Market transparency and Bertrand competition

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Abstract

We investigate the effects of market transparency on prices in the Bertrand duopoly model for both the cases of strategic complementarities and strategic substitutes. For the former class of games "conventional wisdom" concerning prices is confirmed, since they decrease. The consumers are always better off with higher transparency but changes in firm's profits are ambiguous. For the latter class of games, an increase in market transparency may lead to an increase in one of the prices, which implies ambiguity in consumers' utility and firms' profits.

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1 Introduction

In retail economics, a market is said to be transparent if much is known by many about what products and services are available, at what price

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and where. Increasing transparency is often considered as a cure for some imperfections and inefficiencies of the market. From the consumers' point of view it is supposed to increase competition and their surplus, by leading to lower prices and to reductions in price dispersion. Due to the growing popularity of the Internet, which allows for quick spreading of information, in many markets an increase in transparency can be observed.

Empirical findings provide mixed evidence on price comparisons in the Internet and in traditional retailers. For instance, Bailey (1998) shows that Internet commerce may not reduce market friction because prices are higher when consumers buy homogeneous products on the Internet, and price dispersion for homogenous products among Internet retailers is greater than the price dispersion among physical retailers. Lee et al. (2000) found that the average product price in one of the most successful electronic commerce systems (an electronic market system for used-car transactions in Japan) is much higher than in traditional, non-electronic markets. The secondhand cars traded there are usually of much higher quality than those sold in traditional markets, but used-car prices are slightly higher than in traditional markets even for cars of similar quality. Conversely, Brynjolfsson and Smith (2000) observed that books and CDs in the internet are cheaper than in conventional outlets. They find that prices on the Internet are 9-16% lower than prices in conventional outlets and conclude that while there is lower friction in many dimensions of Internet competition, branding, awareness, and trust remain important sources of heterogeneity among Internet retailers.

In the theoretical literature there are studies explaining the phenomenon that prices do not always go down in case of increased transparency. The main argument, recalled in a number of papers, is that increasing transparency might facilitate tacit collusion for the producers (see e.g. Mollgaard and Overgaard, 2001, Nilsson, 1999, and Schultz, 2005).

The problem of market transparency was investigated in many different strands of literature. Varian (1980) showed that in case of homogenous goods and symmetric firms, the expected equilibrium profits decrease in the level of market transparency. This idea was developed in the search literature, for instance Burdett and Judd (1983) or Stahl (1989). If the cost of searching goes down, the consumers search more and inter-firm competition becomes tougher. A contribution to explaining market transparency issues can be found as well in the literature on advertising, with increased advertising typically leading to lower prices, see, for instance, Bester and Petrakis (1995). They consider transparency as a firm's decision variable, while in this paper it is an exogenous parameter.

Another strand of literature studies the demand side of the market under less than full transparency. For the Hotelling model with product differentiation and a fraction of uninformed consumers, Schultz (2004) shows that increasing transparency (measured by the proportion of informed consumers) leads to less product differentiation and lower prices and profits. Moreover, welfare improves for all consumers and total surplus increases. Boone and Potters (2002) analyzed a symmetric Cournot-Nash model, where goods are imperfect substitutes and consumers value variety. They found that more transparency may lead to an increase in total demand and also to higher prices. The level of substitutability is exogenous in their model and, when goods are perfect substitutes, the effect of increasing demand disappears.

The model presented in this paper is closely related to this last strand of literature. We deal with effects of market transparency on prices in the standard Bertrand duopoly model with heterogeneous goods, modified to allow for transparency effects. The analysis is intuitive and simple when we consider two types of strategic interaction between firms' prices in the industry - strategic complementarity and strategic substitutability. We derive our results in the form of equilibrium comparative statics analysis, using the methodology of supermodular games (see Vives, 1999 and Amir, 2005 for surveys of this methodology as applied to oligopoly theory).

In the first case, with prices being strategic complements, the results are close to conventional wisdom, especially, if at the same time products are assumed to be gross substitutes. Namely, equilibrium prices and per-firm profits are always decreasing in the transparency level, while consumer surplus is increasing. On the other hand, when strategic complementarity holds, but products are not gross substitutes, the result on profits is not longer valid.

Considering price competition with strategic substitutes, an ambiguity in the direction of change of prices appears. This leads to ambiguity concerning equilibrium profits and surplus's changes caused by increasing transparency as well.

