Location, Information and Coordination

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March 19, 2006

Abstract. In this paper, we consider a finite population of boundedly rational agents whose preferences differ. The interaction level among agents allows us to partition the population into local networks. In each local network, there exists a fixed agent, as defined by Glaeser and al. [8], who shares, directly or indirectly, her information with all agents within the local network. Time is discrete and in each period agents are paired to play a battle of the sexes game. We show that in the short run, all fixed agent plays a particular strategy, but only neighboring fixed agents need to coordinate on the same strategy. In the long run however, all fixed agents coordinate on the same strategy, leading to a uniform convention, as defined by Young [22]. Our main result shows that location leads to information access and distribution which in turn leads to coordination. In particular, it shows that the outcome that prevails in a population of heterogeneous agents facing asymmetric information is decided by those agents who share the most widely their information.

1. Introduction

Numerous studies have shown the importance of social interactions and neighborhoods effects in explaining phenomenon such as education, income, production, crime, gangs, ghettos and, in a lighter register, restaurant pricing[1]. Therefore, the incorporation of social influences on behavior in theoretical frameworks must be of primary interest. In particular, the influence that some agents exert on others can have a profound impact on the selection of an economic outcome. As Glaeser and al. [8] put it: "There are two classes of agents: (1) agents who influence and are influenced by their neighbors; and (2) agents who influence their neighbors, but who cannot themselves be influenced (*Fixed agents*)". These *fixed agents* are of particular interest when one considers games that are characterized, in accordance with Schelling [19], by a "mixture of mutual dependence and conflict, of partnership and competition".

In this paper, we show how these fixed agents lead a population of individuals who exhibit different preferences and access different information to coordinate in a battle of the sexes game. We also demonstrate why the definition of fixed agents introduced by Glaeser and al. [8] should incorporate the possibility that these agents do get influenced by their peers[2]. Our main result establishes the importance of the mechanisms by which information spreads and explains how the outcome that prevails in a

[1]See e.g Becker [3], Benabou [4], Glaeser and al. [8], Jankowski [12], Venkatesh [21] among many others.

[2]A peer being defined as another fixed agent.

population of heterogeneous agents facing asymmetric information is decided by those fixed agents who share the most widely their information. Whereas in evolutionary game theory literature most coordination problems are solved in environments with homogenous agents, with the exception of Young [23] [24], this paper is the first to address the question of coordination among heterogeneous agents in an evolutionary framework with asymmetric information.

More specifically, we consider an evolutionary model with many populations, each of which with a distinct *preferred strategy.* By *preferred strategy* we mean that if two agents from the same population are paired to play a coordination game, they reach Pareto Efficiency if they both play this *preferred strategy.* Thus, an agent's membership to a population is solely determined by her preferences. This means that agents are homogenous within a population but heterogeneous between populations. Contrarily to what is commonly assumed in the literature on social interactions, we do not impose that neighbors share common preferences. In fact, we do not impose any restriction on the preferences that neighbors have. This is the reason why we make a clear distinction between what we call a population and what we refer to as a local network. Whereas a population is a group of agents who share common preferences, a local network reflects the level of interaction among agents. More precisely, an agent and her neighbors belong to the same local network, as well as the neighbors of her neighbors, and so on. Our definition of a neighbor is similar to the one by Bala and

Goyal [1] and Masson [15]. Hence, a neighbor is an agent from whom one can observe history; whereas a stranger is an agent one does not have any information about. A convenient way to represent these relationships among individuals is as follows. A directed link from agent i to agent j is an information flow from agent i to agent j, as in Bala and Goyal [2]. It means that agent j considers agent i as a neighbor. It also means that agent i considers agent j a stranger. If the link between agent i and agent j is mutual, as in jackson and Wolinsky [11], it simply represents the fact that both agents consider each other as neighbors. Within each local network, there exists at least one *fixed agent* who shares her information, directly or indirectly, with all agents within the local network, like the royal family member of Bala and Goyal [1]. In this paper, we show how these *fixed agents* play a crucial role in the analysis of a coordination game.

