Downstream mergers and producer's capacity choice: why bake a larger pie when getting a smaller slice?*

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Abstract

We study the effect of downstream horizontal mergers on the upstream producer's capacity choice. Contrary to conventional wisdom, we find a non-monotonic relationship: such mergers induce a lower upstream capacity if the cost of capacity is high; a higher upstream capacity if this cost is low. We explain the result by decomposing the total effect into two distinct effects: a *change in hold-up* and a *change in bargaining erosion*.

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1 Introduction

A raising debate in the antitrust arena concerns the *negative* long term effects of downstream horizontal integration on upstream producers' investment incentives. For example, the position held by the European Commission in two recent decisions on the merger of leading retailers, *Kesko/Tuko* and *Rewe/Meinl*, created jurisprudence in merger control: competitive assessment should no longer be restricted to the downstream market but also look for adverse effects on input markets. This concern is also present in the European Commission's guidelines on *purchasing agreements*.[1]

The present paper studies how a downstream horizontal merger affects the capacity choice of an upstream producer. It seems intuitive that following a merger the producer will choose a lower level of capacity since he will be more exposed to the hold-up problem. Perhaps surprisingly then, working from fundamentals, we find a non-monotonic relationship between downstream horizontal integration and upstream equilibrium capacity.

We study a two stage game: in the first stage, the producer chooses capacity and pays for its cost. In the second stage, the producer bargains with downstream firms over input supply. Since the allocation of the bargaining surplus, and therefore investment incentives, depend on the chosen solution concept, this choice is a crucial step. Like other authors studying the effects of

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[1] *Kesko/Tuko*, Case IV/M.784, Commision decision of November 20 1996. *Rewe/Meinl*, Case COM/M.1121, Commision decison of February 3 1999. *Purchasing agreements* see Guidelines on the applicability of the Article 81 (2001).

integration, we use the Shapley value (e.g. Hart and Moore, 1990; Inderst and Wey, 2003; Segal, 2003).

Our main result is that the cost of capacity provides a simple criterion to evaluate claims about the effect of a downstream horizontal merger on the producer's capacity choice: a downstream merger induces a higher equilibrium capacity if the cost of capacity is low; it induces a lower capacity if this cost is high.

We explain this result by decomposing the total effect into two distinct effects. On the one hand, as expected, the merger increases the extent to which downstream firms are able to *hold-up* the producer, i.e. the share of the surplus accruing to the producer decreases.

On the other hand, for any given market configuration, increasing capacity erodes the bargaining power of the producer—"competition" for the input becomes weaker as it becomes more abundant. A downstream merger reduces the rate at which this *bargaining erosion* takes place. The latter effect counteracts the increase in the hold-up, and will dominate when cost of capacity is low.

This result is valid when downstream firms sell on independent markets and on interrelated markets alike, even when "competitive externalities" are taken into account.

Surprisingly, while the hold-up literature has largely studied the effects of vertical integration, there exists little formal analysis on the investment effects of downstream horizontal integration—the growing literature on buyer power has mainly focused on its distributive consequences.

The paper closest to ours is Hart and Moore (1990) which studies similar issues to the one we address here. Assuming that marginal contributions are independent of the level of investment, they find that *"..as one might expect, if two competing traders merge, this will worsen the incentives of the owner-manager of a firm that trades with them" (*Hart and Moore, 1990, p. 1148). Other papers studying (continuous) investment choices have found a similar negative relationship between downstream mergers and upstream investment, e.g. less product diversity (Chae and Heidhues, 1999; Inderst and Shaffer, 2005). The present paper shows precisely that this result may change significantly in a setting where the just mentioned assumption is not verified.

A few other papers look at (discrete) technology choices. Stole and Zwiebel (1996a, 1996b) find that a firm dealing with independent workers has a preference for technologies which give rise to more concave surplus functions since this leverages its bargaining power—this bias is absent when the workforce is unionized. Downstream horizontal mergers reduce a similar bias for an upstream producer and may result in a larger equilibrium surplus (Inderst and Wey, 2003, forthcoming).

While our work shows that downstream horizontal mergers may result in *more* upstream investment—a meaningless statement for a discrete choice—the potential welfare benefit has a similar cause: the merger reduces a strategic bias and induces the producer to focus on value creation itself. (Our paper is also distinct by extending the analysis to a setting with "competitive externalities").

Most of the above mentioned literature uses linear bargaining solutions—e.g. the Shapley value. Recent work showed that results derived using a linear bargaining solution may not be robust in settings which implement nonlinear bargaining solutions (e.g. Chiu, 1998; deMeza and Lockwood, 1998; Inderst and Wey, 2005).

While our results remained valid in examples with nonlinear bargaining solutions, the present paper is, to our knowledge, the first using a linear bargaining solution which finds that integration of "competing" players may actually increase the level of investment made by a complementary player.

The remainder of the paper is organized as follows. In section 2 we illustrate the main ideas of this paper with a simple example. We present the model in section 3 and the analysis in section 4. In section 5 we extend the model to a setting where markets are interrelated. We conclude in section 6. Proofs can be found in the appendix.

2 A simple example

Suppose a producer p chooses, at date 0, a capacity $Q \in \{0, 1, 2\}$. The cost of each unit of capacity is c. There are two outlets, each in a distinct market. At date 1, in *each* outlet one unit can be sold for a net value of 1, but there is no demand for a second unit. Denote the maximal revenue obtained with m outlets a capacity of Q by $\phi(Q, m)$ (e.g. $\phi(1, 1) = \phi(1, 2) = 1$).

There is no discounting and no contracts are signed at date 0. At date 1, gains from trade are split according to each player's Shapley value. Denote the producer's Shapley value by $S_Z(Q)$—subscripts denote the industry configuration. We will compare two situations, A and B, so $Z \in \{A, B\}$.

The producer maximizes its date 1 revenue minus date 0 costs

$$Max\, \{S_Z(Q) - cQ\} \text{ with } Q \in \{0, 1, 2\}$$

He will choose an additional unit of capacity at stage 0 if his *incremental benefit,* defined as

$$\Delta S_Z(Q) \equiv S_Z(Q) - S_Z(Q - 1)$$

is larger than the cost c.

A well known interpretation of the Shapley value has all players in the game being randomly set in an ordered sequence, with each sequence being equally likely. Once a sequence is realized, each player gets his marginal contribution to the coalition formed by those players who precede him in that sequence. The Shapley value is simply the expectation taken over all possible sequences.

In situation A, *each* of two retailers, i and j, serves one outlet. There are six possible sequences: pij, pji, ipj, jpi, ijp and, jip. In two of them the producer comes first, so his marginal contribution is 0. In two other he comes second and therefore his marginal contribution is equal to $\phi(Q, 1)$—the value of allocating all capacity to a single outlet. Finally, there are two sequences where the producer comes last in the ordering and he gets $\phi(Q, 2)$.

Taking expectations we get $S_A(0) = 0$ and,

$$S_A(1) = 2\frac{1}{6}(0) + 2\frac{1}{6}(1) + 2\frac{1}{6}(1) = \frac{2}{3}$$

$$S_A(2) = 2\frac{1}{6}(0) + 2\frac{1}{6}(1) + 2\frac{1}{6}(2) = 1$$

In situation B retailer i serves both outlets. In this case there are only two possible sequences: pi and ip. So $S_B(0) = 0$ and,

$$S_B(1) = \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2}$$

$$S_B(2) = \frac{1}{2}(0) + \frac{1}{2}(2) = 1$$

To study the extent to which the retailer(s) is(are) able to "hold-up" the producer we also define the share of the industry surplus accruing to the producer as

$$\alpha_Z(Q) \equiv \frac{S_Z(Q)}{\phi(Q, 2)}$$

We have

$$\alpha_A(1) = \frac{2}{3} \text{ and } \alpha_A(2) = \frac{1}{2}$$

and

$$\alpha_B(1) = \alpha_B(2) = \frac{1}{2}$$

As expected, the share of date 1 surplus accruing to the producer is (weakly) higher in situation A, so the hold-up is more severe in situation B. We could therefore expect the incremental benefit to be larger in A than in B. While this is true from $Q = 0$ to $Q = 1$ since

$$\Delta S_A(1) = \frac{2}{3} \text{ and } \Delta S_B(1) = \frac{1}{2}$$

it is not true from $Q = 1$ to $Q = 2$ since

$$\Delta S_A(2) = \frac{1}{3} \text{ and } \Delta S_B(2) = \frac{1}{2}$$

The table below represents the equilibrium choice of capacity as a function of c: a downstream horizontal merger may lead to a higher capacity when the cost of capacity is low—and to a lower one when this cost is high.

c	Q_A^*	Q_B^*
$\geq \frac{2}{3}$	0	0
$\in \left[\frac{1}{2}, \frac{2}{3}\right]$	1	0
$\in \left[\frac{1}{3}, \frac{1}{2}\right]$	1	2
$\in \left[0, \frac{1}{3}\right]$	2	2