The paper is organized as follows. In Section 2 we present the general setup of the model of price competition with incomplete transparency. Further there is an overview of definitions of strategic complementarity and substitutability and results concerning these notions, useful in the paper. In Section 3 we study the reaction of prices to increased market transparency, distinguishing cases of strategic complementarity and substitutability. Section 4 is devoted to the analysis of welfare issues - changes in profits and consumers' surplus. In Section 5 we illustrate the results of Sections 3 and 4 by a linear example. Conclusions follow.

2 Setup and definitions

2.1 General setup

We consider a Bertrand price competition game Γ with the following characteristics. Two firms, producing different products, respectively 1 and 2, compete in prices. Both firms have constant marginal costs, respectively c_1 and c_2.

Following Schultz (2004) we consider two different types of consumers. A fraction ϕ are informed about products (both characteristics and prices) and the rest $1-\phi$ are uninformed. $\phi \in [0,1]$ measures the level of transparency of the market.

Schultz considers the Hotelling model with a continuum of consumers uniformly distributed along the interval $[0,1]$ and the demand for firm's 1 product is given by $\phi x + (1-\phi)\frac{1}{2}$, where $x \in [0,1]$ denotes the location of the consumer who is fully informed and indifferent between buying product 1 and 2. The demand for firm's 2 product is $1 - \left(\phi x + (1-\phi)\frac{1}{2}\right)$.

We generalize this approach allowing for other forms of demand functions, but retain the same way of modelling the behavior of informed versus uninformed consumers. We consider a one-shot model with exogenous heterogeneity of the products and firms deciding only on prices.

The full-information demands for goods 1 and 2 are denoted respectively $D^1(p_1,p_2)$ and $D^2(p_2,p_1)$. The uninformed consumers know only one of the products and are not aware of the other. Hence, their demands depend only on one price: $d(p_i)$, $i=1,2$. We assume that half of the uninformed consumers know each good. Thus, the total demand for good i is $\phi D^i(p_i,p_j) + \frac{1-\phi}{2} d(p_i)$.

Moreover, we assume throughout that $D^i(p_i,p_j)$ and $d(p_i)$ are twice continuously differentiable. Goods are *substitutes* (*complements*) when the demand for one of them is increasing (decreasing) in the other's price, hence this implies that $D_j^i > (<) 0$.[1] Moreover, both $D^i(p_i, p_j)$ and $d(p_i)$ can be characterized by their price elasticities: $\varepsilon_D = D_i^i \frac{p_i}{D^i}$ and $\varepsilon_d = d'(p_i) \frac{p_i}{d}$.

Consider the situation if the level of market transparency is zero. Then every firm faces half of the consumers, and there is no relation between firms' pricing decision, hence

$$\pi^i(p_i) = \frac{1}{2}(p_i - c_i)\, d(p_i).$$

The solution of profit maximization problem in this case, $\mathring{p}_i$, is given by the first order condition:

$$d(\mathring{p}_i) - (\mathring{p}_i - c_i)\, d'(\mathring{p}_i) = 0$$

Notice that for any $p_i < \mathring{p}_i$, $\pi_i^i(p_i) > 0$.

When the market is perfectly transparent, i.e. all consumers are informed about prices and characteristics of both goods, the profit of firm i can be expressed by:

$$\begin{aligned} \pi^i(p_i, p_j) &= p_i D^i(p_i, p_j) - c_i D^i(p_i, p_j) \\ &= (p_i - c_i)\, D^i(p_i, p_j), i \neq j, i, j \in \{1, 2\}. \end{aligned}$$

In case of imperfect market transparency, the profit of firm i is given by:

$$\Pi^i(p_i, p_j) = (p_i - c_i)\left(\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d(p_i)\right). \tag{1}$$

We restrict our consideration to prices in $[c_i, \infty), i = 1, 2$, since lower prices are dominated by pricing at marginal cost. Moreover we assume an upper bound on price, $\overline{p}_i$, such that $p_i \in P_i = [c_i, \overline{p}_i]$.

In the next subsection we provide definitions of super- and submodularity, and a number of theorems which will be useful in the remainder.

[1] In case of multivariate functions, lower subscripts denote partial derivative taken with respect to the indicated variable, here e.g. $D_j^i = \frac{\partial D^i}{\partial p_j}$.