The game development is as follows: time is discrete and in each period agents from all local networks are randomly paired to play a $K \times K$ coordination game, where K corresponds to the number of populations. Note that since each population has its own preferred strategy, the number of pure strategies available to each agent is also K. Once matched, each agent faces an opponent who may or may not belong to her population. If the agent considers the opponent as a neighbor, she knows whether or not her opponent shares her preferences. She also knows her opponent's history and can therefore play using a myopic best-response as in Young [22][24]. On the other hand, when the opponent is a stranger the agent does not know anything about her opponent's history, nor does she know whether they share common preferences. In this latter case, the only option an agent has is to refer to her own history or her neighbors', and use the data she collects in order to choose which strategy to play. In this paper we assume that she imitates some of her neighbors who belong to her population. In the case where there is no neighbor from her population within the local network, she considers her own past experiences and imitates the strategy that gave her the highest payoff. The idea that an agent imitates her local network is derived from previous use of the imitation rule in evolutionary game theory, as in Robson and Vega-Redondo [17] or Josephson and Matros [13]. It is also justified by what has been observed in an experiment conducted by Hück and al. [10]. Nonetheless, the imitation rule has been modified to take into account heterogeneity among agents. In particular, an agent imitates other agents only if they share common preferences.

Given this environment, we show that in the short run each *fixed agent* plays a particular strategy, but all *fixed agents* need not to play the same strategy. More precisely, only neighboring connected fixed agents need to agree on the strategy choice. This therefore leads to the existence of segregated local networks reminiscent of the segregated neighborhoods presented by Schelling [20]. In the long run however, all *fixed agents* coordinate on the same strategy, thus leading the population to follow a uniform convention as defined by Young [22].

The paper is organized as follows: Section 2 illustrates the nature of the argument by an example. The detailed description of the model is given in Section 3. Short run predictions are presented in Section 4. Section 5 describes the long run outcome and Section 6 concludes.

2. Example

We illustrate the nature of our argument by the means of an example, regarding the choices of typesetting systems in two economics departments, each having two professors. Let us consider the situation where two of these professors belong to the "TEX Lovers" population, whereas the two others are members of the "SWP Lovers Society". Each economics department contains one member of each group. This means that we consider two populations of homogenous agents, and two local networks of heterogeneous agents, local networks and departments being analogous. Time is discrete, and in each period, professors are randomly paired to co-author a paper. Denote by T the strategy which consists in using TEX, and by S, the strategy which consists in writing in SWP. If both agents belong to the "TEX Lovers" population, they play the following coordination game:

	T	S
T	$10, 10$	$3, 0$
S	$0, 3$	$5, 5$

Figure 1. TEX is matched with TEX.

On the other hand, if they are both members of the SWP Lovers Society, the coordination game becomes:

	T	S
T	$5, 5$	$0, 3$
S	$3, 0$	$10, 10$

Figure 2. SWP is matched with SWP.

Finally, assuming that the row agent belongs to the TEX Lovers population and that the column agent is a SWP Lovers Society member, the coordination game they face is the following:

	T	S
T	$10, 5$	$3, 3$
S	$0, 0$	$5, 10$

Figure 3. TEX is matched with SWP.

This example illustrates the tension faced by agents in coordinating in order to generate a good outcome, and the temptation to defect in order to influence the outcome toward one's preference. Each of these games contain two strict Nash equilibria: (T, T) and (S, S). Note that coordination on the preferred strategy within a population leads to the efficient outcome; whereas coordination on either equilibria is enough to insure Pareto Efficiency, between agents from different populations. Suppose that, up to now, all the professors have always used SWP, leading to a history containing payoffs of 5 for the TEX Lovers and 10 for the SWP Lovers Society. This is a convention as defined by Young [24], and the only way to leave this convention is that one or more agents make the voluntary or involuntary mistake of using TEX instead of SWP. Therefore, suppose that two professors, who do not work in the same department, but who do share the same preferences over TEX are paired and that both use TEX instead of SWP. This gives each of them a payoff of 10, which is better than what they obtained before. If they meet again next period, they might as well play strategy T again since it turned out to be the best strategy for them in the past. Suppose now that the matching is such that every individual faces her own neighbor. The TEX lovers should go back playing S given the history of their neighbors, but their neighbors on the other hand should choose to play T, even though they prefer S. If the matching stays the same next period, one can see that it is possible for the TEX Lovers to impose their preferences on their neighbors, thus leading the whole population to follow the TEX convention.

3. The Model

Suppose that there exist K finite populations of n_k agents such that $n_1+...+n_K = 2n$ and $n_k \geq 2$, for $k = 1, ..., K$. Time is discrete, and in each period, agents are randomly paired to play a $K \times K$ coordination game. Each population i has a preferred strategy i that leads to Pareto Efficiency when two agents from this population are matched.

