Tiebout's Tale in Spatial Economies: Entrepreneurship, Self-Selection and Efficiency

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Abstract

This paper proposes an equilibrium concept based on Rothschild and Stiglitz (1977 QJE) in local public goods economy a la Tiebout (1956 JPE) with spatial elements. Land, location-specific production technology, and wage differences are introduced, and taxes are land taxes (property taxes). Assuming small group effectiveness in the manner of Wooders (1978), Kaneko and Wooders (1986 MASS), Ellickson et al (1999 Econometrica) and Conley and Wooders (1997 mimeo), we show the existence and efficiency of equilibrium. The key requirement is anonymity of land tax, which is attained by imposing Hamilton's (1975 Urban Studies) zoning constraints (otherwise, no equilibrium based on the logic of Rothschild and Stiglitz). Wage differential across locations are allowed, yet we can assure efficiency of equilibrium in our particular economy despite of intuitions by Tiebout (1956 JPE), Buchanan and Wagner (1970 essay) and Flatters, Henderson and Mieszkowski (1974 JPubE). Lastly, our theorem can be directly applicable to the existence and efficiency of a monocentric city equilibrium in urban economics with commuting time costs even if we allow existence of collective residences such as apartments.

Preliminary. Incomplete. No proofs.

1 A Simple Model

There are finite physical locations in the economy. The set of locations is denoted J, and its representative element is $j \in J$. These locations can be heterogeneous in climate or in geographical features. Each location j has quantity land $L_j > 0$, i.e., each location has only limited amount of land to utilize. There is one numeraire commodity which can be produced by labor at each location $j \in J$ with constant returns to scale location specific technology: that is, amount of labor needed to produce one unit of numeraire commodity at location $j \in J$ is constant and is denoted α_j, which can be dependent on j. At location $j \in J$, prices of privately consumed goods, numeraire commodity, land, and leisure are 1, r_j, and w_j, respectively. Given the constant returns to scale technology, $w_j \alpha_j = 1$. There are finite number of possible public projects: the set of public project is denoted G, and its representative element is $g \in G$. Each public project $g \in G$ can be produced by $c(g)$ units of numeraire commodity. We assume $\emptyset \in G$ with $c(\emptyset) = 0$.

A jurisdiction ω provides a public project and imposes a land tax (property tax) on its residents. Each jurisdiction ω is characterized by a list of its location $j_\omega \in J$, public project $g_\omega \in G$, total land size L_ω, and (specific) land tax t_ω. In addition, jurisdictions can impose zoning constraint (Hamilton, 1975 Urban Studies): i.e., the size of land lot for each household can be fixed at ζ_ω (if no zoning constraint, $\zeta_\omega = \emptyset$). In the basic analysis, we assume that a zoning restriction $\zeta_\omega \in \mathbb{R}_+$ ($\zeta_\omega \neq \emptyset$) is placed in each jurisdiction ω. The total land size L_ω is determined by how many residents joins the jurisdiction, and $L_\omega = \zeta_\omega n_\omega$ follows under zoning requirement ζ_ω, where n_ω is the number of households in jurisdiction ω.

There are finite types of consumers. The set of types is denoted Θ, and its representative element is $\theta \in \Theta$. A type θ consumer has a location-specific utility function $u_j^\theta : \mathbb{R}_+ \times [0, \bar{\ell}_j^\theta] \times \mathbb{R}_+ \times G \times \mathbb{Z}_{++} \to \mathbb{R}$ for each $j \in J$, where $u_j^\theta(x, \ell, L, g, n)$ denotes type θ's utility who lives in a juristiction at location j that provides public project g with n residents, consuming private good, land and labor supply by x, L and ℓ. We assume that u_j^θ is a continuous function. Type θ consumer is endowed with land vector $(\bar{L}_j^\theta)_{j \in J}$ That is, a consumer's utility depends on private goods consumption x, L and ℓ, public project g and the level of congestion n as well as her choice of location j itself. Note that we assume that the congestion in public project is **anonymous**. It does not depend on who to share a public project with. In a jurisdiction ω at location j, type θ's budget constraint is denoted

