‘Rising stars’ should shine*

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Abstract

The literature on sabotage in contests allows the players (workers) to exert both productive and negative (sabotaging) activities. By doing so promotion tournaments where the workers’ promotion is based on relative performance are enriched with the common practice of sabotaging in organizations. This literature assumes, however, that workers’ abilities in productive activities are common knowledge among the players.

On the contrary, we assume that worker’s productive ability is *unknown* by his job colleagues. We construct a previous stage where workers signal their productive ability to their job colleagues, and a second stage where workers are called to participate in a promotion tournament with sabotage. We show that, by creating this first stage, the tournament designer (the firm manager) can minimize the occurrence of the so-called ‘raising star’ paradoxes, where the ablest worker is not finally the one promoted. Instead, if there is at least one able worker in the firm, the design of our tournament makes that he will be the player with the highest probability of being promoted. In other words, the ‘raising star’ of the firm should finally shine.

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1 Introduction

Promotion tournaments are a common incentive scheme used by companies and organizations to try to select the ablest member of the group to be promoted to a higher position within the organization. As Bognanno (2001) suggests, promotion tournaments are appropriate for modeling, for example, promotions to very senior leadership positions. Many other situations can also be analyzed using promotion tournaments, such as certain athletic competitions or political elections.

In promotion tournaments, different players (e.g. workers) compete to obtain the prize of being promoted by exerting some effort. One of the main difficulties for the contest designer (the manager of the firm in this case) in these tournaments, however, is that the effort workers exert is unobservable by him, which implies that he will need to rely on some other variable in order to determine who will be the worker promoted. This variable is normally the worker's performance. Hence, in a promotion tournament the worker who achieves the highest performance is the one promoted. In this reasoning, the contest designer (firm manager) considers that ability surely determines the marginal product of effort and as a consequence the worker's optimal amount of exerted effort. Therefore, according to this argument, a promotion tournament based on performance will ultimately lead to the promotion of the worker with highest ability in productive activities.

This reasoning, however, underestimates the fact that promotion in this kind of tournaments is based on *relative* rather than absolute performance. Hence, a worker is interested in obtaining the highest performance among all his colleagues at work, instead of achieving a high performance per se.

The fact that promotion is based on relative terms has lead some authors [Lazear (1989), Chen (2003) and Brown *et al.* (2005)] to suggest that workers actually exert two types of effort: productive effort, which increases his own performance, and negative effort (sabotage) against each of his job colleagues in order to reduce their relative performance. Their results suggest that workers will sabotage the ablest colleague on productive activities, what may lead to the promotion of a worker who is not necessarily the most productive one.

These models are indeed more realistic and clearly illustrate the incentives of workers (or players) in organizations where the probability of promotion is solely based on relative, rather than absolute, performance. Notwithstanding their contribution, they are criticized because of considering a restrictive assumption: workers perfectly know each other's abilities in productive and negative activities at the beginning of the promotion tournament. It seems reasonable to think that workers can't immediately know each other's productive abilities. After working together for some time, workers have much more accurate information about each other's performance and exerted efforts in the firm, which they can use to infer their job colleagues' productive ability.

Hence, we consider that promotion tournaments analyzed by the literature are the last stage of a larger game played, for example, by the workers of a firm when competing for promotion. In particular, we will modify a promotion tournament by adding a previous stage *before* the beginning of the tournament. Specifically, we will assume that the entire game is structured as a tournament, where workers compete in a first stage exerting only productive effort when they do not know each other's productive abilities. At the end of this stage, only the k workers who generated the highest performance are qualified to play the final stage, and the worker who wins this stage becomes, for instance, the new president of the company.

The first stage represents the period of time when workers develop their tasks in the firm but there does not exist any potential promotion in the near future. They only exert productive effort, and some performance results from this effort. We can identify this first stage as the period lasting from the moment that a group of workers is hired until the moment when they learn how to exert sabotaging activities in the context of that particular firm. Hence, productive effort is the only control variable for the worker, who uses it to signal his productive ability to the other workers. Afterwards, in the second stage, a usual promotion tournament with both productive and sabotage activities is conducted, and a winner of this tournament is selected for promotion.

We show that only very talented workers consider to exert some productive effort in the first stage of the promotion tournament. As we will see, this helps to mitigate the so-called 'rising star' paradox, namely, not promoting the worker who everybody considers to be the ablest. As Chen (2003) remarks, this paradox is relatively common in the internal election of a candidate within a political party, or the promotion of some of the top executives of a firm in order to become the president of a company. Indeed, some rising star executive finally 'falls' and the organization ends up promoting the worker who wasn't necessarily the ablest. We will show that by introducing a first stage where workers compete in order to get a place in the final, the organization greatly increases the chances that the member promoted is one of the ablest ones, minimizing the above paradox. In other words, unlike Chen (2003), we show that the 'rising star' of the firm should finally shine.

The paper is organized as follows. In the next section, we present our model and assumptions. In section 3, we solve for the optimal strategies during the final stage of the game, and in section 4 we use these results to find the optimal strategies for the first stage of the tournament. In section 5 we comment on some of the implications of our results, as well as its contributions to the literature on tournaments. Finally, section 6 concludes and suggests some interesting extensions.

2 Model

We will consider a firm consisting of N workers who compete for promotion. Each worker i privately observes his own ability in productive activities, t_i, and in negative (sabotage) activities, s_i. Productive activities are understood in a broad sense, as any task that contributes to improve the worker's absolute level of performance, and as a consequence increases the firm's output. On the contrary, negative activities are conducted to reduce the other workers' performance, which implies an increase in the worker's *relative* performance (relative to all the other workers). Examples of such negative (sabotaging) activities in the firm are, for instance, hiding relevant information to the competitors, spreading rumors about them, and any type of noncooperation in the execution of certain tasks.

We will assume that t_i is identically and independently distributed, and drawn from a uniform distribution $\mathrm{U}[0,1]$. We also assume that all workers are equally able in sabotaging activities. In particular we normalize the workers' sabotaging abilities $s_1 = s_2 = ... = s_N = 1$, which will highlight the results in terms of the probability that the ablest worker is promoted.

Let e_i^1 and e_i^2 be the level of the positive effort of member i during stage 1 and 2, respectively. During the second stage, when the actual promotion tournament is hold, worker i can exert both productive effort, e_i^2, which increases his performance, and sabotaging activities against every member $j \neq i$ of the organization, a_{ij}, which decrease his rivals' performances. Hence, the aggregate amount of effort exerted by worker i in the entire game is given by $e_i^1 + e_i^2 + \sum_{j \neq i} a_{ij}$. All of these types of effort are costly for the worker, causing a disutility measured by $v\left(e_i^1 + e_i^2 + \sum_{j \neq i} a_{ij}\right)$. As usual, we will assume that the disutility of effort is increasing in the amount of any type of effort exerted by the worker $v'(\cdot) > 0$ and it is convex in effort $v''(\cdot) > 0$.

