The Value of Commitment in Contests and Tournaments when Observation is Costly[*]

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Abstract

We study the value of commitment in sequential contests when the follower faces small costs to observe the leader's effort. We show that the value of commitment vanishes entirely in this class of games. By contrast, in sequential tournaments—games where, at a cost, the follower can observe the effectiveness of the leader's effort—the value of commitment is preserved completely provided that the observation costs are sufficiently small.

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1 Introduction

The modeling framework of contests and tournaments—settings where players undertake effort or expend resources in pursuit of some "prize"—has been usefully applied in numerous economic environments. These include patent races, allocating resources in elections, the private funding of public goods, designing incentive contracts in firms, and golf tournaments.[1]

Early modeling of contests focused on equilibrium outcomes when players move simultaneously (see, for example, Tullock 1980, 1985). The usual justification for this modeling choice was that commitment was valuable: if a player can choose between moving at the same time as a rival or moving earlier, he prefers to move earlier. Since all competing parties share the same incentive, they will all race to move at the earliest possible moment. Hence, it was argued that unless some institutional feature allows some parties to move ahead of others, the right model is that of simultaneous moves.

Baik and Shogren (1992) and Leininger (1993) question this argument. In both papers, the order of moves is endogenous and a sequential move contest emerges. The reason is that, even though both players prefer moving first over moving simultaneously, the favorite prefers moving second over moving first, while the underdog prefers moving first over moving second.

While the literature on contests is vast, the literature on *sequential* contests is much smaller.[2] The earliest analysis of sequential contests is by Dixit (1987), who derives conditions under which commitment has value;[3] that is, he shows when a player benefits from moving first rather than moving simultaneously. The central theme of this line of the contest literature is that the timing of moves matters, because actions in earlier stages of the contest have strategic effects on those in later stages and this affects equilibrium outcomes.

A key assumption shared by all models of sequential contests is that moves made in earlier periods are costlessly observed by players who move later in the game. In this paper, we relax this assumption. Instead, we suppose that a player must pay

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[1] See, for instance, Taylor, 1995; Snyder, 1989; Morgan and Sefton, 2000; Lazear and Rosen, 1981; and Ehrenberg and Bognanno, 1990.

[2] See Nitzan (1994) for an excellent survey, particularly with respect to modeling rent-seeking.

[3] See also Baik and Shogren (1992), Baye and Shin (1999), and Dixit (1999) for additional comments on Dixit (1987).

a small cost to observe the prior action of a rival. Thus, our modeling framework follows the one first suggested by Várdy (2004).[4] When strategy spaces are discrete, he showed that there is no value of commitment in pure strategies but that the value of commitment can be completely restored if mixed strategies are permitted.

In contrast to Várdy, we focus on contests with continuous strategy spaces. In this setting, we show that there is *no value to commitment whatsoever* when observation is costly. More precisely, efforts and payoffs in all subgame perfect equilibria of the sequential contest are identical to those in the Nash equilibrium of the standard simultaneous contest. As such, this paper offers a justification for the attention that the existing literature has given to simultaneous contests.

The intuition for our result is quite straightforward. Suppose the simultaneous and the sequential contests are "well-behaved," in the sense that the players' payoff functions are strictly concave in their own levels of effort. In that case, it is easy to show that the first player in a sequential contest with observation costs has a unique best response to any (sequentially rational) beliefs about the second player's strategy. This implies that in any subgame perfect equilibrium, the first player plays a pure strategy. But if player 1 plays a pure strategy, information about player 1's actual choice of effort is of no value to player 2 in equilibrium, because player 2 can already perfectly "solve" what player 1 did. Therefore, the second player will *never* pay to observe the first player's choice, even if the cost of doing so is arbitrarily small. In turn, this destroys any strategic effect player 1's move could have on player 2. Hence, there is no value to commitment.

The key problem for the first player is that, by playing a pure strategy, he destroys the incentive for the second player to pay to observe his action in equilibrium. Indeed, the question arises why the first mover does not create his own "noise" by playing a mixed strategy in order to restore the value of commitment. The reason is that he cannot resist the temptation to "purify" any mixed strategy and play his unique, most profitable action instead, because his optimization problem is "well-behaved" in the sense of strict concavity. Thus, somewhat paradoxically, it ultimately is the "well-behavedness" of the first player's problem that destroys the value of commitment.