2.2 Useful definitions and results

A function $\Pi : P_1 \times P_2 \to R$ is *supermodular* (*submodular*) if for each $p_1^{'} > p_1$ and $p_2^{'} > p_2$

$$\Pi(p_1^{'}, p_2^{'}) - \Pi(p_1, p_2^{'}) \geq (\leq)\, \Pi(p_1^{'}, p_2) - \Pi(p_1, p_2).$$

When the inequality is strict, we have strict supermodularity (submodularity). Supermodularity (submodularity) can be easily detected using differential characterization, namely, if $\Pi_{12}(p_1, p_2) \geq (\leq) 0$, then $\Pi(p_1, p_2)$ is supermodular (submodular).

In R^2, supermodularity (submodularity) is equivalent to *nondecreasing* (*nonincreasing*) *differences* property, in different words, $\Pi(\cdot, p_2^{'}) - \Pi(\cdot, p_2)$ is an nondecreasing (nonincreasing) function.

Π has the (*dual*) *single crossing property* in (p_1, p_2) when for each $p_1^{'} > p_1$ and $p_2^{'} > p_2$

$$\Pi(p_1^{'}, p_2) - \Pi(p_1, p_2) \geq (\leq) 0 \Rightarrow \Pi(p_1^{'}, p_2^{'}) - \Pi(p_1, p_2^{'}) \geq (\leq) 0.$$

If Π is supermodular (submodular) in (p_1, p_2) then it satisfies also the (dual) single crossing property. But the reverse is not generally true. This property is ordinal, thus it is preserved by strictly monotonic transformation. Moreover, if g is strictly monotonic and $g \circ \Pi$ is supermodular (submodular), then Π has the (dual) single crossing property (see Milgrom and Shannon, 1994).

A game is supermodular if the players' strategy spaces are compact and the payoff functions are upper semi continuous in own strategies and supermodular. Such games are also called *games of strategic complementarity.* Intuitively, the latter property expresses the idea that one player's marginal payoff is nondecreasing in his opponent's action, hence, their actions are complementary. These games have the key property of nondecreasing reaction curves defined as

$$r^i(p_j) = \arg\max_{p_i \in P_i} \{\Pi^i(p_i, p_j) : p_j \in P_j\}.$$

Analogously we define *games of strategic substitutability* as being those for which players' payoff functions are submodular. This describes the opposite situation, when one player's marginal payoff is nonincreasing in his opponent's action. These games have nonincreasing reaction curves, since if one player increases his action, the other one reacts by decreasing his.

To determine the direction of change of an equilibrium point as an exogenous parameter changes, we may use the following theorem.

Theorem 1 (Milgrom and Shannon, 1994) *Consider a game, where payoff of player i is given by $\Pi^i(p_i, p_{-i}, \phi)$, where ϕ is a parameter. Assume that for each $\phi \in [0, 1]$ the game is supermodular, and Π^i satisfies the dual single crossing property in (p_i, ϕ) for each p_{-i}. Then, the maximal and minimal equilibria of the game are decreasing functions of ϕ.*

There is no dual version of this theorem for submodular games. When analogous conditions are satisfied for submodular game, the downward sloping reaction curves will both shift down, but it need not mean that both equilibrium actions decrease. It depends on the magnitude of the shifts. This is due to two effects which will be described in detail in the next section.

3 Effect of transparency on prices

In this section we consider the impact of increasing market transparency on equilibrium prices in the model formulated in (1). We distinguish two cases, depending on character of strategic interactions between firms.

3.1 Strategic complementarities

Consider a Bertrand game with perfect transparency and payoffs given by $\pi^i(p_i, p_j) = (p_i - c_i)D^i(p_i, p_j)$. Vives (1990) provides a condition for the Bertrand competition to be a supermodular game. For the linear costs case this condition is as follows:

$$D^i_j + (p_i - c_i)D^i_{ij} \geq 0 \text{ for all } (p_i, p_j) \in P_i \times P_j.$$

It is more easily satisfied when the products are substitutes and when demand is supermodular, but none of these is a necessary condition.[2]

This condition guarantees as well supermodularity of the game with imperfect transparency.

Proposition 2 Π^i *defined in (1) is supermodular in* $(p_i, p_j) \in P_i \times P_j$ *if* D^i *satisfies*

$$D^i_j + (p_i - c_i)D^i_{ij} \geq 0. \tag{2}$$

Proof. The result follows directly from the cross-partial derivative of the profit function, $\Pi^i_{ij}(p_i, p_j) = \phi\left(D^i_j + (p_i - c_i)D^i_{ij}\right)$. ■

The supermodularity of the profit function is useful to establish, how the prices react to a change in the level of market transparency.

