(1)$ . It follows that his IC constraint for truth telling is satisfied, which contradicts the assumption that  $b > \bar{b}$ . Therefore a semi-separating equilibrium exists only when a truth-telling equilibrium exists.

Both players prefer the truth-telling equilibrium to the semi-separating equilibrium. It is clear for the DM since more information makes her better off. Moreover, both types of the expert have (weakly) higher payoffs in the truth-telling equilibrium than in the semi-separating equilibrium. To see why, recall that in a semi-separating equilibrium, the type-0 expert is indifferent between sending 0 (and being identified as type 0) and sending 1 (and pooling with type 1). Since he is identified as type 0 in both equilibria, type 0's expected payoff is the same in the truth-telling equilibrium and the semi-separating equilibrium. As to type 1, he induces a lower action in the semi-separating equilibrium than in the truth-telling equilibrium and therefore he has a strictly higher expected payoff in the truth-telling equilibrium.

**Proposition 2** *A semi-separating equilibrium exists only when a truth-telling equilibrium exists. In this semi-separating equilibrium, the type 0 expert is indifferent between sending message 0 and message 1 and sends both with positive probability. The truth-telling equilibrium Pareto dominates the semi-separating equilibrium, even in the interim.*

## 4 Partially Informed DM: $\frac{1}{2} < q_2 < 1$

Suppose the decision maker also observes an informative signal  $s_2$ , with informativeness parameterized by  $q_2 \in (\frac{1}{2}, 1)$ . Assume that  $s_1$  and  $s_2$  are conditionally (on  $\omega$ ) independent.

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<sup>7</sup>Alternatively, we can assume that  $s_1$  and  $s_2$  are conditionally correlated. But this will complicate analysis without generating much more insight.

Below we analyze the three games  $\Gamma_1, \Gamma_2$  and  $\Gamma_3$  described before.

#### 4.1 Game $\Gamma_1$ : The DM truthfully reveals $s_2$

Since  $s_2$  is revealed before the expert communicates to the DM, the expert's incentives for telling the truth depends on the realization of  $s_2$  as well as his own signal  $s_1$ . To find the conditions for truth telling, we again look at the expert's IC constraints.

Define  $a_j^*(i) = \arg \max(\text{prob}(\omega = 0|s_1 = i, s_2 = j) u^{DM}(0, a) + \text{prob}(\omega = 1|s_1 = i, s_2 = j) u^{DM}(1, a))$  for  $i = 0, 1$  and  $j = 0, 1$ . So  $a_j^*(i)$  is the DM's optimal action if the realization of the expert's signal is  $i$  and the realization of his own signal is  $j$ . In a truth-telling equilibrium in  $\Gamma_1$ , the DM's strategy  $a_1 : S_1 \times S_2 \rightarrow A$  satisfies  $a_1(i, j) = a_j^*(i)$  for  $i = 0, 1$  and  $j = 0, 1$ . The following lemma implies that similar to the baseline case in section 3, type 1's IC constraint is always satisfied and we only need to look at type 0's IC constraint.

**Lemma 2**  $\text{prob}(\omega = 1|s_1 = 0, s_2 = 0) \leq \text{prob}(\omega = 1|s_1 = 1, s_2 = 0)$  and  $\text{prob}(\omega = 1|s_1 = 0, s_2 = 1) \leq \text{prob}(\omega = 1|s_1 = 1, s_2 = 1)$

$$\begin{aligned} \text{Proof. } \text{prob}(\omega = 1|s_1 = 0, s_2 = 0) &= \frac{p(1-q_1)(1-q_2)}{p(1-q_1)(1-q_2) + (1-p)q_1q_2} \\ \text{prob}(\omega = 1|s_1 = 1, s_2 = 0) &= \frac{pq_1(1-q_2)}{pq_1(1-q_2) + (1-p)(1-q_1)q_2} \\ \text{prob}(\omega = 1|s_1 = 0, s_2 = 1) &= \frac{p(1-q_1)q_2}{p(1-q_1)q_2 + (1-p)q_1(1-q_2)} \\ \text{prob}(\omega = 1|s_1 = 1, s_2 = 1) &= \frac{pq_1q_2}{pq_1q_2 + (1-p)(1-q_1)(1-q_2)} \end{aligned}$$

To prove the lemma, we only need to show that  $q_1^2q_2(1-q_2) \geq (1-q_1)^2q_2(1-q_2)$ .

Since  $q_2 > \frac{1}{2}$ , the lemma holds. ■

Suppose the DM reveals that  $s_2 = 0$ . Since  $\text{prob}(\omega = 1|s_1 = 0, s_2 = 0) \leq \text{prob}(\omega = 1|s_1 = 1, s_2 = 0)$ , by Lemma 1,  $a_0^*(0) \leq a_0^*(1)$ . That is, in a truth-telling equilibrium, in the subgame after  $s_2$  is revealed to be 0, the DM responds to message 1 with a higher action than to message 0. Since the expert has an upward bias, type 1's IC constraint is satisfied. Similarly, in the subgame after  $s_2$  is revealed to be 1, the DM responds to message 1 with a higher action than to message 0 ( $a_1^*(0) \leq a_1^*(1)$ ) and type 1's IC constraint is again satisfied. Type 0's IC constraint in the subgame following  $s_2 = 0$  requires that  $Eu^E(a_0^*(0), \omega, b|s_1 = 0, s_2 = 0) \geq Eu^E(a_0^*(1), \omega, b|s_1 = 0, s_2 = 0)$ . Call it  $IC_0$ . The IC constraint for the type 0 expert in the subgame following  $s_2 = 1$  requires that  $Eu^E(a_1^*(0), \omega, b|s_1 = 0, s_2 = 1) \geq Eu^E(a_1^*(1), \omega, b|s_1 = 0, s_2 = 1)$ . Call it  $IC_1$ . Define  $b_0$  to be the value such that  $Eu^E(a_0^*(0), \omega, b_0|s_1 = 0, s_2 = 0) = Eu^E(a_0^*(1), \omega, b_0|s_1 = 0, s_2 = 0)$  and  $b_1$  to be the value such that  $Eu^E(a_1^*(0), \omega, b_1|s_1 = 0, s_2 = 1) = Eu^E(a_1^*(1), \omega, b_1|s_1 = 0, s_2 = 1)$ .

