Search, bargaining and prices in an enlarged monetary union*

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Abstract

This paper studies existence and characterization of monetary equilibria of an enlarged monetary union within a model of search with commodities divisibility. An unbiased degree of integration between each member-country pair ensures existence of accession equilibria, and is a necessary and sufficient condition for both monies to be perfect substitutes for each country's resident, and for no arbitrage to exist from using the same money in different countries. Furthermore, monies are perfect substitutes within each single participating country in every accession equilibrium.

While prices in each country are increasing in the amount of money issued, they are decreasing in the degree of integration between any country-pair.

Keywords: Bargaining, search, prices, monetary enlargement.

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1 Introduction

Price-level differentials across countries within a monetary union are usually explained in terms of divergence of non-tradable goods prices. This explanation, however, arises from models (see, for example, [1] and [7]) where the unrealistic assumption of a cash-less economy is maintained. This paper seeks to overcome this shortcoming by borrowing from the new generation models of monetary economics which combine money, matching and bargaining theory (see, e.g., [3], [4], [8] and [10]).

Traditional models of money (see, e.g, [2], [6] and [9]) have the limitation that too much is taken to be exogenous. As regard the money type used in transactions, for instance, people should choose it – perhaps on the basis of beliefs, social customs, as well as preferences and technology. Namely, what money is accepted where and by who should be endogenously determined.

The first generation of search monetary models is incomplete, however, in the sense that the papers neglect determination of prices and exchange rates. Subsequent works such as [10] and [8] address this issue by introducing bilateral bargaining theory. In a seminal work, [11] combine the original framework of [5] with [10] in order to endogenize prices and exchange rates in a two-country model. This paper extends their analysis to a three-country economy. This enables one to characterize, among other things, the accession of a new member country in an existing two-country monetary union.

The main results of the paper are the following. Equal degree of integration between any country-pair is a necessary and sufficient condition for monies to be perfect substitutes within and across all countries. Equal integration between any country-pair is also a sufficient condition for the existence of the accession equilibrium.

The paper is organized as follow. Section 2 describes the model. Sections 3 and 4 analyze the pre-accession and accession equilibria respectively. The Conclusion ends the paper.

2 The model

The background framework of the paper is borrowed from [11]. Time is discrete and tends to infinity. There are three countries. Two of them, called ICs and labelled $i = \{1, 2\}$, form a monetary union while the third country, called AC and labelled $i = \{3\}$, does not initially belong to the monetary union. Each country begins with a *continuum* of infinitely lived agents and the total population sums to one, that is $\sum_{i=1}^{3} n_i = 1$ where $n_i \in (0, 1)$ is the fraction of population living in country i. There is a unit measure set of each of K specialization types of agents and there are K distinct divisible goods at each date. A specialization-type k agent, $k = \{1, 2, ..., K\}$, produces only good k and consumes only good $k + 1$ (modulo K).

If $q \in \mathbb{R}_+$ units of a good are exchanged, the producer suffers a cost q while the consumer enjoys an utility $u(q)$. The utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave, strictly increasing, continuously differentiable and satisfies $u'(\infty) = 0$, $u'(0) = \infty$ and $u(0) = 0$.

It is assumed both that consumption good is perishable and that $K \geq 3$. This rules out the existence of a *commodity money* and implies the absence of the so called "double-coincidence of wants", respectively. Hence some form of fiat money is needed in transactions. In particular, there are two fiat monies in the economy, the type 1 and the type 2. At the initial date, the ICs government issues one unit of money 1 to a fraction $m_1 \in (0, 1)$ of its population and the AC government issues one unit of money 2 to a fraction $m_2 \in (0, 1)$ of its residents; namely m_1 and m_2 denote the supply of money 1 per ICs agent and the supply of money 2 per AC agent. Let $m_{ij} \in (0, 1)$ be the proportion of the population in country i with a unit of money j. Both types of monies are indivisible. Also, it is assumed that an agent with a unit of money cannot acquire a second unit so that a moneyholder never delivers either less or more than one unit of currency. In each period, the population is partitioned into a group of buyers (consumers), the individuals that hold a unit of money, and a disjoint group of sellers (producers), the agents with no money. The inventory distribution can be summarized by the vector $M_i = (1 - m_{i1} - m_{i2}, m_{i1}, m_{i2})$, where $(m_{i1} + m_{i2})$ is the proportion of buyers and $(1 - m_{i1} - m_{i2})$ is the proportion of sellers in country i. Hence, total supply of money 1 and money 2 amounts to $(n_1 + n_2) m_1 = n_1 m_{11} + n_2 m_{21} + n_3 m_{31}$ and $n_3 m_2 = n_1 m_{12} + n_2 m_{22} + n_3 m_{32}$, respectively.

