Bargaining with Externalities*

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Abstract

This paper studies bargaining between a seller and multiple buyers with externalities. A full characterization of the stationary subgame perfect equilibria in generic games is presented. Equilibria exist for generic parameter values, with delay only for strong positive externalities. The outcome is efficient if externalities are not too positive. Increasing the bargaining power of the seller decreases the set of parameter values for which only efficient equilibria exist.

The paper generalizes the model presented in Jehiel & Moldovanu (1995a and 1995b). Where they find delay equilibria, we find mixed equilibria, except for a region where no stationary equilibria exist. These mixed equilibria entail no delay and the equilibrium strategies converges to pure as the discount factor approaches one. We are able to show existence of stationary equilibria given a reasonable restriction on parameters. We find delay with strong positive externalities, due to a hold-up problem. All equilibria without delay have the property that agreement is with a specific buyer in the limit.

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1 Introduction

Many agreements in society are determined through bargaining. Wages are often described as being determined through a perhaps implicit negotiation. Sales contracts are often bilateral and long term. Standard models often analyze two players bargaining with each other; see e.g., Rubinstein (1982). It is not uncommon, however, to have a seller bargaining with more than one buyer, see e.g., Horn & Wolinsky (1988) and Stole & Zweibel (1996a,b). Also, it seems reasonable that an agreement with one buyer can impose externalities on the other buyers. As an example, a patent right can be sold by a research firm to one of many manufacturing firms. This reasonably imposes externalities on the other manufacturers.

In this paper, we analyze bargaining between a seller and many buyers when externalities are present. We provide a full characterization of generic stationary subgame perfect equilibria.

We find that for almost all parameter values only certain equilibrium types exist. There are *single out* equilibria, where agreement is always with a specific buyer. There are *outside option* equilibria, where there is agreement with two buyers, with the probability of agreement with the second buyer arbitrarily small as the discount factor approaches 1. There are also *hold-up* equilibria, where there is agreement with a subset of buyers, all with a probability less than 1. Thus, as compared to the possible equilibrium type candidates, genericity leads to a dramatic reduction in which equilibrium types exist. If one does not confine attention to generic equilibrium types, many equilibrium types exist. There are parameter values with equilibrium types that have almost any division of the buyers into sets where agreement occurs with probability one, with probability less than one and with zero probability. Equilibria need not exist for all generic parameter values. However, in both examples in Jehiel & Moldovanu (1995a and 1995b) that entail delay, we find mixed equilibria, when parameters are generic. We find a reasonable condition on payoffs that guarantees existence of stationary equilibria for all generic parameter values satisfying this condition. Furthermore, under some conditions, with negative externalities, only efficient equilibria exist.

The bargaining model is a generalization of Jehiel & Moldovanu (1995a and 1995b), allowing for different degrees of bargaining power. Allowing for mixed equilibria simplifies the analysis greatly. In particular, we show that stationary equilibria in mixed strategies always exist, in regions where they do not find equilibria. In the limit, as the discount factor converges to one, these equilibrium strategies converge towards pure strategies. With negative externalities, we do not get delay, in stark contrast with Jehiel & Moldovanu. In the limit as the discount factor converges to one, there will be immediate agreement with a specific buyer.

Only the hold-up equilibria entail substantial delay. These equilibria only exist with strong positive externalities, when all buyers would prefer not to agree.

In section 2 the model is described. Section 3 characterizes the equilibria, section 4 provides sufficient conditions for equilibria to be efficient and finally section 5 concludes. All proofs are relegated to the appendix.

2 The Model

One seller bargains with a finite set N of more than one buyers on the sale of an indivisible good. The surplus of selling to buyer i is $\pi_i > 0$, with all other buyers j receiving their externality $e_{j,i}$, setting $e_{i,i} = 0$. Let $\Omega \subset \mathbb{R}_+^{|N|} \times \mathbb{R}^{|N|(|N|-1)}$ denote the set of possible pies and externalities. We assume that in each round all buyer seller pairs meet with equal probability. Generalizing the model of Jehiel & Moldovanu (1995a and 1995b), the seller makes a bid with probability η, and with $1-\eta$ the buyer does. It is also a generalization in the respect that we allow for equilibria in both pure and mixed strategies.

Let $v_{S,i}$ and $w_{S,i}$ denote the value to the seller in bidding and receiving a bid from buyer i, and $v_{i,S}$ and $w_{i,S}$ denote the value to buyer i of bidding and receiving a bid. Let $p_{S,i}$ be the probability that the seller gives an acceptable bid to i when bidding and $p_{i,S}$ the probability that i gives an acceptable bid. Defining $p_i = (p_{S,i} + p_{i,S})/2$, the value equations are given by:

$$\begin{aligned} v_{S,i} &= (1-p_{S,i})\, w_{S,i} + p_{S,i}\,(\pi_i - w_{i,S}) \\ w_{S,i} &= \delta\left(\frac{\eta}{|N|}\sum_{j\in N} v_{S,j} + \frac{1-\eta}{|N|}\sum_{j\in N} w_{S,j}\right) \\ v_{i,S} &= p_{i,S}\,(\pi_i - w_{S,i}) + (1-p_{i,S})\, w_{i,S} \\ w_{i,S} &= \delta\frac{1}{|N|}\left((1-\eta)\, v_{i,S} + \eta w_{i,S}\right) + \delta\sum_{j\in N\setminus\{i\}}\frac{p_j}{|N|}e_{i,j} + \delta\sum_{j\in N\setminus\{i\}}\frac{1-p_j}{|N|}w_{i,S} \end{aligned} \tag{1}$$

When negotiating with i, in giving an acceptable offer (with probability $p_{S,i}$) it is sufficient to offer $w_{i,S}$ to i. Since $w_{S,i}$ is the continuation value conditional on disagreement, the value $v_{S,i}$ in (1) follows. By similar reasoning $v_{i,S}$ is determined. When rejecting a proposal by i, S gets $v_{S,j}$ with probability η and $w_{S,j}$ with $1-\eta$, giving $w_{S,i}$ in (1). When i rejects a proposal, i is selected to bargain with S with probability $\frac{1}{N}$ giving $(1-\eta)\, v_{i,S} + \eta w_{i,S}$. If some other player j is selected, i will receive $e_{i,j}$ if S and j agree in the next period. With probability $1-p_j$ they do not, giving $w_{i,S}$.

Note that, since the left hand side of the value equation for $w_{S,i}$ is the same for all i, we have $w_{S,i} = w_{S,j}$ for all i and j. Hence we can write

$$w_{S,i} = \frac{\delta\eta}{1-\delta(1-\eta)} \frac{1}{|N|} \sum_{j \in N} v_{S,j} \tag{2}$$

For it to be profitable to make an acceptable offer, it is necessary that

$$\pi_a - w_{a,S} \geq w_{S,a}. \tag{3}$$

Similarly, for the seller and m to bid with $0 < p_m < 1$, they have to be indifferent between bidding and not:

$$\pi_m - w_{m,S} = w_{S,m}. \tag{4}$$

It is also necessary that in negotiations with buyers where $p_r = 0$ that it is profitable to make unacceptable offers, i.e., that

$$\pi_r - w_{r,S} \leq w_{S,r}. \tag{5}$$

2.1 Genericity

As noted in the introduction, it turns out that many equilibrium types exist only for special parameter configurations. More specifically, the set of π_i and e_{ij} that support these equilibrium types have strictly lower dimensionality than the full parameter space as $\delta \to 1$. This implies, that for $\delta \approx 1$, the region where these equilibrium types exist is arbitrarily small.