We examine the robustness of the loss of the value of commitment by considering several variations of the problem. First, we study the case of a discrete strategy

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[4]See also Morgan and Várdy (2004) for experiments relating to the fragility of commitment in this framework.

space on a finite grid. We show that, for any fixed cost of observation, there exists a sufficiently fine grid such that commitment has no value. Next, we study the case where the leader's actions are observed costlessly but noisily as in the 'noisy leader' setup of Bagwell (1995) and Van Damme and Hurkens (1997). We find that for a simple but widely used signaling technology where the second player either observes the first player's action precisely or observes an entirely uninformative signal, strict concavity of the first player's payoff function once again destroys the value of commitment.

Finally, we study sequential rank-order tournaments with observation costs. In a sequential tournament, the second mover can observe the *effectiveness* of the first-mover's effort rather than the effort itself. We show that the value of commitment in tournaments is *completely preserved* provided that the observation costs are sufficiently small. What accounts for this difference? In tournaments, the first player's effectiveness is not deterministic, even if he plays a pure strategy. Thus, the second player derives value from observing the effectiveness of the first player's effort. Hence, for small enough costs, the second player will choose to observe and, therefore, the value of commitment is preserved. This result highlights that it is the observability of the effectiveness of effort—rather than of the effort itself—that creates the value of commitment.

The remainder of the paper proceeds as follows: Section 2 presents the contest model. Section 3 contains the main result, namely, that in sequential contests with observation costs, the value of commitment vanishes completely. Section 4 examines the robustness of this result. Finally, Section 5 concludes. A generalization of the main result is contained in the Appendix.

2   Model

Two risk-neutral players, labeled 1 and 2, are competing to win some object. The players may be thought of as pressure groups and the object as a favorable piece of legislation, a monopoly concession, and so on. Let V_i denote the (positive and finite) value of the object to player i. The valuation that each player places on the object is commonly known. If i does not receive the object, his payoff is normalized to zero (exclusive of contest expenditures—more on this below).

Players compete for the object by making irreversible effort outlays. The effort

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of player i is denoted $x_i \in \mathbb{R}_0^+$. There is a continuously differentiable contest success function $P(x_1, x_2)$, which gives the probability that the object will be awarded to player 1 when efforts x_1 and x_2 are expended. The cost of effort is $C_i(x_i)$. Hence, player 1's expected payoff is:

$$E\pi_1 = P(x_1, x_2) V_1 - C_1(x_1)$$

while player 2's expected payoff is

$$E\pi_2 = [1 - P(x_1, x_2)] V_2 - C_2(x_2)$$

Following Dixit (1987) and much of the contest literature, in the main text we assume that the contest success function takes the Logit form; that is

$$P(x_1, x_2) = \begin{cases} \frac{f_1(x_1)}{f_1(x_1) + f_2(x_2)} & \text{if } (x_1, x_2) \neq (0, 0) \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

where $f_i(0) = 0$, $f_i'(\cdot) > 0$, and $f_i''(\cdot) \leq 0$. We also assume that $C_i(x_i) = x_i$; that is, the cost of effort is equal to the effort itself.

Although, here, the contest success function is assumed rather than derived from some optimization problem, Clark and Riis (1998) and Skaperdas (1996) offer axiomatic foundations for the Logit form of contest success functions. Moreover, in the Appendix, we offer general conditions on payoffs such that the conclusions derived in the main text continue to hold.

Simultaneous Contest

Consider the case where x_1 and x_2 are selected simultaneously. The following facts are well-known in the contest literature (see, e.g., Yildirim, 2005):

Fact 1. Player i's problem is strictly concave in x_i.