The matching technology is the following. There are neither centralized markets nor auctioneers. Agents meet pairwise and at random according to a Poisson process with finite arrival rate. To maintain the notation of [11], let $\alpha_{ii} = \frac{\beta_{ii} n_i}{K}$ be the rate at which a buyer from country i meets a seller of country i who produces his consumption good; α_{ii} is also the rate at which a seller from country i meet a buyer from country i consuming his production good. Similarly, $\alpha_{ix} = \frac{\beta_{ix} n_x}{K}$ is the rate at which a buyer from country i meets a seller from country x holding his consumption good; this is also the rate at which a seller from country i meets a buyer from country x willing to consume the good he produces. The term β_{ix} indicates the frequency of an agent from country i meeting an agent from country x, relative to the frequency of two nationals meeting (it is simply assumed that the chances of nationals meetings are equal across countries). As a result, the term α_{ix} can be seen as a measure of the degree of economic integration between countries i and x.

This paper focuses on pure-strategy and steady-state equilibria, that is agents do not randomize and both the asset distribution and trading strategies are constant over time. Let λ_{ij} be a binary variable which is equal to one if money j circulates in country i, zero otherwise. For example, if euros (money 1) do not circulate in Poland (country 3) then $\lambda_{31} = 0$. It is assumed that a money is always accepted within the country where it was issued, that is money 1 is accepted within ICs and money 2 is accepted within AC (i.e. $\lambda_{11}, \lambda_{21}, \lambda_{32} = 1$). Hence, each equilibrium is identified by the vector $\lambda = (\lambda_{12}, \lambda_{22}, \lambda_{31})$.

Given λ, one can obtain the steady-state conditions $\dot{m}_{ij} = 0$, where

$$\dot{m}_{ij} = \sum_{\substack{x=1 \\ x \neq i}}^{3} \alpha_{ix} \left[(1 - m_{i1} - m_{i2}) m_{xj} \lambda_{ij} - m_{ij} (1 - m_{x1} - m_{x2}) \lambda_{xj} \right], \tag{1}$$

for $i = \{1, 2, 3\}$ and $j = \{1, 2\}$. By construction, if $\lambda_{ij} = 0$ then $m_{ij} = 0$ in steady-state. Namely, if a given type of money is never accepted in a given country then the steady state fraction of agents holding that money is zero in that country. For example, if Poland residents never accept euros

($\lambda_{31} = 0$) they never hold euros ($m_{31} = 0$). Note that transactions among national fellows are not included in (1) since they leave the distribution of buyers unchanged.

Let V_{ij} be the expected lifetime utility for a buyer with money j from country i and V_{i0} the expected lifetime utility for a seller from country i. The term r denotes the discount rate, which is equal for all agents. Then, from dynamic programming, the following Bellman equations are satisfied in steady-state

$$rV_{ij} = \sum_{x=1}^{3} \alpha_{ix} \left(1 - m_{x1} - m_{x2}\right) \lambda_{xj} \left[V_{i0} + u\left(q_{xj}\right) - V_{ij}\right], \tag{2}$$

$$rV_{i0} = \sum_{x=1}^{3} \sum_{j=1}^{2} \alpha_{ix} m_{xj} \lambda_{ij} \left(V_{ij} - q_{ij} - V_{i0}\right), \tag{3}$$

where $q_{xj} \in \mathbb{R}_+$ is the quantity of good produced by a seller from country x in exchange of one unit of money j. Hence, $p_{xj} = 1/q_{xj}$ is a measure of the unit price of output in country x in terms of money j.