Agreement with several sellers cannot be a generic equilibrium type. In order for equilibria with agreement with several buyers with probability 1 to exist for δ close to one, the seller essentially has to be indifferent between who he agrees with. If not, he could simply wait. Indifference as $\delta \to 1$ will imply that parameters are non-generic, however

To illustrate this point, consider the case with two buyers, and conjecture an equilibrium with immediate agreement with both buyers along the lines of Horn & Wolinsky (1988)

In the proposed equilibrium $p_1 = p_2 = 1$. Using this in (1) gives

$$w_{a,S} = \delta \frac{(1-\eta)(\pi_a - w_{S,a}) + e_{a,j}}{2-\delta\eta}$$

for $j \neq a$ and

$$w_{S,a} = \frac{\delta\eta}{2(1-\delta(1-\eta)) - \delta\eta} \frac{1}{2}\left((2-\delta)(\pi_1 + \pi_2) - \delta(e_{1,2} + e_{2,1})\right)$$

Using these in (3) we get

$$\pi_a - \frac{\delta}{2-\delta} e_{a,j} \geq \frac{\delta\eta}{2(1-\delta(1-\eta)) - \delta\eta} \frac{1}{2} \left((2-\delta)(\pi_1 + \pi_2) - \delta(e_{1,2} + e_{2,1})\right)$$

for $j \neq a$. We have, in the limit

$$\pi_1 + e_{2,1} \geq \pi_2 + e_{1,2}$$
$$\pi_2 + e_{1,2} \geq \pi_1 + e_{2,1}$$

This implies that $\pi_1 + e_{2,1} = \pi_2 + e_{1,2}$, implying an additional restriction on the parameters for this equilibrium to exist. Hence, the equilibrium is non-generic.

Let σ denote a stationary strategy profile. Given σ, let $A \subset N$ be the set of buyers that agree with probability one, and let M and R denote the set of buyers that agree with mixed and zero probabilities respectively. Let $\Phi(\sigma) = (|A|, |M|, |R|)$ denote the *equilibrium type* of σ. Let $\Sigma(\omega, \delta)$ denote the correspondence from the set of parameters $\omega \in \Omega$ and δ to the (possibly empty) set of stationary equilibria for these parameters. Define

$$\Omega(u, \delta) = \{\omega \in \Omega : \ \exists \sigma \in \Sigma(\omega, \delta) \text{ such that } \Phi(\sigma) = u\}$$

as the set of parameter values generating the equilibrium type u, given δ. Let λ denote a Lebesque measure of subsets of Ω.

Definition 1 *The equilibrium type u is* generic *if* $\lim_{\delta \to 1} \lambda(\Omega(u, \delta)) > 0$.

Note that genericity for equilibrium types is not defined in the strong sense that it exists for almost all parameter values. It is sufficient that it has positive measure. Non-generic equilibria exists only on sets of measure zero, though.

3 Equilibrium Characterization

In general, there are a large number of equilibrium types, since we can divide buyers into three sets A, M and R where agreement occurs with probability one for $a \in A$, with positive probability less than one form $m \in M$ and with zero probability for $r \in R$. Any partition of the set of players in three such sets is an equilibrium candidate. However, to provide a characterization of the generic SSPE, it turns out that it is sufficient to study the following 3 cases. As we will show in Proposition 5, these are the generic equilibrium types.

1 $p_a = 1$ for some $a \in N$ $\quad p_r = 0$ for $r \neq a$

2 $0 < p_m < 1$ for $m \in M$ $\quad p_r = 0$ for $r \in R = N \backslash M$

3 $p_a = 1$ for some $a \in N$ $\quad 0 < p_m < 1$ for some $m \neq a$ $\quad p_r = 0$ for $r \neq a, m$

Let $E_{R,M}$ be the matrix of externalities $e_{r,m}$ to $r \in R$ when $m \in M$ agree, and $E_{M,M}$ the matrix of externalities to $m \in M$. Let π_M the vector with π_m as the m'th element for $m \in M$ with π_R similarly defined and let Π_M and Π_R be diagonal matrices with π_M and π_R respectively on the main diagonals. Let $J_{x,y}$ be a $|x| \times |y|$ matrix with all elements 1, I_x the $|x|$ dimensional identity matrix, and j_x a $|x|$ vector of ones.

Proposition 2 *There exists a $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$ there exists an equilibrium with $p_a = 1$ for some $a \in N$, and $p_r = 0$ for all $r \neq a$ if*

$$\pi_r - e_{r,a} < \eta \pi_a \tag{6}$$

for all $r \neq a$.

Since $p_a = 1$ and $p_r = 0$ for all other buyers r we can think of the equilibrium payoff as being a situation where the seller only bargains with a and the surplus consists of π_a, giving the seller $\eta \pi_a$. If the seller were to deviate and agree with r instead, the net payoff is $\pi_r - e_{r,a}$. The condition (6) then says that such deviations are unprofitable.

Note that, if externalities are sufficiently positive, an equilibrium of the type in Proposition 2 exists, since then there must be some a for which (6) holds.

Proposition 3 *There exists a $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$ there exists an equilibrium with $p_m > 0$ for $m \in M \subseteq N$ with $|M| > 1$ and $p_r = 0$ for $r \in R = N \backslash M$ if $E_{M,M} - \Pi_M \cdot (J_{M,M} - I_M)$ is invertible,*

$$(E_{M,M} - \Pi_M \cdot (J_{M,M} - I_M))^{-1} \cdot \pi_M \gg 0 \tag{7}$$

and

$$\pi_R \ll (E_{R,M} - \Pi_R \cdot J_{R,M}) \cdot (E_{M,M} - \Pi_M \cdot (J_{M,M} - I_M))^{-1} \cdot \pi_M. \tag{8}$$

The equilibrium does not exist if $e_{i,j} < 0$ for all i, j.

The conditions in the Proposition are rather technical. Equilibrium probabilities is equal to expression (7) times $|N| \frac{1-\delta}{\delta}$ and expression (8) is the condition that all $r \in R$ makes unacceptable proposals, i.e., condition (5). In the case with two buyers, the equilibrium can be more easily illustrated. First, since $|M| > 1$ then R is empty. Second, condition (7) can be rewritten as

$$p_1 = 2\frac{1-\delta}{\delta}\frac{\pi_2}{e_{2,1}-\pi_2},$$
$$p_2 = 2\frac{1-\delta}{\delta}\frac{\pi_1}{e_{1,2}-\pi_1}.$$

First, probabilities are positive only when $e_{2,1} > \pi_2$ and $e_{1,2} > \pi_1$. Thus, externalities have to be larger than the surpluses in case of agreement. Thus if buyers could choose between getting the entire surplus of agreement or getting the externality, they actually prefer the externality. This generates a hold-up problem, which forces probabilities being close to zero. Also, using (1), equilibrium payoffs can be shown to be zero for the seller and π_i for buyer i, implying inefficiencies.

Proposition 4 *There exists a $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$ there exists an equilibrium with $p_a = 1$, $p_m > 0$ for some $a, m \in N$ and $p_r = 0$ for all $r \neq a, m$ if*

$$\infty > \frac{\pi_m - e_{m,a} - \eta\pi_a}{\pi_a + e_{m,a} - \pi_m - e_{a,m}} > 0, \tag{9}$$

$$\pi_m > e_{m,a} \tag{10}$$

and

$$\pi_r - e_{r,a} < \pi_m - e_{m,a} \tag{11}$$

for all $r \neq a, m$.