More precisely, we assume that if two individuals from the same population i are matched to play the game, they face the following payoff matrix:

Player from population i

		1	2	$\cdots$	i	$\cdots$	K
	1	a^i_{11}, a^i_{11}	a^i_{12}, a^i_{21}	$\cdots$	a^i_{1i}, a^i_{i1}	$\cdots$	a^i_{1K}, a^i_{K1}
Player	2	a^i_{21}, a^i_{12}	a^i_{22}, a^i_{22}	$\cdots$	a^i_{2i}, a^i_{i2}	$\cdots$	a^i_{2K}, a^i_{K2}
from	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$
population i	i	a^i_{i1}, a^i_{1i}	a^i_{i2}, a^i_{2i}	$\cdots$	a^i_{ii}, a^i_{ii}	$\cdots$	a^i_{iK}, a^i_{Ki}
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$
	K	a^i_{K1}, a^i_{1K}	a^i_{K2}, a^i_{2K}	$\cdots$	a^i_{Ki}, a^i_{iK}	$\cdots$	a^i_{KK}, a^i_{KK}

where $a^i_{kk} > a^i_{lk}$ for all $i, k, l = 1, ..., K$, $k \neq l$; and $a^i_{ii} > a^i_{kk}$ for any $k \neq i$.

Otherwise, if an agent from population i is matched with an agent from population j, they play the following coordination game:

		Player from population j					
		1	2	$\cdots$	i	$\cdots$	K
	1	a^i_{11}, a^j_{11}	a^i_{12}, a^j_{21}	$\cdots$	a^i_{1i}, a^j_{i1}	$\cdots$	a^i_{1K}, a^j_{K1}
Player	2	a^i_{21}, a^i_{12}	a^i_{22}, a^i_{22}	$\cdots$	a^i_{2i}, a^i_{i2}	$\cdots$	a^i_{2K}, a^j_{K2}
from	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$
population i	i	a^i_{i1}, a^j_{1i}	a^i_{i2}, a^j_{2i}	$\cdots$	a^i_{ii}, a^j_{ii}	$\cdots$	a^i_{iK}, a^j_{Ki}
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\ddots$	$\vdots$
	K	a^i_{K1}, a^j_{1K}	a^i_{K2}, a^j_{2K}	$\cdots$	a^i_{Ki}, a^j_{iK}	$\cdots$	a^i_{KK}, a^j_{KK}

where $a^i_{kk} > a^i_{lk}$ and $a^j_{kk} > a^j_{lk}$ for all $i, j, k, l = 1, ..., K$, $k \neq l$; and $a^i_{ii} > a^i_{kk}$ for any $k \neq i$; and $a^j_{jj} > a^j_{kk}$ for any $k \neq j$.

The first condition $a^i_{kk} > a^i_{lk}$ insures that coordination is favored over defection by every agent. The second condition $a^i_{ii} > a^i_{kk}$ stipulates that an agent from population i prefers to coordinate on strategy i. Thus, an agent is better off coordinating, but she prefers doing so using her preferred strategy. Therefore, Pareto Efficiency is reached when agents coordinate their actions, which means that there exist many possible efficient outcomes (as long as each pair of agents plays such that the outcome belongs to the diagonal of the payoff matrix, the outcome is efficient for all the agents). But populations are not indifferent in which efficient outcome arises.

At time t, each agent from population i chooses a strategy x_i^t from the set $X = \{1, ..., K\}$ according to some behavioral rules (described below) based on past plays' available information. Therefore, the *play* at time t can be defined as $x^t = (x_1^t, x_2^t, ..., x_{2n}^t)$, and the *history of plays* up to time t can be represented by the sequence $h^t = (x^{t-m+1}, ..., x^t)$ of the last m plays.

The information structure we use is similar to Bala and Goyal [1] and Masson[15]. We will call agents from whom agent q can access past plays neighbors of agent q, and any other agent a stranger to agent q. We denote by $Nb(q)$ the set of neighbors of agent q; $St(q)$ is the set of agents considered by agent q as strangers; and $A(q)$ is the set of agents who can access agent q's past plays. Note that agents in $Nb(q)$ need not to share common preferences.