To understand *why* the producer's incremental benefit is lower in A than in B from $Q = 1$ to $Q = 2$, we start by decomposing the incremental benefit as follows:

$$\Delta S_Z(2) = \alpha_Z(2) \cdot \phi(2, 2) - \alpha_Z(1) \cdot \phi(1, 2)$$

We also write the surplus difference as $\Delta\phi(2, 2) = \phi(2, 2) - \phi(1, 2)$ and the change in the share of the surplus accruing to the producer as $\Delta\alpha_Z(2) = \alpha_Z(2) - \alpha_Z(1)$. We have:

$$\Delta\alpha_A(2) = -\frac{1}{6} < 0 \text{ and } \Delta\alpha_B(2) = 0$$

We will call *bargaining erosion* to this (weakly) negative effect of increasing capacity on the producer's share. We now write

$$\Delta S_Z(2) = \Delta\phi(2, 2) \cdot \alpha_Z(1) + \Delta\alpha_Z(2) \cdot \phi(2, 2)$$

and look at how the incremental benefit changes from situation A to situation B, i.e.

$$\Delta S_B(2) - \Delta S_A(2) = \Delta\phi(2, 2)\underbrace{[\alpha_B(1) - \alpha_A(1)]}_{\triangle \text{ Hold-up}} + \underbrace{[\Delta\alpha_B(2) - \Delta\alpha_A(2)]}_{\triangle \text{ Bargaining erosion}}\phi(2, 2)$$

$$\Leftrightarrow \Delta S_B(2) - \Delta S_A(2) = 1 \cdot (-\frac{1}{6}) + \frac{1}{6} \cdot 2 = \frac{1}{6} > 0$$

We find that while downstream horizontal integration increases the hold-up (the first negative term), it also induces the producer to focus more on increasing the industry surplus and less on the effect of the capacity choice on its bargaining position (the second positive term). From $Q = 1$ to $Q = 2$ the second effect dominates the first. Below, we establish these results in a much more general setting.

3 The Model

3.1 Setup

We consider a generic industry with one upstream producer p, a set $N = \{1, ..., n\}$ of downstream firms and m_N identical outlets. Let Z denote a generic allocation of outlets across firms. Each downstream firm i is the single owner of $m_i(Z)$ outlets, so

$$\sum_{i \in N} m_i(Z) = m_N$$

The net revenue at each outlet is described by $R(q_x)$, where q_x is the quantity sold in outlet x—*net* meaning it accounts for any costs of delivery, transforming the intermediate good into a final good, or both. Without loss of generality, $R(0)$ is normalized to zero. R is twice continuously differentiable with $R'' < 0$, $R'(0) > 0$ but $R'(q_x) < 0$ for all large q_x, so there is a unique

$$\arg\max_{q_x} R(q_x) \equiv \overline{q}$$

Here each single market revenue is independent of the quantities allocated to the remaining outlets. This allows us to focus on the vertical interaction aspects of the game and is a good description of environments where downstream firms compete only on the input market—e.g. downstream firms produce in turn different final goods or sell in distinct geographical markets.

The timing of the game is the following. At stage 0 the producer chooses a capacity Q (or stock) and pays its cost described by $C(Q) = cQ$ where $c > 0$. (We discuss affine and convex cost functions in the text.)

At date 1, the producer can produce up to Q at some non-negative constant marginal cost which, without loss of generality, is assumed to be zero. Date 1 bargaining determines the quantities and transfers between the producer and each downstream firm.

The assumption that no supply contracts are signed at date 0 tries to capture the fact that, in general, contracts do not cover the overall economic life of investment.[2] Our model concerns situations where the producer's revenue and investment incentives are mainly determined in post investment bargaining.

3.2 Bargaining

Extensive-form bargaining games between a producer and downstream firms over supply contracts can be found, for example, in Inderst and Wey (2003) and deFontenay and Gans (2004, 2005); equilibrium payoffs of these games coincide with the Shapley value. For brevity and to avoid redundancy, we take here a cooperative approach to bargaining and use as well the Shapley value as our solution concept.[3]

A cooperative game consists of two elements: a set of players $\underline{Y}$ (with a typical subset Y) and a function $v(Y)$ specifying the *value* created by different subsets of players.

In our game $\underline{Y} = N \cup p$. We will write $Y \cup i$ for $Y \cup \{i\}$ and $Y \backslash i$ for $Y \backslash \{i\}$. We denote the cardinality of Y by $|Y|$.

[2] While long term contracts covering the full economic life of capital are used in some energy markets (e.g. Joskow, 1987), the absence of empirical evidence in other markets suggests that this is not the norm. Supply contracts longer than twelve months duration tend to be rare; most capital goods have a longer life. We are unaware of empirical work tying the two aspects together.

[3] With both non-cooperative and axiomatic foundations, the Shapley value has been a benchmark in multilateral bargaining situations (for a general survey see e.g. Winter (2002)).

We derive now the *value* of our game. The highest revenue achieved with m outlets and a capacity of Q is described by

$$\phi(Q,m) \equiv \max\left\{\sum_{x=1}^{x=m} R(q_x)\right\} \text{ subject to } \sum_{x=1}^{x=m} q_x \leq Q$$

The objective is globally concave in the vector of quantities—the Hessian is diagonal with negative elements since R is strictly concave. At the optimum each outlet sells the same fraction of Q. We thus have

$$\phi(Q,m) = \begin{cases} mR(\frac{Q}{m}) \text{ if } Q < \overline{q}m \\ \\ mR(\overline{q}) \text{ if } Q \geq \overline{q}m \end{cases} \tag{1}$$

which is strictly concave in Q if $\frac{Q}{m} \leq \overline{q}$, otherwise linear in m and constant in Q.

Denote by $L \subseteq N$ a generic subset of downstream firms and, with abuse of notation, we will denote the number and the set of outlets owned by L by

$$m_L(Z) = \sum_{i \in L} m_i(Z)$$

For a given Z and capacity Q, the *value* of the game is

$$\begin{aligned} v(\,L \cup p|\, Q, Z) &= \phi(Q, m_L(Z)) \\ v(\,L \backslash p|\, Q, Z) &= \phi(0, m_L(Z)) = 0 \end{aligned} \tag{2}$$

Moreover, for any Z we have

$$v(\,N \cup p|\, Q, Z) = \phi(Q, m_N)$$

that is, the *value* of the industry is independent of how outlets are allocated across firms. It attains a maximum at $Q = \overline{q}m_N$.

Player i's Shapley value is the following linear combination of his marginal contributions[4]

$$S^i = \sum_{Y \subseteq \underline{Y} | i \in Y} \frac{(|Y|-1)!(|\underline{Y}|-|Y|)!}{|\underline{Y}|!}\left[v(Y \cup i) - v(Y \backslash i)\right]$$

Since the *value* of our game is conditional on Q and Z, we denote the dependence of the Shapley value on this pair of variables by $S_Z(Q)$. With (2), the producer's Shapley value simplifies to—for brevity, we also omit the superscript p

$$S_Z(Q) = \sum_{L \subseteq N} \omega_L v(\,L \cup p|\, Q, Z) \text{ where } \omega_L = \frac{|L|!(n-|L|)!}{(n+1)!} \tag{3}$$

It can be checked, with (1), that $S_Z(Q)$ is strictly concave for all $Q \in [0, \overline{q}m_N]$ and constant for all Q larger than $\overline{q}m_N$. As a remark, note that the weights add up to unity.

3.3 The producer's problem

The producer's program is

$$\underset{Q}{Max}\left\{S_Z(Q) - C(Q)\right\}$$

[4] $Y \subseteq \underline{Y} | \, i \in Y$ represents a set $Y \subseteq \underline{Y}$ such that i belongs to Y.

Since S_Z is concave, first order conditions are necessary and sufficient to determine the optimal capacity. Let the superscript ($'$) denote the partial derivative of a function with respect to Q. The optimal capacity level, denoted by Q_Z^*, satisfies

$$S_Z'(Q_Z^*) = c$$

Two remarks are in order here. First, note that the producer's returns to investment are lower than the industry return, i.e.

$$\begin{aligned}
\phi'(Q, m_N) &> S_Z'(Q) > 0 \text{ for } Q \in [0, \overline{q} m_N) \\
\phi'(Q, m_N) &= S_Z'(Q) = 0 \text{ for } Q \geq \overline{q} m_N
\end{aligned}$$

So we will always have underinvestment. Without loss of generality, we focus on the relevant range $Q \in (0, \overline{q} m_N)$. Second, write $S_Z(Q)$ as the share $\alpha_Z(Q)$ of the industry surplus $\phi(Q, m_N)$, i.e.

$$S_Z(Q) = \alpha_Z(Q) \cdot \phi(Q, m_N)$$

The marginal return to capacity can thus be written as

$$S_Z'(Q) = \phi'(Q, m_N) \cdot \alpha_Z(Q) + \alpha_Z'(Q) \cdot \phi(Q, m_N) \tag{4}$$

This equation shows two effects from increasing capacity: it increases date 1 revenue but it will also erode the bargaining power (share) of the producer—we show in proposition 1 that $\alpha_Z'(Q) \leq 0$.