The first condition (9) in the proposition is just the condition that the probability p_m is positive. As $\delta \to 1$ it can be shown that this probability converges to zero. To understand the second condition (10), note that the equilibrium payoff of the seller is $\pi_m - e_{m,a}$, i.e., the payoff of the seller is equal to the payoff if switching and agreeing with m instead of a. Thus, agreeing with m can be seen as the outside option of the seller. The last condition (11) is that the seller would not prefer to deviate and agree with some buyer r, earning the net payoff $\pi_r - e_{r,a}$.

The next proposition not only shows that the equilibrium types in the propositions above exist generically, but also that no other equilibrium types do so.

Proposition 5 *The generic equilibrium types are the following;*

1. *Single out: $u = (1, 0, |N| - 1)$ for all $i \in N$*

2. *Outside option: $u = (1, 1, |N| - 2)$ for all $i \in N$ and $j \neq i$.*

3. Hold-up: $u = (0, i, |N| - i)$ *for all* $1 < i \leq |N|$.

Note that any equilibrium type in Proposition 5, that entails agreement with probability one prescribes agreement with probability one with exactly one buyer. This is proven by using that, in the limit, the conditions for acceptance (3) holds with equality for all buyers in A, rendering additional restrictions on the parameter space if $|A| > 1$. A slightly more complicated argument shows rules out $|M| > 1$ in the outside option equilibrium types. Thus, using that the conditions for acceptance (3) holds with equality in the limit dramatically reduces the equilibrium type candidates.

Now we will show that for almost all parameter values, there exists an equilibrium for δ sufficiently close to 1. Let

$$\bar{\Omega} = \{\omega : \pi_i - e_{i,j} \neq \pi_j - e_{j,i} \text{ and } \pi_j - e_{j,i} \neq \eta\pi_i \text{ for all } i, j\}.$$

As the complement of $\bar{\Omega}$ is the union of a finite set of hyperplanes, it has strictly lower dimension than Ω. Almost always there exists a single out and/or a outside option equilibrium. Note that Proposition 5 above does not show that non-existence is non-generic. It is consistent with the proposition that there is a set of non-zero measure where no stationary equilibrium exists. That this can be the case is illustrated in the following example.[1]

Example 6 *In the following 3 buyer example, there does not exist a hold-up, single out or outside option equilibrium. We assume that*

$$\pi_N = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} \textit{ and } E_{N,N} = \begin{pmatrix} 0 & 1 & 4 \\ 3 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Assume that $\eta < 0.2$. *It is easily verified that (6) is violated for all* $a \in N$ *and all* $r \neq a$. *Also, we have* $\pi_i > e_{i,j}$ *for all* $i \in N$ *and* $j \neq i$ *implying that (7) is violated. Thus, there are no single out and hold-up equilibria. To check whether there are outside option equilibria, since (11) holds and there is agreement with a with probability* 1, *buyer* m *must be the buyer solving*

$$m = \arg\max_i \pi_i - e_{i,a}.$$

[1] The examples in Jehiel & Moldovanu (1995a and 1995b) are nongeneric. However, for nay small perturbation of parameter values leading to genericity, we find stationary equilibria.

The matrix of payoff differences $\pi_i - e_{i,j}$ *is*

$$\begin{pmatrix} 0 & 4 & 1 \\ 1 & 0 & 3 \\ 2 & 2 & 0 \end{pmatrix}.$$

Thus, if $a = 1$ *then* $m = 3$*, if* $a = 2$ *then* $m = 1$ *and if* $a = 3$ *then* $m = 2$*. In addition, (9) must hold. Since* $\eta < 0.2$ *the numerator of the ratio in (9) is positive. However, for all possible choices of* a*, the denominator is negative, implying that there is no outside option equilibrium.*

The following condition guarantees existence, though.

Condition 7 *The parameter vector* $\omega \in \Omega$ *satisfies* ***surplus-monotonicity*** *if when* $\pi_i > \pi_j$ *we have*

$$\pi_i - e_{i,k} > \pi_j - e_{j,k}.$$

Thus, if the own surplus π_i increases the total gain $\pi_i - e_{i,k}$ of agreement for a player i also increases. For example, if firm i is larger than j, profits in case of trade is larger for i than j. Externalities imposed on i might also increase, but not enough to offset the increase in own surplus.

Furthermore, surplus-monotonicity holds when $\pi_i > \pi_j$ implies $e_{i,k} \leq e_{j,k}$. This is clearly reasonable when externalities are negative. A larger surplus in case of agreement implies that there is a larger loss when the seller agrees with some other buyer. This could be the case when the seller sells a patent to goods producers. A firm with a larger plant can gain more when buying the patent, since an increase in unit revenues leads to a larger total surplus. It also looses more than a small firm if somebody else buys the patent, since a decrease in unit revenues leads to a larger loss.

An implication of surplus monotonicity is that, assuming we renumber buyers such that $\pi_i > \pi_j$ when $i < j$, each column in the matrix of payoff differences $\pi_i - e_{i,k}$ is increasing upwards.

Proposition 8 *If* $\omega \in \bar{\Omega}$ *and* ω *satisfies surplus-monotonicity then there is a* $\bar{\delta} < 1$ *such that for all* $\delta > \bar{\delta}$ *there exists an equilibrium.*

To show this result, extensive use is made of : $\pi_i - e_{i,j} \neq \pi_j - e_{j,i}$ and condition (9). To rule out existence of equilibrium, first condition (6) must be violated for all a. Also, since $\pi_m - e_{m,a} - \eta\pi_a$ is the numerator of (9), this has implications of the sign of the denominator of

(9). By repeatedly applying this argument for all a, we are able to show that if an equilibrium as described in Propositions 2 and 4 never exist, a cycle in $\pi_i - e_{i,j}$ must exist.

4 Negative Externalities and Efficiency

In this section, we provide conditions on pies and externalities guaranteeing that equilibria will be efficient for generic parameter values. Too see that this is not generally the case, consider an example with two buyers and assume that externalities are not too negative. Specifically, if $e_{i,j} > \pi_i - \eta\pi_j$ for $i = 1, 2$ and $j \neq i$ then from condition 6 in Proposition 2, there is an equilibrium where $p_1 = 1$ and $p_2 = 0$ as well as an equilibrium where $p_1 = 0$ and $p_2 = 1$. Thus, an inefficient equilibrium exists.

With negative externalities however, we will show that only efficient equilibria exist.

The payoff conditions that we restrict attention to, are the following:

Assumption 9 *For all k, i, j.*

$$\pi_i > \pi_j \Rightarrow e_{k,j} \geq e_{k,i} \tag{12}$$

This assumption seems reasonable, since if the seller and i get a larger surplus than the seller does with j, it is reasonable that the negative externalities imposed on others are also larger. Without loss of generality, we assume that $\pi_1 > \pi_2 > ... > \pi_n$. In addition, we assume that the negative externalities do not outweigh the differences in surplus so that we have

Assumption 10 *For $i < j$ we have*

$$\pi_i + \sum_{k=1}^{n} e_{k,i} > \pi_j + \sum_{k=1}^{n} e_{k,j} \tag{13}$$

Given that payoffs satisfy Assumptions 9 and 10, all generic equilibria are efficient in the limit.