Proposition 3 *If* $\Pi^i, i = 1, 2,$ *is supermodular and* $|\varepsilon_D| > |\varepsilon_d|$, *then an increase in market transparency causes the extremal equilibrium prices of both goods to decrease.*

[2] There is another condition making Bertrand duopoly with linear cost into a game of strategic complementarities. It was given by Milgrom and Roberts (1990) and is equivalent to the cross partial derivative of the log-profit function being positive. In our case it is less useful, since it requires imposing additional conditions on the game with imperfect transparency to secure its log-supermodularity. Amir and Grilo (2003) provide a detailed comparison between the two sufficient conditions.

Proof. The proof uses Theorem 1. Since supermodularity of Π^i is assumed, now we want to check whether $\Pi^i(p_i, p_j, \phi)$ has the dual single crossing property in (p_i, ϕ). To this end, we consider strictly monotonic transformation of Π^i, namely $\ln \Pi^i(p_i, p_j, \phi)$, and show that this is submodular in (p_i, ϕ), so we can conclude that $\Pi^i(p_i, p_j, \phi)$ has the dual single crossing property in (p_i, ϕ). We obtain

$$\frac{\partial^2 \ln \Pi^i(p_i, p_j, \phi)}{\partial p_i \partial \phi} = \frac{1}{2} \frac{D_i^i(p_i, p_j) d(p_i) - D^i(p_i, p_j) d'(p_i)}{\left(\phi D^i(p_i, p_j) + \frac{1-\phi}{2} d(p_i)\right)^2}. \tag{3}$$

This is negative, whenever $D_i^i(p_i, p_j)d(p_i) - D^i(p_i, p_j)d'(p_i) < 0$. Dividing this inequality by $D^i(p_i, p_j)d(p_i)$ and multiplying by p_i gives us $\varepsilon_D - \varepsilon_d < 0$. Since both these elasticities are negative, it is sufficient that $|\varepsilon_D| > |\varepsilon_d|$, to have assumptions of Theorem 1 satisfied, therefore we can conclude that when ϕ goes up both prices go down. ■

The condition $|\varepsilon_D| > |\varepsilon_d|$ is quite natural since it says that the demand for good i is more sensitive to changes in price for those consumers who are aware of existing another good in the market. Reacting to the price increase they may switch to buy another good, whereas those uninformed are not aware of this possibility.

The decrease of equilibrium prices can be interpreted as consisting of two effects. There is a direct effect shifting the reaction curve down as a reaction of the player to the parameter change and an indirect effect of decreasing own price in response to the decrease in opponent's price (see Amir, 2005, 1996).

3.2 Strategic substitutes

Analogously to Proposition 2 we can formulate a condition on D^i to make the game Γ a submodular game.

Proposition 4 Π^i *defined like in (1) is submodular in* $(p_i, p_j) \in P_i \times P_j$ *if* D^i *satisfies*

$$D^i_j + (p_i - c_i)D^i_{ij} \leq 0. \tag{4}$$

Proof. The result follows directly from the cross-partial derivative of the profit function, $\Pi^i_{ij}(p_i, p_j) = \phi\left(D^i_j + (p_i - c_i)D^i_{ij}\right)$. ∎

If condition (4) is satisfied we can conclude that the price competition is of strategic substitutes and hence the best replies are nonincreasing. In this case we cannot use Theorem 1 or any analog to state how the equilibrium prices will react to increased market transparency. From the fact that, as before, (3) is negative, whenever $|\varepsilon_D| > |\varepsilon_d|$, it follows that both reaction curves shift down, but it does not mean that both equilibrium prices decrease. It is possible that one of them may increase. Intuitively, it can be explained by the fact that the two effects mentioned before are conflicting now. The direct effect of the shift in the reaction curve makes the price go down but the indirect effect of adjusting to opponent's price leads in the opposite direction, since in case of strategic substitutes it is profitable to increase own action when the other player decreases his. Thus, the total effect depends on which of these two dominates. Nonetheless, for the special case of symmetric submodular game, we recover the proposition that both equilibrium prices are decreasing in ϕ.

3.2.1 Symmetric games

A Bertrand duopoly is symmetric if $P_i = P_j \equiv P$ and $\Pi^i(p_i, p_j) = \Pi^j(p_j, p_i)$.