The dichotomy neighbor/stranger can be represented by a directed graph, where a directed link from agent q to agent g, $\{q \rightarrow g\}$, means that agent g can access information about past plays of agent q, or $q \in Nb(g)$ and $g \in A(q)$. We will assume that $q \in Nb(q)$ for any q.

Any directed graph is a set of disjoint directed subgraphs, where any subgraph (*local network*) is connected in the following sense.

Definition 1. *A* **local network**, L^z, *is a list of ordered pairs of agents, where*

$\{q \to g\} \in L^z$ indicates that $q \in Nb\,(g)$ and $g \in A\,(q)$; and if $\{q_1 \to g_1\}, \{q_2 \to g_2\} \in L^z$, then there exists a sequence of agents $f_1, ..., f_k$ such that $\{f_m \to f_{m+1}\} \in L^z$ or $\{f_{m+1} \to f_m\} \in L^z$, $m = 1, ..., k-1$ or both and $g_1 = f_1$ and $q_2 = f_k$.

The following example illustrates the definition.

Example 1. Local network of four agents.

$$1 \longrightarrow 2 \longleftrightarrow 3 \longleftarrow 4$$

Figure 1

For each agent $i = 1, 2, 3, 4$ we can define a set of neighbors $Nb(i)$; a set of agents who can access agent i's information about past plays $A(i)$; and a set of strangers, $St(i)$.

$$Nb\,(1) = \{1\}\,,\ A\,(1) = \{2\}\,,\ St\,(1) = \{2,3,4\}\,;$$

$$Nb\,(2) = \{1,2,3\}\,,\ A\,(2) = \{3\}\,,\ St\,(2) = \{4\}\,;$$

$$Nb\,(3) = \{2,3,4\}\,,\ A\,(3) = \{2\}\,,\ St\,(3) = \{1\}\,;$$

$$Nb\,(4) = \{4\}\,,\ A\,(4) = \{3\}\,,\ St\,(4) = \{1,2,3\}\,.$$

All agents can straightforwardly be divided in disjoint local networks, where each local network L^z contains $n\,(L^z) \geq 1$ agents. Since the number of agents is finite, there is a finite number of local disjoint networks. Denote the number of local networks as $\mathcal{L}$. Then there is no directed link between any two local networks

$$L^z \cap L^y = \varnothing \text{ for any } z \neq y$$

and

$$\sum_{k=1}^{K} n_k = \sum_{z=1}^{\mathcal{L}} n\left(L^z\right) = 2n.$$

It is important to note that each agent has her own preferences (belongs to a particular population) as well as her own amount of information (from a set of neighbors). Any two agents can be paired in each round. Each agent chooses her strategy as follows. Fix integers s and m, where $1 \leq s \leq m$. At time $t+1$, each agent q inspects a sample $(x^{t_1}, ..., x^{t_s})$ of size s taken without replacement from her neighbors and her history of size m of plays up to time t, where $t_1, ..., t_s \in \{t-m+1, t-m+2, ..., t\}$. We assume that samples are drawn independently across agents and time.

If an agents is matched with one of her neighbors, she has information about her opponent past plays and plays the best reply against the distribution of strategies in the sample. This approach is rather intuitive: agents are boundedly rational and expect the play of the game to be "almost" stationary. See Kandori, Mailath, and Rob[14] and Young[22] for discussions.

However, the situation is different if an agent is matched with a stranger. Since no information is available about the opponent, we assume that the agent imitates a strategy which gave the highest payoff in the past to an agent with the same preferences (one from the same population).[3,4]

This approach is inspired by Josephson and Matros[13], who consider an evolutionary framework where agents do not have information about their opponents. We believe that imitation among agents sharing common preferences is natural especially in view of the limited amount of information an agent has access to. Of course, one may argue that "Imitation is suicide", as Emerson, or that "No man ever yet became great by imitation", according to Dr. Samuel Johnson, but this type of argumentation is way beyond the scope of this paper. We simply argue that some experimental evidences support that imitation is a sensible rule to use when access to information is limited. See Hück and al. [10] for details.