**Proposition 3** *A truth-telling equilibrium exists in the subgame following  $s_2 = 0$  if and only if  $b \in [0, b_0]$  and a truth-telling equilibrium exists in the subgame following  $s_2 = 1$  if and only if  $b \in [0, b_1]$ .*

**Proof.** Same arguments as in Proposition 1. ■

Analogous to the baseline case, Proposition 3 says that if the interests of the players are close enough, then truth telling can happen in equilibrium in the subgames. Note that a subgame following the revelation of  $s_2$  is equivalent to a baseline game in which the DM is uninformed, with an appropriately chosen prior. The subgame after  $s_2 = 0$  is equivalent to the baseline game with the common prior on  $\omega$  being  $prob(\omega = 0) = \frac{(1-p)q_1}{(1-p)q_1+p(1-q_1)}$  and  $prob(\omega = 1) = \frac{p(1-q_1)}{(1-p)q_1+p(1-q_1)}$ ; the subgame after  $s_2 = 1$  is equivalent to the baseline game with the common prior on  $\omega$  being  $prob(\omega = 0) = \frac{(1-p)(1-q_1)}{(1-p)(1-q_1)+pq_1}$  and  $prob(\omega = 1) = \frac{pq_1}{(1-p)(1-q_1)+pq_1}$ . So the results we derived in section 3 apply to the subgames in  $\Gamma_1$  after  $s_2$  is revealed.

If we keep the informativeness of the DM's signal ( $q_2$ ) constant and vary only the informativeness of the expert's signal ( $q_1$ ), we have

**Remark 1**  $\frac{\partial a_0^*(0)}{\partial q_1} < 0$ ,  $\frac{\partial a_0^*(1)}{\partial q_1} > 0$ ,  $\frac{\partial a_1^*(0)}{\partial q_1} < 0$ ,  $\frac{\partial a_1^*(1)}{\partial q_1} > 0$ .

**Proof.** Appendix. ■

For fixed  $q_2$ , the DM's optimal responses to the expert's messages diverge as the informativeness of the expert's signal increases.

Similarly, if we keep  $q_1$  constant and vary only  $q_2$ , we have

**Remark 2**  $\frac{\partial a_0^*(0)}{\partial q_2} < 0$ ,  $\frac{\partial a_0^*(1)}{\partial q_2} < 0$ ,  $\frac{\partial a_1^*(0)}{\partial q_2} > 0$ ,  $\frac{\partial a_1^*(1)}{\partial q_2} > 0$ .

**Proof.** Appendix. ■

An increase in the informativeness of the DM's signal drives down the DM's optimal responses to either one of the expert's messages if  $s_2 = 0$  and drives up the DM's optimal responses if  $s_2 = 1$ .

Does increasing the informativeness of the expert's signal strengthen his incentives for telling the truth? Does the expert have a stronger incentive to tell the truth when facing a well-informed DM or when facing a DM who has little information? Again, the changes in  $q_1$  and  $q_2$  change the expert's posterior as well as the DM's responses to the messages. The comparative statics results depend on the curvatures of the players' payoff functions. Below, we look at the quadratic case to gain some intuition.

**Example 2** *Quadratic Utilities.*

Suppose  $u^{DM} = -(a - \omega)^2$  and  $u^E = -(a - \omega - b)^2$ .

Similar arguments as those in the baseline case show that whether truth telling can happen in equilibrium depends on the differences in beliefs and therefore actions that the expert's

messages induce. That is, it depends on the distance between  $a_i^*(1)$  and  $a_i^*(0)$  ( $i = 0, 1$ ), relative to the size of  $b$ . In fact,  $b_0 = \frac{1}{2}(a_0^*(1) - a_0^*(0)) = \frac{1}{2}(E(\omega|s_1 = 1, s_2 = 0) - E(\omega|s_1 = 0, s_2 = 1))$  and  $b_1 = \frac{1}{2}(a_1^*(1) - a_1^*(0)) = \frac{1}{2}(E(\omega|s_1 = 1, s_2 = 1) - E(\omega|s_1 = 0, s_2 = 1))$ .

The threshold  $b_0 \geq b_1$  if and only if  $p \geq \frac{1}{2}$ . (See appendix for proof.) Intuitively, if the prior is such that  $\text{prob}(\omega = 1) \geq \text{prob}(\omega = 0)$ , then the difference in posterior that the expert's (truthful) messages induce is larger when the DM's signal is against the prior, i.e., when  $s_2 = 0$ . Therefore, the range of  $b$  for which truth telling exists is larger when  $s_2 = 0$  than when  $s_2 = 1$  if  $p \geq \frac{1}{2}$ .

Remark 1 implies that  $\frac{\partial b_0}{\partial q_1} > 0$  and  $\frac{\partial b_1}{\partial q_1} > 0$ . Independent of the realization of  $s_2$ , an expert with a highly informative private signal expects his messages to have a large impact on the DM's action if the DM believes that the messages are truthful. So the range of  $b$  for which truth telling exists increases with the informativeness of the expert's signal.