Consider a premerger structure A and a postmerger structure B. Using the specification in (4), we decompose the total effect of a downstream horizontal merger on the producer's investment incentives into two effects: the *change in hold-up* and the *change in bargaining erosion*, i.e.

$$S_B'(Q) - S_A'(Q) =$$

$$\phi'(Q, m_N) \underbrace{[\alpha_B(Q) - \alpha_A(Q)]}_{\triangle \text{ Hold-up}} + \underbrace{[\alpha_B'(Q) - \alpha_A'(Q)]}_{\triangle \text{ Bargaining erosion}} \phi(Q, m_N) \tag{5}$$

In proposition 2 we find that a merger increases the hold-up, so the first right hand term is negative. Proposition 3 states that the a merger reduces the rate at which the bargaining erosion takes place, so the second right hand term is positive. The overall effect on the producer's investment incentives and equilibrium capacity depends on the magnitude of these two effects.

In propositions 4 we find that the producer's marginal investment incentives are lower after a merger if Q is low, and higher if Q is high. It follows that a downstream merger induces a higher (lower) equilibrium capacity if the cost of capacity is low (high).

We illustrate these results in the context of a simple example in figures 1 and 2. The example looks at linear pricing in an industry with three outlets. When

$$\phi(Q, 3) = \begin{cases} (1-Q)Q \text{ if } Q \leq \frac{1}{2} \\ \frac{1}{4} \text{ if } Q > \frac{1}{2} \end{cases}$$

each outlet has a linear inverse demand given by $p_x = 1 - 3q_x$. In situation A, each outlet is owned by a distinct downstream firm. In situation B, two outlets are owned by a single firm while another firm owns the remaining one.

3.4 Ownership structures

To study the effect of an exogenous change in the outlet ownership structure, such as a merger, we use the concept of downstream horizontal *integration*. A typical *integration contract* gives one player control over the assets owned by another player.

Definition 1 An *integration contract* allocates those outlets owned by any two firms i and j, with $m_i > 0$ and $m_j > 0$, to a single firm i, while the ownership structure of the remaining outlets is left unchanged.

It can be understood as a merger, an acquisition, or a purchase agreement between two firms. An integration contract changes the game by changing $m_L(Z)$, thereby changing the value of the game. It results in the following relationship:

$$v(L \cup p|\, Q, B) = \begin{cases} v(L \cup j \cup p|\, Q, A) \text{ if } i \in L \wedge j \notin L \\ \\ v(L \backslash j \cup p|\, Q, A) \text{ if } i \notin L \wedge j \in L \\ \\ v(L \cup p|\, Q, A) \text{ otherwise} \end{cases}$$

More generally we have:

Definition 2 B is an *integrated* ownership structure of A if B can be obtained from A by successive *integration contracts.*

Note that *integration* is transitive, i.e. if C is an integrated structure of B, and if B is an integrated structure of A, then C is also an integrated structure of A. Finally, we present a definition that will prove useful below.

Definition 3 A ownership structure is *symmetric* if $m_i = \frac{m_N}{n}$ for each $i \in N$

4 Analysis

4.1 The bargaining erosion effect

We first look at equation (4), namely at the effect of increasing capacity on the producer's bargaining position. The negative relation we find, named *bargaining erosion*, reduces the producer's incentive to invest in capacity—the second right hand term of (4) is negative. As we will see, this effect is ultimately driven by the concavity of R.

Proposition 1 ***Bargaining erosion:*** *If $m_i(Z) \neq m_N$ for all i we have*

$$\alpha'_Z(Q) < 0$$

a) for all Q sufficiently close to $\overline{q}m_N$.
b) for all $Q \in (0, \overline{q}m_N)$ if ϕ is log-supermodular.
If $m_i(Z) = m_N$ for some i we have

$$\alpha'_Z(Q) = 0 \text{ for all } Q$$

The accompanying intuition builds on the following sequence of arguments. In a bargaining situation power is determined by the relative marginal contributions. We measure the marginal contribution of a subset of firms $N\backslash L$ to the industry $N \cup p$ with a first-order difference operator defined as

$$\triangle_{N\backslash L}(Q,Z) \equiv \phi(Q,m_N) - v(L \cup p|\, Q, Z)$$

We add and subtract $\phi(Q,m_N)$ to $S_Z(Q)$ (see (3)) and divide the result by $\phi(Q,m_N)$. Since the weights of the Shapley value add up to unity, rearranging the expression we get:

$$\alpha_Z(Q) = 1 - \sum_{L\subseteq N} \omega_L \frac{\triangle_{N\backslash L}(Q,Z)}{\phi(Q,m_N)} \tag{6}$$

Increasing capacity reduces the bargaining power (share) of the producer when this increases the normalized marginal contribution of each subset of downstream firms.

This last condition is verified if $\phi(Q,m)$ is log-supermodular at Q.[5]

Definition 4 $\phi(Q,m)$ is *log-supermodular* at Q if when $m_1 < m_2 \leq m_N$ and $\phi'(Q,m_2) \neq 0$, we have

$$\frac{\phi'(Q,m_1)}{\phi(Q,m_1)} < \frac{\phi'(Q,m_2)}{\phi(Q,m_2)}$$

First, $\phi(Q,m)$ is log-supermodular at Q sufficiently close to $\overline{q}m_N$—ϕ' continuous and zero for $Q \geq \overline{q}m$ due to the concavity assumptions on R. Second, ϕ is log-supermodular if the elasticity of R is decreasing in q_x, i.e.[6]

$$\frac{\partial}{\partial q_x}\left(\frac{R'(q_x)}{R(q_x)}q_x\right) < 0 \text{ for all } q_x \in (0,\overline{q}) \tag{7}$$

In other words, when the percentage increase in $R(q_x)$ achieved with a one percent increase of q_x decreases as q_x increases. Developing (7) shows that this is the case if R is sufficiently concave.

4.2 Integration and hold-up

This subsection looks at the first effect of integration we identified in (5): the change in holdup. Intuition tells us that following a downstream merge the scope the producer has for playing a downstream firm off against another will be reduced, so the first right hand side element of (5) should be negative. This is proven correct. However, when the industry is not capacity constrained ($Q \geq \overline{q}m_N$) the producer's pay-off is independent of the allocation of outlets across firms, since in this case they do not have to compete for the input.

Proposition 2 ***Integration aggravates the hold-up:*** *If B is an integrated structure of A then*

$$\alpha_B(Q) < \alpha_A(Q)\ \forall Q \in (0,\overline{q}m_N)$$

and

$$\alpha_B(Q) = \alpha_A(Q) = \frac{1}{2} \text{ for } Q = \overline{q}m_N$$

[5] In another interpretation, if the instantaneous growth rate of the industry revenue with respect to Q is increasing in the number of outlets.

[6] Condition under which the cross derivative of (1) with respect to Q and m is positive. With linear pricing, (7) is verified for all concave and linear inverse demand functions. There exists as well a subset of convex inverse demand functions verifying this condition. It contains several commonly used functions.

This result is driven by the substitutability of downstream firms. We capture the notion of substitution with the following second-order difference operator:

$$\Delta^2_{ij} v(L \cup p|\, Q, Z) \equiv \tag{8}$$

$$v(L \cup i \cup j \cup p|\, Q, Z) - v(L\backslash i \cup j \cup p|\, Q, Z) - v(L\backslash j \cup i \cup p|\, Q, Z) + v(L\backslash i\backslash j \cup p|\, Q, Z)$$

Firms i and j are substitutes at Q and L if $\Delta^2_{ij} v(Q, m_L(Z))$ is negative, complements otherwise. We find that downstream firms are substitutes.

Lemma 1 For any two downstream firms i and j we have
a) if $m_i(Z) \neq 0$ and $m_j(Z) \neq 0$ then

$$\Delta^2_{ij} v(L \cup p|\, Q, Z) \begin{cases} < 0 \text{ if } Q \in (0, \overline{q} m_{L\cup i \cup j}) \\ \\ = 0 \text{ if } Q \geq \overline{q} m_{L\cup i\cup j} \end{cases} \tag{9}$$

b) if $m_i(Z) = 0$ for some i then $\Delta^2_{ij} v(L \cup p|\, Q, Z) = 0$ for all Q.