The amounts of exerted efforts in productive or sabotaging activities by worker i are not observable by the firm manager. On the contrary, worker i's performance in the second and first stage are the only two variables the manager can observe. In particular, the manager observes worker i's performance during the first stage, $\omega_i + u_i$, and during the second stage, $W_i + \varepsilon_i$, which are respectively given by

$$\omega_i + u_i = t_i e_i^1 + u_i$$

$$W_i + \varepsilon_i = t_i e_i^2 - g\left(\sum_{j \neq i} a_{ji}\right) + \varepsilon_i$$

In words, worker i's performance is increasing in his productive effort (weighted by his productivity in this type of effort) and is decreasing in the amount of total attacks that every member $j \neq i$ directs against him, $\sum_{j \neq i} a_{ji}$. The function $g(\cdot)$ denotes the effectiveness of these total attacks. We will assume that this effectiveness is increasing in the volume of attacks, $g'(\cdot) > 0$, but at a decreasing rate $g''(\cdot) < 0$. Finally, u_i and ε_i measure a random shock in worker i's performance in the first and second stage, respectively. These shocks eliminate the possibility that (unobserved) effort can be inferred from the observed performance. Specifically, u_i and ε_i are i.i.d. both accross workers and accross time, $\mathrm{E}[u_i] = \mathrm{E}[\varepsilon_i] = 0$, and are distributed according to a commonly known cummulative distribution function $F(\cdot)$ with associated density $f(\cdot)$, which is everywhere positive in their support $u_i, \varepsilon_i \in (-\infty, \infty)$.

Finally, we will assume that the (symmetric) utility that every member derives from being promoted is given by the parameter $\overline{V}$. The structure of the game will be the following.

1. In the first stage every player decides how much positive effort he will exert given his productive ability.

We will analyze the symmetric separating equilibrium, where workers with different productive abilities exert different levels of effort[1].

2. Once the first stage is over, every worker has revealed his productive ability, t_i, to his job colleagues by the level of effort he exerted. Hence, the promotion tournament of the second stage becomes a game of complete information, where the prize of the game is given by $\bar{V}$.

2.1 Observable variables and separating equilibrium

Let's emphasize what players observe at every stage of the game. Firstly, the manager of the firm observes only performance. As a consequence, he cannot make a perfect inference about what can be the productive ability, t_i, of the worker i who generated such performance.

Workers exert some effort in the first stage, which is perfectly observable among them. For instance, worker i can observe both worker j's performance *and* the amount of effort he exerted. Given that we are in a symmetric separating equilibrium with strictly monotonic effort functions, types (productive abilities in this case) can be inferred from observed efforts. This means that productive abilities are perfectly known by all workers at the beginning of the

[1] We are also looking for a strictly monotonic equilibrium for the first stage. That is, workers with higher productive abilities exert higher levels of productive effort during the first stage.

second stage. Notice that this inference cannot be made by the firm manager, since he observes performance, but not effort.

Specifically, the solution concept that we will use is that of a Perfect Bayesian Equilibrium (PBE), and the way we will find it is by firstly solving the second stage of the game. Given the optimal strategy of every player in this stage, in terms of productive and negative efforts, we will find their optimal strategy in the first stage, e_i^1.

Next we define a PBE of this game, emphasizing the characteristics that it must satisfy, and the procedure we will use in finding it, as described above.

Definition 1

A perfect Bayesian equilibrium of the promotion tournament with a previous stage in which productive effort is signalled to all the players is a strategy profile, $\hat{\sigma} = \left\{\hat{e}_i^1, \hat{e}_i^2, \hat{a}_{ij}\right\}_{\forall i, \forall j \neq i}$, *and posterior beliefs* $\mu_i\left(t_j | e_j^1\right)$, *such that the following conditions are satisfied.*

1. *For a given (privately observed) ability in productive activities,* t_i, *and for a given strategy profile in the second stage,* $\left\{\hat{e}_i^2, \hat{a}_{ij}\right\}_{\forall i, \forall j \neq i}$, *worker* i *chooses the amount of optimal effort in the first stage,* $\hat{e}_i^1$ *such that for all* t_i, $\forall i$ *and* $\forall j \neq i$

$$\hat{e}_i^1\left(t_i\right) \in \underset{e_i^1}{\arg\max}\ EU_i^1\left(e_i^1, \hat{e}_i^2, \hat{a}_{ij}, t_i\right)$$

2. *Once the profile of optimal efforts exerted by all workers in stage 1,* $\hat{e}^1$, *is revealed among the workers, each worker* i *optimally chooses the amounts of productive and negative effort to exert in the second stage,* $\hat{e}_i^2$ *and* $\hat{a}_{ij}$ *respectively, given the belief* $\mu_i\left(t_j | e_j^1\right)$ *about which types could have sent* $\hat{e}_j^1$, *such that for all* $\hat{e}^1$, $\forall i$ *and* $\forall j \neq i$

$$\left\{\hat{e}_i^2\left(\hat{e}^1, t_i\right), \hat{a}_{ij}\left(\hat{e}^1, t_i\right)\right\}_{\forall j \neq i} \in \underset{\left\{\hat{e}_i^2, \hat{a}_{ij}\right\}_{\forall j \neq i}}{\arg\max} \int_{t_j} \mu_i\left(t_j | \hat{e}_j^1\right) EU_i^2\left(\hat{e}_i^1, e_i^2, a_{ij}, t_i\right)$$

3. *The workers' posterior beliefs after the first stage must satisfy Bayes' rule*

$$\mu_i\left(t_j | \hat{e}_j^1\right) = \frac{f(t_j)\hat{\sigma}\left(\hat{e}_j^1 | t_j\right)}{\int_{t_j} f(t_j')\hat{\sigma}\left(\hat{e}_j^1 | t_j'\right)} \quad \textit{if} \quad \int_{t_j} f(t_j')\hat{\sigma}\left(\hat{e}_j^1 | t_j'\right) > 0$$

Note that the condition on the Bayesian updating of players' beliefs is satisfied in all separating PBE: a worker's optimal effort on stage 1, when using strictly monotonic strategies, generates point beliefs, what perfectly signals his type. In particular, each worker j's strategy choice in the first stage allows his job colleague (worker i) to totally concentrate his posterior beliefs on a particular realization of t_j. That is,

$$\mu_i\left(t_j|\hat{e}_j^1\right) = 1 \text{ for some } t_j \text{ such that } \hat{e}_j^1\left(t_j\right),$$

since $\hat{e}_j^1\left(t_j\right)$ is symmetric and strictly monotonic, and

$$\mu_i\left(t_j'|\hat{e}_j^1\right) = 0 \text{ for all } t_j' \neq t_j$$

Verbally, in this separating PBE, when worker i observes a particular effort level exerted by worker j in equilibrium, $\hat{e}_j^1$, he is able to perfectly infer the worker j's type, t_j, that generated such effort level $\hat{e}_j^1$, which is represented by this total concentration of beliefs.