If a buyer with money j from country i meets a seller from country x who produce his consumption good (single-coincidence meeting), and they decide to bargain, the level of output q_{xj} to be produced by the latter can be obtained using the general Nash bargaining problem

$$\max_{q_{xj} \in (0,\infty]} \left[V_{i0} + u\left(q_{xj}\right) - V_{ij}\right]^{\theta} \left[V_{xj} - q_{xj} - V_{x0}\right]^{1-\theta}, \tag{4}$$

where θ is the bargaining power of the buyer. Of course, negotiation takes place if and only if both agents can at least have some surplus from switching their status from buyer to seller and vice versa, i.e. if and only if $u\left(q_{xj}\right) + V_{i0} \geq V_{ij}$ and $-q_{xj} + V_{xj} \geq V_{x0}$ both hold. The right-hand-side of each inequality denotes the threat point, that is what an agent gets if he does not trade. It is assumed that $\theta = 1$, which means that buyers make a take-it-or-leave-it offer. In particular, they always demand $q_{xj} = V_{xj} - V_{x0}$ since the seller always accepts when he is indifferent between to trade and not to trade. The case of $\theta = 1$ simplifies things considerably by eliminating the dependence of q on the buyer's nationality (so implying $V_{x0} = 0$) and identifying the buyers' value function with quantities,

$$q_{xj} = V_{xj}. \tag{5}$$

Note that this quantity q_{xj} depends both on the seller's nationality and on the type of money the buyer is delivering. In addition, it is assumed that the seller cannot observe the buyer's nationality. Without this simplifying assumption, as pointed out by [11], regimes with zero or one international money can never be equilibria. Using (5), expression (2) can be rearranged to obtain

$$rq_{ij} = \sum_{x=1}^{3} \alpha_{ix} \left(1 - m_{x1} - m_{x2}\right) \lambda_{xj} \left[u\left(q_{xj}\right) - q_{ij}\right]. \tag{6}$$

Assumptions:

(A1) $n_i = 1/3$ for $i = \{1, 2, 3\}$;

(A2) $2m_1 = m_2 = m$;

(A3) $\alpha_{ix} = \alpha_{xi} = \alpha$ for $i, x = \{1, 2\}$ with $i \neq x$, $\alpha_{3i} = \alpha_{i3} = \alpha_3$ for $i = \{1, 2\}$ with $\alpha_3 \leq \alpha$, and $\alpha_{ii} = \alpha$ for $i = \{1, 2, 3\}$.

Assumptions $(A1)$ and $(A2)$ restrict the analysis to the case in which both the ICs and AC have equal population and money supply. $(A3)$ means that the arrival rate of a meeting among nationals willing to trade is the same within countries, and that ICs residents meets ICs fellows more frequently than they meet AC residents.

By $(A1)$ and $(A2)$, it must hold that

$$m = \sum_{i=1}^{3} m_{ij}, \tag{7}$$

where i denotes the country, and j the money type.

3 Pre-accession equilibrium

The pre-accession equilibrium is meant to be a regime in which each money circulates only in the country where it was initially issued, i.e. money 1 circulates only within ICs and money 2 circulates only within AC. This implies that $\lambda_{12}, \lambda_{22}, \lambda_{31} = 0$ and $\lambda_{11}, \lambda_{21}, \lambda_{32} = 1$ hold in the pre-accession equilibrium. Therefore, using $(A1) - (A3)$, (1) and (7), the relations $m_{11}, m_{21} = \frac{m}{2}$, $m_{32} = m$ and $m_{12}, m_{22}, m_{31} = 0$ hold in steady-state and one can rewrite (6) as follows