Segal (2003) finds that the integration of "two complementary (substitutable) players helps (hurts) players who are indispensable to either" integrating player—here the producer is always indispensable since $v(L\backslash p|\, Q, Z) = 0$ for all L. Formally:[7]

Lemma 2 If B can be obtained from A by an integration contract we have

$$S_B(Q) - S_A(Q) = \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \cdot \Delta^2_{ij} v(L \cup p|\, Q, A) \tag{10}$$

Since downstream firms are substitutes, an integration contract reduces the producer's bargaining surplus, and hence his share. Since integration is transitive, Proposition 2 follows directly from Lemma 1 and 2.

4.3 Integration and bargaining erosion

We now turn to the second effect we identified in (5): how the magnitude of the bargaining erosion changes with integration. We find that the rate at which this effect takes place decreases with integration, so the second right hand side element of (5) is positive.

Proposition 3 ***Integration softens the bargaining erosion:*** *if B is an integrated structure of A, there exists a critical value $\widetilde{Q}_{AB} < \overline{q} m_N$ such that*

$$-\alpha'_B(Q) < -\alpha'_A(Q) \textit{ for all } Q \in \left(\widetilde{Q}_{AB}, \overline{q} m_N\right)$$

$\widetilde{Q}_{AB} = 0$ when B can be obtained by an integration contract from a symmetric structure A and ϕ is log-supermodular.

Figure 1 shows the relationship between Q and $\alpha_Z(Q)$, illustrating the previous propositions, in the context of the example introduced in subsection 3.3.

[7] Adapted from Segal (2003, p. 452-453); our integration contract corresponds to his collusive contract.

Figure 1

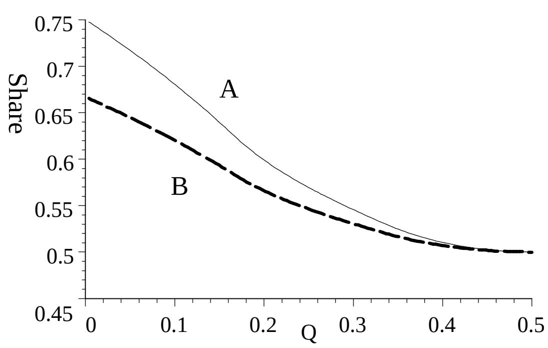
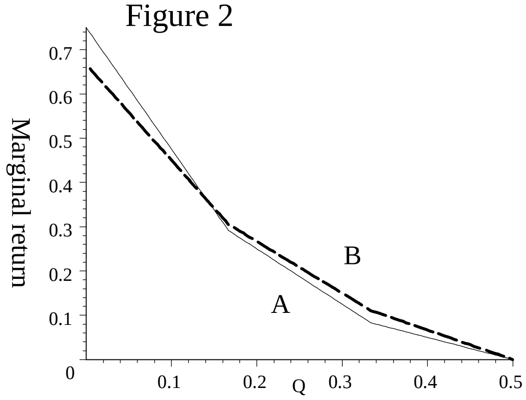


Figure 2



4.4 Integration and capacity choice

Proposition 2 and 3 show that the two effects we identified in (5) counteract each other. Here we look at the total effect of integration on the producer's investment incentives and equilibrium capacity choice.

We find that marginal returns *always* cross. So no single ownership structure provides unambiguously higher investment incentives. Moreover, the producer's investment incentives are lower in an integrated ownership structure if Q is low, and higher if Q is high.

Proposition 4 ***Marginal returns to capacity are non-monotonic:*** *if A is an integrated structure of B there exists a critical value $\underline{Q}_{AB}$ such that*

$$S_B'(Q) - S_A'(Q) < 0 \text{ for } Q \in \left[0, \underline{Q}_{AB}\right)$$

and critical value $\overline{Q}_{AB} \geq \underline{Q}_{AB}$ such that

$$S_B'(Q) - S_A'(Q) > 0 \text{ for } Q \in \left(\overline{Q}_{AB}, \overline{q}m_N\right)$$

Figure 2 shows this relationship between Q and marginal investment incentives in the context of the example introduced in subsection 3.3.

We look now at the optimal choice of capacity. Let

$$\overline{c} = S_A'(0), \widetilde{c} = S_A'(\underline{Q}_{AB}) \text{ and } \underline{c} = S_A'(\overline{Q}_{AB})$$

Solving for the producer's investment problem we find that marginal cost of capacity provides a simple criterion to determine the effect of capacity in the equilibrium capacity:

Proposition 5 ***Integration induces a higher (lower) equilibrium capacity if the cost of capacity is low (high):***

$$\begin{aligned} Q_A^* &= Q_B^* = 0 \text{ if } c \geq \overline{c} \\ Q_A^* &> Q_B^* \text{ if } c \in (\widetilde{c}, \overline{c}) \\ Q_A^* &< Q_B^* \text{ if } c \in (0, \underline{c}) \\ \lim_{c \to 0} Q_A^* &= \lim_{c \to 0} Q_B^* = \overline{q}m_N \end{aligned}$$

With affine and convex capacity cost functions we still find that the equilibrium capacity is higher after integration if fixed costs are low and marginal costs of capacity remain low for all $Q \in [0, \overline{q}m_N]$. Only when the cost of capacity is high—but not too high, profits must be positive—will the conventional wisdom go through, and the hold-up effect dominate.

It is important to highlight that consumers may benefit from downstream integration when the cost of capacity is low, since higher output levels are associated with lower prices and a higher consumer surplus.

5 Interrelated markets

In this section we look at the case where each outlet's revenue depends negatively on the quantities sold in the remaining outlets, as when firms compete on both the input and output market.

We consider two cases. In the first bargaining results in an efficient outcome. In the second we take into account the problem of "competitive externalities". In both cases we find, once again, that if the cost of capacity is low (high), a downstream merger induces a higher (lower) equilibrium capacity.

5.1 Setup

The setup is similar to the one presented in section 3, except that outlets' revenue is now described by a function $R(q_x, Q_{-x})$, where q_x is the quantity sold in outlet x and Q_{-x} the aggregate quantity sold in the remaining outlets, i.e.

$$Q_{-x} = \sum_{y \in m_N \backslash x} q_y$$

R is twice continuously differentiable, decreasing in the quantities sold in the remaining outlets[8]

$$R_2 \begin{cases} = 0 \text{ if } q_x = 0 \\ < 0 \text{ if } q_x > 0 \end{cases}$$

$R(0, Q_{-x}) = 0$ for all Q_{-x}, $R_1(0,0) > 0$ but $R_1 < 0$ for sufficiently large q_x, Q_{-x} or both.

R_{11}, R_{12} and R_{22} are such that, for all non-negative q_x and Q_{-x}, the Hessian matrix of

$$\sum_{x \in m_L(Z)} R(q_x, Q_{-x})$$

is negative definite for all $L \subseteq N$.

A well known example satisfying the above conditions is the case of linear pricing with a linear inverse demand at each outlet given by

$$a - b(q_x + \gamma Q_{-x}) \tag{11}$$

where $a, b > 0$ and $\gamma \in (0,1)$ measures the degree of differentiation—outlets are perfect substitutes for $\gamma = 1$. We will call it the *linear* model (and *invariant linear* model to the particular case where the first best level of capacity is invariant in γ, i.e. when $b = \frac{m_N}{a+\gamma(m_N-1)}$).

[8] For the remainder of the section, for a given function $f(z_1, z_2)$ we denote by $f_h = \partial f / \partial x_h$; similarly $f_{hg} = \partial^2 f / \partial x_h \partial x_g$.

5.2 Bargaining

We take again a cooperative approach to bargaining and the Shapley value as the solution concept. The new revenue function alters the game by changing its value. We consider two values: a *committed* value and an *opportunistic* value

We start by defining, once again, the value that can be achieved with m outlets for a given Q as

$$\phi(Q,m) = \max\left\{\sum_{x=1}^{m} R(q_x, Q_{-x})\right\} \text{ subject to} \sum_{x\in m} q_x \leq Q \text{ with } q_y = 0 \text{ for all } y \notin m$$

Let $Q^m \equiv \arg\max \phi(Q,m)$. Due to our concavity assumptions, $\phi(Q,m)$ simplifies to

$$\phi(Q,m) = \begin{cases} mR(\frac{Q}{m}, \frac{m-1}{m}Q) \text{ if } Q < Q^m \\ \\ mR(\frac{Q^m}{m}, \frac{m-1}{m}Q^m) \text{ if } Q \geq Q^m \end{cases}$$

$\phi(Q,m)$ is increasing in m and, for a given m, increasing and strictly concave in Q for $Q < Q^m$.

