Reputation for Toughness in Bargaining with Incomplete Information

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Ella Segev*

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1 Introduction

Abreu and Gul's seminal paper "Bargaining and Reputation" (Abreu and Gul (2000)) combines the terms and knowledge of two widely investigated fields - bargaining theory and reputation theory. They study a complete information bargaining game and amend it to accommodate for irrational (or behavioral) players. Irrational players demand a certain share of the pie and are never willing to accept any smaller share. Irrational players are therefore nonstrategic. Rational players can benefit from imitating (at least to a certain point) this behavior and attain the reputation of being irrational. They identify the unique equilibrium, which is independent of the bargaining procedure. Moreover, they emphasize the effect of irrational types on the equilibrium by showing that the inefficiency (i.e. delay) entailed in it is influenced by the ex-ante probability of different irrational types.

In an earlier work (Heifetz and Segev (2004)) we were able to show how an "endowment effect" or a toughness bias in bargaining may be evolutionary viable. We studied the terms under which a positive toughness bias, i.e. overestimating the object for the seller and underestimating it for the buyer, will be optimal in a bargaining setting and showed that this bias will take over the population in the evolutionary process.

This paper links our earlier work with bargaining with committed types as in Abreu and Gul by expanding the notion of behavioral types and incorporating it into an incomplete information bargaining game. In this study an irrational

*William Davidson Faculty of Industrial Engineering and Management, The Technion - Israel Institute of Technology. E-mail: esegev@ie.technion.ac.il

player has a toughness bias, a bias in how she perceives the value of the object to her (as in Heifetz and Segev (2004)).

There is a large body of literature in experimental economics (see Horowitz and McConnell (2002) and Brown and Gregory (1999) for surveys) that documents misperceptions of one's valuation as they appear in this study. These misperceptions might be the explanation for the large disparity between one's willingness to pay (WTP) and one's willingness to ask (WTA) for the same inexpensive market goods that we observe in these experiments. In some cases the authors refer to this phenomenon as the endowment effect. Some of these papers suggest that a plausible explanation for the disparity between WTA and WTP is the uncertainty of the players regarding the value of the object - both to them and to their opponent. Alternatively we suggest that they know their own valuation but this valuation is not the one they would have in a market situation - they misperceive their valuation when they become sellers or buyers in a bargaining situation. The presence of even a small portion of individuals in the population who have such a bias and always misperceive the value of the object (either as sellers - upwards or as buyers - downwards) can explain why all other players state a higher WTA and a lower WTP.

In this paper I address the question whether one can achieve reputation for being a tough bargainer. I make the assumption (common in the reputation literature), that a small portion of the players are irrational players who indeed act tough while bargaining - sometimes above and beyond rational decision making. Given the existence of such players I show here that a rational player can pretend to be tough and earn the desired reputation and consequently a better agreement.

This paper generalizes Abreu and Gul's results for incomplete information bargaining games. This generalization is a natural expansion for this new research field of bargaining and reputation and will allow us to reach meaningful conclusions on the reputational effects in bargaining. The results of the analysis are compared with existing experiments' results that document players' behavior in these bargaining games. In many of these experiments we observe a behavior, which is different than the expected equilibrium behavior (even in games in which the equilibrium is unique and the equilibrium strategies are dominant strategies). I claim that players' assumption that a small share of the population is irrational might explain why they deviate from equilibrium strategies (which are found under the assumption that there is common knowledge that all players are rational) in these games.

In this paper an irrational player has a toughness bias, a bias in how she perceives the value of the object to her. An irrational player in the role of a seller perceives the object to be worth more than its actual value and an irrational player in the role of a buyer perceives the object to be worth less than its actual value. The true value of the object for the irrational seller (buyer) is the minimal (maximal) price for which she would be willing to sell (buy) the object as a price taker in a market. When she enters the bargaining situation she can no longer identify this true value and believes that her valuation is different. This misperception is a line of character for the player, therefore it is true for every object she sells or buys and for every interaction in which she bargains. The size, in absolute value, of her toughness bias is independent of the specific object (when normalizing her valuation for all the objects, in the eyes of her opponent, to be distributed in the interval [0,1]) or her role in the bargaining - seller or buyer. Thus in this work (in contrast with Abreu and Gul (2000)) an irrational player is a strategic player who maximizes her perceived payoff. The only irrational aspect of her behavior is her misperception of the value of the object - her toughness.