For comparative statics results with respect to  $q_2$ , we can show that  $\frac{\partial b_0}{\partial q_2}$  has the same sign as  $(p - q_2)$  and  $\frac{\partial b_1}{\partial q_2}$  has the same sign as  $(1 - p - q_2)$ . (See appendix for proof.) Therefore, when the prior is symmetric ( $p = \frac{1}{2}$ ), both  $b_0$  and  $b_1$  are decreasing in  $q_2$ ; when the prior is asymmetric ( $p \neq \frac{1}{2}$ ), at least one of  $b_0$  and  $b_1$  is decreasing in  $q_2$ . So an increase in the informativeness of the DM's signal may prevent truth telling from happening in equilibrium. This happens when an increase in  $q_2$  lowers the impact of the expert's messages on the DM's choice of action.

We have shown in section 3 that when  $q_2 = \frac{1}{2}$ , a mixed strategy non-babbling equilibrium exists only when there exists a truth-telling equilibrium. The same arguments apply in  $\Gamma_1$ . In the two subgames after  $s_2$  is revealed to the expert, an equilibrium in which the expert's private information is partially revealed to the DM exists only when there exists another equilibrium in which the expert truthfully reveals  $s_1$ . The most informative equilibrium is either truth telling or babbling, depending on the parameters.

## 4.2 Game $\Gamma_2$ : The DM keeps $s_2$ private

If the DM keeps her information private, the expert does not know for certain what action the DM will choose in response to his messages in equilibrium, even though the DM always plays a pure strategy. From the expert's point of view, his messages induce probability distributions of actions by the DM.

For the type 0 expert, the IC constraint for truth telling requires that  $\text{prob}(s_1 = 0, s_2 = 0) Eu^E(a_0^*(0), \omega, b|s_1 = 0, s_2 = 0) + \text{prob}(s_1 = 0, s_2 = 1) Eu^E(a_1^*(0), \omega, b|s_1 = 0, s_2 = 1) \geq \text{prob}(s_1 = 0, s_2 = 0) Eu^E(a_0^*(1), \omega, b|s_1 = 0, s_2 = 0) + \text{prob}(s_1 = 0, s_2 = 1) Eu^E(a_1^*(1), \omega, b|s_1 = 0, s_2 = 1)$ . Call this IC constraint  $IC_{private}$ . Note that  $IC_{private}$  in  $\Gamma_2$  is a convex combination of  $IC_0$  and  $IC_1$  in  $\Gamma_1$ . Similarly, type 1's IC constraint in  $\Gamma_2$  is a convex combination of type 1's IC constraints in  $\Gamma_1$ . Since type 1's IC constraints in  $\Gamma_1$  are always satisfied, their convex

combination is always satisfied as well. Again, only type 0's IC constraint may be binding in  $\Gamma_2$ .

Let  $b_{private}$  be the value of  $b$  such that  $IC_{private}$  is binding.

**Proposition 4** *The game  $\Gamma_2$  has a truth-telling equilibrium if and only if  $b \leq b_{private}$ .*

**Proof.** Same arguments as in Proposition 1. ■

**Example 3** *Quadratic utilities. Suppose  $u^{DM} = -(a - \omega)^2$ ,  $u^E = -(a - \omega - b)^2$ . We can show that  $\frac{\partial b_{private}(p, q_1, q_2)}{\partial q_2} < 0$ . (See appendix for proof.) This means that the more informative the DM's signal is, the smaller the range of bias for which a truth-telling equilibrium exists, when the DM keeps  $s_2$  private. As we explained before, when the utilities are quadratic, whether or not a truth-telling equilibrium exists depends on the difference in the actions that the DM chooses in response to messages 0 and 1. When it is common knowledge that the DM has a relatively informative private signal, the expert expects that his messages do not weigh much in the DM's choice of action. This means that the DM's (expected) responses to different messages are not far apart. Hence type 0 has an incentive to deviate from reporting honestly and truth telling fails to be an equilibrium.*

*One interesting implication is that having a more informative signal does not necessarily benefit the DM. It is possible that the loss of information from the expert due to the change in strategic incentives more than offsets the gain from the increase in the quality of the DM's own signal. This will be illustrated in the example we provide in section 4.3.*

Using a similar argument as in section 3, we can show that in  $\Gamma_2$ , if there exists a mixed strategy equilibrium in which the expert's information is partially transmitted to the DM, then it must be the case that  $b < b_{private}$  and a truth-telling equilibrium exists as well. Therefore, depending on the parameters, the most informative equilibrium is either truth telling or babbling.

### 4.3 Comparison of equilibria in $\Gamma_1$ and $\Gamma_2$

Sometimes the DM may choose the communication environment. For example, if  $s_2$  comes from the report of a neutral third party, the DM may decide whether or not to make the third party's report public. Alternatively, if  $s_2$  is a public signal, then it may be possible for the DM to choose the timing of the communication: she could decide whether to ask the expert to report his private observation of  $s_1$  before or after the arrival of the public signal  $s_2$ . In yet another interpretation,  $s_2$  could be the DM's private information that can be verified by the expert at no cost once revealed. Then, the choice between playing  $\Gamma_1$  and  $\Gamma_2$  is the same as the decision of whether or not to reveal  $s_2$  to the expert.<sup>8</sup>

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<sup>8</sup>In this interpretation, we assume that the DM's decision of whether or not to reveal  $s_2$  is made before she observes  $s_2$  so that the choice of "not revealing" has no signaling effect.