$$
\begin{aligned}
rq_1 &= 2\alpha\left(1-\tfrac{m}{2}\right)\left[u\left(q_1\right)-q_1\right], \\
rq_{31} &= 2\alpha_3\left(1-\tfrac{m}{2}\right)\left[u\left(q_1\right)-q_{31}\right], \\
rq_2 &= \alpha_3\left(1-m\right)\left[u\left(q_{32}\right)-q_2\right], \\
rq_{32} &= \alpha\left(1-m\right)\left[u\left(q_{32}\right)-q_{32}\right],
\end{aligned} \tag{8}
$$

where $q_{11} = q_{21} = q_1$ and $q_{12} = q_{22} = q_2$. That is, the unit price of good within ICs only depends on the type of money held by the buyer, no matter what seller's nationality. Using $(A3)$, expressions in (8) imply

$$
\begin{aligned}
q_{31} &\leq q_1 \leq u\left(q_1\right), \\
q_2 &\leq q_{32} \leq u\left(q_{32}\right).
\end{aligned} \tag{9}
$$

The pre-accession equilibrium exists when an ICs buyer wants to exchange his money with ICs sellers but not with AC sellers while an AC buyer wants to spend his money to buy goods from AC sellers but not from ICs sellers, i.e.

$$
\begin{aligned}
u\left(q_{31}\right) &\leq q_1 \leq u\left(q_1\right), \\
u\left(q_2\right) &\leq q_{32} \leq u\left(q_{32}\right).
\end{aligned} \tag{10}
$$

Expressions (10), which by (9) collapse to $u\left(q_{31}\right) \leq q_1$ and $u\left(q_2\right) \leq q_{32}$, can be rewritten as

$$
\begin{aligned}
q_1 &\geq u\left[\tfrac{\alpha_3 r+2\alpha\alpha_3\left(1-\frac{m}{2}\right)}{\alpha r+2\alpha\alpha_3\left(1-\frac{m}{2}\right)}q_1\right], \\
q_{32} &\geq u\left[\tfrac{\alpha_3 r+2\alpha\alpha_3(1-m)}{\alpha r+2\alpha\alpha_3(1-m)}q_{32}\right],
\end{aligned} \tag{11}
$$

using (8). By (11), the pre-accession equilibrium exists if the degree of integration α among ICs is high enough relative to the degree of integration α_3 between ICs and the AC.

4 Accession equilibrium

The accession equilibrium is characterized by the fact that each money circulates both within ICs and AC, i.e. monies 1 and 2 are international currencies. For this equilibrium to exist it is required that $u\left(q_{ij}\right) \geq q_{xj}$ for any $i, x = \{1, 2, 3\}$ and $j = \{1, 2\}$. This means that $\lambda_{ij} = 1$ for any $i = \{1, 2, 3\}$ and $j = \{1, 2\}$.

Proposition 1 *In the accession equilibrium, monies are perfect substitutes within each country, i.e.* $V_{i1} = V_{i2}$ *for any* $i = \{1, 2, 3\}$.

Proof. From (6), one can write the subsystem

$$\begin{aligned} rq_1 &= 2\alpha\left(1-\tfrac{2m}{3}\right)\left[u\left(q_1\right)-q_1\right]+\alpha_3\left(1-\tfrac{2m}{3}\right)\left[u\left(q_{31}\right)-q_1\right], \\ rq_{31} &= 2\alpha_3\left(1-\tfrac{2m}{3}\right)\left[u\left(q_1\right)-q_{31}\right]+\alpha\left(1-\tfrac{2m}{3}\right)\left[u\left(q_{31}\right)-q_{31}\right], \end{aligned} \tag{12}$$

which can be solved for q_1 and q_{31} independently of q_2 and q_{32}. Alternatively, (6) implies a subsystem function of q_2 and q_{32} which can be solved independently of q_1 and q_{31}. Since this subsystem is identical to (12), then $q_1 = q_2 = Q_1$ and $q_{31} = q_{32} = Q_3$. Using (5) ends the proof. ■

By Proposition 1, the quantity of good produced by an agent living in country i for a unit of money j, q_{ij}, is the same regardless the money type held by the buyer. That is, the value of monies coincides *within* each country. Therefore, in an hypothetical market for currency exchange, the market clearing price would equal one, though the ratio Q_1/Q_3 may or may not.