Fact 1 implies that for each x_j, player i has a unique best response $x_i(x_j)$ satisfying $\partial E\pi^i / \partial x_i = 0$. Together with the earlier assumptions made on f_i and C_i, this implies that $x_i(x_j)$ is continuously differentiable and bounded. Hence, a pure strategy Nash

equilibrium is a pair (x_1^*, x_2^*) satisfying

$$\frac{f_2\left(x_2^*\right) f_1'\left(x_1^*\right)}{\left(f_1\left(x_1^*\right) + f_2\left(x_2^*\right)\right)^2} V_1 - 1 = 0$$

$$\frac{f_1\left(x_1^*\right) f_2'\left(x_2^*\right)}{\left(f_1\left(x_1^*\right) + f_2\left(x_2^*\right)\right)^2} V_2 - 1 = 0$$

Fact 2. The best-response function $x_i\left(x_j\right)$ is strictly increasing when efforts x_i and x_j are such that $f_i\left(x_i\right) > f_j\left(x_j\right)$, reaches its maximum when $f_i\left(x_i\right) = f_j\left(x_j\right)$, and is strictly decreasing when $f_i\left(x_i\right) < f_j\left(x_j\right)$.

Fact 2 implies that the best-response functions $x_1\left(x_2\right)$ and $x_2\left(x_1\right)$ cross the locust where $f_i\left(x_i\right) = f_j\left(x_j\right)$ exactly once. Therefore, the Nash equilibrium (x_1^*, x_2^*) exists and is unique.

Sequential Contest

Next, suppose that the contest is played sequentially. That is, player 1 chooses x_1, player 2 costlessly and perfectly observes x_1, and then chooses x_2.

Note that player 2's best-response function, $x_2\left(x_1\right)$, is identical to his best-response function in the simultaneous contest. Player 1's optimization problem is to choose x_1 to maximize $E\pi_1$, recognizing that player 2 will be playing a best response to x_1.

Fact 3. Given that player 2 is playing a best response, player 1's problem is strictly concave in x_1.

From Fact 3 it now follows that there exists a unique subgame perfect equilibrium in the sequential contest, which we denote by $(x_1^{**}, x_2\left(x_1^{**}\right))$.

Finally, following Dixit (1987), we say that there is *value to commitment* if the profits of the first mover in the subgame perfect equilibrium of the sequential contest are higher than in any pure strategy Nash equilibrium of the simultaneous contest. It is easily checked that this condition corresponds to $f_1\left(x_1^*\right) \neq f_2\left(x_2^*\right)$ for the Nash equilibrium pair (x_1^*, x_2^*). We assume that this condition holds.

For the remainder of the paper, consider the sequential contest but suppose that, prior to deciding on x_2, player 2 must decide whether to pay a cost $\varepsilon > 0$ to observe player 1's choice of effort, x_1. If player 2 pays this cost, then player 1's choice is revealed to him. If not, then player 2 obtains no information about 1's choice.[5]

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[5]Note that the limiting case where $\varepsilon = 0$ is not entirely equivalent to the standard sequential

3   Main Result

We now present the main result of the paper: the value of commitment vanishes completely when observation is costly.

Proposition 1 *Fix $\varepsilon > 0$. In any subgame perfect equilibrium of the sequential contest with observation costs, there is no value to commitment.*

To establish Proposition 1, we show that all subgame perfect equilibria of the sequential contest with observation costs correspond to the Nash equilibrium of the simultaneous contest.

First, we prove the following lemma.

Lemma 1 *Fix $\varepsilon > 0$. In any pure strategy subgame perfect equilibrium of the sequential contest with observation costs, player 2 never pays to observe player 1's choice.*

Proof. By definition of pure strategy equilibrium, the leader takes a specific action with probability one. The follower holds some beliefs about this action even without observing it. In equilibrium, these beliefs must be correct and, therefore, also degenerate. Degeneracy of correct beliefs implies that the follower can perfectly predict the leader's equilibrium action. In that case, it can never be optimal for the follower to spend any amount, no matter how small, to merely confirm his (correct) beliefs. Therefore, in any pure strategy equilibrium, the follower never observes the leader's action. ∎

For the moment, we continue to restrict attention to pure strategies. Given that player 1 anticipates that player 2 never observes player 1's effort, player 1's choice of x_1 satisfies the first order condition

$$\frac{f_2\left(\hat{x}_2\right) f_1'\left(x_1\right)}{\left(f_1\left(x_1\right) + f_2\left(\hat{x}_2\right)\right)^2} V_1 - 1 = 0$$

Here, $\hat{x}_2$ is player 1's conjecture about player 2's choice, where player 2's choice cannot depend on x_1 because he does not observe x_1.

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contest. The reason is that in the limiting case, the follower still has the option of not observing the leader's choice whereas in the base game no such option exists. If, however, one eliminates weakly dominated strategies, then the limiting case becomes identical to the base game.

