Conversion Rates in Auctions for Sponsored Search

[Extended Abstract]

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Abstract

The generalized second-price auction (GSP) is used predominantly for sponsored search in search engines like Google, MSN-Live Search and Yahoo!. It has been shown by [1, 2] that GSP possesses an efficient pure Nash equilibrium. The model studied by [1, 2] assumed that all the clicks on the search engine ads gain the advertiser the same benefit. In practice, when an ad can be shown in one of several positions (a.k.a., slots) on the search results page, often the lower slots have higher acquisition rates. We study the implications of relaxing the identical acquisition rate assumption for GSP. In this case, we show that GSP does not always admit an efficient equilibrium anymore (neither pure nor mixed), even in the special case where ordering the advertisers by bid remains optimal. We show that when the bid space is discrete, an (inefficient) pure Nash equilibrium always exists, and we characterize the equilibria as a function of the parameters of the bidders' preferences. Finally, we quantify the inefficiency of these equilibria.

1 Introduction

Popular search engines like Google, MSN-Live search and Yahoo! sell advertisements alongside their search results. These advertisements are often called *sponsored search*, and search engines gain multi-billion dollars from selling such ads each year. The auctions used by the leading search engines are variants of the *Generalized Second-Price (GSP)* auction, that essentially sells the ith slot to the bidder with the ith highest bid, and each bidder pays, only when his ad is clicked, the minimal bid for which he would still win the same slot.

Two important factors in the design of sponsored-search auctions are the Click-Through-Rate (CTR) and the Acquisition (or Conversion) Rate (AR). CTR captures the probability that an ad is clicked by a user, once this ad is presented to this user in the search engine. AR stands for the probability that a user makes an acquisition, given that he already clicked on the relevant ad. The meaning of an "acquisition" changes according to the relevant industry, or even between advertisers, and may be translated, for instance, to the purchase of an airline ticket, a book, or successfully filling some form on the advertiser's web page. GSP allows, both in theory and in practice, individual CTR's and AR's for each advertiser, but only allows the CTR to differ over slots. It is a common assumption, however, that the AR changes between slots. A possible reason for that is that many clicks on the first slot are made by mistake, thinking that it is the best search result. Actually, users that click on lower slots were probably attracted by the content of the ad and are probably more serious about making an acquisition.

Incorporating this into the analysis of GSP derives that GSP does not admit an efficient equilibrium any more. Nonetheless, GSP does admit pure Nash equilibria in this new model, and in this paper we characterize these equilibria and measure their inefficiency.

2 Our Contribution

The model we study is essentially equivalent to the model studied by [2, 1], when non-identical conversion rates over slots are allowed.

Consider a set of n advertisers competing for k slots for positioning ads for a single keyword (e.g., "Las Vegas hotel"). The probability that a user clicks on slot j is CTR_j, and after slot j was clicked, an acquisition will be made with probability AR_j.[1] Each advertiser i has a value per conversion v_i, therefore his value-per-click on slot j is $AR_j \cdot v_i$, and his value-per-impression is $CTR_j \cdot AR_j \cdot v_i$. The value for no acquisition is zero for each bidder, and the bidders gain no value from impressions or clicks that are not followed by an acquisition.

We study this model as a full-information game, where the action of each bidder is her bid, and the payoffs for each profile of actions is determined by the GSP auction:

Generalized Second-Price (GSP) auction:

Consider a profile of bids $b_1, ..., b_n$, and, w.l.o.g., $b_1 \geq b_2 ... \geq b_n$.

- Allocation: Allocate slots $1, ..., k$ to bidders $1, ..., k$, respectively. Bidders with tied bids are ordered uniformly at random.
- Payments: For each $1 \geq i \geq k$, bidder i pays b_{i+1} per click.

The analysis of the game as a full information game, rather than a game with incomplete information, tries to capture the behavior of the bidders in the dynamic game in which the auction is repeated numerous times. See [2, 1] for further discussion on the full-information model for sponsored search auctions.