An important assumption here is that rational players incur a cost when imitating an irrational player (for example the cost of not blushing when pretending to be tough). In general, without this cost, the payoffs of rational players would exceed those of the most profitable irrational type at any nondegenerate population composition. Thus, in this case only rational players can survive in the long run. When rational players pretend to be irrational one cannot observe their rationality, and that's why we would get that irrational types cannot survive unless rationality comes at a cost. Therefore by attaching a cost to rational behavior we mean that when a rational player pretends to be an irrational one, it is impossible to distinguish between them at first sight.

Another assumption is that an irrational player's type (degree of toughness) is observable by her opponent. This is because toughness in bargaining is a line of character observable by others and moreover it is possible to distinguish players with different degrees of toughness. Thus, in a population with more than one irrational type, a rational player must decide whether or not to pretend to be irrational and moreover - which irrational type she pretends to be (what degree of toughness she pretends to have) out of a finite set of types. We assume that different costs are involved with pretending to be irrational of different types and the cost increases with the degree of toughness the player chooses to pretend to have.

The rational player's decision whether or not to pretend to be irrational and at which degree of toughness is taken at the outset of the game - before the player knows her own valuation but after she knows her role - seller or buyer - in the game. Thus, when two players confront each other they observe the others' toughness degree (or toughness mask if the other is rational who pretends to be irrational of a specific type), find out the realization of the value of the object to them and then play the bargaining game. A strategy of a rational player is hence a decision whether or not to pretend to be irrational and of which type (degree of toughness) together with a full strategy for the following game. Irrational players here don't have the ability to pretend to have types other than their own.

In Abreu and Gul (2000) the only source of incomplete information is the question of whether the opponent is behavioral or rational. Therefore a strategy of a player consists only of the time she chooses to concede. While bargaining the players are already aware of their partner's demand and can only choose when or whether to accept it. In this paper however, the players are unaware both of their partner's valuation for the object (and therefore of their demand) and whether their partner is rational or not. I follow a well-analyzed equilibrium behavior in several such bargaining games with incomplete information and show under what conditions the rational players choose to imitate the equilibrium behavior of irrational types and how does that changes the equilibrium characters.

2 The Model

We have a large population of individuals who are continuously and repeatedly matched at random to bargain and assume the role of seller or buyer with equal probabilities. A small fraction of the population is irrational. At any point in time there is a finite set of irrational types denoted $C = \{\varepsilon_1, ..., \varepsilon_k\}$. A player is irrational of type ε_i if when she has the seller's role, she misperceives her valuation of the object to be higher by ε_i than it really is. She believes and acts as if her valuation for the object is $s = S + \varepsilon_i$ when the objective worth (the one which determines her fitness) of the object to her is S. Similarly, when she has the buyer's role, she misperceives her valuation of the object to be lower by ε_i than it really is. She believes and acts as if her valuation of the object is $b = B - \varepsilon_i$ when the objective worth of the object to her is B.

A player is either rational - knows the true value of the object - or irrational throughout all her interactions and being rational or irrational is independent of the true value of the object to her and of her role in the game (seller or buyer). The true valuations of the buyer and of the seller are always drawn at random and independently from a uniform distribution on the interval $[0, 1]$. Thus if a player is confronted with an irrational opponent of degree ε_i (or a rational player who pretends to be irrational of that degree) she believes her opponent's valuation is drawn from the interval $[-\varepsilon_i, 1 - \varepsilon_i]$ if her opponent is the buyer and the interval $[\varepsilon_i, 1 + \varepsilon_i]$ if her opponent is the seller.

The probability that any given player is irrational of type ε_i is denoted x_i and $x_0 = 1 - \sum_{i=1}^{k} x_i$ is the probability that the player is rational. We interpret the probabilities as population shares of the types in the large population from which the players are both drawn. We refer to $x = (x_1, ...x_k)$ as the population composition. If a rational player chooses to pretend to be irrational of type ε_i she pays a cost of c_i and if $\varepsilon_i > \varepsilon_j$ then $c_i > c_j$.

2.1 The take-it-or-leave-it-offer

Consider for a start the take-it-or-leave-it bargaining game in which an uninformed seller makes an offer to an informed buyer who can either accept it in the proposed price, or reject it, in which case no trade takes place. The seller valuation S for the object is known and normalized to be zero. The buyer's valuation is her own private information and is drawn from a uniform distribution on the interval $[0, 1]$. With no irrational players the seller simply chooses the offer that maximizes her payoff $\max_t U(S = 0) = \max_t \int_t^1 t dB$ therefore she chooses $t = \frac{1}{2}$. A buyer with a valuation $B \geq \frac{1}{2}$ accepts the offer and buyers with other valuations reject it.