Player 2's optimal choice of x_2 satisfies the first order condition

$$\frac{f_1\left(\hat{x}_1\right) f_2'\left(x_2\right)}{\left(f_1\left(\hat{x}_1\right) + f_2\left(x_2\right)\right)^2} V_2 - 1 = 0$$

where $\hat{x}_1$ is player 2's conjecture about player 1's choice.

Now notice that the resulting first-order conditions, in combination with the equilibrium restrictions $\hat{x}_1 = x_1$ and $\hat{x}_2 = x_2$, are identical to those for the unique Nash equilibrium of the simultaneous move contest. Thus, we have:

Lemma 2 *The effort levels in any pure strategy subgame perfect equilibrium of the sequential contest with observation costs are identical to the effort levels in the unique pure strategy Nash equilibrium of the simultaneous move contest.*

Next, we turn to mixed strategies.

Suppose player 1 believes that player 2 pays to observe player 1's effort with probability p. Conditional on observing, subgame perfection implies that player 2 plays his (unique) best response $x_2\left(x_1\right)$. Represent player 1's beliefs about player 2's action x_2 conditional on *not* observing by the cumulative distribution function $H\left(\cdot\right)$. Then, player 1's optimization problem is

$$\max_{x_1} E\pi_1 = p\left[\frac{f_1\left(x_1\right)}{f_1\left(x_1\right) + f_2\left(x_2\left(x_1\right)\right)} V_1 - x_1\right] + (1-p)\int_{x_2}\left[\frac{f_1\left(x_1\right)}{f_1\left(x_1\right) + f_2\left(x_2\right)} V_1 - x_1\right] dH\left(x_2\right)$$

This optimization problem is strictly concave in x_1, because it is a convex combination of expressions that are strictly concave in x_1 by assumption. Therefore, for all $H\left(\cdot\right)$, there is a unique x_1 that solves this optimization problem. Hence, player 1 plays a pure strategy in any subgame perfect equilibrium which, in turn, implies that $p = 0$. Therefore, player 2 also plays a pure strategy. We conclude:

Lemma 3 *All subgame perfect equilibria of the sequential contest with observation costs are in pure strategies.*

Now, Lemmas 2 and 3 imply Proposition 1. This completes the proof of the main result.

Discussion

In the case of finite games, Bagwell (1995) first observed that if one restricts attention to pure strategy equilibria, the value of commitment in sequential move games of complete information is fragile to small perturbations of the game where the second player only imperfectly observes the first player's move. Várdy (2004) made a similar observation for finite sequential move games with observation costs. However, it is well known that the value of commitment in these settings can be restored if one allows for mixed strategies.

Sequential contests—games with continuous strategy spaces—lead to a stronger result. Proposition 1 shows that the value of commitment vanishes, regardless of whether one includes mixed strategies.

Why is it that allowing for mixed strategies does not restore the value of commitments in contests? In games with finite strategy spaces, it is always possible to find two or more actions for player 1 over which he is indifferent in equilibrium. This allows player 1 to "commit" to playing a mixed strategy which, in turn, creates positive value to observing player 1's action. Indeed, if the cost is sufficiently small, it induces player 2 to observe players 1's choice with strictly positive probability, thus restoring the "transmission path" for commitment to have value.

In the continuous case by contrast, precisely because the game is "well-behaved" in the sense that the first player's payoff function is strictly concave regardless of the (sequentially rational) strategy profile of the second player, the first player cannot credibly commit to anything other than his *unique* best response. In other words, mixing is not incentive compatible for the first player.

One may wonder how robust the result in Proposition 1 really is. In the next section, we investigate several variations of the model and show in which directions the result is robust and which it is not. In the Appendix, we offer general conditions on contest success functions and cost of effort functions such that Proposition 1 continues to hold. These conditions illustrate that our main result does not crucially depend on the Logit form of the contest success function or the linear cost of effort.

4 Robustness

Discrete Strategy Spaces

A key difference between our model and the extant literature is that in our model

strategy spaces are continuous rather than discrete. Thus, one might speculate that the loss of the value of commitment is purely an artifact of this modelling choice. As we show below, this is not the case.