Proposition 2 *The unit price of good in each country is a decreasing function of the degree of integration between any country-pair.*

Proof. Using the relations $Q_1 = q_1 = q_2$ and $Q_3 = q_{31} = q_{32}$, and differentiating (12) one gets

$$\begin{aligned} &\begin{bmatrix} 2\alpha\left(\frac{u_1}{Q_1}-u_1'\right)+\alpha_3\frac{u_3}{Q_1} & -\alpha_3 u_3' \\ -2\alpha_3 u_1' & 2\alpha_3\frac{u_1}{Q_3}+\alpha\left(\frac{u_3}{Q_3}-u_3'\right) \end{bmatrix}\begin{bmatrix} dQ_1 \\ dQ_3 \end{bmatrix} \\ &= \begin{bmatrix} u_3 - Q_1 \\ 2\left(u_1 - Q_3\right) \end{bmatrix} d\alpha_3 \end{aligned} \tag{13}$$

where $u_i = u\left(Q_i\right)$ by ease of exposition. Premultiplying (13) by the inverse of the left-hand-side square matrix and then multiplying by $1/d\alpha_3$ one gets

$$\begin{aligned} &\frac{\Delta}{d\alpha_3}\begin{bmatrix} dQ_1 \\ dQ_3 \end{bmatrix} \\ &= \begin{bmatrix} 2\alpha_3\frac{u_1}{Q_3}+\alpha\left(\frac{u_3}{Q_3}-u_3'\right) & \alpha_3 u_3' \\ 2\alpha_3 u_1' & 2\alpha\left(\frac{u_1}{Q_1}-u_1'\right)+\alpha_3\frac{u_3}{Q_1} \end{bmatrix}\begin{bmatrix} u_3 - Q_1 \\ 2\left(u_1 - Q_3\right) \end{bmatrix} \end{aligned} \tag{14}$$

where Δ, the determinant of the square matrix in (13), is equal to

$$\begin{array}{c}\Delta = 2\alpha^2\left(\frac{u_1}{Q_1}-u_1'\right)\left(\frac{u_3}{Q_3}-u_3'\right)+4\alpha\alpha_3\left(\frac{u_1}{Q_1}-u_1'\right)\frac{u_1}{Q_3} \\ +\alpha\alpha_3\left(\frac{u_3}{Q_3}-u_3'\right)\frac{u_3}{Q_1}+2\alpha_3^2\left(\frac{u_1}{Q_3}\frac{u_3}{Q_1}-u_1'u_3'\right)>0\end{array} \tag{15}$$

since $\frac{u_i}{Q_i} > u_i'$ by concavity of $u\left(Q_i\right)$. Thus,

$$\begin{array}{l}\Delta\frac{dQ_1}{d\alpha_3} = \left[\alpha\left(\frac{u_3}{Q_3}-u_3'\right)+2\alpha_3\frac{u_1}{Q_3}\right]\left(u_3-Q_1\right)+2\alpha_3 u_3'\left(u_1-Q_3\right)\geq 0 \\ \Delta\frac{dQ_3}{d\alpha_3} = 2\alpha_3 u_1'\left(u_3-Q_1\right)+2\left[2\alpha\left(\frac{u_1}{Q_1}-u_1'\right)+\alpha_3\frac{u_3}{Q_1}\right]\left(u_1-Q_3\right)\geq 0,\end{array} \tag{16}$$

since both $u_1 \geq Q_3$ and $u_3 \geq Q_1$ always hold in the accession equilibrium. Similarly, Q_1 and Q_3 are both increasing in α. Hence, p_1 and p_3 are decreasing in α_3 and α. ■

The intuition underlying Proposition 2 is the following. More likely the meetings between residents of different nationality the higher the value of a particular money accepted in transactions, as that money facilitates transactions between agents and thus consumption. Since the buyers' demand of goods is increasing in the value of the money they hold (see expression (5)), the level of prices p_{xj} are decreasing in the degree of integration between any country-pair.