Proposition 11 *For all $\omega \in \bar{\Omega}$ satisfying Assumptions 9 and 10, any equilibrium $\sigma \in \lim_{\delta \to 1} \Sigma(\omega, \delta)$ is efficient.*

5 Conclusion

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Restricting attention to generic equilibria and allowing mixed strategies makes the analysis of equilibria in the general setting of this model simple. Generically, there are only three types of equilibria: single out, outside option and hold-up. In single out equilibria, agreement is with a specific buyer. In the outside option equilibria, in the limit as $\delta \to 1$, there is also only agreement with a specific buyer. These are the only equilibrium types that exist, unless externalities are positive and sufficiently large. Then there also exist hold-up equilibria, with significant delay, in stark contrast with Jehiel & Moldovanu (1995a and 1995b).

A Proofs

In negotiations with $r \in R$, $m \in M$ and $a \in A$ we have, from the value equations (1) and (4) we have

$$\begin{aligned} v_{S,r} &= w_{S,r} \\ v_{r,S} &= w_{r,S}, \end{aligned} \tag{14}$$

$$\begin{aligned} v_{S,m} &= w_{S,m} \\ v_{m,S} &= \pi_m - w_{S,m} \end{aligned} \tag{15}$$

and

$$\begin{aligned} v_{S,a} &= \pi_a - w_{a,S} \\ v_{a,S} &= \pi_a - w_{S,a}. \end{aligned} \tag{16}$$

Using (14), (15) in (4) and (16) gives

$$\begin{aligned} w_{r,S} &= \frac{\delta\left(\sum_{j\in A} e_{r,j} + \sum_{j\in M} p_j e_{r,j}\right)}{|N|\,(1-\delta) + \delta\,|A| + \delta\sum_{j\in M} p_j} \\ w_{m,S} &= \frac{\delta\,(1-\eta)\,(\pi_m - w_{S,m}) + \delta\left(\sum_{j\in A} e_{m,j} + \sum_{j\in M} p_j e_{m,j}\right)}{|N|\,(1-\delta) + \delta - \delta\eta + \delta\,|A| + \delta\sum_{j\in M\setminus\{m\}} p_j} \\ w_{a,S} &= \frac{\delta\,(1-\eta)\,(\pi_a - w_{S,a}) + \delta\left(\sum_{j\in A} e_{a,j} + \sum_{j\in M} p_j e_{a,j}\right)}{|N|\,(1-\delta) - \delta\eta + \delta\,|A| + \delta\sum_{j\in M} p_j} \end{aligned} \tag{17}$$

Using (17), (14), (15) and (16) in (2) gives, using some straightforward but tedious algebra,

$$w_{S,i} = \frac{\delta\eta\left(\sum_{a\in A}\left(\pi_a\left(|N|\left(1-\delta\right)+\delta\left(|A|-1+\sum_{m\in M}p_m\right)\right)-\delta\left(\sum_{j\in A\setminus\{a\}}e_{a,j}+\sum_{m\in M}p_m e_{a,m}\right)\right)\right)}{\left(|N|\left(1-\delta\right)+\delta\,|A|\right)\left(|N|\left(1-\delta\right)+\delta\eta\left(|A|-1\right)\right)+\delta\left(|N|\left(1-\delta\right)+\delta\eta\,|A|\right)\sum_{m\in M}p_m} \tag{18}$$

A.1 Single Out Equilibria

Proof of Proposition 2: With $|A| = 1$ and $|M| = 0$ the condition for acceptance (3) for a is satisfied for all δ since, using (17) and (18)

$$v_{S,a} - w_{S,a} = (\pi_a - w_{S,a})\frac{|N|\left(1-\delta\right)}{|N|\left(1-\delta\right)+\left(1-\eta\right)\delta} = \frac{|N|\left(1-\delta\right)}{|N|\left(1-\delta\right)+\delta}\pi_a > 0. \tag{19}$$

There will not be acceptance in negotiations with r when (5) is fulfilled. Inserting solutions for $w_{r,S}$ and $w_{S,i}$ gives

$$\pi_r - \frac{\delta e_{r,1}}{|N|\left(1-\delta\right)+\delta} \leq \frac{\delta\eta}{|N|\left(1-\delta\right)+\delta}\pi_a. \tag{20}$$

From condition (6) in the statement of the proposition, there exists a $\bar{\delta} < 1$ such that (20) holds for all $\delta > \bar{\delta}$. As $w_{S,a} > 0$, firms also make a non-negative profit. Thus the conditions for the equilibrium to exist are satisfied for $\delta > \bar{\delta}$. ■

A.2 Hold-up Equilibria

Proof of Proposition 3: Using $|A| = 0$ and $|M| > 1$ in (17), (14) and (15), expression (2) is

$$w_{S,i} = \frac{\delta\eta}{1-\delta\left(1-\eta\right)}\frac{1}{|N|}\sum_{j\in N}w_{S,j} \iff w_{S,i} = 0. \tag{21}$$

Using (15), (4) and $w_{S,m} = w_{S,r} = 0$ in the value equation for $w_{m,S}$ gives

$$|N|\frac{1-\delta}{\delta}\pi_m = \sum_{j\in M\setminus\{m\}}e_{m,j}p_j - \pi_m\sum_{j\in M\setminus\{m\}}p_j.$$

Note that the condition $|M| > 1$ in the proposition follows, since when $|M| = 1$ we have $\left(1-\delta\right)\pi_m = 0$, contradicting $\pi_m > 0$ by assumption.

In matrix form, the above expression is

$$|N|\frac{1-\delta}{\delta}\pi_M = \left(E_{M,M} - \Pi_M\cdot\left(J_M - I_M\right)\right)\cdot p_M \tag{22}$$

Since the matrix on the right hand side is invertible by assumption, we get

$$p_M = |N| \frac{1-\delta}{\delta} \left(E_{M,M} - \Pi_M \cdot (J_M - I_M)\right)^{-1} \cdot \pi_M. \tag{23}$$

Thus from assumption (7) $p_M \gg 0$ for all $\delta < 1$. Note that, as δ converges to one, p_M converges to zero. Thus there exists a $\bar{\delta} < 1$ such that $p_m < 1$ for all $m \in M$ and $\delta > \bar{\delta}$.

If externalities are negative then $E_{M,M} - \Pi_M \cdot (J_M - I_M)$ is a matrix with zero on the main diagonal and negative off diagonal elements. Then, for any $p_M \geq 0$, the right hand side of (22) is non-positive and hence no hold-up equilibrium exists.

In order for all r not to make acceptable bids, (5) holds. As $w_{S,r} = 0$ by (21) and using $w_{r,S}$ in (17) when $|A| = 0$ and $|M| > 1$ we have

$$|N| \frac{1-\delta}{\delta} \pi_r \leq \sum_{k \in M} p_k e_{r,k} - \pi_r \sum_{k \in M} p_k.$$

In matrix form this condition becomes, using the solution for probabilities (23)

$$\pi_R \leq (E_{R,M} - \Pi_R \cdot J_{R,M}) \cdot \left(E_{M,M} - \Pi_M \cdot (J_M - I_M)\right)^{-1} \cdot \pi_M$$

which holds by condition (8) in the proposition.■

A.3 Outside Option Equilibria

Proof of Proposition 4: From indifference (4) for m, we get, using (17) with $|A| = 1$ and $|M| = 1$

$$\pi_m - w_{S,m} = \frac{\delta}{|N|(1-\delta) + \delta} e_{m,a}$$

and hence using (18) with $|A| = 1$ and $|M| = 1$ and solving for p_m gives

$$p_m = (1-\delta)\Delta$$

$$\Delta = \frac{|N|\left(|N|(1-\delta)+\delta\right)\left(\pi_m\left(|N|(1-\delta)+\delta\right) - \delta\eta\pi_a - \delta e_{m,a}\right)}{\delta\left(\left(|N|(1-\delta)+\delta\eta\right)\delta e_{m,a} + \left(\eta\delta\left(\pi_a - e_{a,m}\right) - \pi_m\left(|N|(1-\delta)+\delta\eta\right)\right)\left(|N|(1-\delta)+\delta\right)\right)}$$

Note that

$$\lim_{\delta \to 1} \Delta = \frac{|N|}{\eta} \frac{\pi_m - e_{m,a} - \eta\pi_a}{\pi_a + e_{m,a} - \pi_m - e_{a,m}} \tag{24}$$

Since $p_m = (1-\delta)\Delta$, by condition (9) and the continuity of Δ, there exists a $\bar{\delta} < 1$ such that $0 < p_m < 1$ for all $\bar{\delta} < \delta < 1$.