Define grid $G(m)$ on $\mathbb{R}_0^+$ to be $\left\{0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, ...\right\}$, where parameter m is some positive integer. The larger m is, the finer the grid. We now show that

Proposition 2 *For discrete strategy spaces on a sufficiently fine grid, Proposition 1 continues to hold.*

Formally: fix $\varepsilon > 0$ and constrain players' efforts, x_i where $i \in \{1,2\}$, to values on the grid $G(m)$. For m sufficiently large, there is no value to commitment in any subgame perfect equilibrium of the sequential contest with observation costs.

Proof. We prove the result by showing that, when m is sufficiently large (i.e., the grid is sufficiently fine), the second player chooses to never observe the first player's effort.

Clearly, in any equilibrium where player 1 plays a pure strategy, player 2 never observes. Now suppose player 1 is mixing in equilibrium. In that case, we claim that player 1 must be mixing between exactly two adjacent actions. If player 1 mixes between more than two actions, by strict concavity, he cannot be indifferent between them. Similarly, if the two actions are not adjacent, strict concavity implies that any action in between is strictly more profitable. Now, by increasing m, the distance between adjacent actions becomes arbitrary small. This implies that the value to player 2 of learning which one of the two possible actions player 1 has actually played becomes arbitrarily small as well. By continuity, player 2 is then strictly better off not expending ε to observe player 1's action for m sufficiently large. This proves the result. ∎

How can we reconcile the result in Proposition 2 with the apparently opposite result obtained in Várdy (2004)? The key is the difference in what we keep fixed and what we make small. In Proposition 2, we keep the size of the observation cost ε fixed while we make the grid size sufficiently small. In contrast, Várdy considers the case of a fixed grid size and studies what happens when we make ε sufficiently small. A mixed strategy equilibrium that preserves player 1's value of commitment exists only in the latter case. Thus, whether the value of commitment is lost or (potentially) preserved depends on the size of the observation cost ε relative to the coarseness of the grid: if

the cost of observation is small relative to the grid, the value of commitment can be preserved; if the grid is sufficiently fine relative to the cost of observation, then the value of commitment is lost.

Noisy Sequential Contest

An alternative modeling approach to study the fragility of commitment was offered by Bagwell (1995). He studies finite leader-follower games where, with small but positive probability, the follower receives the wrong signal about the leader's action. The pure strategy Nash equilibrium outcomes of such a 'noisy leader game' turn out to be equal to the pure strategy Nash equilibrium outcomes of the simultaneous game. In other words, the leader's value of commitment may not be robust to noise in the communication technology.[6] Van Damme and Hurkens (1997) partly salvage the value of commitment in finite noisy leader games by showing that these games always have a mixed strategy equilibrium in which the value of commitment is preserved asymptotically when the noise vanishes.[7]

In view of our main result, one may wonder whether the value of commitment survives in noisy leader games when strategy spaces are continuous. In this section we show that, for a simple but widely used signaling structure, concavity arguments analogous to those in Proposition 1 imply that there is no value to commitment either.

To see this, suppose that the contest success function satisfies all of the assumptions given above and that the strategy space is $X_i \equiv \left[0, C_i^{-1}(V_i)\right]$; that is, effort levels are restricted to undominated strategies. Suppose further that player 2 faces the following signal structure. With probability p, player 2 observes a signal $s_1 = x_1$; that is, player 1's effort is perfectly revealed to player 2. With probability $1 - p$, player 2 receives a signal s_1 that is uniformly distributed on X_1; that is, the signal is completely uninformative. Now suppose that player 1 chooses effort according to the cdf $H_1(\cdot)$. If player 2 observes a signal s_1 whose value lies in the support of H_1, then Bayes' rule implies that player 2 must believe that $x_1 = s_1$ with probability 1. If player 2 observes a signal s_1 whose value does not lie in the support of H_1, then Bayes' rule implies that player 2's posterior belief is equal to H_1. In that case, denote the cumulative distribution of player 2's effort choice as $H_2(\cdot)$. Now, player

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[6]See Huck and Müller (2000) for experiments relating to Bagwell's game.

[7]See also Güth *et al.* (1998) for restoration of commitment with n players, as well as Maggi (1999) for a restoration of commitment under private information. Oechssler and Schlag (2000), however, question the reasonableness of mixed strategy equilibria restoring commitment, while Bhaskar (2005) shows they may not exist in certain economic environments.