$$x + (r_j + t_\omega)\zeta_\omega \leq w_j(\bar{\ell}_j^\theta - \ell) + \sum r_j \bar{L}_j^\theta,$$

where $L = \zeta_\omega$ if ω has a zoning restriction. Thus, type θ's utility by choosing jurisdiction ω is

$$U^\theta(\omega) \equiv \max_{x,\ell} u_j^\theta(x, \ell, \zeta_\omega, g_\omega, n_\omega) \quad s.t. \quad x + (r_j + t_\omega)\zeta_\omega \leq w_j(\bar{\ell}_j^\theta - \ell) + \sum r_j \bar{L}_j^\theta,$$

and thus, type θ's jurisdiction choice correspondence is

$$\omega^*(\theta) \equiv \arg\max_{\omega \in \Omega} U^\theta(\omega).$$

Each jurisdiction has a manager who maximizes its fiscal surplus, $t_\omega \zeta_\omega n_\omega - c(g_\omega)$ (tax revenue minus expenditure) by choosing a policy $(j_\omega, g_\omega, t_\omega, \zeta_\omega, n_\omega)$. A manager knows consumers' utility functions and other jurisdictions' policy choices, and chooses a profit maximizing policy that can attract consumers (for its residents, that jurisdiction policy gives the highest payoffs). This setup allows a jurisdiction manager to attract potential residents to her jurisdiction to raise fiscal surplus instead of taking her resident profile as given. This setup take goes back to Rothschild and Stiglitz (1977) in insurance market model, but in local public goods economy, it was first used in Epple and Romer (1991) (and Bewley (1981) to some extent uses the same idea).[1] It is important to use this setup in order to attain the efficiency of equilibrium.

We impose a few key assumptions.

Assumption 1. There are a continuum of consumers. The measure (population) of type θ consumer is denoted $m^\theta > 0$ and $\sum_{\theta \in \Theta} m^\theta = 1$.

This assumption is standard in local public goods economy in order to avoid integer problems that cause nonexistence of equiliubrium. The next assumption is the key for our result.

Assumption 2. Each jurisdiction can have only a finite number of residents, and the number is bounded above. That is $n_\omega \leq \bar{n}$.

This assumption is formulated in a various ways with various names. However, the simplest way to state is the above one. Wooders (1978 JET) is the first paper that introduced this assumption in a large finite economy. Kaneko and Wooders (1986 MASS) extended it in a continuum economy in oder to dismiss a small scale integer problem. Bewley (1981 Econometrica) made many critical comments on Tiebout's tale, but his negative results are partly from not adopting this assumption. Finiteness of residents in each jurisdiction together with a continuum of consumers (and finite types) guarantee the integer problems to vanish. Note that Assumption 2 necessarily implies that there are a continuum of jurisdictions in the economy. Ellickson et al. (1999 Econometrica) and Conley and Wooders (1998 mimeo) prove the existence of equilibrium and the first welfare theorem in local public goods economies with passive jurisdiction managers under completeness of markets.[2] Here, we add spatial structure and

[1] Caplin and Nalebuff (1997) distinguished these two setups clearly by calling position-based and membership-based equilibria.

[2] The model in Ellickson et al. (1999 Econometrica) is a club economy and consumers are allowed to join multiple clubs, but there is not big difference with local public goods economy since the number of clubs each consumer can join is bounded above. They also allow infinite number of consumer types, while the number of public project is finite.

land into their model, and analyze an equilibrium concept in which jurisdiction managers are entrepreneurs.

The equilibrium is described as follows. Since there will be a continuum of jurisdictions that use the same policies, we use ω to represent a policy of its jurisdiction $(j_\omega, g_\omega, t_\omega, \zeta_\omega, n_\omega)$ instead of a jurisdiction itself (there will be many jurisdictions that use the same policies). That is, we set $\omega = (j_\omega, g_\omega, t_\omega, \zeta_\omega, n_\omega)$, and Ω is the set of available policies.