We start by analyzing the case of three players competing in the first round of the game, where only the two players generating the highest performance are qualified to play the final. This is a very common situation, such as electoral contests where normally only two candidates with significant probabilities of being elected run for election, or in many firms, where the main competition to become the next president of the company is only among two well-known candidates. As commented above, we will start by finding the optimal strategies for the second stage of the game. Afterwards, given these optimal strategies for the final, we will find workers' optimal effort for the first stage of the tournament.

3 Optimal strategies in the final stage

Recall that worker i's observed performance during the second (final) stage of the promotion tournament against worker k is given by

$$W_i + \varepsilon_i = t_i e_i^2 - g\left(\sum_{k\neq i} s_k a_{ki}\right) + \varepsilon_i$$

That is, performance is increasing in the worker's effort in productive activities and decreasing in the sabotaging attacks he receives from all the other workers. Therefore, the expected utility of player i when participating in the promotion tournament of stage 2 is given by

$$EU_i\left(e_i^2, a_i\right) = prob_i(win\ stage\ 2)\bar{V} - v(e_i^2 + \sum_{k\neq i} a_{ik})$$

where $\sum_{k\neq i} a_{ik}$ represents the total amount of attacks that worker i directs against his opponents in the final.

Notice that since N=2, the probability that worker i wins the promotion tournament is, in fact, given by the chances that his observed performance is higher than the observed performance of worker $k \neq i$. Hence, the probability that worker i wins can be written as

$$prob\,(W_i + \varepsilon_i \geqslant W_k + \varepsilon_k) = prob\,(\varepsilon_i - W_{ki} \geqslant \varepsilon_k) \text{ where } k \neq i \text{ and } i\text{=1,2}$$

where $W_{ki} = W_k - W_i$. Additionally, since both ε_i and ε_k are i.i.d. in the interval $(-\infty, \infty)$, then rearranging the last term of the above expression and inserting the result back into $EU_i\left(e_i^2, a_i\right)$, we obtain worker i's maximization problem for the second stage

$$\max_{\left\{e_i^2, a_{ik}\right\}_{\forall k \neq i}} EU_i\left(e_i^2, a_i\right) = \left[\int_{-\infty}^{\infty} f\left(\varepsilon_i\right)\left[\int_{-\infty}^{\varepsilon_i - W_{ki}} f\left(\varepsilon_k\right) d\varepsilon_k\right] d\varepsilon_i\right] \bar{V} - v(e_i^2 + a_{ik})$$

where we have simplified the last term by using the fact that when N=2, $\sum_{k \neq i} a_{ik} = a_{ik}$.

Now we want to obtain the optimal vector of positive and negative effort against all the other $(N-1)$ players in stage 2, $\{\hat{e}_i^2, \hat{a}_{ik}\} \in \arg max\ EU_i\left(e_i^2, a_i\right)$. Note that this vector is, generally, of order $N = 1+(N-1)$, i.e. in this case, this vector is of order N=2, $\left\{\hat{e}_i^2, \hat{a}_{ik}\right\}$. Applying first order conditions with respect to e_i^2 and a_{ik} we find the following expressions, which generally characterize the N different first order conditions for each worker.

$$t_i\left[\int_{-\infty}^{\infty} f\left(\varepsilon_i\right) f\left(\varepsilon_i - W_{ki}\right) d\varepsilon_i\right] \bar{V} = v_e'(\hat{e}_i^2 + \hat{a}_{ik}) \qquad (\partial e_i^2)$$

$$s_i g'\left(s_i \hat{a}_{ik}\right)\left[\int_{-\infty}^{\infty} f\left(\varepsilon_i\right) f\left(\varepsilon_i - W_{ki}\right) d\varepsilon_i\right] \bar{V} = v_a'(\hat{e}_i^2 + \hat{a}_{ik}) \qquad (\partial a_{ik})$$

Isolating $\bar{V}$ in both expressions, equating both results and considering that $s_i = s_k = 1$, we obtain

$$g'\left(\hat{a}_{ik}\right) = \frac{v_a'\left(\hat{e}_i^2 + \hat{a}_{ik}\right)}{v_e'\left(\hat{e}_i^2 + \hat{a}_{ik}\right)} t_i$$

In order to obtain an explicit expression for the candidates for optimal effort $\left\{\hat{e}_i^2, \hat{a}_{ik}\right\}$ resulting from the above first order conditions, we will thereafter construct a numerical approximation. In particular, we will assume a convex quadratic cost function $v\left(e_i^2 + a_{ik}\right) = \left(e_i^2 + a_{ik}\right)^2$ and a concave function representing the effectiveness of the sabotaging activities $g\left(a_{ik}\right) = \left(a_{ik}\right)^{0.5}$. If we insert these two functions into the above first order conditions and assume that $\varepsilon_i \sim$U$(-\infty, \infty)$, then we obtain an optimal amount of sabotaging activities $\hat{a}_{ik} = \frac{1}{4t_i^2}$, which is clearly decreasing in t_i. As a consequence, we obtain the following Lemma.

Lemma 1

When two players compete in the promotion tournament of the second stage, and additionaly, we assume convex disutility of effort and concave effectiveness of sabotage, then the optimal amount of sabotaging activities worker i exerts against worker k is decreasing in worker i's productive ability.

By inserting the optimal value of $\hat{a}_{ik}$ back into the first order condition for ∂e_i^2, we can obtain the optimal value of $\hat{e}_i^2(t_i, t_k)$. That is, the optimal effort in productive activities to be exerted during the final as a function of worker i's own ability, t_i, and his opponent's ability, t_j. In order to make the explicit solution for the optimal effort $e_i^2(t_i, t_k)$ clearer, we assume again the above convex function for effort $v(\cdot)$ and the concave function for the effectiveness of sabotaging activities, $g(a_{ik})$. We present the results for $\hat{e}_i^2(t_i, t_k)$ in the following figure.

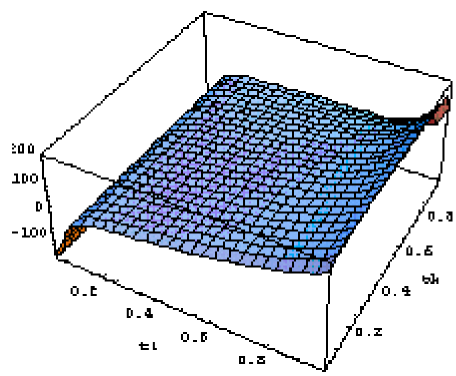

Figure 1. Candidate for optimal effort in the second stage, $\hat{e}_i^2$.