If the DM could choose between  $\Gamma_1$  and  $\Gamma_2$ , what would she choose? With the analysis of  $\Gamma_1$  and  $\Gamma_2$  in previous sections, we can compare their effectiveness as mechanisms to facilitate information transmission. Since the DM's expected utility is increasing in the amount of the information she extracts from the expert, she favors the environment that is most conducive to information transmission.

We find that whether the DM extracts more information from the expert (in the most informative equilibrium) in  $\Gamma_1$  or in  $\Gamma_2$  depends on the parameters.

First, let's fix  $p, q_1, q_2, \sigma$  and vary the expert's bias  $b$ . Without loss of generality, suppose  $b_1 \leq b_0$ .

If  $b \leq b_1$ , then both  $IC_0$  and  $IC_1$  are satisfied. As a convex combination of  $IC_0$  and  $IC_1$ ,  $IC_{private}$  is also satisfied. This means that a truth-telling equilibrium exists in both  $\Gamma_1$  and  $\Gamma_2$  and the DM extracts the maximal amount of information in both games.

If  $b_1 < b \leq b_{private}$ , then  $IC_0$  and  $IC_{private}$  are satisfied while  $IC_1$  is violated. This implies that if the DM reveals  $s_2$  to the expert before he reports, then in the most informative equilibrium in  $\Gamma_1$ , the expert truthfully reveals his signal in the subgame after  $s_2$  is revealed to be 0 but babbles in the subgame after  $s_2$  is revealed to be 1. In contrast, if the DM does not reveal  $s_2$  to the expert, then there exists a truth-telling equilibrium. Therefore, the DM can extract more information from the expert by keeping her signal private when  $b$  lies in this range.

If  $b_{private} < b \leq b_0$ , then only  $IC_0$  is satisfied. In the most informative equilibrium in  $\Gamma_1$ , the expert truthfully reveals his signal in the subgame after  $s_2$  is revealed to be 0. However, only babbling can happen in equilibrium in  $\Gamma_2$  since  $IC_{private}$  is violated. Therefore, when  $b$  lies in this range, by revealing  $s_2$  to the expert, the DM can extract useful information from him in the event that  $s_2 = 0$  while she can extract no information if she keeps  $s_2$  private.

Finally, if  $b_0 < b$ , then all three incentive constraints are violated and the expert babbles in both  $\Gamma_1$  and  $\Gamma_2$ .

Suppose the DM can choose between  $\Gamma_1$  and  $\Gamma_2$ , then we have the following result, given the above comparison. The DM is indifferent between the two games if the bias of the expert is extreme (either very small or very large) but she strictly prefers one over the other if the bias is in the intermediate range. Specifically, as the bias increases from a small value to a moderate value, the DM prefers keeping her signal private to revealing it to the expert. However, as the bias gets even larger, she prefers revealing the signal to keeping it private.

It is also interesting to compare  $\Gamma_1$  and  $\Gamma_2$  as we vary the informativeness of the DM's signal,  $q_2$ , while fixing  $p, q_1$  and  $b$ . Assuming that the players have quadratic utilities, we have the following numerical example.

**Example 4** Suppose  $u^{DM} = -(a - \omega)^2$  and  $u^E = -(a - \omega - b)^2$ .

Let  $p = 0.7$ ,  $q_1 = 0.8$ ,  $\sigma = 0$  and  $b = 0.2$ .

Note that  $b_1 = 0.2$  if  $q_2 = 0.67839$ ,  $b_0 = 0.2$  if  $q_2 = 0.91990$ , and  $b_{\text{private}} = 0.2$  if  $q_2 = 0.89711$ .

The following figure shows the DM's expected payoff as a function in  $q_2$  in the most informative equilibrium in  $\Gamma_1$  and  $\Gamma_2$ . The thick plot is for  $\Gamma_1$  and the thin one is for  $\Gamma_2$ . The two plots coincide for extreme values of  $q_2$ , but they are different for intermediate values of  $q_2$ .

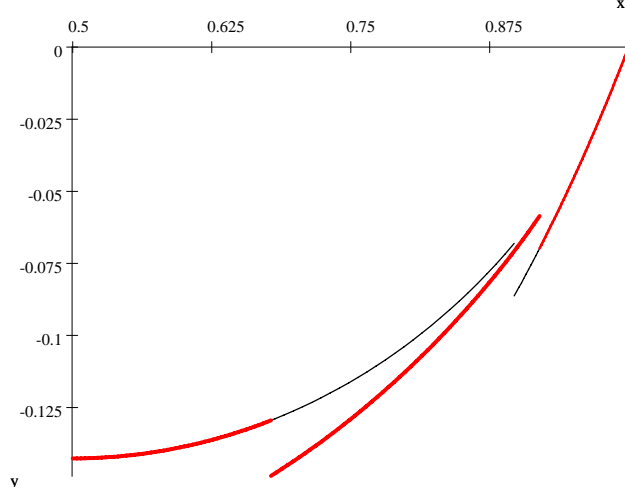


Figure 1

The DM's payoff depends on how much information she has when making a decision. The increase in  $q_2$  has two effects on her payoff. On the one hand, it benefits the DM since the signal she directly observes is more informative. On the other hand, it could be disadvantageous because the increase in  $q_2$  may prevent the expert from revealing his information credibly, as shown earlier. Combining these two effects, we see that the DM's expected payoff is NOT monotonically increasing in  $q_2$  in both  $\Gamma_1$  and  $\Gamma_2$ . The thick curve has two discontinuities at  $q_2 = 0.67839$  and  $q_2 = 0.91990$ . These are the two points at which the information content of the expert's communication changes in  $\Gamma_1$ . They divide the interval  $[0.5, 1]$  into three ranges. Within each of the three ranges, the DM's payoff is increasing in  $q_2$ . The thin curve has only one discontinuity at  $q_2 = 0.89711$ . If  $q_2 \in [0.5, 0.89711]$ , a truth-telling equilibrium exists in  $\Gamma_2$ ; if  $q_2 \in (0.89711, 1]$ , only babbling equilibrium exists in  $\Gamma_2$ .