A **Tiebout equilibrium with entrepreneural jurisdictions** is a list of $((r_j^*, w_j^*)_{j\in J}, \Omega^*, (j_\omega, g_\omega, t_\omega, \zeta_\omega, n_\omega)_{\omega\in\Omega^*}, (m_\omega^\theta, x_\omega^\theta, \ell_\omega^\theta)_{\theta\in\Theta,\omega\in\Omega^*})$ such that

1. (Optimality of Private Consumption Choice) For all $\omega \in \Omega^*$, and all $\theta \in \Theta$ with $m_\omega^\theta > 0$, $(x_\omega^\theta, \ell_\omega^\theta) \in \arg\max_{x,\ell} u_{j_\omega}^\theta(x, \ell, \zeta_\omega, g_\omega, n_\omega) \quad s.t. \quad x + (r_{j_\omega}^* + t_\omega)\zeta_\omega \leq w_{j_\omega}^*(\bar{\ell}_j^\theta - \ell) + \sum r_{j_\omega}^* \bar{L}_{j_\omega}^\theta$,

2. (Optimality of Jurisdiction Choice) For all $\omega \in \Omega^*$, and all $\theta \in \Theta$ with $m_\omega^\theta > 0$, we have $\omega \in \arg\max_{\omega'\in\Omega^*} U^\theta(\omega')$,

3. (Land Market Clearing) $\sum_{\theta\in\Theta}\sum_{\omega\in\Omega^*,\ j_\omega=j} m_\omega^\theta \zeta_\omega = \bar{L}_j$ for all $j \in J$,

4. (Labor Market Clearing) $w_j^* = \frac{1}{\alpha_j}$ for all $j \in J$,

5. (Numeraire Commodity Market Clearing) $\sum_{\theta\in\Theta}\sum_{\omega\in\Omega^*,\ j_\omega=j} m_\omega^\theta \alpha_j \ell_\omega^\theta = \sum_{\theta\in\Theta}\sum_{\omega\in\Omega^*} m_\omega^\theta x_\omega^\theta + \sum_{\omega\in\Omega^*}(\sum_{\theta\in\Theta} m_\omega^\theta)\frac{c(g_\omega)}{n_\omega}$,

6. (Jurisdiction's Zero Profit Condition) $t_\omega \zeta_\omega n_\omega = c(g_\omega)$ for all $\omega \in \Omega^*$,

7. (Exhausted Profit Opportunities by Entrepreneural Jurisdictions) For all $\omega \in \Omega\backslash\Omega^*$with $t_\omega \zeta_\omega n_\omega > c(g_\omega)$, we have for all $\theta \in \Theta$,

$$\begin{aligned} &\max_{\omega'\in\Omega^*} U^\theta(\omega') \\ > \quad &\max_{x,\ell} u_{j_\omega}^\theta(x, \ell, \zeta_\omega, g_\omega, n_\omega) \quad s.t. \quad x + (r_{j_\omega}^* + t_\omega)\zeta_\omega \leq w_{j_\omega}^*(\bar{\ell}_j^\theta - \ell) + \sum r_{j_\omega}^* \bar{L}_{j_\omega}^\theta. \end{aligned}$$

The key of the above definition is that we distinguish Ω^* (observable jurisdiction policies) and $\Omega\backslash\Omega^*$ (unobservable jurisdiction policies). Jurisdiction managers can easily observe how profitable a policy is as long as there is a jurisdiction that chooses that policy. However, if a policy is not chosen by any jurisdiction, a manager needs to make a guess how profitable it would be by utilizing her information on consumers' utilities (in the manner of Rothschild and Stiglitz, 1977 QJE). This entrepreneurship is captured in equilibrium condition 7. In contrast, if managers are passive, there can be many inefficient equilibria

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Conley and Wooders (1998 mimeo) and Allouch, Conley and Wooders (2005 mimeo) allow infinite number of public projects, while the number of consumer types is finite.