Then, using $p_m = (1-\delta)\Delta$ in (18) and (17) with $|A| = 1$ and $|M| = 1$

$$\begin{aligned}
&\pi_r - w_{r,S} - w_{S,r} \\
&= \pi_r - \frac{\delta\left(e_{r,a} + p_m e_{r,m}\right)}{|N|(1-\delta) + \delta + \delta p_m} - \frac{\delta\eta\left(\pi_a\left(|N| + \delta\Delta\right) - \delta\Delta e_{a,m}\right)}{\left(|N|(1-\delta) + \delta\right)|N| + \delta\left(|N|(1-\delta) + \delta\eta\right)\Delta}
\end{aligned}$$

In the limit, this expression is, using (24),

$$\pi_r - w_{r,S} - w_{S,r} = \pi_r - e_{r,a} - \left(\pi_m - e_{m,a}\right)$$

By (11), this expression is strictly positive. Then, there exists a $\bar{\delta} < 1$ such that (5) holds for all $\delta > \bar{\delta}$.

Using (16) and (18) with $|A| = 1$ and $|M| = 1$ we have

$$\begin{aligned}
&v_{S,a} - w_{S,a} \\
&= \frac{\pi_a\left(|N|(1-\delta) + \delta p_m\right) - \delta p_m e_{a,m}}{|N|(1-\delta) - \delta\eta + \delta + \delta p_m} \frac{|N|(1-\delta)\left(|N|(1-\delta) + \delta(1-\eta)\right) + \delta\left(|N|(1-\delta)\right)p_m}{|N|(1-\delta)\left(|N|(1-\delta) + \delta\right) + \delta\left(|N|(1-\delta) + \delta\eta\right)p_m}
\end{aligned}$$

Note that the second ratio and the denominator of the first ratio are positive for $\delta < 1$.The numerator of the first ratio can be rewritten as

$$(1-\delta)\left(\pi_a\left(|N| + \delta\Delta\right) - \delta\Delta e_{a,m}\right)$$

If

$$\pi_a\left(|N| + \delta\Delta\right) - \delta\Delta e_{a,m} > 0$$

then (3) is satisfied. As $\delta \to 1$, the left hand side of this expression is, using (24),

$$\frac{|N|}{\eta}\left(\frac{\pi_m - e_{m,a} - \eta\pi_a}{\pi_a + e_{m,a} - \pi_m - e_{a,m}} + 1\right)\left(\pi_m - e_{m,a}\right)$$

>From (10), there exists a $\bar{\delta} < 1$ such that (3) holds for all $\delta > \bar{\delta}$. ■

A.4 Genericity

Proof of Proposition 5: To prove the proposition, we need to show both that the equilibrium types stated in the proposition are generic, and that any other equilibrium type only exists non-generically. We begin by showing that the equilibrium types stated in the proposition are generic. Then we continue to show that any other equilibrium type only exists non-generically.

A.4.1 Genericity of Single Out equilibria

Consider the single out equilibrium type u_S where $|A| = 1$, $|M| = 0$ and $|R| = N - 1$. Given that condition (6) in Proposition 2 holds for some parameter value $\bar{\omega} \in \Omega$, there exists a closed ball $B(\bar{\omega})$ with radius ε around the parameter vector $\bar{\omega}$ such that the condition holds for all $\omega \in B(\bar{\omega})$. Also, from the proof of Proposition 2, the condition for acceptance (19) for a holds for all δ. Rewriting (20) we get

$$|N| \frac{1-\delta}{\delta} \pi_r \leq \eta \pi_a - (\pi_r - e_{r,1}) \tag{25}$$

For a given ω, let $\delta(\omega)$ denote the largest δ such that (25) holds for all $\delta > \delta(\omega)$ and all $r \neq a$. Let $\hat{\delta}$ denote the largest $\delta(\omega)$ for all $\omega \in B(\bar{\omega})$. By continuity of (19) and (6), ε can be chosen such that $\hat{\delta} < 1$. Then $B(\bar{\omega}) \subseteq \Omega(u_S, \delta)$ for all $\delta > \hat{\delta}$ and hence $\lim_{\delta \to 1} \lambda(\Omega(u_S, \delta)) \geq \lambda(B(\bar{\omega})) > 0$ establishing genericity of u_S.

A.4.2 Genericity of Hold-up Equilibria

To show that the hold-up equilibrium type is generic, consider the case where $e_{j,i} = e_j$ for all $i \neq j$ and assume that $e_i > \pi_i$ for all $i \in N$. In addition let all m have identical pies and externalities $\pi_m = \alpha$ and $e_m = \beta$ for all $m \in M$ and similarly $\pi_r = \theta$ and $e_r = \tau$ for all $r \in R$. Let $\bar{\omega}$ denote this parameter vector. We also assume

$$\frac{\theta}{\tau - \theta} < \frac{|M|}{|M| - 1} \frac{\alpha}{\beta - \alpha}. \tag{26}$$

Note that, for any parameter vector ω, there exists a $\delta(\omega) < 1$ such that probabilities are smaller than one for all $\delta > \delta(\omega)$.

Then the invertibility condition in proposition 3 is satisfied and, using $E_{M,M} = \beta(J_{M,M} - I_M)$

$$(E_{M,M} - \Pi_M \cdot (J_{M,M} - I_M))^{-1} \cdot \pi_M = \frac{1}{|M| - 1} \frac{\alpha}{\beta - \alpha} j_M$$

This is positive as $\beta > \alpha$ by assumption and hence condition (7) is satisfied. Furthermore, using $E_{R,M} = \tau J_{R,M}$

$$\pi_R = \theta j_R \ll (\tau - \theta) \frac{|M|}{|M| - 1} \frac{\alpha}{\beta - \alpha} j_R.$$

Then, since (26) holds, condition (8) is satisfied.

Since the invertibility condition is satisfied for the parameter vector $\bar{\omega}$ and the determinant is a continuous function of $\omega \in \Omega$, there exists a ball $B(\bar{\omega})$ with radius ε around the parameter vector $\bar{\omega}$ such that the matrix $E_{M,M} - \Pi_M \cdot (J_{M,M} - I_M)$ is invertible and the conditions (7) and (8) still hold. Let $\hat{\delta}$ denote the largest $\delta(\omega)$ such that probabilities are smaller than one for all $\delta > \delta(\omega)$ for all $\omega \in B(\bar{\omega})$. Then a hold-up equilibrium exists for all $\omega \in B(\bar{\omega})$ for all $\hat{\delta} < \delta < 1$, implying genericity.