Proposition 5 *Consider a symmetric Bertrand duopoly such that* $\Pi^i(p_i, p_j)$ *is strictly quasi-concave in own action, submodular and* $|\varepsilon_D| > |\varepsilon_d|$. *Then* *an increase in market transparency causes the equilibrium prices of both goods to decrease.*

Proof. Quasi-concavity of Π^i in p_i guarantees that the reaction curve of player i, $r^i(p_j) = \arg\max\{\Pi^i(p_i, p_j) : p_j \in P\}$ is continuous. Then, there must exist a symmetric equilibrium and it is unique. Let $r^i(p^*(\phi)) = p^*(\phi)$ be the equilibrium of the game.

From submodularity it follows that $r^i(p_j)$ is decreasing. Consider now $\Pi^i(p_i, p_j, \phi)$. Since $\frac{\partial^2 \ln \Pi^i(p_i,p_j,\phi)}{\partial p_i \partial \phi}$, given by (3) is negative, as showed in the proof of Proposition 3, we know that $\Pi^i(p_i, \phi)$ satisfies the dual single crossing property, and the reaction curve $r^i(p_j)$ shifts down when ϕ goes up.

Take $\phi_1 < \phi_2$. Consider situation when ϕ increases from ϕ_1 to ϕ_2 and denote $\widehat{r^i}(p_j)$ the reaction curve after the increase of ϕ. We want to show that the equilibrium prices both decrease. Since the unique equilibrium is still symmetric, we conclude that both prices change in the same direction. Proceed by contradiction and assume that $p^*(\phi_1) < p^*(\phi_2)$. Then

$$r^i(p^*(\phi_1)) = p^*(\phi_1) < p^*(\phi_2) = \widehat{r^i}(p^*(\phi_2)) \leq \widehat{r^i}(p^*(\phi_1)) \geq r^i(p^*(\phi_1))$$

where the second inequality comes from the fact that the reaction curve is decreasing and the third one comes from the negative shift of the reaction curve when ϕ increases. This is a contradiction, hence we conclude that $p^*(\phi_1) > p^*(\phi_2)$. ∎

3.2.2 Asymmetric games

To deal with asymmetric supermodular games we need to impose an additional assumption on Π^i. Since we have to relay now on traditional comparative statics method, we need Π^i to be concave, at least locally at equilibrium, throughout the rest of the paper. The analysis presented below covers as well all the cases when the strategic interactions of the game are neither strategic complements nor substitutes.

Here we investigate sufficient conditions for equilibrium prices to be decreasing in transparency level. To simplify the notation below we omit the argument of all functions, hence we denote $p_i^{*\prime}(\phi) = p_i^{*\prime}$ and $D^i(p_i^{*\prime}(\phi), p_j^{*\prime}(\phi)) = D^i$. To avoid misunderstanding and keep the notation compact we denote here $d(p_i) = d^i$ and $d(p_j) = d^j$.

Proposition 6 *The equilibrium price of good i is decreasing in the level of transparency if*

$$\begin{aligned} &[d^{i\prime}(p_i - c_i) + d^i][2D_j^j + (p_j - c_j)(D_{jj}^j - \frac{1-\phi}{2\phi} d^{j\prime\prime})] \\ &< [d^{j\prime}(p_j - c_j) + d^j][D_j^i + (p_i - c_i) D_{ij}^i]. \end{aligned} \tag{5}$$

Proof. To establish this result we use the classical method of comparative statics, thus we differentiate the first order conditions of both firms with respect to ϕ and solve the system of equations to find $p_1^{*\prime}(\phi)$ and $p_2^{*\prime}(\phi)$.

The first order conditions for firms' profit maximization are following:

$$\phi D^1 + (p_1 - c_1)\left(\phi D_1^1 + \frac{1-\phi}{2} d^{1\prime}\right) + \frac{1-\phi}{2} d^1 = 0$$
$$\phi D^2 + (p_2 - c_2)\left(\phi D_2^2 + \frac{1-\phi}{2} d^{2\prime}\right) + \frac{1-\phi}{2} d^2 = 0$$