As we can see from the figure, when  $q_2 \in (0.67839, 0.89711]$ , the DM extracts more information in  $\Gamma_2$  by keeping  $s_2$  private and therefore enjoys a higher expected utility. However, when  $q_2 \in (0.89711, 0.91990]$ , she could extract more information in  $\Gamma_1$  by revealing  $s_2$  to the expert.



#### 4.4 Game 3 $\Gamma_3$ : No commitment – The DM strategically communicates to the expert

Now suppose we add an extra round of communication in which the DM strategically and costlessly communicates her observation of  $s_2$  to the expert before the expert reports to her.<sup>9</sup> How does this extra round of communication affect the DM’s ability to extract information from the expert?

Define equilibrium outcomes in all three games on the payoff relevant space  $S_1 \times S_2 \times A$ . Denote by  $EO(\Gamma_i)$  the set of equilibrium outcomes for game  $\Gamma_i$ ,  $i = 1, 2, 3$ . The set depends on the parameter values  $p$ ,  $q_1$ ,  $q_2$  and  $b$ .

**Lemma 3** *Fix  $p$ ,  $q_1$ ,  $q_2$  and  $b$ .  $EO(\Gamma_2) \subseteq EO(\Gamma_3)$ .*

The intuition behind this lemma is simple. Suppose  $E(\Gamma_2)$  is a PBE in  $\Gamma_2$ . Since the messages used by the DM in the first round of communication in  $\Gamma_3$  are “cheap talk,” there always exists an equilibrium in  $\Gamma_3$  in which the DM babbles in the first round of communication and in effect keeps her signal  $s_2$  private and the players follow the strategies prescribed in  $E(\Gamma_2)$  in the continuation of  $\Gamma_3$ . Therefore, when given an opportunity to costlessly communicate to the expert first, the DM can do at least as well as when she keeps her information private.

The following result says that the converse is true.

**Proposition 5** *Fix  $p$ ,  $q_1$ ,  $q_2$  and  $b$ . The DM cannot extract more information from the expert in  $\Gamma_3$  than in  $\Gamma_2$ .*

See appendix for proof.

This proposition says that the decision maker’s expected payoffs in the most informative equilibria in  $\Gamma_2$  and  $\Gamma_3$  are the same. So the extra round of cheap talk does not help the DM.

What is the intuition behind this result? In  $\Gamma_3$ , the sole purpose of the DM’s cheap talk in the first round is to elicit information from the expert in the second round. Previous analysis shows that the informativeness of the expert’s equilibrium strategies can be ranked

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<sup>9</sup>This is different from Aumann and Hart’s (2003) “long cheap talk.” They consider two-person games in which one player is better informed but both players take payoff-relevant actions and they allow the players to talk as long as they wish. In their model, different rounds of cheap talk can help the players agree on compromises as well as reveal substantive information.

It is also different from Krishna and Morgan’s (2005) study on multiple rounds of communication. In their setting, the uninformed DM and the perfectly-informed expert exchange cheap-talk messages in the first round of communication. The two-sided simultaneous communication induces a joint lottery on the DM’s equilibrium actions. Because the players are risk-averse, this allows more information to be transmitted in equilibrium.

in the sense of Blackwell (1953). If one particular message from the DM induces the expert to play a strategy that is strictly more informative, the other message cannot be sent by the DM in equilibrium and pooling happens in the first round. In this case, the DM’s information is effectively kept private in  $\Gamma_3$  and the set of equilibrium outcomes in  $\Gamma_3$  is the same as that in  $\Gamma_2$ . If both messages 0 and 1 are sent in equilibrium in the first round of communication in  $\Gamma_3$ , then it must be the case that the expert reveals the same amount of information in the most informative equilibrium in the continuation of  $\Gamma_3$ , no matter what message he receives from the DM. Since the expert’s strategy is constant in the DM’s signal, it follows that he will reveal the same information even if the DM keeps her signal private. Again, in this case, the DM does no better in  $\Gamma_3$  than in  $\Gamma_2$ . Since the DM’s preference for information is monotone, she cannot effectively communicate her private information to the expert without the commitment to revealing  $s_2$  truthfully.

## 5 Equilibrium Selection

So far we have focused on finding the conditions for the existence of truth-telling equilibria. In our comparative statics analysis and the comparison of different games, we have implicitly focused on the “most informative” equilibrium. But is there any reason why we should select it among the multiple equilibria that may exist? When a truth-telling equilibrium exists as well as a babbling equilibrium, why should the former be considered more “reasonable”? Below we use two approaches to see whether we can rule out any equilibria.

The first is Farrell’s (1993) “neologism-proofness.” Fix an equilibrium of a cheap talk game. Consider an announcement by the sender “my type is in the set  $X$ .” This announcement is a neologism if it is not sent in the candidate equilibrium. A neologism is deemed credible if it is precisely those types in the set  $X$  that receive strictly higher payoffs than their equilibrium payoffs if the neologism is believed by the receiver. An equilibrium is neologism-proof if there does not exist any credible neologism relative to it.

If we apply the neologism-proofness criterion to games  $\Gamma_1 - \Gamma_3$ , we have the following proposition.