A.4.3 Genericity of Outside option equilibria

Consider the outside option equilibrium type u_O where $|A| = 1$, $|M| = 1$ and $|R| = N - 2$. Given that condition (9) in Proposition 4 holds for some parameter values $\bar{\omega} \in \Omega$, there exists a closed ball $B^m(\bar{\omega})$ with radius ε^m around the parameter vector $\bar{\omega}$ such that the condition holds for all $\omega \in B^m(\bar{\omega})$. From the proof of Proposition 4 we have $p_m = (1-\delta)\Delta$ where $\lim_{\delta\to 1}\Delta = \frac{|N|}{\eta}\frac{\pi_m - e_{m,a} - \eta\pi_a}{\pi_a + e_{m,a} - \pi_m - e_{a,m}}$. Since Δ is continuous in δ there is some $\delta(\omega)$ such that probabilities are smaller than one for all $\delta > \delta(\omega)$. Let δ^m denote the largest $\delta(\omega)$ such that probabilities are smaller than one for all $\delta > \delta(\omega)$ for all $\omega \in B^m(\bar{\omega})$.

Similarly, given that (10) holds at $\bar{\omega}$, here exists a closed ball $B^a(\bar{\omega})$ with radius ε^a such that the condition holds for all $\omega \in B^a(\bar{\omega})$. Let δ^a denote the largest $\delta(\omega)$ such that the condition for acceptance (3) holds for all $\delta > \delta(\omega)$ for all $\omega \in B^a(\bar{\omega})$. A similar argument using (11) establishes the existence of δ^r and $B^r(\bar{\omega})$ where δ^r is the largest $\delta(\omega)$ such that, for all r, the condition for rejection (5) holds for all $\delta > \delta(\omega)$ for all $\omega \in B^r(\bar{\omega})$.

Letting $\hat{\delta} = \max\{\delta^a, \delta^m, \delta^r\}$ and $B(\bar{\omega}) = B^a(\bar{\omega}) \cap B^m(\bar{\omega}) \cap B^r(\bar{\omega})$ then, for $\delta > \hat{\delta}$ and $\omega \in B(\bar{\omega})$ the conditions (3) and (5) hold with $0 < p_m < 1$, establishing genericity. By continuity of (**??**) and the solutions for the values in Proposition 4, ε can be chosen such that $\hat{\delta} < 1$. Then $B(\bar{\omega}) \subseteq \Omega(u_O, \delta)$ for all $\delta > \hat{\delta}$ and hence $\lim_{\delta\to 1}\lambda(\Omega(u_O,\delta)) \geq \lambda(B(\bar{\omega})) > 0$ establishing genericity of u_O.

A.4.4 Non-generic Single Out Equilibria

To show that single out equilibria with $|A| > 1$ are non-generic,note that $w_{S,i}$ in (18) is well defined for $\delta = 1$. Using $|M| = 0$ in (17), to eliminate $w_{a,S}$ and $w_{r,S}$ in (3) and (5)

$$\pi_a - w_{S,a} \geq \frac{\delta}{(|N|(1-\delta) - \delta + \delta|A|)}\sum_{k\in A} e_{a,k} \tag{27}$$

$$\pi_r - w_{S,r} \leq \frac{\delta\sum_{k\in A} e_{r,k}}{|N|(1-\delta) + \delta|A|}.$$

For $\omega \in \Omega$ and $\delta \leq 1$ let $\psi : D \twoheadrightarrow R_+$ where $D \subset \Omega \times [0,1]$ be the correspondence satisfying (27) and (18) with $|M| = 0$. Thus, ψ maps payoff parameters ω and δ to the set of equilibrium agreement probabilities and seller respondent values. The correspondence ψ is upper-hemicontinuous (uhc), see Border 1985: If for $n = 1, 2, \ldots$ we have $p_n \in \psi(\omega_n, \delta_n)$ and $(\omega_n, \delta_n) \to (\omega, \delta)$ as $n \to \infty$, and $p = \lim_{n\to\infty} p_n$ then, since (27) and (18) with $|M| = 0$ define closed sets, we have $p \in \psi(\omega, \delta)$, establishing that ψ is uhc.

For $\delta \leq 1$, let the correspondence $\varphi(\delta)$ be the set of ω, such that $\psi(\omega, \delta)$ is non-empty. $\varphi(\delta)$ is uhc. Let $\delta_n \to \delta$ and $\omega_n \to \omega$ such that $\omega_n \in \varphi(\delta_n)$. Then there exist p_n such that $p_n \in \psi(\omega_n, \delta_n)$ and since ψ is uhc $p_n \to p \in \psi(\omega, \delta)$. Thus $\omega \in \varphi(\delta)$.

Using the solution for $w_{S,i}$ from (18) with $|M| = 0$ in (27) gives, when $|A| > 1$

$$\pi_a - \frac{1}{|A|-1}\sum_{k\in A} e_{a,k} \geq \frac{1}{|A|}\sum_{h\in A}\left(\pi_h - \frac{1}{|A|-1}\sum_{k\in A} e_{h,k}\right) \tag{28}$$

Since (28) holds for all a, it holds for the a that minimizes the LHS. As the minimal element is weakly greater than the average over all a, then $\pi_a - \frac{1}{|A|-1}\sum_{j\in A} e_{a,j}$ is the same for all a. Then $\varphi(1)$ is defined as, for all $a \in A$ and $r \in R$,

$$\pi_a - \frac{1}{|A|-1}\sum_{k\in A} e_{a,k} = K$$
$$\pi_r - w_{S,r} \leq \frac{\sum_{k\in A} e_{r,k}}{|A|}.$$

Thus, $\lambda(\varphi(1)) = 0$. Suppose that $\lim_{\delta\to 1}\lambda(\varphi(\delta)) > 0$. Then there exists a sequence $(\delta_n, \omega_n) \to (1, \omega)$ such that $\omega_n \in \varphi(\delta_n)$ for all δ_n but $\omega \notin \varphi(1)$. This contradicts the upper-hemicontinuity of φ.

A.4.5 Non-generic Outside Option Equilibria

Show that $|A| > 1$ is non-generic. To show that outside option equilibria with $|A| > 1$ are non-generic, using (17) in the indifference equation (4) and rearranging gives

$$(\pi_m - w_{S,m})\left(|N|(1-\delta) + \delta|A| + \delta\sum_{j\in M\setminus\{m\}} p_j\right) = \delta\left(\sum_{j\in A} e_{m,j} + \sum_{j\in M} p_j e_{m,j}\right) \tag{29}$$

When $|A| > 1$ $w_{S,i}$ is well defined for $\delta = 1$. Using (17) in (3) and (17) in (5) and (29) gives

$$\begin{aligned}
\pi_a - w_{S,i} &\geq \frac{\delta\left(\sum_{j\in A} e_{a,j} + \sum_{j\in M} p_j e_{a,j}\right)}{|N|(1-\delta) + \delta(|A|-1) + \delta\sum_{j\in M} p_j} \\
\pi_m - w_{S,i} &= \frac{\delta\left(\sum_{j\in A} e_{m,j} + \sum_{j\in M} p_j e_{m,j}\right)}{|N|(1-\delta) + \delta|A| + \delta\sum_{j\in M\setminus\{m\}} p_j} \\
\pi_r - w_{S,i} &\leq \frac{\delta\left(\sum_{j\in A} e_{r,j} + \sum_{j\in M} p_j e_{r,j}\right)}{|N|(1-\delta) + \delta|A| + \delta\sum_{j\in M} p_j}.
\end{aligned} \tag{30}$$

Then for $\omega \in \Omega$ and $\delta \leq 1$ let $\psi : D \twoheadrightarrow [0,1]^M \times R_+$ where $D \subset \Omega \times [0,1]$ be the correspondence satisfying (30) and (18). The correspondence ψ is upper-hemicontinuous: If for $n = 1, 2, \ldots$ we have $p_n \in \psi(\omega_n, \delta_n)$ and $(\omega_n, \delta_n) \to (\omega, \delta)$ as $n \to \infty$, and $p = \lim_{n\to\infty} p_n$ then, since (30) and (18) define closed sets, we have $p \in \psi(\omega, \delta)$, establishing that ψ is uhc.

