Contest when the winner gets her effort reimbursed

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Abstract

Many important economics problems can be modeled as a contest. The contest literature includes among others Loury (1979), Tullock (1980), Lazear and Rosen (1981), Rosen (1981, 1986), Dixit (1987, 1999), Baik and Shogren (1992), Baye and Shin (1999), and Moldovanu and Sela (2001, 2006).

In this paper we analyze n-player contests with one main prize. Suppose that the design of the contest is fixed, but the contest designer can transfer players' contributions from one player to another. For example, in lotteries, one of the players can get a wager of the winner. How can the designer extract the most rent from the players in the contest such that players still want to participate in the contest? We find all such mechanisms if players have the same value for the prize. In particular, one of these mechanisms is the contest in which the winner gets her effort reimbursed.

We analyze contests in which the winner gets her effort reimbursed in detail.

First, we consider stochastic (Tullock's) asymmetric model. *All* equilibria in pure strategies are found. It turns out that equilibria can be of two types: i-type and internal type. i-type equilibria are such that only player i exerts (high) positive effort and all other players exert zero effort. Internal equilibria are such that at least two players exert positive effort. Some n-player contests can have an internal equilibrium where *all* players exert positive effort. The simplest example of this kind is a contest where all players have the same value for the main prize.

Then, we consider properties of equilibria. In particular, we demonstrate that the players' expected payoffs are zero in any internal equilibrium. Moreover, a higher value

(stronger) player *always* exerts less effort than a lower value (weak) player and therefore has a lower chance to win the contest in *any* internal equilibrium.

In the recent paper Cohen and Sela (2005) consider an asymmetric contest with one main prize in which the winner gets her effort reimbursed. They show that in a two-player case there exists a unique internal equilibrium where the weak contestant wins with higher probability than the stronger one. In 3-player contests, Cohen and Sela (2005) demonstrate that each one of the 2 underdogs may win with the highest probability.

Second, we consider deterministic (auctions with winner's reimbursement) model. The players' valuations for the main prize are commonly known among the players. Since players' strategy space is unbounded, the existence of pure and mixed strategy equilibria is not guaranteed by any of the existing theorems for discontinuous games since they all rely on the *compactness* of the strategy space.[1] Moreover, it is obvious that there are no pure strategy Nash equilibria in auctions with winner's reimbursement. We derive a class of mixed strategy equilibria. It turns out that these equilibria have "similar" properties with the internal equilibria in the stochastic model. In particular, the players' expected payoffs are zero in any mixed-strategy equilibrium. Moreover, a higher value (stronger) player exerts less expected effort than a lower value (weak) player and therefore has a lower expected chance to win the auction.

Keywords: *Tullock's contest, Contest, Winner's reward, Auctions*

[1] See, for instance, Baye et al. (1993), Dasguspta and Maskin (1986), Reny (1999).