Differentiating them with respect to ϕ, we obtain following system of equations:

$$
\begin{aligned}
& p_1^{*\prime}(2\phi D_1^1 + (p_1 - c_1)\,(\phi D_{11}^1 - \frac{1-\phi}{2} d^{1\prime\prime})) + \\
& +\phi p_2^{*\prime}\left(D_2^1 + (p_1 - c_1)\, D_{12}^1\right) \\
= & \frac{1}{2} d^1 - D^1 - (p_1 - c_1)\,(D_1^1 - \frac{1}{2} d^{1\prime}) \\
& \phi p_1^{*\prime}\left(D_1^2 + (p_2 - c_2)\, D_{21}^2\right) + \\
& +p_2^{*\prime}(2\phi D_2^2 + (p_2 - c_2)\,\phi(D_{22}^2 - \frac{1-\phi}{2} d^{2\prime\prime})) \\
= & \frac{1}{2} d^2 - D^2 - (p_2 - c_2)\,(D_2^2 - \frac{1}{2} d^{2\prime})
\end{aligned}
$$

From first order conditions it follows that

$$
\begin{aligned}
\frac{1}{2} d^1 - D^1 - (p_1 - c_1)\left(D_1^1 - \frac{1}{2} d^{1\prime}\right) &= \frac{1}{2\phi}(d^{1\prime}\,(p_1 - c_1) + d^1) \\
\frac{1}{2} d^2 - D^2 - (p_2 - c_2)\left(D_2^2 - \frac{1}{2} d^{2\prime}\right) &= \frac{1}{2\phi}(d^{2\prime}\,(p_2 - c_2) + d^2)
\end{aligned}
$$

so that we replace it in the system.

The solution to this system is following:

$$
\begin{aligned}
p_i^{*\prime} = & \frac{1}{2\phi} \frac{(d^{i\prime}\,(p_i - c_i) + d^i)(2\phi D_j^j + (p_j - c_j)\,(\phi D_{jj}^j - \frac{1-\phi}{2} d^{j\prime\prime}))}{k} \\
& -\frac{1}{2\phi} \frac{\phi(d^{j\prime}\,(p_j - c_j) + d^j)(D_j^i + (p_i - c_i)\, D_{ij}^i)}{k}
\end{aligned}
$$

where

$$
\begin{aligned}
k = & (2\phi D_i^i + (p_i - c_i)\,(\phi D_{ii}^i - \frac{1-\phi}{2} d^{i\prime\prime})) \\
& \times(2\phi D_j^j + (p_j - c_j)\,(\phi D_{jj}^j - \frac{1-\phi}{2} d^{j\prime\prime})) \\
& -\phi^2\left(D_i^j + (p_j - c_j)\, D_{ji}^j\right)\left(D_j^i + (p_i - c_i)\, D_{ij}^i\right)
\end{aligned}
$$

Since the denominator is positive for stable equilibria, we restrict our attention to the latter so that sign of $p_i^{*\prime}$ depends solely on the sign of the numerator:

$$
\begin{aligned}
& sign(p_i^{*\prime}) \\
= & sign\{[d^{i\prime}(p_i - c_i) + d^i][2\phi D_j^j + (p_j - c_j)(\phi D_{jj}^j - \frac{1-\phi}{2} d^{j\prime\prime})] \\
& -\phi[d^{j\prime}(p_j - c_j) + d^j][D_j^i + (p_i - c_i) D_{ij}^i]\}.
\end{aligned}
$$

For simplicity we can divide this expression by ϕ, since it is positive and we obtain condition (5). ■

Note the interpretation of the components of condition (5). Expression $d^{i\prime}(p_i - c_i) + d^i = \pi_i^i(p_i) > 0$ for all $p_i < \mathring{p}_i$, where $\mathring{p}_i$ can be interpreted as a monopoly price, if there is zero level of transparency (compare Section 2). The second term of the left hand side captures the concavity of the opponent's profit, hence it is always locally negative at equilibrium, and the second term of the right hand side describes the strategic complementarity or substitutability of players' actions in the game from the point of view of player i.[3]

Is is easy to observe that convexity of the demand in own actions, its submodularity and complementary character of goods are in favor to one of the prices being increasing in ϕ.

In the next section we analyze how an increase in market transparency influences firms' profits and consumers' surplus, taking into account the behavior of the equilibrium prices.

4 Effects of transparency on profits

In this section we study how the transparency level influences consumers and firms welfare in equilibrium.

[3] Although for each demand system $D_2^1 = D_1^2$, it may happen that one of the demands is strictly supermodular, while the other is strictly submodular. The reaction curves have in such a case the opposite slopes. For an example, see Amir et al. (1999).