Proposition 3 *Equal degree of integration between any country-pair is a necessary and sufficient condition for monies to be perfect substitutes within and across all countries.*

Proof. (*Sufficiency.*) Assume $\alpha_{ix} = \alpha$ for any $i, x = \{1, 2, 3\}$. Therefore, $\alpha_3 = \alpha$ by ($A3$). Then, (12) becomes

$$\begin{array}{l}rQ_1 = 2\alpha\left(1-\frac{2m}{3}\right)\left[u\left(Q_1\right)-Q_1\right]+\alpha\left(1-\frac{2m}{3}\right)\left[u\left(Q_3\right)-Q_1\right], \\ rQ_3 = 2\alpha\left(1-\frac{2m}{3}\right)\left[u\left(Q_1\right)-Q_3\right]+\alpha\left(1-\frac{2m}{3}\right)\left[u\left(Q_3\right)-Q_3\right],\end{array} \tag{17}$$

or

$$\begin{array}{l}\left[r-3\alpha\left(1-\frac{2m}{3}\right)\right]Q_1 = \alpha\left(1-\frac{2m}{3}\right)\left[2u\left(Q_1\right)+u\left(Q_3\right)\right], \\ \left[r-3\alpha\left(1-\frac{2m}{3}\right)\right]Q_3 = \alpha\left(1-\frac{2m}{3}\right)\left[2u\left(Q_1\right)+u\left(Q_3\right)\right],\end{array} \tag{18}$$

which implies $Q_1 = Q_3$. Using (5), V_{ij} is the same for any $i = \{1, 2, 3\}$ and $j = \{1, 2\}$.

(*Necessity.*) Assume V_{ij} is the same for any $i = \{1, 2, 3\}$ and $j = \{1, 2\}$. Then, by (5), $Q_1 = Q_3 = Q$. Then, the system of equations (12) can be rewritten as

$$\begin{aligned} rQ &= 2\alpha \left(1 - \tfrac{2m}{3}\right) \left[u\left(Q\right) - Q\right] + \alpha_3 \left(1 - \tfrac{2m}{3}\right) \left[u\left(Q\right) - Q\right], \\ rQ &= 2\alpha_3 \left(1 - \tfrac{2m}{3}\right) \left[u\left(Q\right) - Q\right] + \alpha \left(1 - \tfrac{2m}{3}\right) \left[u\left(Q\right) - Q\right], \end{aligned} \tag{19}$$

or

$$\begin{aligned} rQ &= \left(1 - \tfrac{2m}{3}\right) \left[u\left(Q\right) - Q\right] \left(2\alpha + \alpha_3\right), \\ rQ &= \left(1 - \tfrac{2m}{3}\right) \left[u\left(Q\right) - Q\right] \left(2\alpha_3 + \alpha\right), \end{aligned} \tag{20}$$

which implies $\alpha_3 = \alpha$. Using $(A3)$ the necessity is proved. ■

By proposition 3, the quantity of output produced by an agent in exchange for a unit of money is the same within and across all countries if and only if the degree of integration coincides for any country-pair. In this case, the ICs currency gives its holders as equal trading opportunities as the AC currency. Hence, a seller wants to produce the same quantity of output no matter what the money the counterpart is delivering. This holds for all countries. It turns out that the ratio Q_1/Q_3 equals unity and coincides with the price that would arise if there was a market for currencies.

Proposition 4 *The unit price of good within all countries is an increasing function of the initial proportion of money m.*

Proof. Substituting Q_1 and Q_3 into (12) and differentiating it follows

$$\begin{aligned} &\begin{bmatrix} 2\alpha\left(\frac{u_1}{Q_1} - u_1'\right) + \alpha_3 \frac{u_3}{Q_1} & -\alpha_3 u_3' \\ -2\alpha_3 u_1' & 2\alpha_3 \frac{u_1}{Q_3} + \alpha\left(\frac{u_3}{Q_3} - u_3'\right) \end{bmatrix} \begin{bmatrix} dQ_1 \\ dQ_3 \end{bmatrix} \\ &= -\frac{2}{3\left(1 - \frac{2m}{3}\right)} \begin{bmatrix} 2\alpha\left(u_1 - Q_1\right) + \alpha_3\left(u_3 - Q_1\right) \\ 2\alpha_3\left(u_1 - Q_3\right) + \alpha\left(u_3 - Q_3\right) \end{bmatrix} dm \end{aligned} \tag{21}$$