The above figure represents the effort schedule during the second stage of the game (final) for a player with own productive ability t_i, when his opponent's ability is t_k.

Notice that, since $\hat{e}_i^2$ is monotonic in t_i, then the above candidate of effort schedule obtained from the first order conditions of the maximization problem is indeed the optimal effort to be exerted by worker i if, in addition, the usual individual rationality condition holds. That is, in our process of confirming that the above $\hat{e}_i^2$ is indeed the optimal effort for worker i, we need to check that

$$\widehat{EU_k^2} = EU^2\left(\hat{e}_i^2, \hat{a}_{ik}\right) \geqslant 0$$

This condition is confirmed, as we describe in the following lemma.

Lemma 2

When worker i plays strategies $\hat{e}_i^2$ and $\hat{a}_{ik}$, resulting from the solution to the above maximization problem for EU^2, he gets an expected utility level of $\widehat{EU_k^2}$ in equilibrium, represented by the following figure

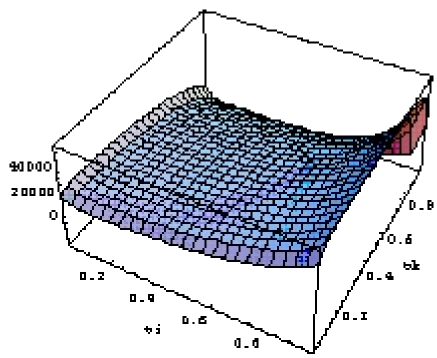

Figure 2. Expected utility from
playing equilibrium strategies, $\widehat{EU_k^2}$

which is everywhere positive. Therefore, individual rationality is satisfied.

Hence, we confirmed that figure 1 indeed represents the optimal effort schedule of worker i during the final stage of the game. This effort, however, can be better understood from its first-order derivatives with respect to the worker's own ability, t_i, with respect to the ability of his opponent in the final, t_k, and with respect to the amount of the prize, $\overline{V}$, respectively; what we do next.

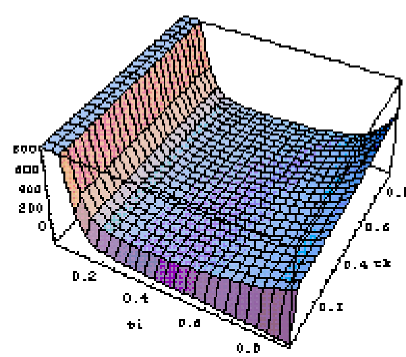

Figure 3. First order derivative of
$\hat{e}_i^2$ with respect to t_i.

The above figure just represents how worker i's effort at the final stage changes as a result of changes in his own ability, t_i. Initially, for a marginal increase in his own ability, the rates at which he changes his exerted effort are huge, i.e. $\left.\frac{\partial \hat{e}_i^2}{\partial t_i}\right|_{t_i \to 0} \simeq 800$. Afterwards, he is not so influential to improvements in his own ability, and as a consequence he positively increases his effort, but at lower rates than before. From approximately t_i=0.3, any increase in his ability induces him to change his exerted effort, but approximately in the same proportion for any further increase until t_i=1. The intuition of this result is that worker i is very confident at the beginning (i.e. when he has an ability close to zero and he is told that his ability increased), but he becomes much more constant in his reaction to these announcements when his ability increases enough.

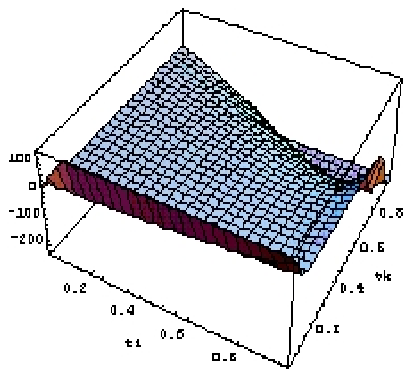

Figure 4. First order derivative of $\hat{e}_i^2$ with respect to t_k.

The above figure represents how the worker i's effort at the final stage changes as a result of changes in t_k, the ability of the opponent: as we further increase his opponent's ability, we see that worker i reduces his effort more (i.e. the rate of change of e_i^2 changes for greater t_j). This change is not the same for all values of worker i's abilities. When he is specially talented, he gets specially frightened by the announcement that his opponent is also very able, and decreases his effort in the final much more than if he wasn't so talented (low values of t_i).

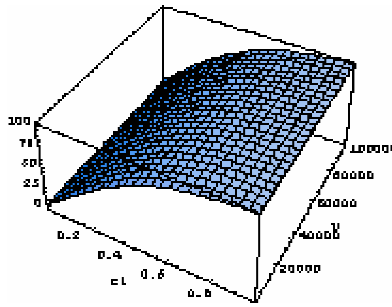

Figure 5. First order derivative
of $\hat{e}_i^2$ with respect to $\bar{V}$.

Finally, this figure represents how the worker i's optimal effort in the final changes as a result of changes in the prize attached to being promoted, $\overline{V}$. Opponent's ability, t_j, has been fixed at 0.5 in order to highlight the effects of a change in $\overline{V}$. The figure shows that, when increasing the prize, $\overline{V}$, worker i always increases his effort at the same rate. However, abler workers decide to increase his effort at a higher rate than unable workers, when a change in the prize is announced. This could be understood as an overreaction of abler workers in terms of getting confident about their chances of winning.

In the following proposition we summarize all of the patterns for the optimal effort that worker i will exert in the final. As expected, it is similar to propositions 3 and 4 in Chen (2003), but with some differences given the particular simulation we applied above.

Proposition 1
When only N=2 workers are classified to play the final stage of the tournament, sabotage among the workers is allowed and, additionally, we assume convex disutility of effort and concave effectiveness of sabotage, the following results are applicable for the final stage of the game. Worker i's productive effort in the final stage, $\hat{e}_i^2$, is

- *increasing in his own ability, t_i.*
- *decreasing in his opponent's ability, t_k.*
- *increasing in the prize attached to being promoted, $\overline{V}$.*

Regarding worker i's sabotaging effort, $\hat{a}_{ik}$, we obtain that it is decreasing in worker i's own productive ability, t_i, and independent on worker k's ability, t_k.

Notice that the positive response of $\hat{e}_i^2$ to changes in t_i is especially important for very low or very high values of t_i; and the negative response of $\hat{e}_i^2$ to changes in t_k is particularly high when worker i is very able and he observes that his opponent is very talented as well. Finally, the positive slope of $\hat{e}_i^2$ to changes in $\overline{V}$ is especially steep when worker i is very able.