Clearly, consumers are better off, when both prices decrease. Hence, consumers' surplus rises if the game is of strategic complementarities and if it is symmetric of strategic substitutes and if condition (5) is satisfied for $i = 1, 2$.

Consider now equilibrium profits of the firms.

Proposition 7 $\Pi^i(p_i^*(\phi), p_j^*(\phi))$ *is decreasing in* ϕ *if*

- *goods are substitutes and* $p_j^{*\prime}(\phi) < 0$*;*

  or

- *goods are complements and* $p_j^{*\prime}(\phi) > 0$*.*

Proof. We compute the derivative of $\Pi^i(p_i^*(\phi), p_j^*(\phi))$ to state its reactions to increasing transparency.

$$\begin{aligned} \frac{d}{d\phi}\Pi^i(p_i^*(\phi), p_j^*(\phi)) &= \Pi_i^i p_i^{*\prime} + \Pi_j^i p_j^{*\prime} \\ &= (p_i^* - c_i)\phi D_j^i p_j^{*\prime} \end{aligned}$$

since the component $\Pi_i^i p_i^{*\prime}$ is equal to zero from the first order condition of the firm. This is negative if exactly one of D_j^i and $p_j^{*\prime}$ is negative. ■

In case of complementary goods, when $D_j^i < 0$, and decreasing equilibrium prices both firms are better off. If one of the product's prices increases in ϕ, its producer is better off, contrary to his opponent.

In case of substitute goods, when $D_j^i > 0$, and both prices decreasing, we conclude that both profits decrease as well. But in general case substitute goods do not imply strategic complementarity in the Bertrand model (which guarantees that both equilibrium prices decrease with ϕ), so it may happen, for strongly supermodular demand functions to form strategic substitutes. In this case firm's profit increase in ϕ, whenever the opponent's price increases.

Hence, in case of complementary goods and both prices decreasing, both firms and consumers gain in consequence of transparency increase.

5 Linear example

This section contains a numerical example, based on liner demand functions, illustrating main findings of the paper.

Consider the following demand of an representative informed consumer for product i:

$$D^i(p_i, p_j) = \alpha_i - \beta_i p_i + \gamma p_j.$$

We assume that $\alpha_i > 0$. This is needed for demand function to be well-defined - otherwise the autonomous demand (when both prices are zero) would be negative. Moreover, we assume that each demand reacts more to changes of own price than to changes of the opponent's price. Hence, $|\gamma| < \beta_i, i = 1, 2$.

Goods are substitutes whenever $\gamma > 0$ and complements if $\gamma < 0$. It will be shown that also γ is responsible for character of strategic interaction between firms.

Consider also a linear form of the demand of an uninformed consumer:

$$d(p_i) = a - bp_i.$$

We assume that $a > 0$ for the same reason as before. Moreover, we assume that $|\varepsilon_D| > |\varepsilon_d|$, since the consumer who is aware of the existence of two goods, reacts more to changes in price of one of them.

Firm's i profit is given by:

$$\Pi^i(p_i, p_j) = (p_i - c_i)(\phi(\alpha_i - \beta_i p_i + \gamma p_j) + \frac{1-\phi}{2}(a - bp_i))$$

Its cross-partial derivative

$$\Pi^i_{ij}(p_i, p_j) = \gamma\phi$$

is positive whenever γ is positive. Hence, we can conclude that reaction curve of one firm to price of the other one is increasing if goods are substitutes and decreasing if they are complements.

From Proposition 2 follows that for positive γ both equilibrium prices are decreasing in ϕ. For negative γ, this is guaranteed only for symmetric demand functions. The numerical example below illustrates situation, when this is not the case.