More formally, let the sampling process begin in period $t = m + 1$ from some arbitrary initial sequence of m plays, h^m. We define a finite Markov chain (call it $BI^{m,s,0}$) on the state space $(X)^{2n} = H$ of sequences of length m drawn from the strategy space X, with an arbitrary initial state h^m. The process $BI^{m,s,0}$ moves from the current state h to a successor state h' in each period, according to the following

[3]Since an agent can sample from herself, this is always possible.
[4]Our results will still hold if the agents imitate the stategy with highest *average* payoff.

transition rule. For each $x_i \in X$, let $p_i(x_i \mid h)$ be the conditional probability that agent i chooses x_i, given that the current state is h. We assume that $p_i(x_i \mid h)$ is independent of t and $p_i(x_i \mid h) > 0$ if and only if there exists a neighbor and a sample s such that x_i is a best reply to the sample drawn from this neighbor, or there exists a sample s such that x_i is the action which gave the highest payoff in the sample.

The perturbed version of the above process can be described as follows. In each period, there is a small probability $\varepsilon > 0$ that any agent experiments by choosing a random strategy from X instead of applying the described rule. The event that one agent experiments is assumed to be independent from the event that another agent experiments. The resulting perturbed process is denoted by $BI^{m,s,\varepsilon}$. As we will see below, the resulting process $BI^{m,s,\varepsilon}$ is *ergodic*, making the initial state irrelevant in the long run.

4. Short Run

In order to characterize the short-run outcomes of the model, we need to adopt some terminology. In particular, we want first to explicitly define what a *fixed agent* is in our context. Formally,

Definition 2. *q^* is a* **fixed** *agent, if*

(i) $\{q^*\} = Nb\,(q^*)$, *or*

(ii) if there exist an agent g and a sequence of agents $g_1, ..., g_k$ such that $\{g_m \rightarrow g_{m+1}\}$,

$m = 1, ..., k-1$, and $q^* = g_k$ and $g = g_1$, then there must exist a sequence of agents $f_1, ..., f_l$ such that $\{f_m \to f_{m+1}\}$, $m = 1, ..., l-1$, and $q^* = f_1$ and $g = f_l$.

Fixed agents propagate information within local networks. For example, agents 1 and 4 are fixed agents and agents 2 and 3 are not in Figure 1. Indeed, $Nb(2) = \{1, 3\}$ and $Nb(1)$ and $Nb(3) = \{2, 4\}$. Therefore, in order for agent 2 to be a fixed agent, there must be a sequence of directed links from agent 2 to agents 1, 3 and 4. But such sequence does not exist from agent 2 to agent 1. Such sequence does not exist either from agent 3 to agent 4, even though $Nb(3) = \{2, 4\}$, which means that agent 3 is not a fixed agent. On the other hand , agents 1 and 4 are fixed agents since they do not have any neighbors.

We establish that there must be at least one fixed agent for each local network.

Lemma 1. *Each local network has a fixed agent.*

Proof. Since each local network is finite, the claim follows immediately. **End of proof.**

Some local networks can have several fixed agents. For example, all agents are fixed agents in complete networks.[5] Therefore, two fixed agents are connected if they share some information with each other. Formally,

[5] Each agent is a neighbor of every other agent in the complete network.

Definition 3. *Two fixed agents* q^* *and* g^* *in local network* L^z *are* **connected***, if there exists a sequence of agents* $f_1, ..., f_k$ *such that* $\{f_m \rightarrow f_{m+1}\} \in L^z$, $m = 1, ..., k-1$, *and* $q^* = f_1$ *and* $g^* = f_k$.

It is obvious that each agent in the sequence of agents $f_1, ..., f_k$ from the previous definition is a fixed agent too. The following example illustrates the definition.

Example 2. Agents 2 and 3 are two connected fixed agents in Figure 2.

$$1 \leftarrow 2 \longleftrightarrow 3 \rightarrow 4$$

FIGURE 2

It is of importance to note that there may exist many distinct groups of connected fixed agents within the population. We therefore need to make a distinction between these groups, by introducing the following definition:

Definition 4. *Two connected fixed agents are* **neighboring connected fixed agents** *if there exists a direct link between them.*

Whereas the previous definitions were focused on the informational structure, the following definitions describe the actions played by the fixed agents or the entire population of agents.

Definition 5. *A* **partial convention** *is a state where:*

(1) All neighboring connected fixed agents play the same unique strategy.

(2) Fixed agents who are not connected play a unique strategy. All fixed agents who are not connected do not have to play the same strategy.

The definition of partial convention describes a state where all fixed agents always play the same strategy, but fixed agents do not have to agree on the strategy played, as long as they are not neighboring connected fixed agents.