or, rearranging,

$$\begin{aligned} &\frac{\Delta}{dm} \begin{bmatrix} dQ_1 \\ dQ_3 \end{bmatrix} = -\frac{2}{3\left(1 - \frac{2m}{3}\right)} \begin{bmatrix} 2\alpha_3 \frac{u_1}{Q_3} + \alpha\left(\frac{u_3}{Q_3} - u_3'\right) & \alpha_3 u_3' \\ 2\alpha_3 u_1' & 2\alpha\left(\frac{u_1}{Q_1} - u_1'\right) + \alpha_3 \frac{u_3}{Q_1} \end{bmatrix} \\ &\times \begin{bmatrix} 2\alpha\left(u_1 - Q_1\right) + \alpha_3\left(u_3 - Q_1\right) \\ 2\alpha_3\left(u_1 - Q_3\right) + \alpha\left(u_3 - Q_3\right) \end{bmatrix} \end{aligned} \tag{22}$$

where Δ, the determinant of the square matrix in (21), is equal to (15). Thus, using (22), one obtains

$$\Delta \frac{dQ_1}{dm} = -\frac{2}{3\left(1-\frac{2m}{3}\right)} \{\alpha_3 u_3' \left[2\alpha_3 \left(u_1 - Q_3\right) + \alpha \left(u_3 - Q_3\right)\right] + \left[2\alpha_3 \frac{u_1}{Q_3} + \alpha \left(\frac{u_3}{Q_3} - u_3'\right)\right] \left[2\alpha \left(u_1 - Q_1\right) + \alpha_3 \left(u_3 - Q_1\right)\right]\} \leq 0, \quad (23)$$

and

$$\Delta \frac{dQ_3}{dm} = -\frac{2}{3\left(1-\frac{2m}{3}\right)} \{2\alpha_3 u_1' \left[2\alpha \left(u_1 - Q_1\right) + \alpha_3 \left(u_3 - Q_1\right)\right] + \left[2\alpha \left(\frac{u_1}{Q_1} - u_1'\right) + \alpha_3 \frac{u_3}{Q_1}\right] \left[2\alpha_3 \left(u_1 - Q_3\right) + \alpha \left(u_3 - Q_3\right)\right]\} \leq 0. \quad (24)$$

since both $u_1 \geq Q_3$ and $u_3 \geq Q_1$ always hold in the accession equilibrium. Hence, (23) and (24) imply that both Q_1 and Q_3 are decreasing functions of the quantity of money m or, equivalently, that both $p_1 = \frac{1}{Q_1}$ and $p_3 = \frac{1}{Q_3}$ are increasing functions of m. ■

Proposition 5 *Equal integration between any country-pair is a sufficient condition for the existence of the accession equilibrium.*

Proof. Assume $\alpha_{ix} = \alpha$ for any $i, x = \{1, 2, 3\}$. By virtue of $(A3)$ this implies $\alpha_3 = \alpha$ which implies, in turn, $Q_1 = Q_3$ from Proposition 3. Since by assumption a money type is always accepted in the country where it is issued, i.e. $u(Q_1) \geq Q_1$ and $u(Q_3) \geq Q_3$, then it must be that also $u(Q_1) \geq Q_3$ and $u(Q_3) \geq Q_1$ are both satisfied. The assumption $(A3)$ ends the proof. ■

It can be shown that for small deviations from $\alpha_{ix} = \alpha$ both currencies are still internationally accepted, that is the accession equilibrium exists, but they are no longer perfect substitutes *across* countries.

5 Conclusion

This paper studied the accession of a third country in a two-country monetary union within a model of search with output divisibility. The main results of the paper are the following: equal degree of integration between any country-pair is a necessary and sufficient condition for monies to be perfect substitutes within and across all countries. Equal integration between any country-pair is also a sufficient condition for the existence of the accession equilibrium.

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