Communication through Noisy Channels

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Abstract

We study two-player coordination games in which one player is better informed than the other and where the costs of miscoordination are different in distinct states of nature. Specifically, let $\Omega=\{\omega_1,\ldots, \omega_k\}$ be the states of nature, and for each $l=1,2,\ldots k,$ let $\Pi^l=(A_1,A_2,u^l{}_1,u^l{}_2)$ be the strategic form game associated with ω_l, where A_i and $u^l{}_i$ are the set of actions and payoff function of Π^l , respectively, of player i. In this model, nature chooses one state ω_l (and hence the game Π^l) using a commonly known probability distribution and informs only to player 1 of its choice. Then, the informed player transmits his private information to the uninformed one, though a noisy channel, and actions are chosen and payoff realized.

We assume that there is a unique optimal play, $(\hat{a}^l{}_1, \hat{a}^l{}_2) \in A_1xA_2$ in each Π^l, and that players communicate by using repeatedly n times a discrete memory less noisy channel. Thus, we assume that signals or messages can be distorted in the communication process. A discrete channel is a system consisting of an input alphabet X and output alphabet Y, and a probability transition matrix $p(y/x)$, that expresses the probability of observing the output symbol y given that the symbol x was sent. The channel is memory less if the probability of the output depends only on the input at that time and is conditionally independent of previous inputs or outputs.

Our aim is to analyze and characterize the set of achievable equilibrium outcomes for a given channel and a given *finite* number of "uses" (repetitions of the channel). We bound the efficiency loss of noisy communication in terms of the properties of the noisy channel such as the transition probability $p(y/x)$ and the channel capacity.

We define a *noisy communication protocol* which consists of a codification of the states of nature into a channel input sequences, $X^n(\omega_l)$, a noisy channel, $(X, p(y/x), Y)$, and a decodification rule mapping channel outputs into the uninformed player's actions. The noisy channel transforms the input sequence into an output sequence, that is random but it has a distribution that depends on the input sequence. From the realized output sequence, y^n, the uninformed player attempts to recover the transmitted message. Each of the possible input sequences induces a probability distribution on the output sequences. Since two different input sequences may give rise to the same output sequence, the inputs are confusable. The informed player would like to choose a "non-confusable" subset of input sequences so that with high probability, there is only one highly likely input that could have caused the particular output. The number of possible decodification functions is exponential with respect to n, since it is given by the number of partitions of the k subsets associated to each element of Ω, over the set of output sequences of length n.

To cope with this problem, we establish a simple deterministic coding rule -*the "block coding"*-. Then, we define a partition of the output set Y^n, with the property that each set of the partition verifies a closeness condition with respect to the feasible input sequences -where 'closeness' here is defined in terms of a 'Hamming'-like distance function-. Elements of the partition are mapped into the uninformed player's actions in the following way: the action corresponding to state ω_l is played if and only he gets an output sequence belonging to the set of the partition with sequences "close" enough to those input sequences coding state ω_l,.

Although these coding /decoding rules are suboptimal for a finite number of repetitions of the channel, they are a very simple procedure (polynomial with respect to n) of designing noisy communication protocols. Thus, our protocol is much less complex than that of checking all pairs of feasible codification/decodification rules to find the optimal ones, but still allows the players to transmit enough information to achieve coordination outcomes.