COALITION-PROOF BARGAINING

Extended Abstract

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It is well-known that a natural generalization of the Rubinstein bilateral bargaining game to the N-player case leads to indeterminacy, by which every agreement can be sustained as a subgame perfect equilibrium outcome. Different authors have explored alternative bargaining procedures that give a unique SPE, for example, Jun (1987), Chae and Yang (1988, 1994), and Krishna and Serrano (1996). A common feature among these bargaining procedures is that the final agreement consists of a series of bilateral agreements that are reached either sequentially or simultaneously. In other words, unanimity is not respected in these procedures. It makes them vulnerable to a certain form of collusion. Specifically, two players can form a coalition and take away the whole pie when one of them is in the role of proposer.

The goal of this project is to search for bargaining procedures that can provide sharp game theoretic prediction and discourage collusion at the same time. We explore three different approaches. The main ideas are illustrated in the following with the three-player case. Extension to $N(>3)$-player case seems to be straightforward. For simplicity, we assume that players have linear utility function and common discount factor $\delta \in (0,1)$.

1 "Self-Interest" Players

The first approach can be viewed as a refinement of the SPE of the generalized Rubinstein game with three players. The game is obviously coalition-proof. Although any agreement can be supported as a SPE outcome in this game, there is only one SPE, in which players' strategies are stationary (or, history-independent). It leads to the division $\left(\alpha, \delta\alpha, \delta^2\alpha\right)$ of the pie, where $\alpha\left(1+\delta+\delta^2\right) = 1$. However, the restriction to stationary strategies is too strong and unappealing.

We explore an alternative restriction on strategies, namely, the "self-interest" restriction. A player's strategy is "self-interest" if, whenever in the role of responder, he specifies a threshold value, and accepts a proposal if and only if it gives him a share not lower than the threshold value. Note that the threshold value may vary from round to round, thus our restriction is much weaker than stationarity. More specifically, a stationary strategy must be "self-interest", but the converse is not true. The "self-interest" restriction captures the intuition that a player should only care his own share, not how other players split the rest of the pie. We find it intuitive and plausible.

Our main result is rather surprising. There is only one SPE that satisfies the "self-interest" restriction, and it is exactly the same equilibrium under the restriction of stationarity. The proof is straightforward. Denote by M (m) the supremum (infimum) of the share which player 1 can obtain in any SPE. Under the "self-interest" restriction, it can be shown that in any possible SPE, player 2's final share $x_2 \in [\delta m, \delta M]$, and player 3's final share $x_3 \in \left[\delta^2 m, \delta^2 M\right]$.[1] After this being established, we can apply the technique due to Shaked and Sutton (1984). Specifically, we have

$$\begin{aligned} m &= 1 - \delta M - \delta^2 M \\ M &= 1 - \delta m - \delta^2 m, \end{aligned}$$

that is,

$$m = M = \frac{1}{1 + \delta + \delta^2}.$$

The rest of the Proof follows easily.

Our result conveys the following message: what induces indeterminacy in N-player Rubinstein game is not the non-stationarity of strategies, it is instead the non-monotonicity in the acceptance rule specified in players' strategies.

2 Exit with Unanimous Endorsement

In the second approach, we propose a bargaining procedure that is similar to the one studied by Krishna and Serrano (1996) (henceforth KS). In their model, players take turns to make proposal. After a proposal is made, responders who accept the proposal can exit the game with proposed shares. The remaining players then bargain over the rest of the pie.

We argue that the KS bargaining procedure is vulnerable to collusion. Note that the exit of a player is indeed a bilateral agreement between that player and the proposer. Hence two

[1] Note that this does not hold without the "self-interest" restriction.

players can form a coalition and take away the whole pie when one of them is in the role of proposer.

We consider a variation of the KS's exit game which respects unanimity. The exit of any player should be unanimously endorsed by all players. Specifically, after a proposal is made, each responder specifies all players who may exit the game with proposed shares. Only those who get unanimous endorsement can exit. Note that if a player is satisfied with his own share, then he has no say on others' share; and if he is not satisfied, he has to identify at least one other player with whom he will negotiate in next round. The proposer can also exit the game if the share he specifies for himself is approved unanimously.

The game appears to be more strategically complicated than KS's exit game. We manage to show that it also has a unique SPE, which generates the same outcome as that in KS's game. The equilibrium agreement is $(\alpha, \delta\alpha, \delta\alpha)$, with $\alpha(1+2\delta)=1$, and it is unanimously approved immediately. Since unanimity is well respected, our bargaining procedure is robust to the form of collusion mentioned above.

3 Sequential Demand Game with Commitment

We also consider a sequential demand game with commitment. In each round, players take turns to announce their demands. The game ends if it is feasible to grant all demands; otherwise the game proceeds to next round, in which the order of announcements is rotated. The commitment assumption is similar to that in the subscription game studied by Admati and Perry (1991). Specifically, once a player has made a demand, he cannot subsequently increase it.

We are working on a complete characterization of SPE outcome(s) in this game. Two question are of particular interest. The first one is whether it has a unique SPE. The second one is whether there exists an equilibrium which involves delay. Our conjecture is that the game has a unique and efficient SPE.