Hence, worker i's expected utility of playing stage 2 is just given by inserting the above results for the optimal productive and sabotaging activities, $\hat{e}_i^2(t_i, t_k)$ and $\hat{a}_{ik}(t_i)$, into the expression $EU_i^2(e_i^2, a_i)$,

$$\widehat{EU_k^2}\left(\hat{e}_i^2(t_i,t_k), \hat{a}_{ik}(t_i)\right) = \left[\int_{-\infty}^{\infty} f(\varepsilon_i)\left[\int_{-\infty}^{\varepsilon_i - \hat{W}_{ki}} f(\varepsilon_k)\, d\varepsilon_k\right] d\varepsilon_i\right]\overline{V} - v(\hat{e}_i^2(t_i,t_k) + \hat{a}_{ik}(t_i))$$

where $\hat{W}_{ki}$ is specified in equilibrium as follows

$$\hat{W}_{ki} = \hat{W}_k - \hat{W}_i = t_k\hat{e}_k^2(t_i,t_k) - g(s_i\hat{a}_{ik}(t_i)) - t_i\hat{e}_i^2(t_i,t_k) + g(s_k\hat{a}_{ki}(t_k))$$

and where $\widehat{EU_k^2}$ is the expected utility from playing the equilibrium strategies $\hat{e}_i^2$ and $\hat{a}_{ik}$ against worker k. Notice that all elements of this expression are known in stage 2, since t_k became common knowledge after stage 1. The above expression of $\widehat{EU_k^2}$ will become relevant in the next subsection, when we analyze the optimal effort that worker i will decide to exert at the first stage of the game.

4 Optimal strategy in round 1

On stage 1 worker i exerts some positive effort $\hat{e}_i^1$ without knowing the productive ability of the two players who compete with him in the first stage of the game.

At the beginning of the game, worker i's expected utility from playing the game, EU^1, given his optimal choice of productive and sabotaging activities for stage 2, $\hat{e}_i^2(t_i,t_j)$ and $\hat{a}_{ij}(t_i)$, is defined by three different situations. Let's analyze each of them.

1. If during the first stage of the game worker i's performance, $\omega_i + u_i$, is greater than the performance of worker j, $\omega_j + u_j$, but smaller than that of worker k, $\omega_k + u_k$, then worker i and k are qualified to compete in the final stage of the game. Worker i's expected utility from this case will be represented by the probability that he beats worker j (but not worker k) times the expected utility from playing his optimal strategies, $\hat{e}_i^2(t_i,t_k)$ and $\hat{a}_{ik}(t_i)$, in the final against worker k, $\widehat{EU_k^2}$. That is,

$$prob\left(\omega_i + u_i \geqslant \omega_j + u_j \text{ and } \omega_i + u_i \leqslant \omega_k + u_k\right) \times \widehat{EU_k^2}$$

$$\Longleftrightarrow prob\left(u_i - \omega_{ji} \geqslant u_j \text{ and } u_i - \omega_{ki} \leqslant u_k\right) \times \widehat{EU_k^2}$$

where $\omega_{ji} = \omega_j - \omega_i = t_j e_j^1 - t_i e_i^1$.

Which we can start writing as

$$\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \int_{u_i - \omega_{ki}}^{\infty} f(u_k)\, du_k \right) du_i$$

The above expression, however, does not represent the probability specified in the previous formula. Indeed, since worker i does not know his opponents' abilities in the first stage, he will also take expectations over $t_j, t_k \in [0, 1]$, resulting in the following expression for the expected utility from beating worker j but not worker k.

$$\int_0^1 f(t_j) \int_0^1 f(t_k) \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \int_{u_i - \omega_{ki}}^{\infty} f(u_k)\, du_k \right) du_i \times \widehat{EU_k^2} \right] dt_k dt_j$$

2. If during the second stage, worker i obtains a greater performance than worker k, but a smaller than worker j, we just have a situation symmetric to the one specified above. That is,

$$prob\left(\omega_i + u_i \geqslant \omega_k + u_k \text{ and } \omega_i + u_i \leqslant \omega_j + u_j\right) \times \widehat{EU_j^2} =$$

$$\int_0^1 f(t_k) \int_0^1 f(t_j) \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ki}} f(u_k)\, du_k \int_{u_i - \omega_{ji}}^{\infty} f(u_j)\, du_j \right) du_i \times \widehat{EU_j^2} \right] dt_j dt_k$$

3. Finally, if worker i has a greater performance than both worker j and k, we can observe two different cases. If worker j produces more than worker k, then worker i will end up competing with worker j in the final, with an associated expected utility of $\widehat{EU_j^2}$. If, on the contrary, worker k outperforms worker j, the former will be the player competing against worker i in the final stage of the game, generating an expected utility of $\widehat{EU_k^2}$ for worker i. Hence, the expected utility that worker i will obtain in these two cases will be given by

$$prob\left(\omega_i + u_i \geqslant \omega_j + u_j \text{ and } \omega_i + u_i \geqslant \omega_k + u_k\right) \times$$

$$\left[prob\left(\omega_j + u_j \geqslant \omega_k + u_k\right) \times \widehat{EU_j^2} + prob\left(\omega_j + u_j \leqslant \omega_k + u_k\right) \times \widehat{EU_k^2} \right]$$

Where $\widehat{EU_j^2}$ and $\widehat{EU_k^2}$ represent the expected utility of worker i when competing in the final of the second stage against workers j and k, respectively.

Rearranging the above expression we find,

$$\int_0^1 f(t_k) \int_0^1 f(t_j) \left\{ \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i-\omega_{ji}} f(u_j)\, du_j \int_{-\infty}^{u_i-\omega_{ki}} f(u_k)\, du_j \right) du_i \right] \times \right.$$
$$\left[\int_{-\infty}^{\infty} f(u_j) \left(\int_{-\infty}^{u_j-\omega_{kj}} f(u_k)\, du_k \right) du_j \times \widehat{EU_j^2} + \right.$$
$$\left. \left. + \int_{-\infty}^{\infty} f(u_k) \left(\int_{-\infty}^{u_k-\omega_{jk}} f(u_j)\, du_j \right) du_k \times \widehat{EU_k^2} \right] \right\} dt_k dt_j$$

Once we have described the situations that worker i can face in the final, and the probabilities of reaching each of them from the stage 1's perspective, we can succinctly write the expression of the expected utility of playing the entire game from stage 1's point of view,

$$EU^1 = \text{Situation}_1 + \text{Situation}_2 \ - \ \text{Situation}_3 - \ v\left(e_i^1\right)$$

where the two first signs are due to the additive law of probability.