Our first result shows that GSP does not possess an efficient equilibrium when the value-per-click differs over slots.

Theorem 1. *In the full-information game, GSP does not always possess an efficient Nash equilibrium.*

This can be shown by a simple example for a 3-bidder 2-slot auction. This negative result is robust in several aspects. First, no efficient equilibrium exists, neither in pure strategies nor in mixed strategies. Second, this claim holds for every bid space, both discrete and continuous. Such inefficiency result can be shown even when the product $CTR_j \cdot AR_j$ decreases with the slot number, and, finally, it holds even when the CTR's and AR's are identical among all advertisers.

The theorem is proved by a simple example. Consider the auction instance described in Figure 1. The figure describe an auction with 3 bidders, with values per conversion of 5,3.5 and 2 respectively. We also set $CTR_1 = 0.4, CTR_2 = 0.2$ and $AR_1 = 0.3, AR_2 = 0.2$. The figure draws two curves per each advertiser, describing their payoff for each slot they receive as a function of the price they pay. We will show that no efficient equilibrium exists for this setting.

Proof. (of Theorem 1)

Consider the above 3-bidder 2-slot auctions. Since $v_1 > v_2 > v_3$, for efficiency we must have that $b_1 > b_2 > b_3$. We will now use the fact that bidder 3 will be willing to be assigned to the second slot with any payment less than 0.6 and bidder 1 prefers slot 2 over slot 1 for payments

[1] In the full paper we also consider a more general model where each advertiser has individual CTR's and AR's for the slots.

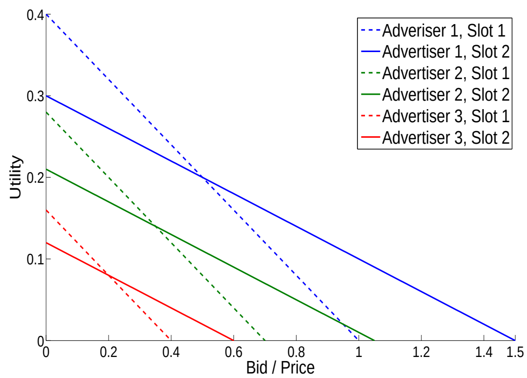

Figure 1: The figure describes the utility for three advertisers in GSP as a function of the price they pay. The auction sells two slots, and each advertiser has a different curve for each slot. In this example, no efficient Nash equilibrium exists.

higher than 0.6. If $b_2 < 0.6$, then bidder 3 is better off by bidding b_2 since his utility for the 2nd slot is non-negative. If $b_2 \geq 0.6$, then bidder 1 is better off by bidding b_2, since for such prices he prefers a lottery of the 2nd and 1st slots over receiving the 1st slot for sure. $\square$

The proof implies that the game will lack efficient equilibria whenever the crossing point of the two curves of bidder 1 (that is, the payment p for which $CTR_1(AR_1 v_1 - p) = CTR_2(AR_2 v_1 - p)$) is smaller than 0.6 (the highest payment for which bidder 3 desires slot 2). In other words, when the values of the bidders are not very far apart, no efficient equilibrium exists. Actually, we show that the above condition is also necessary for the existence of an efficient equilibrium.

2.1 Equilibria in GSP Auctions

We then show that when the AR's decrease with the slot number, pure Nash equilibria do exist. We characterize these equilibria as a function of the parameters of the problems (the CTR's, the AR's and the values of the bidders) for any number of bidders. In these equilibria, some subset of the bidders bid the same bid although they have different values, and therefore they are indistinguishable and the inefficency follows. This analysis holds when the bid space that is allowed for the bidders is discrete (we argue that no pure Nash equilibrium exists when continuous bid space is allowed). Finally, we quantify the inefficiency of these pure Nash equilibria (the "price of anarchy"). More details appear in the full paper.

References

[1] Michael Ostrovsky, Benjamin Edelman, and Michael Schwarz. Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords. *Forthcoming in American Economic Review*, 2006.

[2] Hal Varian. Position auctions. *To appear in International Journal of Industrial Organization*, 2006.