For $\delta \leq 1$, let the correspondence $\varphi(\delta)$ be the set of ω, such that $\psi(\omega, \delta)$ is non-empty. $\varphi(\delta)$ is uhc. Let $\delta_n \to \delta$ and $\omega_n \to \omega$ such that $\omega_n \in \varphi(\delta_n)$. Then there exist p_n such that $p_n \in \psi(\omega_n, \delta_n)$ and since ψ is uhc $p_n \to p \in \psi(\omega, \delta)$. Thus $\omega \in \varphi(\delta)$.

Using (17) and (18) in (3) gives, when $|A| > 1$ and $\delta = 1$,

$$\pi_a - \frac{\sum_{j\in A\setminus\{a\}} e_{a,j} + \sum_{m\in M} p_m e_{a,m}}{|A| - 1 + \sum_{m\in M} p_m} \geq \frac{1}{|A|}\sum_{a\in A}\left(\pi_a - \frac{\sum_{j\in A\setminus\{a\}} e_{a,j} + \sum_{m\in M} p_m e_{a,m}}{|A| - 1 + \sum_{m\in M} p_m}\right). \tag{31}$$

Using the same argument following (28), the left-hand side of (31) are the same for all a. Thus for all $a, b \in A$

$$\pi_a P_1 - \left(\sum_{j\in A\setminus\{a\}} e_{a,j} + \sum_{m\in M} p_m e_{a,m}\right) = \pi_b P_1 - \left(\sum_{j\in A\setminus\{b\}} e_{b,j} + \sum_{m\in M} p_m e_{b,m}\right) \tag{32}$$

Let $P_1 = |A| - 1 + \sum_{m\in M} p_m$ and $P_m = |A| + \sum_{j\in M\setminus\{m\}} p_j$. Combining (32) for some $b \neq a$ with the system of $|M|$ equations obtained from substituting (18) into (29), setting $\delta = 1$ and rearranging, we can define the following system $0 = F(p_M, \omega)$, where $F : [0,1]^{|M|} \times \Omega \to \mathbb{R}^{|M|+1}$

$$\begin{aligned}
0 &= \pi_a P_1 - \left(\sum_{j\in A\setminus\{a\}} e_{a,j} + \sum_{m\in M} p_m e_{a,m} \right) \\
&\quad - \left(\pi_b P_1 - \left(\sum_{j\in A\setminus\{b\}} e_{b,j} + \sum_{m\in M} p_m e_{b,m} \right) \right) \\
0 &= \left(\pi_m P_1 - \frac{1}{|A|} \sum_{k\in A} \left(\pi_k P_1 - \left(\sum_{j\in A\setminus\{k\}} e_{k,j} + \sum_{m\in M} p_m e_{k,m} \right) \right) \right) P_m \\
&\quad - \left(\sum_{j\in A} e_{m,j} + \sum_{j\in M} p_j e_{m,j} \right) P_1
\end{aligned}$$

The derivative of the system above with respect to π_a and π_m is

$$Z = \begin{pmatrix}
z_{aa} & 0 & \cdots & \cdots & 0 \\
z_{ma} & z_{mm} & 0 & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & 0 \\
z_{ma} & 0 & \cdots & 0 & z_{mm}
\end{pmatrix}$$

where

$$\begin{aligned}
z_{mm} &= P_1 P_m \\
z_{ma} &= \frac{z_{mm}}{|A|} \\
z_{aa} &= P_1 \left(1 - \frac{1}{|A|} \right).
\end{aligned}$$

Since $|A| > 1$ and $p_m \geq 0$, we have $P_1 > 0$ and $P_2 > 0$. Then since $\det(Z) = z_{aa}(z_{mm})^{|M|} \neq 0$, Z is invertible. Using the Transversality Theorem 8.3.1 in Mas-Colell (1985), the equation system is regular on the set $\hat{\Omega}$ with $\lambda(\hat{\Omega}) = 1$. Using Proposition H.2.2 in Mas-Colell (1985), since the number of equations is larger than $|M|$, there are no solution when the system is regular. Since probabilities have to satisfy (32) for all $a, b \in A$, the set of parameter values for which an equilibrium exists $\Omega^* \subset \hat{\Omega}$, establishing $\lambda(\varphi(1)) = 0$. Suppose that $\lim_{\delta\to 1} \lambda(\varphi(\delta)) > 0$. Then there exists a sequence $(\delta_n, \omega_n) \to (1, \omega)$ such that $\omega_n \in \varphi(\delta_n)$ for all δ_n but $\omega \notin \varphi(1)$. This contradicts the upper-hemicontinuity of φ.

Showing non-genericity with $|A| = 1$ and $|M| > 1$. We claim that for buyer a (3) must hold with equality in the limit.

Lemma For buyer a (3) holds with equality as $\delta \to 1$ when $|A| = 1$.

Proof: Consider the following strategy: When initially negotiating with 1, the firm makes unacceptable proposals and rejects proposals. Following this, the firm makes acceptable proposals only when meeting 1 and rejects all proposals. The payoff when following this strategy is , where $\hat{w}_{S,a}$ denotes the payoff

$$\begin{aligned}\hat{w}_{S,a} = {} & \delta\frac{\eta(v_{S,1} + (|N| - |M|)\hat{w}_{S,a} + (|M| - 1)\hat{w}_{S,a})}{|N|} \\ & + \delta\frac{(1-\eta)(\hat{w}_{S,a} + (|N| - |M|)\hat{w}_{S,a} + (|M| - 1)\hat{w}_{S,a})}{|N|}\end{aligned} \tag{33}$$

Since $w_{S,a}$ is optimal for S we have $w_{S,a} \geq \hat{w}_{S,a}$. Then we have

$$v_{S,a} - w_{S,a} \leq v_{S,a} - \hat{w}_{S,a}.$$

Also, note that $v_{S,a} - w_{S,a}$ is the surplus from making an acceptable proposal when bargaining with a. Solving for $\hat{w}_{S,a}$ in (33) gives

$$\hat{w}_{S,a} = \frac{\delta\eta v_{S,a}}{|N|(1-\delta) + \delta\eta}$$

and hence

$$v_{S,a} - \hat{w}_{S,a} = \frac{|N|(1-\delta)v_{S,a}}{|N|(1-\delta) + \delta\eta}$$

which is zero in the limit, implying that

$$v_{S,a} = \pi_a - w_{a,S} = w_{S,a}$$

■

In equilibrium, we have

$$\pi_a - w_{a,S} = \pi_m - w_{m,S}$$

in the limit. Using (17) gives

$$w_{S,i}$$
$$= \frac{\pi_m(1+k-p_m)(1-\eta+k) - (1-\eta+k)\left(e_{m,a} + \sum_{j\in M} p_j e_{m,j}\right) - (1-\eta+1+k-p_m)\left(\pi_a k - \sum_{j\in M} p_j e_{a,j}\right.}{(1-\eta)(1-p_m)}$$