1's optimization problem is:

$$\max_{x_1} E\pi_1 = p\left[\frac{f_1(x_1)}{f_1(x_1) + f_2(x_2(x_1))}V_1 - x_1\right] + (1-p)\int_{x_2}\left[\frac{f_1(x_1)}{f_1(x_1) + f_2(x_2)}V_1 - x_1\right]dH_2(x_2)$$

Notice that it is identical to the optimization problem of player 1 in the sequential contest with costly observation. Therefore, by the same strict concavity arguments that underlie Proposition 1 it follows that H_1 and H_2 are degenerate in equilibrium. In other words, all equilibria are in pure strategies. Now, one may directly apply the proof in Bagwell (1995) with respect to pure strategy equilibria to conclude that the value of commitment is lost in all equilibria.

While this demonstrates that for a simple signal generating function the value of commitment vanishes completely, it remains for future research to determine if the result holds more generally.

Tournament

Returning to the intuition behind Proposition 1, it seems clear that what the first player needs in order to restore the value of commitment is a mechanism to credibly commit to unpredictable behavior, and thereby induce player 2 to pay to observe player 1's actions. Here we show that sequential rank-order tournaments offer an avenue to do this.

Our model of tournaments closely follows that of Lazear and Rosen (1981). In Lazear and Rosen, effort x_i has an effectiveness $y_i = x_i + \delta_i$, which may be interpreted as output. That is, the output generated by a player's effort depends on the effort itself as well as on a random component δ_i. The object is then awarded to the player with the greater output. In a simultaneous tournament, both players choose their effort at the same time. Subsequently, the random variables δ_1 and δ_2 are realized and the player with the higher output is the winner. Lazear and Rosen (1981) show that, with sufficient structure, there exists a pure strategy equilibrium (x_1^*, x_2^*).

In a sequential tournament, player 1 chooses x_1 and immediately thereafter δ_1 is realized. The effectiveness $y_1 = x_1 + \delta_1$ is then revealed to player 2. Upon observing y_1, player 2 chooses his effort x_2 and δ_2 is realized. Finally, the players receive their payoffs.[8] One can show that, with sufficient structure, there exists a subgame perfect

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[8]Dixit (1987) studies a different kind of sequential tournament. In his model, δ_1 and δ_2 are realized after both x_1 and x_2 have been chosen. In that case, a sequential tournament simply is a sequential contest with a particular functional form for the success function. However, for

equilibrium in pure strategies $(x_1^{**}, x_2^{**}(y_1))$ where commitment has value.[9]

Now consider the case of a sequential tournament with costly observation. Suppose that player 2 observes player 1's output. Then, even if player 1 is playing a pure strategy x_1, player 2's best response varies depending on the realization of player 1's output y_1. This means that, for sufficiently small costs of observation, player 2 will indeed find it optimal to observe y_1. Together, these considerations imply that in a sequential tournament the value of commitment is preserved when observation is costly.

Proposition 3 *For sufficiently small costs of observation, the value of commitment is preserved completely in the sequential tournament with observation costs.*

Formally: there exists a $k > 0$, such that, for all $\varepsilon < k$, strategies where effort levels $(x_1^{**}, x_2(y_1))$ are chosen and where player 2 observes with probability one constitute a subgame perfect equilibrium of the sequential tournament with observation costs.

5 Conclusions

Previously, Bagwell and others have shown that if one restricts attention to pure strategies, the value of commitment is fragile in leader-follower games with discrete strategy spaces. However, the value of commitment in these games can be restored by allowing for mixed strategies.

In this paper we have shown that, for a general class of contests, mixed strategies do not restore the value of commitment. In fact, the same conditions that ensure that a sequential contest is well behaved destroy the value of commitment when the follower is obliged to pay even arbitrarily small costs to observe the action of the leader. Specifically, if the leader's payoff function is strictly concave, the value of commitment is destroyed.

Why is this so? For commitment to have value, the second player must choose to observe the first player's action with strictly positive probability. This only happens if

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tournaments such as the Olympic long jump, or a sequential version of the sales "contest" described in Glengarry Glenn Ross (Mamet, 1984), modeling δ_i as occurring contemporaneously with x_i seems to us more reasonable.