To $\phi(Q, m_L(Z))$ we will call the *committed value*, i.e.

$$\begin{aligned} v(L\cup p|\, Q,Z) &= \phi(Q, m_L(Z)) \\ v(L\backslash p|\, Q,Z) &= \phi(0, m_L(Z)) = 0 \end{aligned}$$

Once again, both $v(N\cup p|\, Q,Z)$ and its maximizing capacity Q^{m_N} are independent of Z. Denote a vector of quantities available to L downstream firms by

$$Q^L = (Q_1^L, .., Q_n^L) \text{ where } Q_i^L = 0 \text{ if } i \notin L$$

For a given Z, the vector Q^L inducing $v(L\cup p|\, Q^{m_L(Z)}, Z)$ has

$$Q_i^L = \frac{Q^{m_L(Z)}}{m_L(Z)} \text{ for all } i \in L$$

At this particular vector, each single retailer can increase its own revenue by negotiating a higher supply. To see this, note that the derivative of firm i's revenue with respect to Q_i^L, i.e.[9]

$$R_1(\frac{Q_i^L}{m_i(Z)}, \sum Q_j^L - \frac{Q_i^L}{m_i(Z)}) + (m_i(Z)-1)R_2(\frac{Q_i^L}{m_i(Z)}, \sum Q_j^L - \frac{Q_i^L}{m_i(Z)}) \qquad (12)$$

is strictly positive when evaluated at this vector for each $i \in L$ when $m_i(Z) \notin \{0, m_L\}$.

This additional revenue comes, in part, at the expense of a lower revenue to the remaining firms: for each firm $j \neq i$, with $m_j(Z) \neq 0$, the derivative of its total revenue with respect to Q_i^L evaluated at the vector inducing the vertically integrated revenue is strictly negative, i.e.

$$m_j(Z)R_2\left(\frac{Q^{m_L(Z)}}{m_L(Z)}, (\frac{m_L(Z)-1}{m_L(Z)})Q^{m_L(Z)}\right) < 0$$

So there is scope for opportunistic behavior, a problem addressed in the "competitive externalities" literature (e.g. McAfee and Schwartz, 1994; Segal and Whinston, 2003; deFontenay and Gans, 2004, 2005). As we will see, aggregate revenue may be lower than the *committed value.*

[9]Given our concavity assumption, each downstream firm finds it optimal to sell a fraction $\frac{1}{m_i(Z)}$ of Q_i^L at each of its outlets.

deFontenay and Gans (2004, 2005) show that, in our setting, the producer's pay-off of a particular bargaining game corresponds to its Shapley value over a value $\widehat{v}$ which is the highest revenue maximizing the bilateral surplus between each downstream firm and the producer—taking the remaining quantities as given, i.e.[10]

$$\widehat{v}(L \cup p|\, Z) = \max_{Q^L} \delta(Q^L, Z) \text{ subject to } (12) = 0 \text{ for all } i \in L$$

where

$$\delta(Q^L, Z) \equiv \sum_{i \in N} m_i(Z) R\left(\frac{Q_i^L}{m_i(Z)}, \sum_{j \in N} Q_j^L - \frac{Q_i^L}{m_i(Z)}\right)$$

Notice that the condition $(12) = 0$ defines implicitly the best response correspondence of a firm i in a Cournot setting when the set of firms is L. So $\widehat{v}(L \cup p|\, Z)$ is simply the highest aggregate Cournot revenue when the set of downstream firms is L. We assume that, for each L, the Cournot outcome is unique—this is the case e.g. in the *linear* model.[11]

It is important to note that the aggregate Cournot quantity of L for a given Z, which we denote by C_Z^L, may exceed the producer's capacity Q. We extend $\widehat{v}$ to take this constraint into account (a similar issue is addressed in Segal and Whinston (2003)).

We obtain the value $\widehat{v}(L \cup p|\, Q, Z)$, which now depends on Q and, which is still the highest revenue satisfying the condition that the bilateral surplus between each downstream firm and the producer is maximized—but taking *both* remaining quantities and capacity as given, i.e.

$$\widehat{v}(L \cup p|\, Q, Z) = \begin{cases} \widehat{\phi}(Q, m_L(Z)) \text{ if } Q < C_Z^L \\ \\ \max\left\{\widehat{v}(L \cup p|\, Z), \widehat{\phi}(Q, m_L(Z))\right\} \text{ if } Q \geq C_Z^L \end{cases}$$

with

$$\widehat{\phi}(Q, m_L(Z)) = \max_{Q^L} \left\{\delta(Q^L, Z)\right\} \text{ subject to } \sum Q_j^L = Q$$

The vector Q^L inducing $\widehat{v}(L \cup p|\, Q, Z)$ is immune to profitable bilateral renegotiations between any firm in L and the producer. We call $\widehat{v}(L \cup p|\, Q, Z)$ the *opportunistic value* since it captures the oversupply problem associated with producer's opportunism: for all $Q > Q^{m_L(Z)}$ we have

$$\widehat{v}(L \cup p|\, Q, Z) < v(L \cup p|\, Q, Z) \text{ if } m_L(Z) \neq m_i(Z) \text{ for all } i \in L$$

The above mentioned literature suggests that commitment mechanisms may eliminate the negative effects of producer's opportunism —e.g. public contracting or assigning downstream firms nonoverlapping territories. The choice of a particular value should depend on the availability of such commitment mechanisms. We study both. We denote the Shapley value in the *committed* case by $S_Z(Q)$, and in the *opportunistic* case by $\widehat{S}_Z(Q)$.

5.3 Integration and capacity choice

While $S_Z(Q)$ is concave for all Q, and we can use first order conditions to solve the producer's problem, $\widehat{S}_Z(Q)$ is not. Nevertheless, we can still use first order conditions provided that $\widehat{S}_Z(Q)$

[10] deFontenay and Gans (2005, 2006) study Bayesian equilibria of an extensive form bargaining game where players hold "passive beliefs" on unobserved actions. The result holds if the bi-core is non-empty.

[11] The problem becomes a linear system of equations in $|L|$ variables and $|L|$ unknowns. Since the equations are linearly independent, by Cramer's rule there is a unique solution (not for publication).

is concave in Q when $\widehat{S}_Z(Q)$ is non-decreasing in Q. So we assume that for all $L \subset N$ such that $m_L(Z) > 0$ we have

$$Q^{m_N} < C_Z^L \text{ if } m_L(Z) \neq m_i(Z) \text{ for all } i \in L$$

In the context of the *invariant linear* model this condition is satisfied if product differentiation is low, i.e. for

$$\gamma \geq \frac{2(m_N - 2)}{3m_N - 4}$$

Precisely the case contrasting with the independent market set-up studied in the previous section (corresponding to $\gamma = 0$).

Denote by Q_Z^* and $\widehat{Q}_Z^*$ the equilibrium capacity choice in the *committed* and the *opportunistic* case respectively. With the previous assumption, the optimal capacity levels satisfy

$$S_Z'(Q_Z^*) = c$$

and

$$\widehat{S}_Z'(\widehat{Q}_Z^*) = c$$

We have once again a problem of underinvestment since

$$\begin{array}{rcl} \phi'(Q, m_N) & > & S_Z'(Q) \geq \widehat{S}_Z'(Q) \ \forall Q \in [0, Q^{m_N}) \\ \phi'(Q, m_N) & = & S_Z'(Q) = 0 \geq \widehat{S}_Z'(Q) \ \forall Q \geq Q^{m_N} \end{array}$$

We therefore focus on the relevant range $Q \in [0, Q^{m_N}]$. In this range, we also have that

$$v(N \cup p | Q, Z) = \widehat{v}(N \cup p | Q, Z) = \phi(Q, m_N)$$

We now look at the overall effect of integration on the producer's capacity choice. For tractability we focus on the case where downstream firms are symmetric before the integration contract. In this particular case we have:

Lemma 3: If B can be obtained with an integration contract from a symmetric structure A then, for all $Q \in [0, Q^{m_N}]$, we have

$$S_B(Q) - S_A(Q) = \frac{1}{N(N-1)}(\phi(Q, m_N) - 2S_A(Q))$$

and

$$\widehat{S}_B(Q) - \widehat{S}_A(Q) =$$

$$\frac{1}{N(N-1)}\left[(\phi(Q, m_N) - 2\widehat{S}_A(Q)) + \frac{2}{N+1}(v(\{i,j\} \cup p | Q, A) - \widehat{v}(\{i,j\} \cup p | Q, A)\right] \tag{13}$$

The main difference between the *committed* and the *opportunistic* case lies in the second right hand term. This term captures the fact that when $L = \{i, j\}$, if these two firms integrate the opportunism problem disappears—since with a retail monopoly supply level is always expost efficient. This effect is positive when Q is sufficiently high ($Q > Q^{m_{\{i,j\}}(A)}$)—it is zero when Q is low.