Using the solution for (18) gives

$$\pi_a\left(\left(1-\eta\right)\left(1-p_m\right)+\left(1-\eta+1+k-p_m\right)k\right)-\left(1-\eta\right)\left(1-p_m\right)\frac{\sum_{m\in M}p_m e_{a,m}}{k} \tag{34}$$
$$-\left(\pi_m\left(1+k-p_m\right)\left(1-\eta+k\right)-\left(1-\eta+k\right)\left(e_{m,a}+\sum_{j\in M}p_j e_{m,j}\right)+\left(1-\eta+1+k-p_m\right)\sum_{j\in M}p_j e_{a,j}\right)$$

When $\lim_{\delta\to 1}\sum_{k\in M}p_k = k>0$ we get, using the solutions for $w_{m,S}$ in (17) and $w_{S,m}$ in (18), in the limit

$$\pi_m-\frac{\pi_a k-\sum_{j\in M}p_j e_{a,j}}{k}=\frac{1}{1+k-p_m}\left(e_{m,a}+\sum_{j\in M}p_j e_{m,j}\right) \tag{35}$$

Combining (34) and (35) gives

$$Z=\begin{pmatrix} z_{aa} & z_{am} & . & z_{am} \\ -1 & 1 & 0 & 0 \\ . & 0 & . & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

where

$$z_{aa}=\left(1-\eta\right)\left(1-p_m\right)+\left(1-\eta+1+k-p_m\right)k$$
$$z_{am}=-\left(1+k-p_m\right)\left(1-\eta+k\right)$$

We have linear dependence iff

$$z_{aa}+|M|\,z_{am}=0$$

which is true only if $|M|=1$, since $z_{aa}=-z_{am}\neq 0$.

When $\lim_{\delta\to 1}\sum_{k\in M}p_k=0$ we get, in the limit, using (29)

$$w_{S,m}=\pi_m-e_{m,a}$$

for all m. Hence, since $w_{S,m}$ is the same for all m, we have, for any $m,s\in M$:

$$\pi_m-e_{m,a}=\pi_s-e_{s,a},$$

establishing non-genericity.■

Using the argument when $|A|>1$ and $|M|=0$ gives Then $\varphi\left(1\right)$ is defined as, for all $a\in A$

and $r \in R$,

$$\pi_m - e_{1,m} = K$$

$$\pi_r - w_{S,r} \leq e_{r,a}.$$

A.5 Existence

Proof of Proposition 8:

Renumber buyers such that $\pi_i > \pi_j$ when $i < j$. If condition (6) holds for some a, a single out equilibrium exists. Suppose (6) is violated for all $a \in N$. Then, for each a there is some $r \neq a$ such that

$$\pi_r - e_{r,a} > \eta\pi_a. \tag{36}$$

Also, when solving

$$m = \arg\max_i \pi_i - e_{i,a}.$$

we have, when $a = 1$ that $m = 2$ and when $a > 1$ that $m = 1$. Then, by (36) and the choice of m we have, for all a,

$$\pi_m - e_{m,a} > \eta\pi_a,$$

and hence the numerator of the ratio in (9) is positive. Consider $a = 1$ and $a = 2$. Since $\omega \in \bar{\Omega}$, suppose without loss of generality that

$$\pi_1 - e_{1,2} > \pi_2 - e_{2,1}.$$

Then, for $a = 1$ and $m = 2$, the ratio in (9) is positive. Finally, if (10) is violated, then

$$\pi_2 - e_{2,1} < 0,$$

implying that (6) holds for $a = 1$. ■

A.6 Efficiency

Proof of Proposition 11: consider first the equilibrium of Proposition 2. Suppose that there exist an inefficient equilibrium satisfying Proposition 2. Then, by (12) and (13), there are a and r such that $\pi_r > \pi_a$ and satisfying (6). Since externalities are negative, we have

$$\pi_r < \pi_r - e_{r,a} \leq \eta\pi_a,$$

contradicting $\pi_r > \pi_a$.

Now consider the equilibrium of Proposition 4. Note that condition $\pi_1 + \sum_{k=1}^{n} e_{k,1} > \pi_2 + \sum_{k=1}^{n} e_{k,2}$ can be rewritten as

$$\pi_1 + e_{2,1} > \pi_2 + e_{1,2} + \sum_{k>2} e_{k,2} - \sum_{k>2} e_{k,1}.$$

With the assumptions above, this implies that

$$\pi_1 + e_{2,1} > \pi_2 + e_{1,2}. \tag{37}$$

Repeating the same argument for 2 and i gives

$$\pi_2 + e_{i,2} > \pi_i + e_{2,i} + \sum_{k\neq 2} e_{k,i} - \sum_{k\neq i} e_{k,2}.$$

Then

$$\pi_2 + e_{i,2} > \pi_i + e_{2,i}$$

by the same argument.

We also have $e_{i,2} > e_{i,1}$ and $e_{2,1} < e_{2,i}$ establishing that

$$\pi_i + e_{2,1} \leq \pi_2 + e_{i,1} \tag{38}$$

which is the condition for existence of the efficient equilibrium.

If some other equilibrium is to exist, we require that

$$\pi_i + e_{1,i} > \pi_1 + e_{i,1}$$

or

$$\pi_1 + e_{j,k} \leq \pi_j + e_{1,k}.$$

The first condition (i.e., when i is first and 1 is second) is violated by the argument that showed that $\pi_1 + e_{2,1} > \pi_2 + e_{1,2}$, replacing 1 with i and 2 with 1.

The second condition takes care of the case when the seller agrees with zero probability with 1 and with probability 1 with k and mixed probability with j. First, if it is more efficient to agree with j than with k a similar argument establishes a contradiction of the first ε condition ($\varepsilon_{k,j}$ instead of $\varepsilon_{1,2}$) Thus, it must be more efficient to agree with k than with j. Noting that we have $\pi_1 + \sum_{i=1}^n e_{i,1} > \pi_j + \sum_{i=1}^n e_{i,j}$ and rearranging gives

$$\pi_1 + e_{j,1} > \pi_j + e_{1,j} + \sum_{i \neq 1}^n e_{i,j} - \sum_{i \neq j}^n e_{i,1}.$$

We then get

$$\pi_1 + e_{j,1} > \pi_j + e_{1,j}.$$

Using that $e_{j,1} < e_{j,k}$ and $e_{1,j} > e_{1,k}$ establishes that

$$\pi_1 + e_{j,k} > \pi_j + e_{1,k}$$

a contradiction.

If (6) holds for $a = 1$and $r \neq 1$ then the equilibrium in Proposition 2 exists and if (6) is violated then the equilibrium in Proposition 4 exists. If

$$\pi_2 - e_{2,1} > \eta\pi_1$$

then, from (37) and (9), the equilibrium in Proposition 4 exists. If

$$\pi_2 - e_{2,1} \leq \eta\pi_1$$

then, from (38) we have $\pi_r - e_{r,1} \leq \pi_2 - e_{2,1}$ and hence

$$\pi_r - e_{r,1} \leq \eta\pi_1,$$

establishing that the equilibrium in 2 exists.

Finally, the equilibrium of Proposition 3 does not exist with negative externalities.■

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