    [9]Following Lazear and Rosen, consider the case where $\delta_i \backsim \mathcal{N}(0, \sigma^2)$, $V_1 \neq V_2$, and $C_i(x_i) = \frac{\gamma}{2}x_i^2$, where γ is a parameter describing the degree of convexity of the cost function. It may be verified that for sufficiently large values of γ and σ^2, there indeed exists a pure strategy subgame perfect equilibrium where commitment has value.

observing the first player's move is indeed valuable. When the first mover is playing a pure strategy however, in equilibrium, his move is perfectly predictable. Thus, actually observing the move adds no value. In principle, the first mover in a sequential contest could inject noise into his strategy to restore the value of observation for the second mover. But while this would clearly be desirable, because of the strict concavity of his payoff function the first mover cannot overcome the temptation to "purify" his behavior and always play the unique pure strategy that maximizes his profits given his beliefs. Of course, the second mover anticipates this and the scenario unravels.

While in this paper we have focused on contests, the same techniques can be applied to other games where the first mover's problem is strictly concave. An important example is Cournot/Stackelberg quantity competition. It is easy to see that the same conclusion as in Proposition 1 applies: in all subgame perfect equilibria of the sequential Stackelberg game with observation costs, quantities produced are identical to those in the unique Cournot-Nash equilibrium of the simultaneous move game regardless of the size of the observation costs.

To conclude, this paper offers a justification for the attention that the existing literature has given to simultaneous contests. Indeed, in settings where effectiveness of lobbying is unobservable even though the lobbying itself can be observed at some small cost, the sequential contest is outcome equivalent to the simultaneous contest. As is illustrated by the radically different conclusions for sequential tournaments, another way to interpret our results is that the value of commitment depends crucially on modeling assumptions that might seem innocuous or are even mathematically equivalent in the simultaneous case.

A Appendix: Contests with General Payoff Structures

In this section, we derive general sufficient conditions for contest success and cost of effort functions such that Proposition 1 continues to hold.

First, suppose that the contest success function, P, is continuously differentiable and that $P_1 \equiv \frac{\partial P(x_1, x_2)}{\partial x_1} > 0$ and $P_2 \equiv \frac{\partial P(x_1, x_2)}{\partial x_2} < 0$, for all x_1, x_2. That is, player 1's chances of winning are increasing in his own effort and decreasing in his rival's effort. Let $C_i(x_i)$ denote the cost to player i of expending effort x_i and assume that $C_i(\cdot)$ is continuously differentiable, strictly increasing, and (weakly) convex. We also assume that $\lim_{x_i \to \infty} C_i(x_i) = \infty$. Since V_i is bounded, this ensures that players undertake finite effort levels.

Simultaneous Contest

Suppose that the two players compete simultaneously. To ensure that this problem is well-behaved we make the following regularity assumption.

Assumption 1. Player i's problem is strictly concave. That is, for all x_1, x_2 :

$$P_{11}(x_1, x_2) V_1 - C_1''(x_1) < 0$$

and

$$-P_{22}(x_1, x_2) V_2 - C_2''(x_2) < 0$$

Assumption 1 is satisfied if C_i is sufficiently convex or $P_{11} < 0$ and $P_{22} > 0$. It guarantees that the first-order conditions are both necessary and sufficient for characterizing the best-response functions of the players. Hence, a pure strategy Nash equilibrium is a pair (x_1^*, x_2^*) satisfying

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\begin{aligned}
P_1(x_1^*, x_2^*) V_1 - C_1'(x_1^*) &= 0 \\
-P_2(x_1^*, x_2^*) V_2 - C_2'(x_1^*) &= 0
\end{aligned}
$$

Note that at least one such equilibrium exists, since the best-response functions are bounded and continuous.

Sequential Contest

Next, suppose that the contest is played sequentially. That is, player 1 chooses x_1, player 2 costlessly and perfectly observes its value, and then chooses x_2.