Solving the producer's investment problem we find:

Proposition 6 ***Integration induces a higher (lower) equilibrium capacity if the cost of capacity is low (high):*** *when B can be obtained with an integration contract from a symmetric structure A, there exist positive values* $\overline{c}^{+}, \widetilde{c}^{+}$ *and* $\underline{c}^{+}$ *such that*

$$\begin{aligned}
Q_A^* &= Q_B^* = \widehat{Q}_A^* = \widehat{Q}_B^* = 0 \textit{ if } c \geq \overline{c}^{+} \\
Q_A^* &> Q_B^* \textit{ and } \widehat{Q}_A^* > \widehat{Q}_B^* \textit{ if } c \in \left(\widetilde{c}^{+}, \overline{c}^{+}\right) \\
Q_A^* &< Q_B^* \textit{ and } \widehat{Q}_A^* < \widehat{Q}_B^* \textit{ if } c \in \left(0, \underline{c}^{+}\right) \\
\lim_{c \to 0} Q_A^* &= \lim_{c \to 0} Q_B^* = Q^{m_N} \geq \lim_{c \to 0} \widehat{Q}_B^* > \lim_{c \to 0} \widehat{Q}_A^*
\end{aligned}$$

In a setting where market revenues are interrelated, and firms are symmetric before the merger, the cost of capacity provides once again a simple criterion to evaluate claims about the effect of a downstream merger on the producer's capacity choice, both in the *committed* and *opportunistic* cases.

To illustrate the role of product differentiation, we consider now the *invariant linear* model in the context of the example presented in subsection 3.3. In this case, marginal incentives cross once and we have:

$$\widetilde{c}^{+}(\gamma) = \underline{c}^{+}(\gamma) = \frac{5}{2}\left(\frac{1-\gamma}{7-\gamma}\right)$$

This critical value is decreasing in γ (and converges to 0 as $\gamma \to 1$), suggesting that an integrated structure is more likely to induce a lower upstream capacity when product differentiation is low— and a higher one when product differentiation is high.

6 Conclusion

This paper studied how integration of players who compete for an input affects the capacity choice (or stock) of a monopolist supplying that input. We showed and explained why there exists a non-monotonic relationship between downstream integration and the producer's level of investment.

We also found that the cost of capacity provides a simple criterion to evaluate claims about the effect of a downstream merger on the producer's capacity choice: if the cost of capacity is low (high), a downstream merger induces a higher (lower) equilibrium capacity. These results are valid when downstream firms sell on independent markets and in interrelated markets alike.

At a practical level, these results should be useful to merger analyses since they help sign the effect of downstream integration on the producers' capacity choice and, indirectly, welfare.

Our work also suggests that further research on the effect of investment on the substitutability of firms should lead to more general results on investment effects of mergers.

At a more theoretical level, these results provide theoretical support to Galbraith's (1952) idea of Countervailing Power. One of the main ideas is that, when competition fails on both sides of the market, allocative efficiency may be nurtured not by increased competition but by a process of concentration on the most competitive side. Our results support this idea.

However, Galbraith's informal argument is that one position of power may be neutralized by another[12]. In our model, the mechanism at play is different. Bargaining power arises from controlling a scarce resource. The producer has therefore a strategic incentive to maintain the resource he controls relatively scarce in order to leverage his bargaining power. This effect, which

[12] "The fact that a seller enjoys a measure of monopoly power (..) means that there is an inducement to those firms from whom he buys or those to whom he sells to develop the power with which they can defend themselves against exploitation, (..). In this way the existence of market power creates an incentive to the organization of another position of power that neutralizes it." Galbraith (1952) p. 119.

distorts supply downwards, is particularly important when the retail sector exhibits a low level of concentration.

These results are of interest for other settings. For example, they cast doubt on the usual claim that unionization reduces a firm's incentive to invest in labour-complementary capital. Another application concerns public procurement. As an example, we have in mind Roche's recent reluctance to upscale its stock of the Tamiflu antiviral. Our results suggest this decision could have been made earlier had European governments been negotiating through a single entity. However, to rigorously address these issues we would have to work with different set-ups.

7 Appendix

Proof Proposition 1: From (6) the share of the surplus accruing to the producer can be written as follows

$$\alpha_Z(Q) = 1 - \sum_{L \subseteq N} \omega_L \frac{\triangle_{N \backslash L}(Q, m_L(Z))}{\phi(Q, m_N)}$$

Thus

$$\alpha'_Z(Q) = - \sum_{L \subseteq N} \omega_L \frac{\partial}{\partial Q} \left(\frac{\triangle_{N \backslash L}(Q, m_L(Z))}{\phi(Q, m_N)} \right) \tag{14}$$

and for each $L \subseteq N$

$$\frac{\partial}{\partial Q} \left(\frac{\triangle_{N \backslash L}(Q, m_L(Z))}{\phi(Q, m_N)} \right) = \frac{\phi'(Q, m_N) \cdot \phi(Q, m_L(Z)) - \phi'(Q, m_L(Z)) \cdot \phi(Q, m_N)}{(\phi(Q, m_N))^2} \tag{15}$$

We consider first those ownership structures where $m_i(Z) \neq m_N$ for all i (Step 1) and then those where $m_i(Z) = m_N$ for some i (Step 2).

Step 1 ($m_i(Z) \neq m_N$ for all i): it follows from (15) together with (14) that $\alpha'_Z(Q) < 0$ if for all $L \subset N$

$$\frac{\phi'(Q, m_L(Z))}{\phi(Q, m_L(Z))} \leq \frac{\phi'(Q, m_N)}{\phi(Q, m_N)}$$

and the inequality is strict for at least one L.

Since $m_i(Z) < m_N$, and $m_L(Z) \leq m_N$ it follows that $\alpha'_Z(Q) < 0$ if $\phi(Q, m)$ is log-supermodular at Q—see definition in the text. So:

a) $\alpha'_Z(Q) < 0$ for all Q close to $\overline{q}m_N$, since $\phi(Q, m)$ is log-supermodular at Q—$\phi'(Q, m_N) > \phi'(Q, m_L(Z)) = 0$ if $m_L(Z) < m_N$.

b) $\alpha'_Z(Q) < 0$ for all $Q \in (0, \overline{q}m_N)$ if $\phi(Q, m)$ is log-supermodular at all $Q \in (0, \overline{q}m_N)$.

Step 2 ($m_i(Z) = m_N$ for some i): for all $L \subseteq N$ we have $m_L \in \{0, m_N\}$. The normalized second order difference operator is constant and equal to either 1 if $m_L(Z) = 0$ or 0 if $m_L(Z) = m_N$. We have $\alpha'_Z(Q) = 0$.

Proof Proposition 2: Lemma 1 (see proof. below) together with lemma 2 (from Segal (2003)) imply that the share of the surplus accruing to the producer is lower after an integration contract (note that all the elements of (10) are negative, and strictly negative for L such that $m_{L \cup i \cup j}(A) = m_N$. It remains unchanged for $Q = \overline{q}m_N$).

We now use transitivity of integration to complete the proof. An integrated structure results from successive integration contracts. If the share is reduced, for all $Q \in (0, \overline{q}m_N)$, from B to C and from A to B, then it is also reduced from A to C. It follows that:

a) The share accruing to the producer is reduced after integration for all $Q \in (0, \overline{q}m_N)$.

b) It remains unchanged for all Z when $Q = \overline{q} m_N$.

When $m_i(Z) = m_N$ (a bilateral monopoly situation) the producer gets one half of the surplus. It follows that $\alpha_Z(\overline{q} m_N) = \frac{1}{2}$ for all Z.

Proof Lemma 1 (For brevity, we drop here the notational dependence on Z.) For all L, i and j

$$m_{L\backslash i\backslash j} + m_{L\cup i\cup j} = m_{L\cup i\backslash j} + m_{L\cup j\backslash i}$$

and therefore exists a $\widehat{\lambda} \in [0,1]$ such that (adding up vertically the expression below we obtain the expression above):

$$\begin{aligned} \widehat{\lambda} m_{L\backslash i\backslash j} + (1-\widehat{\lambda}) m_{L\cup i\cup j} &= m_{L\cup j\backslash i} \\ (1-\widehat{\lambda}) m_{L\backslash i\backslash j} + \widehat{\lambda} m_{L\cup i\cup j} &= m_{L\cup i\backslash j} \end{aligned} \tag{16}$$

If $m_i \neq 0$ and $m_j \neq 0$ then $\widehat{\lambda} \in (0,1)$. If $m_i = 0$ for some i then $\widehat{\lambda} = 0$. We consider the two cases in turn.

a) $m_i \neq 0$ and $m_j \neq 0$: recall from (1) that $\phi(Q, m)$ is increasing and concave in m if $\frac{Q}{\overline{q}} < m$. It is linear if $\frac{Q}{\overline{q}} \geq m$. So for all $\lambda \in (0,1)$ (and thus for $\widehat{\lambda}$) and $m_1 < m_2$

$$\lambda \phi(Q, m_1) + (1-\lambda)\phi(Q, m_2) \leq \phi(Q, \lambda m_1 + (1-\lambda) m_2) \tag{17}$$

The inequality is strict if $\frac{Q}{\overline{q}} < m_2$.