**Proposition 6** *In game  $\Gamma_2$  and the subgames of  $\Gamma_1$  and  $\Gamma_3$  after the DM sends the expert a message, a truth-telling equilibrium is neologism-proof for any parameter values; a mixed strategy non-babbling equilibrium is never neologism-proof; a babbling equilibrium is neologism-proof if the following two conditions hold: (1) type 0 expert strictly prefers being perceived as type 1 than pooling with type 1 and (2) type 0 expert weakly prefers pooling with type 1 than being identified as type 0.*

The only neologism relative to a truth-telling equilibrium is “my type is in  $\{0, 1\}$ .” Since type 1 expert would never want to make this announcement, it is not credible and therefore

a truth-telling equilibrium is neologism-proof.

In a mixed strategy non-babbling equilibrium, type-0 expert randomizes between sending 0 and 1 and type-1 expert sends only one message. Therefore, “my type is 1” is a neologism. It is also credible because only type 1 gets a strictly higher payoff than the equilibrium payoff if the neologism is believed.<sup>10</sup> Hence the equilibrium is not neologism-proof.

Relative to a babbling equilibrium, there are two neologisms: “my type is 1” and “my type is 0.” Type 1 strictly benefits from announcing “my type is 1” if it is believed. So if type 0 weakly prefers playing the babbling equilibrium than being perceived as type 1, then “my type is 1” is a credible neologism. As to the other neologism “my type is 0,” type 1 would not want to make the announcement and type 0 would if and only being identified as type 0 gives him a (strictly) higher payoff than babbling. In that case, “my type is 0” is a credible neologism and babbling equilibrium is not neologism-proof.

One well-recognized problem with neologism-proofness is that “neologism-proof” equilibria do not always exist. In fact, take any of the three games  $\Gamma_1 - \Gamma_3$ , there are parameters with which “neologism-proof” equilibria fail to exist. To see this, consider a game in which only babbling on the expert’s part can happen in equilibrium. The two neologisms relative to the equilibrium are: “my type is 0” and “my type is 1.” As we have shown before, when babbling is the unique equilibrium outcome, type 0’s *IC* constraint fails and he prefers being perceived as type 1 to being identified as type 0. So he prefers babbling to being identified as type 0 as well and “my type is 0” is not a credible neologism. However, “my type is 1” is credible if type 0 prefers babbling to being perceived as type 1. Hence, when the expert’s bias is sufficiently large (so that babbling is the unique equilibrium outcome) but not too large (so that “my type is 1” is a credible neologism), no equilibrium passes the “neologism-proofness” test.

The second approach we take is to use the equilibrium robustness condition “no incentive to separate” (NITS), discussed in details in Chen, Kartik and Sobel (2007). The condition, originally applied to the Crawford-Sobel (1982) model of cheap talk, requires that a certain type of sender’s equilibrium payoff is at least as high as the payoff if he could fully reveal his type (hence no incentive to separate). The justifications for NITS is similar in the Crawford-Sobel model and the games in this paper. So we direct the readers to Chen, Kartik and Sobel (2007).

In our games, we apply the condition to the type-0 expert since he is the “lowest” type: the type that no other type would want to mimic but may itself have an incentive to mimic other types. Accordingly, an equilibrium in  $\Gamma_i$  satisfies NITS if and only if type-0 expert’s equilibrium payoff is weakly higher than the payoff he gets if the DM believes that  $s_1 = 0$ .

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<sup>10</sup>In the candidate equilibrium, type 0 is indifferent between being identified as type 0 and (partially) pooling with type 1. So the action that the DM takes when believing that he is type 1 is too high to be beneficial. And obviously, with an upward bias, type 1 expert would be better off if identified as type 1.

As we have shown earlier, in a truth-telling equilibrium and a mixed-strategy non-babbling equilibrium, type-0 expert's payoff is equal to his payoff if the DM believes that  $s_1 = 0$ . So NITS holds. A babbling equilibrium may or may not satisfy NITS. In particular, if the DM's equilibrium actions are so high that type 0 prefers to be identified as such than pooling with type 1, then NITS fails.

**Proposition 7** *The equilibrium  $\hat{E}$  in game  $\Gamma_i$  ( $i = 1, 2, 3$ ) satisfies NITS if and only if one of the following conditions holds:*

1.  $\hat{E}$  is a truth-telling equilibrium.
2.  $\hat{E}$  is a mixed strategy non-babbling equilibrium.
3.  $\hat{E}$  is a babbling equilibrium in which the type-0 expert prefers babbling to being identified as type 0.

As we see, the above two approaches of equilibrium selection generate different predictions.

The concept of “neologism-proofness” yields stronger predictions in our games. However, sometimes it is too strong that no equilibrium is neologism-proof. In contrast, the NITS condition does not have the non-existence problem. To see this, fix a game in which a truth-telling equilibrium fails to exist (which implies that a mixed strategy non-babbling equilibrium does not exist either), so the unique equilibrium outcome involves babbling. Since type-0 expert's IC constraint for truth telling is violated, type-0 expert prefers being (incorrectly) perceived as type 1 to being identified as type 0 by the DM. This implies that he strictly prefers pooling with type 1 to being identified as type 0.<sup>11</sup> According to Proposition 7, the babbling equilibrium where both type of the expert send the high message satisfies NITS.

Although the problem of non-existence does not arise, the NITS condition is not fully satisfactory for equilibrium selection either because for certain parameter values, it still admits multiple equilibria.<sup>12</sup> Interestingly, if there are multiple equilibria that pass NITS, then among them, only the truth-telling equilibrium is neologism-proof. Recall that the

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<sup>11</sup>The action that type 0 expert induces the DM to take when pooling with type 1 is between the actions he induces when identified as type 0 and when perceived as type 1. Since type 0's IC constraint for truth telling doesn't hold, the action that he induces when perceived as type 1 is NOT too high to be profitable. Accordingly, type 0 expert must prefer the higher action that he induces when pooling with type 1 to the action he induces when identified as type 0.