Definition 6. *A* **convention** *is a state where all agents in each local network play the same strategy.*

Our definition of a convention is similar to Young [22] [24], but at a local level. However, the next definition will be proven later to correspond exactly to what Young defines as a convention.

Definition 7. *A* **uniform convention** *is a state where all fixed agents play the same strategy.*

Note that there exist K uniform conventions. Using the terminology that we have introduced, we are now in position to characterize all recurrent classes - short-run outcomes.

Theorem 1. *If $s/m \leq 1/2n$, the process converges with probability one to a partial convention.*

Proof. It is evident that any convention is a recurrent class. We shall prove that if $s/m \leq 1/2n$, then conventions are the only recurrent classes of the unperturbed process. The idea behind the proof is simple. First, we show that there is a positive probability that a fixed agent can play the same strategy s times in a row. Then we show how she can spread this strategy to the whole local network.

Consider an arbitrary initial state $h^t = (x^{t-m+1}, ..., x^t)$ and a fixed agent q^* in a local network L^z. There is a positive probability that agent q^* is matched with the same agent and samples $x^{t-s+1}, ..., x^t$ in every period from $t+1$ to $t+s$ inclusive. Without loss of generality, assume that agent q^* plays a unique best reply, a pure strategy x_q^*. With positive probability, agent q^* can be matched with one of the agents in $A(q^*)$, and this agent samples $x_{q^*}^{t+1}, ..., x_{q^*}^{t+s+1}$ in every period from $t+s+1$ to $t+2s$ inclusive. Since the agents in $A(q^*)$ samples only strategy x_q^*, she plays strategy x_q^* in the coordination game form Figure 2 in every period from $t+s+1$ to $t+2s$ inclusive. In general, for any agent g in the local network L^z there exists a sequence of agents $f_1, ..., f_k$ such that $\{f_m \to f_{m+1}\} \in L^z$, $m = 1, ..., k-1$, and $q^* = f_1$ and $g = f_k$. Each of the agents in the sequence of agents $f_1, ..., f_k$ has a positive probability to have a string size s of strategy x_q^* in her history in the way described above. Since $s/m \leq 1/2n$, there is a positive probability that *all* agents in the local network L^z will have a string size s of strategy x_q^* in their history before period $t+m$. Suppose that all agents (or all but one is the number of agents is odd) are matched with the agents from their local network for the next m periods. There is a positive probability that each agent samples a string size s of strategy x_q^* inclusive for the next m periods. As the result, local convention will be obtained. **End of proof.**

The following example illustrates how the strategies of non-fixed agents can vary in a recurrent class.

Example 3. Assume that $m = 2$, $s = 1$, and consider the four agents represented in Figure 1. Suppose that agents 1 and 2 are from population A, and agents 3 and 4 are from population B. Agents are matched at random to play 2×2 coordination games.

Note that agents 1 and 4 are fixed agents. Therefore, they could either play only strategy A or only strategy B. If both fixed agents coordinate on strategy A, then the following state is absorbing

$$h_{(A,A)} = ((A, A), (A, A), (A, A), (A, A)),$$

where the first bracket represents the strategy choices of agent 1 in the last two periods, the second bracket shows the strategy choices of agent 2 in the last two periods, and so on. If both fixed agents coordinate on strategy B, then the following state is absorbing:

$$h_{(B,B)} = ((B,B),(B,B),(B,B),(B,B)).$$

Fixed agents do not need to coordinate between them in order for the process to be in an absorbing state. For example, agent 1 played strategy A in the last two periods, and agent 4 played strategy B in the last two periods, the following vector of plays is a recurrent class:

$$\{((A,A),(w,x),(y,z),(B,B))\},$$

where $w,x,y,z \in \{A,B\}$. Analogously, if agent 1 played strategy B in the last two periods, and agent 4 played strategy A in the last two periods, the following is also a recurrent class:

$$\{((B,B),(w,x),(y,z),(A,A))\},$$

where $w,x,y,z \in \{A,B\}$. Note that non-fixed agents 2 and 3 can switch from strategy A to strategy B (and vice versa) in the short run. Therefore, only partial coordination represented by segregated local networks arises in the short run.

In the case where all fixed agents coordinate on the same strategy, we have the following result.