From (16) and (17), together with the definition of $\Delta^2_{ij}\phi$ in (8), we have

$$\phi(Q, m_{L\cup i\cup j}) + \phi(Q, m_{L\backslash i\backslash j}) \leq \phi(Q, m_{L\cup j\backslash i}) + \phi(Q, m_{L\cup i\backslash j})$$

which is equivalent to

$$\Delta^2_{ij} v(\, L \cup p |\, Q, Z) \leq 0$$

The inequality is strict if $\frac{Q}{\overline{q}} < m_{L\cup i\cup j}$—since $m_{L\cup i\cup j} \neq \left\{ m_{L\cup i\backslash j}, m_{L\cup j\backslash i} \right\}$. It is verified with equality if $\frac{Q}{\overline{q}} \geq m_{L\cup i\cup j}$.

b) $m_i = 0$ for some i: in this case we have $m_{L\cup i\cup j} = m_{L\cup j\backslash i}$ and $m_{L\backslash i\backslash j} = m_{L\cup i\backslash j}$ so by definition $\Delta^2_{ij} v(\, L \cup p |\, Q, Z) = 0$.

Proof Proposition 3: We proceed in 3 steps. In step 1 we show that the rate at which "bargaining erosion" takes place decreases if the second order difference operator increases with Q. This is verified if Q is high (step 2). In step 3 we look at the particular case where the ownership structure is symmetric before the integration contract.

Step 1: Dividing (10) from lemma 1 by $\phi(Q, m_N)$ we write

$$\alpha'_B(Q) - \alpha'_A(Q) = \sum_{L \subseteq N | j \in L \wedge i \notin L} \omega_L \cdot \frac{\partial}{\partial Q} \left(\frac{\Delta^2_{ij} v(\, L \cup p |\, Q, A)}{\phi(Q, m_N)} \right)$$

and we have

$$\frac{\partial}{\partial Q} \left(\frac{\Delta^2_{ij} v(\, L \cup p |\, Q, A)}{\phi(Q, m_N)} \right) = \frac{\frac{\partial}{\partial Q} \Delta^2_{ij} v(\, L \cup p |\, Q, A) \cdot \phi(Q, m_N) - \phi'(Q, m_N) \cdot \Delta^2_{ij} v(\, L \cup p |\, Q, A)}{(\phi(Q, m_N))^2}$$

From Lemma 1, for $Q \in [0, \overline{q} m_N)$ we have $\Delta^2_{ij} v(L \cup p |\, Q, A) \leq 0$ for all L. This inequality is strict for L such that $m_{L\cup i}(A) = m_N$ (there exists at least one such L). Because $\phi'(Q, m_N) > 0$ we have

$$-\alpha'_B(Q) < -\alpha'_A(Q) \text{ if } \frac{\partial}{\partial Q} \Delta^2_{ij} v(\, L \cup p |\, Q, A) \geq 0 \text{ for all } \; L \subseteq N |\, j \in L \wedge i \notin L$$

i.e. when the *normalized substitutability of i and j in L* is decreasing in Q.

Step 2: With (2), we write the derivative of the second order difference operator as

$$\frac{\partial}{\partial Q}\Delta_{ij}^2 v(L\cup p|\,Q,A)=\phi'(Q,m_{L\cup i\cup j}(A))-\phi'(Q,m_{L\backslash i\cup j}(A))-\phi'(Q,m_{L\backslash j\cup i}(A))+\phi'(Q,m_{L\backslash i\backslash j}(A)) \tag{18}$$

From (1), $\phi'(Q,m_2)>\phi'(Q,m_1)$ for any $m_2>m_1$ if $Q<\overline{q}m_2$ and $\phi'(Q,m)=0$ for all m if $Q\geq\overline{q}m$ (since R has a maximum at $\overline{q}$). Because $m_i(A)\neq$ and $m_j(A)\neq 0$, for $Q\geq\overline{q}\min\left\{m_{L\backslash j\cup i}(A),m_{L\backslash i\cup j}(A))\right\}$ we have:

$$\frac{\partial}{\partial Q}\Delta_{ij}^2 v(L\cup p|\,Q,A)=\phi'(Q,m_{L\cup i\cup j}(A))-\phi'(Q,\max\left\{m_{L\backslash i\cup j}(A),m_{L\backslash j\cup i}(A)\right\})$$

and therefore

$$\frac{\partial}{\partial Q}\Delta_{ij}^2 v(L\cup p|\,Q,A)\begin{cases}>0\text{ if }Q\in\left[\overline{q}\min\left\{m_{L\backslash j\cup i}(A),m_{L\backslash i\cup j}(A))\right\},\overline{q}m_{L\cup i\cup j}\right)\\=0\text{ if }Q\geq\overline{q}m_{L\cup i\cup j}\end{cases}$$

So, for all $L\subseteq N$ such that $j\in L$ and $i\notin L$ we have

$$\frac{\partial}{\partial Q}\left(\frac{\Delta_{ij}^2 v(L\cup p|\,Q,A)}{\phi(Q,m_N)}\right)\geq 0\text{ for all }Q\in\left[\overline{q}\min\left\{m_{N\backslash i}(A),m_{N\backslash j}(A))\right\},\overline{q}m_N\right) \tag{19}$$

This inequality is strict for L such that $m_{L\cup i}(A)=m_N$. We conclude that

$$-\alpha_B'(Q)<-\alpha_A'(Q)\text{ for all }Q\in(\widetilde{Q}_{AB},\overline{q}m_N)$$

where $\overline{q}\min\left\{m_{N\backslash i}(A),m_{N\backslash j}(A))\right\}$ is an upper bound of $\widetilde{Q}_{AB}$. (Once again, by transitivity of integration, for all Q sufficiently close to Q we have that $\alpha_C'(Q)-\alpha_B'(Q)>0$ and $\alpha_B'(Q)-\alpha_A'(Q)>0$ so $\alpha_C'(Q)-\alpha_A'(Q)>0$.)

Step 4: Assume A is symmetric. If B can be obtained from A by an integration contract then (see Lemma 3):

$$S_B(Q)-S_A(Q)=\frac{1}{N(N-1)}(\phi(Q,N)-2S_A(Q))$$

therefore

$$\alpha_B'(Q)-\alpha_A'(Q)=-\frac{2}{N(N-1)}\alpha_A'(Q)$$

But if ϕ is log-supermodular

$$-\alpha_B'(Q)<-\alpha_A'(Q)\text{ for all }Q\in(0,\overline{q}m_N)$$

since then $\alpha_A'(Q)<0$ for all $Q\in(0,\overline{q}m_N)$ (see Proposition 1).

Proof proposition 4: With Lemma 2, we write

$$S_B'(Q)-S_A'(Q)=\sum_{L\subseteq N|j\in L\wedge i\notin L}\omega_L\cdot\frac{\partial}{\partial Q}\Delta_{ij}^2 v(L\cup p|\,Q,A)$$

We first show that the producer's marginal return to capacity is always higher before integration for Q is close to 0 (step 1). We then show the opposite for Q close to $\overline{q}m_N$ (step 2).

Step 1: From (1), $\phi'(0,m) = R'(0)$ for all $m > 0$. With (18) we see that

$$\frac{\partial}{\partial Q}\Delta^2_{ij} v(L \cup p|\, 0, A) \begin{cases} = 0 \text{ if } m_{L\backslash i\backslash j}(A) \neq 0 \\ \\ -R'(0) \text{ if } m_{L\backslash i\backslash j}(A) = 0 \end{cases}$$

Since $m_{L\backslash i\backslash j} = 0$ for $L = \{j\}$, we have

$$S'_B(0) - S'_A(0) < 0$$

An integrated structure results from successive integration contracts. If the marginal return to capacity at 0 is higher in B than in C and in A than in B, then it is also higher in A than in C. By transitivity and continuity we conclude that for any two ownership structures A and B, such that B is an integrated structure of A, there exists $\underline{Q}_{AB} > 0$ such that

$$S'_B(0) - S'_A(0) < 0 \text{ for all } Q \in \left[0, \underline{Q}_{AB}\right)$$

Step 2: From (19), for all $L \subseteq N$ with $j \in L$ and $i \notin L$ we have

$$\frac{\partial}{\partial Q}\Delta^2_{ij} v(L \cup p|\, Q, A) \geq 0 \text{ for all } Q \in \left(\overline{q}\min\left\{m_{N\backslash i}(A), m_{N\backslash j}(A))\right\}, \overline{q}m_N\right)$$

The inequality is strict for L such that $m_{L\cup i}(A) = m_N$. So the difference in marginal returns induced by an integration contract is strictly negative for all $Q \in \left(\overline{q}\min\left\{m_{N\backslash i}(A), m_{N\backslash j}(A))\right\}, \overline{q}m_N\right)$.