Now suppose we have a population of players with a fraction of irrational types who have a bias in a finite set of biases C. We have $C = \{\varepsilon_1, ..., \varepsilon_k\}$ and $x = (x_1, ...x_k)$.

Note that since the strategy of the seller is only the price she offers she has no incentive to pretend to be irrational. She has nothing to gain from her opponent not knowing whether she is rational or not. Moreover she certainly would not want to adopt the irrational players' strategy (offer) since by doing so she will be maximizing something different than her own expected payoff. Therefore a rational seller will introduce herself as rational and will make the offer that maximizes her payoff. The buyer on the other hand might have something to gain from pretending to be irrational, conditional on the cost for that pretension not being too high. After introducing himself as irrationally tough, the rational buyer will accept the offer if and only if it is below his valuation.

We have the following proposition:

Proposition 1 *If there exists an index for which*

$$c_l - \frac{x_l^2 \varepsilon_l^2}{8\left(x_0 + x_l\right)^2} - \frac{x_l \varepsilon_l}{4\left(x_0 + x_l\right)}\left(1 - \sum_{j=1}^{k} x_j \varepsilon_j\right) < \min\left\{0, \min_{i=1,\ldots k, i \neq j}\left\{c_i - \frac{\varepsilon_i^2}{8} - \frac{\varepsilon_i}{4}\left(1 - \sum_{j=1}^{k} x_j \varepsilon_j\right)\right\}\right\}$$

then there exists a symmetric equilibrium of the game in which rational sellers choose not to pretend to be irrational while rational buyers choose to pretend to be irrational of type ε_l. A rational seller when confronted with an irrational buyer of type ε_l will make the offer

$$t_l = \frac{1}{2} - \frac{x_l \varepsilon_l}{2\left(x_0 + x_l\right)} < \frac{1}{2}$$

while when confronted with an irrational buyer of type $\varepsilon_i, i \neq l$, will make the offer

$$t_i = \frac{1}{2} - \frac{\varepsilon_i}{2} < \frac{1}{2}$$

. An irrational seller of type ε_j when confronted with an irrational buyer of type ε_l will make the offer

$$t_l^j = \frac{1 + \varepsilon_j}{2} - \frac{x_l \varepsilon_l}{2\left(x_0 + x_l\right)}$$

while when confronted with an irrational buyer of type $\varepsilon_i, i \neq l$, will make the offer

$$t_i^j = \frac{1 + \varepsilon_j}{2} - \frac{\varepsilon_i}{2}$$

. Buyers will accept the offer if and only if it is below their valuation (the true one for rational players and the perceived one for irrational players).

In the case with only one irrational type the condition in the proposition above becomes:

$$c_1 < \frac{1}{8} x_1 \varepsilon_1 \left(2 - x_1 \varepsilon_1\right)$$

If this condition holds (i.e. the cost for pretending is not too high) then in the unique equilibrium all rational buyers will pretend to be irrational. Thus in this case our model predicts that a small fraction of the offers that are made (those made by irrational sellers) will be larger than one half and a larger share of the offers will be smaller than one half (i.e. a better price for the buyer). The overall efficiency (total probability of trade) of the game would reduce from $\int_{\frac{1}{2}}^{1} dB = \frac{1}{2}$ to (recall that in this case $x_0 = 1 - x_1$)

$$(1-x_1)^2 \int_{t_1}^{1} dB + (1-x_1)\, x_1 \left(\int_{t_1}^{1-\varepsilon_1} db + \int_{t_1^1}^{1} dB \right) + x_1^2 \int_{t_1^1}^{1-\varepsilon_1} db = \frac{1}{2} - x_1\varepsilon_1$$

where $t_1 = \frac{1}{2} - \frac{x_1\varepsilon_1}{2}$ and $t_1^1 = \frac{1+\varepsilon_1}{2} - \frac{x_1\varepsilon_1}{2}$.

Thus we get less trade than expected by the regular model. Moreover the overall gains from trade also reduce from $\int_{\frac{1}{2}}^{1} BdB = \frac{3}{8}$ to

$$(1-x_1)^2 \int_{t_1}^{1} BdB + (1-x_1)\, x_1 \left(\int_{t_1}^{1-\varepsilon_1} bdb + \int_{t_1^1}^{1} BdB \right) + x_1^2 \int_{t_1^1}^{1-\varepsilon_1} bdb = \frac{3}{8} - \frac{1}{8} x_1\varepsilon_1 \left(8 - 3\varepsilon_1 - x_1\varepsilon_1\right)$$

3 References

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