Now, in order to find a candidate for solution that determines the optimal amount of effort, $\hat{e}_i^1$, to be exerted by worker i during stage 1, we need to take first order conditions in the above expression of EU^1. We included the analytical solution of the first order conditions in the appendix, as well as some comments about its intuition. Notwithstanding this analytical solution, and with the goal of clarifying the results, we assume here the same convex function for the disutility of effort and the same concave function for the effectiveness of sabotaging activities we used in the previous section. Inserting them into EU^1, taking first order conditions with respect to e_i^1, and solving for e_i^1, we find the following effort schedule $\hat{e}_i^1(t_i)$ as a candidate for the optimal effort to be exerted by worker i in the first stage.

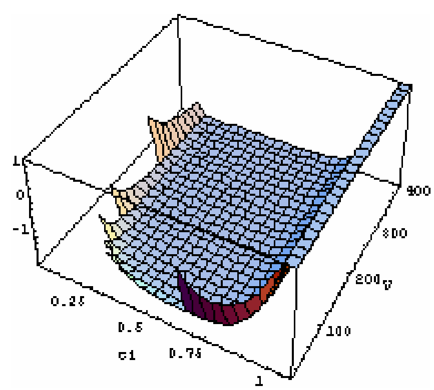

Figure 6. Candidate for optimal effort in the first stage, $\hat{e}_i^1$.

The above figure represents the candidate for the optimal effort of worker i during the first stage of the game, as a function of his own productive ability, t_i, and the prize of being promoted, $\overline{V}$, which he can obtain only if he is classified to play in the next stage and he is finally the winner of this final stage. We will comment on the pattern of this effort schedule $\hat{e}_i^1$ below. Firstly, we will concentrate on proving that this candidate for solution is indeed the optimal effort to be exerted by worker i in the first stage.

This effort schedule, as is clear from the figure, is monotonic in worker i's productive ability, t_i. This fact reaffirms the validity of this candidate for solution of the maximization problem associated with EU^1. As commented in the previous subsection, the only remaining condition that we need to check, in order to ensure that the proposed candidate for solution is indeed the optimal effort, $\hat{e}_i^1$, is just the individual rationality (or voluntary participation) condition

$$\widehat{EU^1} = EU^1\left(\hat{e}_i^1\left(t_i\right)\right) \geqslant 0$$

where $\widehat{EU^1}$ represents the expected utility that worker i obtains when playing the equilibrium strategies specified above. As we describe in the following lemma, this condition is satisfied.

Lemma 3

When worker i plays the equilibrium strategy $\hat{e}_i^1$ resulting from the solution to the above maximization problem for EU^1, and the equilibrium strategies for stage 2, $\hat{e}_i^2$ and $\hat{a}_{ik}$, specified in the previous subsection, he obtains an expected utility level in equilibrium of $\widehat{EU^1}$, given by the following figure

Insert here figure 7 of $\widehat{EU^1}$

which is always positive. Hence, (Bayesian) individual rationality is satisfied.

Therefore, the above lemma confirms that the expression $\hat{e}_i^1$ represented in figure 6 is indeed the optimal effort schedule for worker i in the first stage. Let's now comment on the behavior of $\hat{e}_i^1$.

As is clear from figure 6, worker i's optimal effort is increasing in the worker's type (his productive ability, t_i). This increase, however, is very small for workers with very low productive abilities (very low values of t_i), and drastically increases for high values of t_i. That is, workers with very low abilities exert almost zero effort during the first stage, while those workers with high productive abilities (specially from t_i=0.75 approximately) exert high efforts during the first stage of the game. The reason for unable workers to exert such small effort in the first stage can be explained using the following reasoning. Firstly, a very unable worker does not win anything for sure if he is classified to participate in the final. Being classified just gives him an expected payoff, which can be extremely small if his chances of winning the final are close to zero, something that is specially likely when his ability t_i is small.

Unlike unable workers, very productive workers exert large amounts of effort during the first stage. The reason for this behavior can be understood as follows. Firstly, exerting a great effort in the first stage is a method of ensuring their probabilities of being classified for the final. That is, their great abilities multiplied by a greater effort makes that their performance, $\omega_i = t_i e_i^1$, is higher than his competitors', even if they take into account the possibility of some random shocks affecting all workers' performance. Secondly, when worker i exerts a high effort in the first stage, he signals to their future competitor during the final that he is a 'hard' player. In other words, if an unable('weak') player gets into the final and observes such past behavior from this 'hard' player, he will introduce into his optimal effort function for the second stage the fact that his competitor's productive ability is high. As we analyzed in section 3, this will lead to a reduction in the productive effort exerted by this unable player. And this fact ultimately provokes an increase in the probability that the 'hard' player's performance is higher than that of the 'weak' player. In other words, by signalling his ability, the 'hard' player softens the competition in the final round, what increases his probabilities of winning the final promotion.

In order to get a clearer understanding of the above intuition, we found the first order derivative of worker i's optimal effort in the first stage, $\hat{e}_i^1$, with respect to his own productive ability, t_i. Below, we represent this derivative, evaluated at different points of t_i and $\overline{V}$.

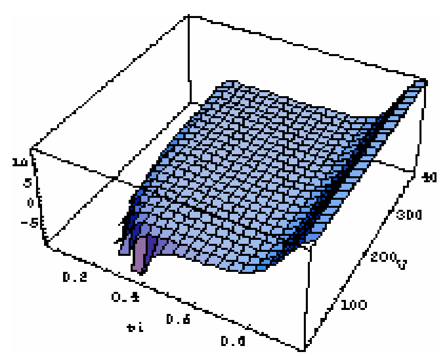

Figure 8. First order derivative of $\hat{e}_i^1$ with respect to t_i.

Intuitively, the above figure expresses how a change in the worker i's productive ability, t_i, changes the productive effort that he optimally exerts during the first stage, $\hat{e}_i^1$. As is clear from the figure, if a very unable worker i is told that his productive ability, t_i, has marginally increased, he will not modify his productive effort very much. In the figure, this fact is represented by the flat surface until t_i=0.75 approximately, where the first order derivative of $\hat{e}_i^1$ is close to zero.

For greater values of worker i's productive ability, however, we observe a different pattern. Now, announcing to this very able worker that his productive ability has marginally increased induces him to greatly increase his exerted effort during the first stage of the game. This fact can be understood by applying the same reasoning as above. In particular, very able workers get really optimistic about their chances of winning the final. Increasing their effort today increases their probabilities of being classified to play the final and signals their opponent in the final that they are dealing with a 'hard' type, so that they would rather decrease their effort when competing against such a 'hard' type in the final.

We summarize the results of this subsection dealing with the first stage of the game in the following proposition.