<sup>12</sup>When applied to the Crawford and Sobel (1982) model with a continuous state space, the perturbation approach generates strong results. However, when the state space is binary, the results are weaker. In both cases, we find that in a monotonic equilibrium of a perturbed game, the expected equilibrium payoff for the lowest type of expert (type 0) must be at least as high as the payoff he gets if identified by the DM as the lowest type. This condition implies uniqueness in the case of a continuous state space, but not necessarily so when the state space is binary.

mixed strategy non-babbling equilibrium is not neologism-proof. Consider the babbling equilibrium in which both types of expert send message 1 and type 0 prefers pooling with type 1 to being identified as type 0. Since a truth-telling equilibrium exists, type 0 must prefer being identified as type 0 to being perceived as type 1. It follows that type 0 prefers pooling with type 1 to being perceived as type 1. This implies that relative to the babbling equilibrium, “my type is 1” is a credible neologism and therefore the babbling equilibrium is not neologism-proof.

## 6 Conclusion

In order to make good decisions, people often seek advice and information from others (usually referred to as experts in the literature), who do not necessarily share the same interests. In a communication game between an expert and a DM, the expert’s incentives for information transmission are determined by the actions that the DM takes in response to his messages, which are in turn affected by the private information that the DM possesses, as we assume that the DM is partially informed in this paper.

When taking the strategic interaction into consideration, we find that the decision maker does not necessarily benefit from having more accurate information of her own since it may have the adverse effect of preventing the expert from revealing his private information credibly in equilibrium.

Even in the simple setting that we study, the decision maker’s choice between making her information public before the expert reports to her or keeping it private is not trivial. Which alternative yields a higher expected payoff to the decision maker depends on how aligned the players’ interests are. Furthermore, given that the decision maker always wants to extract as much information as possible from the expert, allowing her to communicate (without committing to telling the truth) to the expert before the expert reports to her does not help the decision maker because the first round of communication cannot be effective.

## Appendix

### Proof of Remark 1 - Remark 2 .

$$\text{prob}(\omega = 1 | s_1 = 0, s_2 = 0) = \frac{p(1-q_1)(1-q_2)}{p(1-q_1)(1-q_2) + (1-p)q_1q_2}$$

$$\text{prob}(\omega = 1 | s_1 = 1, s_2 = 0) = \frac{pq_1(1-q_2)}{pq_1(1-q_2) + (1-p)(1-q_1)q_2}$$

$$\text{prob}(\omega = 1 | s_1 = 0, s_2 = 1) = \frac{p(1-q_1)q_2}{p(1-q_1)q_2 + (1-p)q_1(1-q_2)}$$

$$\text{prob}(\omega = 1 | s_1 = 1, s_2 = 1) = \frac{pq_1q_2}{pq_1q_2 + (1-p)(1-q_1)(1-q_2)}$$

We can compute that

$$\frac{\partial \left( \frac{p(1-q_1)(1-q_2)}{p(1-q_1)(1-q_2) + (1-p)q_1q_2} \right)}{\partial q_1} = -\frac{(1-p)pq_2(1-q_2)}{(p(1-q_1)(1-q_2) + (1-p)q_1q_2)^2} < 0 \text{ and}$$

$$\frac{\partial \left( \frac{p(1-q_1)(1-q_2)}{p(1-q_1)(1-q_2) + (1-p)q_1q_2} \right)}{\partial q_2} = -\frac{(1-p)pq_1(1-q_1)}{(p(1-q_1)(1-q_2) + (1-p)q_1q_2)^2} < 0.$$

$$\begin{aligned} \frac{\partial \left( \frac{pq_1(1-q_2)}{pq_1(1-q_2)+(1-p)(1-q_1)q_2} \right)}{\partial q_1} &= \frac{(1-p)pq_2(1-q_2)}{(pq_1(1-q_2)+(1-p)(1-q_1)q_2)^2} > 0 \text{ and} \\ \frac{\partial \left( \frac{pq_1(1-q_2)}{pq_1(1-q_2)+(1-p)(1-q_1)q_2} \right)}{\partial q_2} &= -\frac{(1-p)pq_1(1-q_1)}{(pq_1(1-q_2)+(1-p)(1-q_1)q_2)^2} < 0. \\ \frac{\partial \left( \frac{p(1-q_1)q_2}{p(1-q_1)q_2+(1-p)q_1(1-q_2)} \right)}{\partial q_1} &= -\frac{(1-p)pq_2(1-q_2)}{(p(1-q_1)q_2+(1-p)q_1(1-q_2))^2} < 0 \text{ and} \\ \frac{\partial \left( \frac{p(1-q_1)q_2}{p(1-q_1)q_2+(1-p)q_1(1-q_2)} \right)}{\partial q_2} &= \frac{(1-p)pq_1(1-q_1)}{(p(1-q_1)q_2+(1-p)q_1(1-q_2))^2} > 0. \\ \frac{\partial \left( \frac{pq_1q_2}{pq_1q_2+(1-p)(1-q_1)(1-q_2)} \right)}{\partial q_1} &= \frac{(1-p)pq_2(1-q_2)}{(pq_1q_2+(1-p)(1-q_1)(1-q_2))^2} > 0 \text{ and} \\ \frac{\partial \left( \frac{pq_1q_2}{pq_1q_2+(1-p)(1-q_1)(1-q_2)} \right)}{\partial q_2} &= \frac{(1-p)pq_1(1-q_1)}{(pq_1q_2+(1-p)(1-q_1)(1-q_2))^2} > 0. \end{aligned}$$