Example 8 *Lets $P_1 = P_2 = [0, 1]$. Consider following parameter values: $\alpha_1 = 2$, $\beta_1 = 3$, $\gamma = -2.99$, $c_1 = 0.05$, $a = 6$, $b = 4$, $\alpha_2 = 4$, $\beta_2 = 3$, $c_2 = 0.02$, $\phi = 0.5$. We left to the reader checking if all the assumptions mentioned in this chapter are satisfied.*

Firms profits are given by

$$\Pi^1(p_1, p_2) = (p_1 - 0.05)(0.5(2 - 3p_1 - 2.99p_2) + 0.25(6 - 4p_1))$$
$$\Pi^2(p_1, p_2) = (p_2 - 0.02)(0.5(4 - 3p_2 - 2.99p_1) + 0.25(6 - 4p_1))$$

and reaction curves are

$$r^1(p_2) = 0.525 - 0.299p_2$$
$$r^2(p_1) = 0.71 - 0.299p_1.$$

They are depicted by solid lines in figure 1. Equilibrium prices, given by the point where the reaction functions cross, are $p_1^* = 0.343\,41$, $p_2^* = 0.607\,32$. Firms equilibrium profits are, respectively, $\Pi^1(p_1^*, p_2^*) = 0.21523$ and $\Pi^2(p_1^*, p_2^*) = 0.86236$

When the transparency level ϕ increases to reach 0.8, all the best responses of both firms decrease. New reaction curves are given by

$$\hat{r}^1(p_2) = 0.417\,86 - 0.427\,14p_2$$
$$\hat{r}^2(p_1) = 0.688\,57 - 0.\,427\,14p_1$$

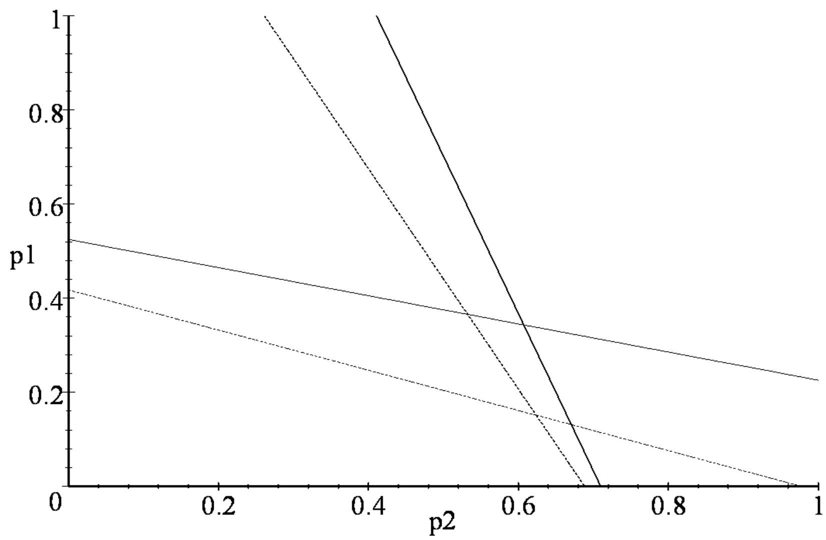

Figure 1: Reaction curves for $\phi = 0.5$ depicted as solid lines, reaction curves for $\phi = 0.8$ depicted as dashed lines. If ϕ increases, one of the equilibrium prices goes down, when another one goes up.

and are depicted by dashed lines in figure 1. They cross at $\hat{p}_1^* = 0.151\,35$*,* $\hat{p}_2^* = 0.623\,92$*, which are new equilibrium prices. Notice that the first one decreased, but the second one increased with the level of transparency.*

Since goods are complements and p_2^* *increases in* ϕ*, it follows from Proposition 7 that* $\Pi^1(p_1^*, p_2^*)$ *decreases and* $\Pi^2(p_1^*, p_2^*)$ *increases in* ϕ*. In fact* $\Pi^1(\hat{p}_1^*, \hat{p}_2^*) = 0.028763$ *and* $\Pi^2(\hat{p}_1^*, \hat{p}_2^*) = 1.0212$*.*

6 Conclusions

We analyzed market transparency issue in the context of the Bertrand price competition model. In case of strategic complementarity we directly generalize the results of Schultz (2004) in terms of equilibrium prices and consumer surplus, but not for firms' profits. While prices are decreasing in the transparency level and consumers are better off, firms are worse off only in case of substitute goods. Otherwise, one of them may gain, and even, in some cases of complementary goods, both profits can increase.

Allowing for strategic substitutability leads to ambiguous results even for prices - one of them may increase in the level of transparency. Such a situation precludes general conclusions about profits and consumers' surplus. However, when both prices decrease, consumers gain and in case of complementary goods, both firms gain as well.

We provide also a condition allowing for a definite conclusion in cases when strategic complementarity or substitutability is not satisfied, but this requires knowing functional forms of the demands and equilibrium prices. Its derivation relies on regularity conditions, since it is derived from the traditional comparative statics approach.

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