Proposition 2

When only $N=2$ workers are classified to play the final stage of the tournament, sabotage among the workers is allowed and, additionally, we assume convex disutility of effort and concave effectiveness of sabotage, the following results are applicable for the first stage of the game.

Worker i's productive effort in the first stage, $\hat{e}_i^1(t_i)$, is increasing in the worker's own ability, t_i. Moreover, there exists a cutoff level $\bar{t} \in (0,1)$ such that all workers with $t_i < \bar{t}$ exert almost zero and those workers with $t_i > \bar{t}$ exert higher amounts of effort.

In the next section we elaborate on the implications of our results, and compare them with the existing literature.

5 Implications and contributions

Notice some of the important implications of the above results. Firstly, the workers who exert high amounts of effort in the first stage will very likely be classified to play the final. That is, if the firm manager initially hired a very able worker (recall that he couldn't observe his ability), then this worker will exert so much effort that his probabilities of being in the final are close to one, especially when competing against unskilled workers. This is good news for the manager. Additionally, if this able worker faces one or more able workers during the first stage of the game, then all able workers will exert high efforts during stage one, and the final will probably be played by two very able workers.

Secondly, if an unable worker is hired by the firm, he will probably not be classified to play the final, since he does not exert any (or almost any) effort during stage one. This are again good news for the firm manager, because there are larger probabilities that the final is won by a player who is, at least, one of the 'top' workers in terms of productive ability. That is, by introducing the first stage, the firm manager minimizes the probability of promoting an unable worker.

We summarize the above results, derived from propositions 1 and 2, in the following proposition.

Proposition 3

When a firm organizes a promotion tournament that satisfies all the above assumptions and one or more able workers start their competition at the first stage of the game, then the worker with the highest ability has the highest probability of being promoted.

Proof

Let's start by denoting as H a worker with a high productive ability, i.e. with $t_i > \bar{t}$, and by L a worker with $t_i < \bar{t}$.

Let's analyze each of the cases involved.

1. If the profile of workers at the first stage is HHL, then both H-type workers will exert a high effort, while the L-type worker will exert almost no productive effort whatsoever. Hence, both H-type workers have more probabilities of being qualified to play the final than the L-type does. Thus, an H-type wins the final almost surely (i.e. a very talented worker is promoted). For this particular profile of workers, the first stage of the game helped in disqualifying unable workers.

2. If the profile of workers during the first stage is, instead, HHH, then we obviously have an strenghtened version of the above results.

3. Finally, if the inital profile of workers is HLL, then we have that the H-type worker will exert a great effort during the first stage, what almost guarantees that he will be in the final (but not for sure, since random shocks in performance are always present). On the other hand, both L-type workers will exert an effort close to zero. Thus, the only reason why one of the L-types will also be classified to play the final is because he received the highest random shock among all the L-type workers. Hence, the final in this case is most probably played by the unique H-type who started the game and one of the L-types. In this final stage, the H-type optimally increases his effort, since now he knows that he is competing with an unable type, while the L-type worker decreases his effort for the opposite reason. This leads to the similar result as above: if only an H-type starts the tournament, then he is the worker with the highest probabilities of being promoted.□

This result is clearly different from Chen's (2003). In his model, having three players could lead to the 'rising star' paradox. Namely, the ablest worker is identified by his job colleagues as the player who mostly endangers their probabilities of being promoted. This leads them to coordinate their sabotaging attacks towards him. In fact, for $N \geqslant 3$, Chen (2003) shows that there are many chances that the ablest worker (the 'star') is not finally the one promoted. Therefore, we have shown that this 'rising star' paradox can be greatly minimized if the firm manager clearly splits the tournament into two separated stages, as the ones described in this paper.

This result has clear implications for the tournament designer (e.g. firm manager). If his goal is to promote the ablest member of the organization, but hires workers without observing their abilities, then running a qualifying stage in the first period can greatly reduce the probabilities of promoting an unable worker, since they will probably be disqualified and will not get to play the final. In terms of policy implications, the firm manager should announce that, from a set of workers who are hired at the same moment in time, only those two workers generating the highest performance are going to be considered for promotion. By doing so, he reduces the chances that any unable worker reaches the final stage and is finally promoted.

Notice also an indirect benefit from reducing the chances that unable workers get into the final. As we analyzed in section 3, optimal sabotaging activities, $\hat{a}_{ij}(t_i)$, are decreasing in the productive talent of the worker, t_i. Hence, by decreasing the probability that workers with low talents reach the final, the firm manager also reduces the amount of sabotaging activities that (specially unable workers) will exert against the performance of their job colleagues. And this, in turn, increases the overall output of the firm.

6 Conclusions

In this paper we analyzed a promotion tournament with two stages: in the first one players do not observe each other's types (productive abilities), whereas in the second stage this private information becomes perfectly revealed to the workers.

We showed that, for the case of dealing with only two finalists, unable workers are very reluctant to exert any productive effort in the first stage. Intuitively, when a worker knows that his chances of being classified to the final are small, and that his probabilities of winning the final stage are even more scarce, he will decide not to exert any effort during the first stage. His chances of being classified to play the final depend, as a consequence, only on some positive random shock on performance he may experience.

On the other hand, we showed that only the ablest workers of the company will exert high amounts of effort during the first stage in order to ensure their probabilities of being classified to play the final. Finally, we proved that the 'rising star' paradox is minimized by introducing the first stage of the tournament, where players compete between each other in order to qualify to play the final. In other words, the firm manager increases the probability of promoting one of the ablest workers, if there is at least one, by introducing the first (qualification) stage. Additionally, by reducing the probabilities that unable workers get to the final, the manager reduces the amount of sabotaging activities that, specially unable workers, exert against the performance that other workers generate, what turns into a greater output for the firm.

Notice, however, that we assumed that the final stage of the tournament is only played by two workers. This can be appropriate for many real life settings, but eliminates the possibility of potential coordination among the unable workers in order to sabotage the 'rising star' during the final stage of the game. If this coordination in attacks occurs for $N \geqslant 3$ workers in the final, we could find some differences in our above results. Now, the ablest worker will probably hide his productive ability and refrain from exerting positive effort at the first stage of the game, i.e. positive effort for a higher cutoff level $\bar{t}$. Hence, an interesting extension of this paper is its application to $N \geqslant 3$ workers in the final.

In addition, we assumed that all workers are equally able in sabotaging activities, $s_i = s_j$ for all $i \neq j$. This assumption helped in emphasizing the results in terms of the probabilities that the ablest workers are very likely to be promoted in the firm. Allowing for different abilities on sabotage, however, may enrich the results by conditioning the optimal efforts on both productive and sabotage abilities.