Player 2's best-response function, which we denote by $x_2(x_1)$, is identical to his best-response function in the simultaneous contest. It is characterized by the first order condition

$$P_2(x_1, x_2) V_2 - C_2'(x_1) = 0$$

Player 1's optimization problem is to choose x_1 to maximize $E\pi_1$, recognizing the dependence of x_2 on x_1. To ensure that player 1's optimization problem is well behaved, we make the following assumption which is again satisfied provided that C_i is sufficiently convex:

Assumption 2. Player 1's problem is strictly concave. That is, for all x_1,

$$V_1 \left[\begin{array}{c} P_{11}(x_1, x_2(x_1)) + 2P_{12}(x_1, x_2(x_1)) \frac{\partial x_2}{\partial x_1} + \\ P_{22}(x_1, x_2(x_1)) \left(\frac{\partial x_2}{\partial x_1} \right)^2 + P_2(x_1, x_2(x_1)) \frac{\partial^2 x_2}{(\partial x_1)^2} \end{array} \right] - C_1''(x_1) < 0$$

If Assumptions 1 and 2 hold, then there exists a *unique* subgame perfect equilibrium, $(x_1^{**}, x_2(x_1^{**}))$, in the sequential contest. This equilibrium is characterized by the following first order conditions:

$$V_1 \left[P_1(x_1^{**}, x_2(x_1^{**})) + P_2(x_1^{**}, x_2(x_1^{**})) \frac{\partial x_2(x_1^{**})}{\partial x_1} \right] - C_1'(x_1^{**}) = 0$$
$$-P_2(x_1, x_2) V_2 - C_2'(x_2) = 0$$

where
$$\frac{\partial x_2(x_1^{**})}{\partial x_1} = \frac{-P_{12}(x_1^{**}, x_2(x_1^{**})) V_2}{P_{22}(x_1^{**}, x_2(x_1^{**})) V_2 - C_2'(x_2(x_1^{**}))}$$

Value of Commitment

Fix the effort levels of the two players at some pure strategy Nash equilibrium, (x_1^*, x_2^*), of the simultaneous contest. The following assumption guarantees that there is positive value of commitment.

Assumption 3. For all (x_1^*, x_2^*),

$$V_1 \left[P_2 \left(x_1^*, x_2 \left(x_1^* \right) \right) \frac{\partial x_2 \left(x_1^* \right)}{\partial x_1} \right] \neq 0$$

Here, $x_2 \left(x_1^* \right) = x_2^*$, by definition, and

$$\frac{\partial x_2 \left(x_1^* \right)}{\partial x_1} = \frac{-P_{12} \left(x_1^*, x_2 \left(x_1^* \right) \right) V_2}{P_{22} \left(x_1^*, x_2 \left(x_1^* \right) \right) V_2 - C_2' \left(x_2 \left(x_1^* \right) \right)}$$

Together, Assumptions 1-3 ensure that the class of contests we study excludes "pathological cases," where the first mover's problem is ill-behaved, equilibrium only exists in mixed strategies, or the follower is non-reactive.

Sequential Contest with Costly Observation

Recall that our main result was:

In any subgame perfect equilibrium of the costly leader contest, there is no value to commitment.

To establish the result for the current, more general set up, we again show that all subgame perfect equilibria of the costly leader contest correspond to Nash equilibria of the simultaneous contest.

First, note that Lemmas 1 and 2 carry over immediately to the more general setting without requiring any changes in their proofs. Thus, we need only worry about mixed strategy equilibria.

Suppose player 1 believes that player 2 pays to observe player 1's effort with probability p. Conditional on observing, subgame perfection implies that player 2 plays his (unique) best response $x_2 \left(x_1 \right)$. Represent player 1's beliefs about player 2's action x_2 conditional on *not* observing by the cumulative distribution function $H \left(\cdot \right)$. Then, player 1's optimization problem is

$$\max_{x_1} E\pi_1 = p \left[P \left(x_1, x_2 \left(x_1 \right) \right) V_1 - C_1 \left(x_1 \right) \right] + (1 - p) \int_{x_2} \left[P \left(x_1, x_2 \right) V_1 - C_1 \left(x_1 \right) \right] dH \left(x_2 \right)$$

This optimization problem is strictly concave in x_1, as it is a convex combination of expressions that we know to be strictly concave in x_1 by Assumptions 1 and 2. Hence, player 1 plays a pure strategy in any subgame perfect equilibrium. Finally, this implies that $p = 0$. Therefore, all subgame perfect equilibria in the sequential

contest with observation costs are in pure strategies and Proposition 1 follows.

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