Here, the completeness of markets means that it is known for jurisdiction managers that how much t each jurisdiction policy (j, g, ζ, n) can raise through price mechanism (completeness of prices). We do not deal with incompleteness associated with time and uncertainty (c.f. Magill and Shafer, 1991, essay in Handbook of Math Econ).

if no jurisdiction choose potentially profitable policies that are not observable. The equilibrium concept here differs from Ellickson et al. (1999 Econometrica) and Conley and Wooders (1998 mimeo) since they assume that markets for all $\omega \in \Omega$ exist. In contrast, we assume that there are markets only for observable policies $\Omega \backslash \Omega^*$ may not be empty. If it were the case in their paper, equilibrium may be inefficient since there may be a jurisdiction that can do better than the existing ones.[3] We managed to eliminate this possibility by adopting an equilibrium concept in Rothschild and Stiglitz (1977 QJE). Wooders (1978) and Bewley (1981 Econometrica) have similar ideas, but instead of exhausting possible policies proposable by jurisdiction managers, they consider a coalitional deviation constructed by a group of consumers. The main difference is that coalitional deviation is initiated by consumers, while profit opportunities are seeked by jurisdiction managers having information on consumers' preferences.

The main results are stated below. The key observation is the following lemma.

Lemma 1 *In all Tiebout equilibria with entrepreneural jurisdictions, we have* $U^\theta(\omega) = \max_{\omega' \in \Omega} U^\theta(\omega')$ *for all* $\theta \in \Theta$ *and all* $\omega \in \Omega^*$ *with* $m_\omega^\theta > 0$.

This result contrasts with Rothschild and Stiglitz (1977 QJE). In Rothschild and Stiglitz (1977 QJE), there is no equilibrium due to nonanonymous crowding. However, in our Tiebout economy, as long as zoning constraint exists then property trax payment is common to all residents, and there is no free riding. *Hamilton's (1975 Urban Studies) idea is essential in obtaining a positive result in both existence and efficiency of equilibrium.*

Theorem 2 *There exists a Tiebout equilibrium with entrepreneural jurisdictions, and it is Pareto efficient under the following assumptions on utility functions:*

1. *for all* $\theta \in \Theta$, *all* $j \in J$, *all* $g \in G$, *all* $n \in \{1, ..., \bar{n}, \}$, $u_j^\theta(x, \ell, L, g, n)$ *is continuous and strictly monotonic in* (x, ℓ, L),

2. *for all* $\theta \in \Theta$, *all* $j \in J$, *all* $\ell_j \in [0, \bar{\ell}_j^\theta]$, *all* $L \in \mathbb{R}_+$, *all* $g \in G$, *all* $n \in \{1, ..., \bar{n}\}$, $u_j^\theta(0, \ell, L, g, n) = \min_{j', x', \ell', L', g', n'} u_{j'}^\theta(0, \ell', L', g', n')$ *(essentiality of private good).*

Remark 3 *We do not need convexity of preferences. Condition 1 makes prices of private goods strictly positive, and condition 2 assures upper hemi continuity of consumers' jurisdiction choice correspondence (for existence).*

Remark 4 *The method of the proof of existence theorem is to utilize a poll tax. First, we assume that each jurisdiction charge a poll tax* $\tau = c(g)/n$ *for a policy* $(g, n) = \omega \in \Omega$. *Assuming complete price system for all possible* ω*s,*

[3] The same remark applies to Sonstelie and Portney (1976 JUE), Scotchmer (1994 essay) and Wildasin (1994 mimeo).

we can find an equilibrium, by using Ellickson et al.'s argument for local public goods together with Berliant and Konishi's (2000, RSUE) arguments for spatial aspects. (Since we need to accommodate zoning aspects, it is hard to dispense finite types of consumers. Given finiteness of Θ*, we can simply use Konishi's (1996, JET) simple fixed point mapping.) For each* $\omega \in \Omega^*$*, there is at least a* $\theta \in \Theta$ *with* $m_\omega^\theta > 0$*. For them, construct a zoning policy* $\zeta_\omega = L_\omega^\theta$ *with* $t_\omega = c(g_\omega)/n_\omega \zeta_\omega$*. This works as a Hamilton's zoning policy. This shows that property tax is distortionary unless zoning policy is placed (*MRS *at the zoning land consumption level is not the same as* $r_{j_\omega}^* + t_\omega$*). Finally, drop complete price assumption, and place requirement 7.*