An integrated structure results from successive integration contracts. If the marginal return to capacity is lower in B than in C and in A than in B, then it is also lower in A than in C. By transitivity and continuity we conclude that for any two ownership structures A and B, such that B is an integrated structure of A, there exists a $\overline{Q}_{AB} < \overline{q}(m_N - 1)$ such that

$$S'_B(Q) - S'_A(Q) > 0 \text{ for all } Q \in (\overline{Q}_{AB}, \overline{q}m_N)$$

Proof Proposition 5: Since S is strictly concave for $Q \in [0, \overline{q}m_N]$, equilibrium capacity is given by

$$S_Z(Q^*_Z) = c$$

and $S'_z(Q)$ is high when Q is low and low when Q is high—and equal to zero for $Q = \overline{q}m_N$.

For any Z, if $c \geq S'_z(0)$ we have $Q^*_Z = 0$. From proposition 4 we know that

$$S'_B(0) - S'_A(0) < 0 \text{ for all } Q \in \left[0, \underline{Q}_{AB}\right)$$

But then, $Q^*_A = Q^*_B$ for all $c \geq S'_A(0)$ and $Q^*_A > Q^*_B$ for all $c \in (S'_A(0), S'(\underline{Q}_{AB}))$. Since

$$S'_B(0) - S'_A(0) > 0 \text{ for all } Q \in (\overline{Q}_{AB}, \overline{q}m_N)$$

a similar argument shows that when $c \in (S'_A(\overline{Q}_{AB}), \overline{q}m_N)$ we have $Q^*_A < Q^*_B$. Finally, $S'_Z(Q) = 0$ for all Z therefore $\lim_{c\to 0} Q^*_A = \lim_{c\to 0} Q^*_B = \overline{q}m_N$.

Proof Lemma 3: We proceed in three steps. In the first step we restate a known result for symmetric ownership structures. In the second step we prove the result for S; in the third for $\widehat{S}$.

Step 1: When A is symmetric, the producer's Shapley value simplifies to

$$S_A(Q) = \sum_{h=1}^{h=n} \frac{1}{n+1}\phi(Q, h\frac{m_N}{n})$$

since there are $\frac{n!}{(n-h)!h!}$ sets L such that when $|L| = h$ we have

$$m_L(A) = h\frac{m_N}{n}$$

Step 2: Recall from Lemma 2 that

$$S_B(Q) - S_A(Q) = \sum_{L\subseteq N|j\in L\wedge i\notin L} \omega_L \cdot \Delta^2_{ij} v(\, L \cup p|\, Q, A) \text{ with } \omega_L = \frac{|L|!(n-|L|)!}{(n+1)!} \tag{20}$$

With symmetry, for all L such that $|L| = h$ we have

$$\Delta^2_{ij} v(\, L\cup p|\, Q, A) = \phi(Q, (h+1)\frac{m_N}{|N|}) - 2\phi(Q, h\frac{m_N}{|N|}) + \phi(Q, (h-1)\frac{m_N}{|N|})$$

and in A there are

$$\frac{(n-2)!}{(n-h-1)!(h-1)!}$$

sets $L \subseteq N$ such that $j \in L$ and $i \notin L$ for which $|L| = h$.

Since

$$\omega_L \frac{(n-2)!}{(n-h-1)!(h-1)!} = \frac{h(n-h)}{(n+1)n(n-1)}$$

we can write (20) as

$$S_B(Q) - S_A(Q) = \frac{1}{(n+1)n(n-1)} \sum_{h=1}^{h=n-1} h(n-h) \left[\phi(Q, (h+1)\frac{m_N}{n}) - 2\phi(Q, h\frac{m_N}{n}) + \phi(Q, (h-1)\frac{m_N}{n})\right]$$

The sum can be rewritten as

$$(n-1)\phi(Q, n\frac{m_N}{n}) - 2(n-1)\phi(Q, (n-1)\frac{m_N}{n}) + 2(n-2)\phi(Q, (n-1)\frac{m_N}{n}) +$$

$$\sum_{h=2}^{h=n-2} \left[(n-(h+1))(h+1) - 2(n-h)h + (n-(h-1))(h-1)\right]\phi(Q, h\frac{m_N}{n})$$

$$+2(n-2)\phi(Q, \frac{m_N}{n}) - 2(n-1)\phi(Q, \frac{m_N}{n})$$

Finally, since

$$(n-(h+1))(h+1) - 2(n-h)h + (n-(h-1))(h-1) = -2$$

we write

$$S_B(Q) - S_A(Q) = \frac{1}{n(n-1)} \left[\frac{n-1}{n+1}\phi(Q, m_N) - 2\sum_{h=1}^{h=n-1} \frac{1}{n+1}\phi(Q, h\frac{m_N}{n})\right]$$

$$= \frac{1}{n(n-1)}\left[\phi(Q,m_N) - 2\sum_{h=1}^{h=n}\frac{1}{n+1}\phi(Q,h\frac{m_N}{n})\right] = \frac{1}{n(n-1)}\left[\phi(Q,m_N) - 2S_A(Q)\right]$$

Step 3: We now look at $\widehat{S}$. First, we assumed that

$$Q^{m_N} < C_Z^L \text{ if } m_L(Z) \neq m_i(Z) \text{ for all } i \in L$$

Thus, for all L such that $m_i(Z) \neq m_L(Z)$ we will have

$$\widehat{v}(L \cup p|\, Q, Z) = \widehat{\phi}(Q, m_L(Z)) = m_L(Z) R\left(\frac{Q}{m_L(Z)}, \frac{m_L(Z)-1}{m_L(Z)}Q\right)$$

So the value depends only on the number of $m_L(Z)$, and not on the particular allocation of outlets. So the proofs of step 1 and 2 apply to $\widehat{S}$ as well, except for those L such that $m_i(A) \neq m_L(A)$ when $m_i(B) = m_L(B)$ for some i. In the case of symmetric firms there is only one such $L = \{i,j\}$, where $\widehat{v}(\{i,j\} \cup p|\, Q, A)$ becomes $v(\{i,j\} \cup p|\, Q, A)$. Since this set has a weight

$$\frac{2}{N(N-1)(N+1)}$$

in the Shapley value, we obtain in this way our result.

Proof Proposition 6: We proceed in two steps. In the first step we prove the result for the case where c is high; in the second for c low.

Step 1: Note that for Q close to 0 we have

$$\widehat{v}(L \cup p|\, Q, Z) = v(L \cup p|\, Q, Z) = \phi(Q, m_L(Z))$$

So $S_Z(Q) = \widehat{S}_Z(Q)$ and the proof of Proposition 5 applies.

Step 2: For high levels of Q, we still have

$$v(L \cup p|\, Q, Z) = \phi(Q, m_L(Z))$$

and proof of Proposition 5 applies as well to $S_Z(Q)$. However, this is not true for $\widehat{S}_Z(Q)$ since

$$\widehat{v}(L \cup p|\, Q, Z) \neq \phi(Q, m_L(Z))$$

for some L when Q is high (except when $m_i(Z) = m_N$ for some i). In fact for all Z, such that $m_i(Z) \neq m_N$, there exists a critical $\widehat{Q}_Z^0 \in (0, Q^{m_N})$ such that

$$\widehat{S}'_Z(Q) \begin{cases} > 0 \text{ if } Q < \widehat{Q}_Z^0 \\ = 0 \text{ if } Q = \widehat{Q}_Z^0 \\ < 0 \text{ if } Q > \widehat{Q}_Z^0 \end{cases}$$

So, $\widehat{S}'_Z(Q^{m_N}) < 0$ and we have $\lim_{c\to 0}\widehat{Q}_Z^* < Q^{m_N}$ (for Z such that $m_i(Z) = m_N$ for some i we still have $\lim_{c\to 0}\widehat{Q}_Z^* = Q^{m_N}$). Moreover, while by definition

$$\widehat{S}'_A(\widehat{Q}_A^0) = 0$$

from (13) we have that

$$\widehat{S}'_B(\widehat{Q}_A^0) = \frac{1}{N(N-1)}\left[\phi'(\widehat{Q}_A^0, m_N) + \frac{2}{N+1}(v'(\{i,j\} \cup p|\, \widehat{Q}_A^0, A) - \widehat{v}'(\{i,j\} \cup p|\, \widehat{Q}_A^0, A)\right] > 0$$

since $\phi'(Q, m_N) > 0$ and $v'(\{i,j\} \cup p|\, Q, Z) - \widehat{v}'(\{i,j\} \cup p|\, Q, Z) \geq 0$ for all $Q \in (0, Q^{m_N})$. By continuity, there exists a $\underline{c}^+ \geq \widehat{S}'_B(\widehat{Q}_A^0)$ such that $\widehat{Q}_A^* < \widehat{Q}_B^*$ if $c \in (0, \underline{c}^+)$.

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