So, by Lemma 1,  $\frac{\partial a_0^*(0)}{\partial q_1} < 0$  and  $\frac{\partial a_0^*(0)}{\partial q_2} < 0$ ;  $\frac{\partial a_0^*(1)}{\partial q_1} > 0$  and  $\frac{\partial a_0^*(1)}{\partial q_2} < 0$ ;  $\frac{\partial a_1^*(0)}{\partial q_1} < 0$  and  $\frac{\partial a_1^*(0)}{\partial q_2} > 0$ ;  $\frac{\partial a_1^*(1)}{\partial q_1} > 0$  and  $\frac{\partial a_1^*(1)}{\partial q_2} > 0$ . ■

**Proof that with quadratic utilities,  $b_0 \geq b_1$  if and only if  $p \geq \frac{1}{2}$ .**

$$b_0 = \frac{1}{2} \frac{p(1-p)q_2(1-q_2)(2q_1-1)}{(pq_1(1-q_2)+(1-p)(1-q_1)q_2)(p(1-q_1)(1-q_2)+(1-p)q_1q_2)}$$

and

$$b_1 = \frac{1}{2} \frac{p(1-p)q_2(1-q_2)(2q_1-1)}{(pq_1q_2+(1-p)(1-q_1)(1-q_2))(p(1-q_1)q_2+(1-p)q_1(1-q_2))}.$$

The numerators are the same. If we compare the denominators, we see that

$$\begin{aligned} &(pq_1(1-q_2) + (1-p)(1-q_1)q_2)(p(1-q_1)(1-q_2) + (1-p)q_1q_2) - \\ &(pq_1q_2 + (1-p)(1-q_1)(1-q_2))(p(1-q_1)q_2 + (1-p)q_1(1-q_2)) \\ &= q_1(1-q_1)(2q_2-1)(1-2p) \end{aligned}$$

Given that  $q_1, q_2 \in (\frac{1}{2}, 1)$ , we have  $b_0 \geq b_1$  if and only if  $p \geq \frac{1}{2}$ . ■

**Proof that with quadratic utilities,  $\frac{\partial b_0}{\partial q_2}$  has the same sign as  $(p-q_2)$  and  $\frac{\partial b_1}{\partial q_2}$  has the same sign as  $(1-p-q_2)$ .**

$\frac{db_0}{dq_2} = \frac{q_1(1-q_1)(p(1-q_2)+(1-p)q_2)(p-q_2)}{(pq_1(1-q_2)+(1-p)(1-q_1)q_2)^2(p(1-q_1)(1-q_2)+(1-p)q_1q_2)^2}$ . Since all the other terms are positive, the sign of  $\frac{db_0(p, q_1, q_2)}{dq_2}$  is the same as the sign of  $(p-q_2)$ .

$\frac{db_1}{dq_2} = \frac{q_1(1-q_1)(pq_2+(1-p)(1-q_2))(1-p-q_2)}{(pq_1q_2+(1-p)(1-q_1)(1-q_2))^2(p(1-q_1)q_2+(1-p)q_1(1-q_2))^2}$ . Similarly, since all the other terms are positive, the sign of  $\frac{db_1(p, q_1, q_2)}{dq_2}$  is the same as the sign of  $(1-p-q_2)$ . ■

**Proof of Proposition 5.**

Fix  $p, q_1, q_2$ . WLOG, assume that  $b_0 \geq b_{private} \geq b_1$ .

If  $b \leq b_{private}(p, q_1, q_2)$ , then a truth-telling equilibrium exists in  $\Gamma_2$ . Since the DM extracts the maximal amount of information from the expert in  $\Gamma_2$ , he cannot do better in  $\Gamma_3$ .

Now suppose  $b > b_{private}(p, q_1, q_2)$ . So there is no information transmitted in  $\Gamma_2$ .

Suppose  $b > b_0 \geq b_1$ . Then clearly the DM cannot extract any information from the expert in  $\Gamma_3$  either. If  $b > b_0$ , for any belief that the expert may have over the distribution

of  $s_2$ , the bias  $b$  is too high for the IC constraint to hold. As shown before, when truth telling fails to be an equilibrium, babbling is the unique equilibrium outcome.

Suppose  $b_0 \geq b > b_{private} \geq b_1$ . We can show by contradiction that no information can be transmitted from the expert to the DM in  $\Gamma_3$ . First, observe that in a PBE in  $\Gamma_3$ , the expert has to reveal the same amount of information in response to any of the messages sent with positive probability by the DM (and the information revealed by the expert on the equilibrium path has to be at least as much the information revealed off the equilibrium path). Also, recall that an equilibrium where partial information is transmitted from the expert to the DM exists only if a truth-telling equilibrium exists. Suppose in  $\Gamma_3$ , an equilibrium exists in which the expert truthfully reveals  $s_1$  upon receiving either message 1 or message 0 from the DM. Then, for the IC constraints to hold, type 0 expert's posterior belief on the distribution of  $s_2$  upon hearing any one of the DM's messages, i.e.,  $prob(s_2 = 0|l)$  ( $l = 0, 1$ ), must be strictly higher than  $prob(s_2 = 0|s_1 = 0)$ . Obviously, this can never be satisfied with any admissible (mixed) strategy of the DM. Therefore, the expert babbles in equilibrium in  $\Gamma_3$  if  $b_0 \geq b > b_{private} \geq b_1$ . The DM cannot extract more information by strategically sending a message about her signal than by keeping it private.

■

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