Remark 5 *Note that wage rates in different locations are different due to productivity difference. Tiebout (1956 JPE) says that restrictions due to employment opportunities are not considered (his assumption 4) in stating his conjecture. Buchanan and Wagner (1970 essay) and Flatters, Henderson and Mieszkowski (1974 JPubE) elaborate Tiebout's statement mentioning that spatial employment opportunity cannot be allowed to attain efficiency of equilibrium since voting with feet is a utility equalization process instead of marginal product equalization one. In their finite number of jurisdiction case, their assertion is completely true. However, in our model, spatial wage differentials play no role in achieving efficiency of equilibrium. The proof of the first welfare theorem needs to utilize McKenzie's trading sets instead of consumption sets.*

Remark 6 *Our theorem can easily extended to more general setting with many private goods, general (CRS) production technologies, and many different occupations consumers can choose from. Conley and Wooders (1998 Reserch in Economics) consider occupation choice with occupation-dependent crowding type, and show that equilibrium is efficient if occupation-dependent nonanonymous tax can be imposed. In contrast, if we just allow utility and wage are different by occupation choices then we do not need such occupation-dependent taxes to attain efficiency.*

Remark 7 *Our theorem can be regarded as a spatial version of Conley and Wooders (1997), Ellickson et al. (1999) and Allouch, Conley and Wooders (2006), or a formalization of Sonstelie and Portney (1976, JUE). Although our result is positive, we need to be careful about how to interpret the theorem. Note that our theorem holds only in an idealized situation, since it requires that there are very many jurisdictions for each land type, which is not very realistic. If there are limited number of jurisdictions in each location, then two problems can occur: Monopoly powers by jurisdictions, and insufficient choice sets provided by jurisdictions. Moreover, the model is static, so our theorem does not answer how to rearrange jurisdiction borders when new a jurisdiction is set up, or how to ask the current residents to move from an existing jurisdiction that is not profitable. Thus, there are more frictions in the presence of a spatial structure, the Tiebout's tale is harder to be justified.*

Despite of the above cautious remark, our theorem may find a useful application. We can directly apply our theorem to prove the existence and efficiency of a closed-economy monocentric city equilibrium with transport cost (in commuting time) with possible collective residential buildings (apartments instead of houses) as long as there are finite number of rings of heterogeneous land (dstinguished by the distance from the CBD. In our model, let us order locations $j_0, j_1, ..., j_K$, and assume that production can be made only in the CBD location j_0: i.e., $\alpha_j = \infty$ for $j \neq j_0$, while $\alpha_{j_0} < \infty$. As an index k increases, the distance from the CBD increases. As a result, for all $\theta \in \Theta$, $\bar{\ell}^{\theta}_{j_0} > \bar{\ell}^{\theta}_{j_1} > ... > \bar{\ell}^{\theta}_{j_K}$. If the geography is one-dimensional (linear city), then we may assume $\bar{L}_{j_k} = \bar{L}_{j_{k'}}$ for all $k, k' \in \{0, 1, ..., K\}$. If it is two dimensional, then we may assume $\bar{L}_{j_0} < \bar{L}_{j_1} < ... < \bar{L}_{j_K}$. We interpret $\omega = (g_\omega, n_\omega) \in \Omega$ be a building that can be a house or an apartment: g_ω is a type of building (say, high quality, low quality, with a swiming pool, or with a nicely landscaped garden, etc.) and n_ω is the number of households living in the building (if $n_\omega = 1$ then it is a single household house, and n_ω is large it is an apartment complex). Our theorem says that there is an equilibrium sorting with various housing qualities including collective housing such as apartments.[4] Assuming that land at each location is physically the same and that land is a normal good, it is easy to see that land price goes down as index k increases, since choosing a smaller index location means more income for all $\theta \in \Theta$. However, other characteristics of equilibrium allocations need more assumptions on consumers' preferences over Ω and distribution of their land endowments.

Actually, "monocentric" assumption is not important for the existence and efficiency results. Even if a consumer can choose her locations of residence and work freely, our results are not affected. The only modification needed is that now we need to assume that each consumer (say type θ) chooses a pair of locations $(j, j') \in J \times J$, and leisure endowment for the choice is $\bar{\ell}^{\theta}_{jj'}$ since her commuting time depends on her residential and work locations (see Konishi, 1996 JET).

[4]Finiteness can be dropped if collective residential buildings are assumed away (see La Fountain, 2005 accepted in JPET).