

A Subsidized Vickrey Auction for Cost Sharing

Jesse A. Schwartz*
Kennesaw State University

Quan Wen†
Vanderbilt University

March 2006

Abstract

In a cost sharing situation, a group of players jointly produce some good and must decide how much each player consumes and how much each player pays. Two well-known solutions are the average and serial cost mechanisms. Although both mechanisms raise exactly enough money to pay the production costs, they involve complicated equilibrium strategies and are not allocatively efficient. On the other hand, the standard Vickrey auction induces dominant strategies and allocative efficiency, but generates revenue in excess of production costs. Our paper amends the Vickrey auction so that some of the surplus revenue subsidizes additional production of the good, but in such a way that preserves the dominant strategies. This subsidized Vickrey auction is allocatively inefficient, but Pareto dominates the standard Vickrey auction.

JEL Classification Numbers: C72, D44, H42

Keywords: Cost sharing, dominant strategy implementation, serial cost, Vickrey auction, subsidized Vickrey auction

*Department of Economics, Finance, and Quantitative Analysis, Kennesaw State University, 1000 Chastain Road, Box 0403, Kennesaw, GA 30144, U.S.A. Email: jschwar7@kennesaw.edu

†Department of Economics, Vanderbilt University, VU Station B #351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, U.S.A. Email: quan.wen@vanderbilt.edu

Extended Abstract

A group of players who jointly produce some private good must decide how much each player obtains and how much each player pays. There are many examples of this cost sharing situation, including farmers using an irrigation system, office workers sharing a secretary, and divisions of a corporation using an internal advertising department or training facility. Cost sharing is similar to a typical auction situation, where quantity is to be allocated among players who value the good differently. The key distinction, however, is that with cost sharing there is no opposing seller producing the good, choosing the rules of the auction, and pocketing any resulting profit, as there typically would be in an auction. Rather, the players can design for themselves a mechanism in any way they like to strive for such goals as efficiency and dominant strategies.

The model in our paper has players with private values (represented by demand curves). Since our paper will focus on games with dominant strategies, it will not be important whether players have complete or incomplete information about their opponents' demand curves. To motivate our paper, we first show that the average and serial cost sharing mechanisms featured prominently in the literature are not efficient and do not induce dominant strategy equilibria. We then consider the well-known Vickrey (1961) auction, which is allocatively efficient and does induce each player to submit its true demand curve as a dominant strategy. But with no seller to pocket the surplus revenue over cost, the Vickrey auction leaves money on the table. We know of no other mechanism that allocates quantity efficiently and induces players to bid their true demand curves as a dominant strategy. Thus, the players may decide to look for mechanisms that forgo dominant strategies in order to achieve allocative efficiency and budget-balancedness (revenue generated equals production cost); mechanisms introduced in Arrow (1979) and d'Aspremont and Gerard-Varet (1979a, 1979b) and summarized in Krishna (2002). But with no practical implementation of these complicated budget-balanced mechanisms, the players may instead pursue ways to reduce the surplus revenue generated by a Vickrey auction while maintaining the dominant strategies.

Our paper distills from the Vickrey auction the key conditions that ensure that players bid their true demand curves as weakly dominant strategies, and shows that allocative efficiency—though certainly a desirable property—is not among these conditions. We call these conditions that ensure dominant strategies: (1) *independent supplies* and (2) *perfect price discrimination*. The independent supplies condition requires that each player faces a supply curve that does not depend on the demand curve bid by this player. The perfect price discrimination condition says that for whatever quantity the player obtains, it will pay the area under this supply curve up to this quantity, and the player obtains the quantity where its submitted demand curve intersects this independent supply curve. The intuition is that since the supply curve a player faces is not at all influenced by the demand curve it submits, and since the player must pay the area under this supply curve, the best a player can do is to obtain the quantity where its true demand curve intersects this supply curve. But this is accomplished by bidding one’s true demand curve.

With these conditions in mind, we introduce the subsidized Vickrey auction, which uses some of the surplus generated in the Vickrey auction to subsidize production costs, allowing players to consume more than the efficient quantity. Nevertheless, the subsidized Vickrey auction satisfies the independent supplies and perfect price discrimination conditions, and so each player has a dominant strategy to bid its true demand curve. We show that the extra consumption in the subsidized Vickrey auction benefits the players more than leaving surplus revenue on the table, or put more precisely, the subsidized Vickrey auction Pareto dominates the standard Vickrey auction. To make sure the subsidized Vickrey auction is feasible, we show that enough money is raised to cover the production costs.