Proposition 1. *In any uniform convention ALL agents will coordinate on the same* *strategy.*

Proof. In any uniform convention all fixed agents coordinate on one and the same strategy. Since only fixed agents can spread information, all other agents must coordinate on this strategy in the short run. **End of proof.**

A particular case of interest is when ALL directed edges are double-sided. If we follow our terminology, ALL agents are therefore fixed agents. But if we follow Glaeser and al. terminology, the fact that we only have double sided links means that we have no fixed agent at all. Indeed, if the entire population is composed only by fixed agents, they all can influence their neighbors, which means that they all can be influenced. This is the reason why the definition of fixed agents: "Agents who influence their neigbors but who cannot themselves be influenced", should become "Agents who influence their neigbors but who cannot themselves be influenced by non-fixed agents".

5. Long Run

As we saw in the previous section, many conventions are candidates for the long-run outcome. The following result demonstrates that only K long-run outcomes are possible. These long-run outcomes are uniform conventions.

Lemma 2. *Suppose that the number of any connected fixed agents is less than a half of the total number of fixed agents. Then there exists a sample size s^*, such that for* *any* $s \geq s^*$ *and for each non-uniform convention, there exists a uniform convention with lower stochastic potential.*

Proof. First, note that conventions are the only recurrent classes from Theorem 1. Second, it takes at least two mistakes to leave any uniform convention. It is enough to show now that it takes just one mistake to leave a non-uniform convention. It becomes obvious once we see how to leave a convention where all but one (connected) fixed agent(s) coordinate on one strategy.

There is a positive probability that all connected fixed agents who are not coordinated with all other fixed agents are matched with other (coordinated) star agents for $s-1$ periods. As the result, all uncoordinated fixed agents have $s-1$ miscoordination play. Suppose that one of this fixed agents, agent i, makes a mistake and coordinates on the other strategy. There is a positive probability that she and every member from the set $A(i)$ sample her last s plays for the next s periods. Suppose all connected fixed agents from $A(i)$ and agent i are matched with fixed agents they consider strangers. This will lead to a uniform convention. **End of proof.**

From Lemma 2 it follows that coordination must be reached (a uniform convention) in the long-run. We can now state the main theorem of this paper.

Theorem 2. *Suppose that the number of all connected fixed agents is less than a half of the total number of fixed agents and among any connected fixed agents there* *exists at least one fixed agent from population i. Then there exists a sample size s^*, such that for any $s \geq s^*$, the perturbed process $BI^{m,s,\varepsilon}$ puts a positive probability on the uniform convention, where all agents play strategy i.*

Proof. It takes at least two mistakes to leave a uniform convention, from Lemma 2. Note that it takes exactly two mistakes to leave a uniform convention j, if there is at least one agent in each of two disconnected local networks from population $i \neq j$. The underlying reasoning is similar to the proof of the previous lemma and is therefore omitted.

Hence, following Ellison[5], if there are enough agents (at least one in any set of connected stars) from population i, the uniform convention i is stochastically stable. **End of proof.**

The presence of fixed agents within each local network facilitates the coordination among all agents, even though they do not share common preferences. Since information gathered by fixed agents can be distributed, directly or indirectly, to any agent within a local network, it appears quite intuitive that the selection of the uniform convention that prevails in the long run depends on the number of fixed agents each population has. But since only a fixed agent can influence another fixed agent, the long-run outcome selection also depends on the links these fixed agents possess with other fixed agents. Therefore, a population that presents a high number of connected fixed agents can easily impose its preferences on other populations.

6. Conclusion

In this paper we show that heterogeneous agents can coordinate on the same strategy even though they evolve in an asymmetric information environment. The short run outcome is influenced by all fixed agents whose behavior determines the absorbing state to which local networks conform. However, the long run outcome is determined by the number of fixed agents within each population as well as the links these fixed agents have with other fixed agents. In particular, we show that the long run outcome is not imposed by the population with the largest number of agents. The long run outcome is determined by the population that has the largest number of agents who can influence others (fixed agents connected to other fixed agents).

Our main result shows that location leads to information distribution and access which in turns leads to coordination. In particular, it shows that the outcome that prevails in a population of heterogeneous agents facing asymmetric information is decided by those agents who share the most widely their information.

Possible extensions of this paper include the design of an experiment in order to test the prediciton of the model. Also, options other than imitation are available, as for example in Matros [16], and could be considered in future research built upon this model.

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