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7 Appendix

Let's rewrite here the entire expression of worker i's expected utility at the very beginning of the game, EU^1, by summing up the expected utility from each of the three situations described in the text.

$$EU^1 = \int_0^1 f(t_j) \int_0^1 f(t_k) \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \int_{u_i - \omega_{ki}}^{\infty} f(u_k)\, du_k \right) du_i \times \widehat{EU_k^2} \right] dt_k dt_j +$$

$$+ \int_0^1 f(t_k) \int_0^1 f(t_j) \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ki}} f(u_k)\, du_k \int_{u_i - \omega_{ji}}^{\infty} f(u_j)\, du_j \right) du_i \times \widehat{EU_j^2} \right] dt_j dt_k$$

$$- \int_0^1 f(t_k) \int_0^1 f(t_j) \left\{ \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \int_{-\infty}^{u_i - \omega_{ki}} f(u_k)\, du_j \right) du_i \right] \times \right.$$

$$\left[\int_{-\infty}^{\infty} f(u_j) \left(\int_{-\infty}^{u_j - \omega_{kj}} f(u_k)\, du_k \right) du_j \times \widehat{EU_j^2} + \right.$$

$$\left. \left. + \int_{-\infty}^{\infty} f(u_k) \left(\int_{-\infty}^{u_k - \omega_{jk}} f(u_j)\, du_j \right) du_k \times \widehat{EU_k^2} \right] \right\} dt_k dt_j - v(e_i^1)$$

where $\omega_{ji} = \omega_j - \omega_i = t_j e_j^1 - t^i e_i^1$.

Taking first order conditions with respect to e_i^1, we obtain the following expression. In order to make the intuition as clear as possible, we splitted the first order derivative of each of the above three situations. Note that this is possible since the objective function is additively separable.

Hence, we firstly write the first order derivative corresponding to the situation where worker i wins worker j but not k, and as a consequence, worker i plays the final against worker k.

$$\int_0^1 f(t_j) \int_0^1 f(t_k) \left[t_i \int_{-\infty}^{\infty} f(u_i) f(u_i - \omega_{ji}) \left(\int_{u_i - \omega_{ki}}^{\infty} f(u_k)\, du_k \right) du_i + \right.$$

$$\left. - \int_{-\infty}^{\infty} f(u_i) f(u_i - \omega_{ki}) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \right) du_i \times \widehat{EU_k^2} \right] dt_k dt_j +$$

$$\int_0^1 f(t_j) \int_0^1 f(t_k) \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \int_{u_i - \omega_{ki}}^{\infty} f(u_k)\, du_k \right) du_i \times \frac{\partial \widehat{EU_k^2}}{\partial e_i^1} \right] dt_k dt_j$$

Let's now continue the first order conditions for the situation where worker i beats worker k but not j, and thus worker i and j go to the final,

$$\int_0^1 f(t_k) \int_0^1 f(t_j) \left[t_i \int_{-\infty}^{\infty} f(u_i) f(u_i - \omega_{ki}) \left(\int_{u_i - \omega_{ji}}^{\infty} f(u_j)\, du_j \right) du_i + \right.$$

$$- \int_{-\infty}^{\infty} f(u_i) f(u_i - \omega_{ji}) \left(\int_{-\infty}^{u_i - \omega_{ki}} f(u_k)\, du_k \right) du_i \times \widehat{EU_j^2} \Bigg] dt_j dt_k +$$

$$\int_0^1 f(t_k) \int_0^1 f(t_j) \left[\int_{-\infty}^{\infty} f(u_i) \left(\int_{-\infty}^{u_i - \omega_{ki}} f(u_k)\, du_k \int_{u_i - \omega_{ji}}^{\infty} f(u_j)\, du_j \right) du_i \times \frac{\partial \widehat{EU_j^2}}{\partial e_i^1} \right] dt_j dt_k$$

And finally, let's now find the first order derivative for the third situation, where worker i is the top player during the first stage, and he considers the possibility of competing against either worker j or k in the final.

$$\int_0^1 f(t_k) \int_0^1 f(t_j) \Bigg\{ \Bigg[t_i \int_{-\infty}^{\infty} f(u_i) f(u_i - \omega_{ji}) \left(\int_{\infty}^{u_i - \omega_{ki}} f(u_k)\, du_k \right) du_i +$$

$$- \int_{-\infty}^{\infty} f(u_i) f(u_i - \omega_{ki}) \left(\int_{-\infty}^{u_i - \omega_{ji}} f(u_j)\, du_j \right) du_i \Bigg] \times$$

$$\Bigg[\int_{-\infty}^{\infty} f(u_j) \left(\int_{-\infty}^{u_j - \omega_{kj}} f(u_k)\, du_k \right) du_j \times \widehat{EU_j^2} +$$

$$+ \int_{-\infty}^{\infty} f(u_k) \left(\int_{-\infty}^{u_k - \omega_{jk}} f(u_j)\, du_j \right) du_k \times \widehat{EU_k^2} \Bigg] \Bigg\} dt_k dt_j +$$

$$\int_0^1 f(t_k) \int_0^1 f(t_j) \Bigg\{ \Bigg[\int_{-\infty}^{\infty} f(u_j) \left(\int_{-\infty}^{u_j - \omega_{kj}} f(u_k)\, du_k \right) du_j \times \frac{\partial \widehat{EU_j^2}}{\partial e_i^1} +$$

$$+ \int_{-\infty}^{\infty} f(u_k) \left(\int_{-\infty}^{u_k - \omega_{jk}} f(u_j)\, du_j \right) du_k \times \frac{\partial \widehat{EU_k^2}}{\partial e_i^1} \Bigg] \Bigg\} dt_k dt_j - v'(\hat{e}_i^1)$$

Note that the integrals involved in the last two rows are treated as constants, since neither ω_{kj} nor ω_{jk} include e_i^1.

The reader might have probably noticed that each of the above terms considers both the first order *and* the second order effects of marginally increasing e_i^1 in the first stage of the game. Firstly, an increase in e_i^1 increases the probability that worker i is one of the players selected to play the final, either against worker k or j. These are not all the effects of marginally increasing e_i^1, though. Indeed, increasing e_i^1 makes worker i's opponent in the final to concentrate his beliefs about t_i in a higher value of t_i. This leads to a smaller effort of worker i's opponent in the final, what increases worker i's probability of winning the final stage.

Note, additionaly, that the first order effect, expressing the increased probability of being classified to play the final, is scaled down by the probability of winning this final. Similarly, the second order effect, reflecting the higher chances of winning the final, is also scaled down by the probability of being classified for the final. These two (scaled down) marginal benefits, in equilibrium, must equal the marginal costs of increasing e_i^1 in terms of a greater disutility of effort, measured by the term $v'(\hat{e}_i^1)$.