A Model of Interim Information Sharing under Incomplete Information*

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Abstract

We propose a two-person game-theoretical model to study information sharing decisions at an *interim* stage when information is incomplete. The two agents have pieces of private information about the *state of nature*, and that information is improved by combining the pieces. Agents are both senders and receivers of information. There is an institutional arrangement that fixes a transfer of wealth from an agent who lies about her private information. In our model we show that (i) there is a positive relation between information revelation and the amount of the transfers, (ii) information revelation has a collective action structure, in particular, the incentives of an agent to reveal are decreasing with respect to the amount of information disclosed by the other.

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1. Introduction

Consider a two-person game in which each agent has some private *payoff-relevant* information, the combination of the pieces of information being consistent with the true *state* and refining the information of each agent. Each agent sends a message to the other and they take an action after drawing inferences from the messages received. Many economic encounters under incomplete information are characterized by this kind of informational exchange between the agents before they decide their actions. The decision over messages changes the priors of the agents who receive them so that an agent with such a possibility of affecting the *beliefs* of the others will choose strategically the amount of private information that she discloses.

The purpose of this paper is two-fold. First, we develop a game-theoretical framework to study, in a tractable manner, the problem described above. Second, we use the model to obtain insights on strategic information sharing in an environment of *conflict*.

We should be interested in this problem since, as has been extensively analyzed, many economic *phenomena* are driven by how incomplete is the information available to the agents, and in many situations that is determined ultimately by their decisions on information sharing.

Most of the models on strategic information sharing used in applications (see, for example, Gal-Or [6], Li [11], Millon and Thakor [12], Pagano and Jappelli [14], Raith [15], and Shapiro [17]) assume that agents decide first on an informational regime and, after that, they get to know their private information, i.e., they consider that agents make *ex-ante* calculations of expected payoffs. Thus, agents are supposed not to know their private information *before* they decide on its revelation. Instead, we are interested in the question of *interim* information revelation; an essential feature of our model is that it allows agents to condition their decisions about information revelation on their *realized* private information.

We propose a model in which two agents have to choose one out of four possible *paths*, one of which leads to a prize. A path consists of two coordinates, and, initially, each agent knows a coordinate of the *correct* path. Before deciding over paths, the agents are asked to name the coordinate they know, i.e., their private information. Messages are sent simultaneously, and actions too are taken simultaneously. An agent obtains utility from both her decision over paths and her choice over messages. The utility obtained by the agents if both find the prize is strictly smaller than that which accrues when only one of them finds it, i.e., the model captures a "competitive" environment. Also, we consider *costs of lying* by assuming that an agent has to pay a non-negative penalty to the other if she "declares" not the coordinate she knows. This provides us with a game in which agents are allowed to condition their actions on the messages they receive, and in which payoffs depend on both messages and actions; our model is related to those of *signaling*[1] and *cheap talk*.[2]

[1]See, e.g., Rothschild and Stiglitz [16], Spence [18], and Wilson [19].

[2]The seminal work on cheap talk is due to Crawford and Sobel [5]. A characterization of equilibria for a general class of cheap talk games has been provided recently by Aumann and Hart [1].

Our results show the existence of key features of the problem of information sharing that cannot be analyzed by using a standard signaling game with a one-way flow of information. We use sequential equilibrium as solution concept. Under mild assumptions on the *fundamentals* of the model, equilibria in which agents reveal in a symmetric way are characterized in Section 3. We obtain that the incentives of the individuals to reveal information are increasing with respect to the amount of the penalties. In Section 4 we analyze equilibria in which the agents transmit their private information in a asymmetric way. Under slightly stronger assumptions on priors, we show that there is a class of equilibria such that an agent is more willing to provide information as her opponent reveals less, Proposition 5 combined with the results in Section 3. This suggests that the fact that the flow of information is bilateral plays an important role in shaping the decisions on information revelation. The important point is that the incentives of an agent to disclose her information do not depend only on exogenous costs (as has been traditionally studied by the signaling literature), but are also affected by the information revelation decisions of the other.

The model presented belongs to a class of models, of importance for research in game theory, that analyze strategic interactions under incomplete information; the decision variable subject matter of our model is the information that the agents provide. In principle, the exercise is not easy as agents update their priors by using the information that they receive and this must be taken into account to compute the effects induced on payoffs by possible strategy deviations. As a consequence, analyzing a model with a more general specification than ours of possible "states" and/or of the information structure might be a extremely complicated task since, even for a finite *state space*, the number of information sets at which the agents decide on information transmission, each of them containing possibly many nodes, may be too large. Therefore, it is not evident that the use of belief-based backwards induction solution concepts, such as sequential equilibrium, can, in fact, be managed. We wanted to work with a theoretical structure that be the the simplest one that allows for (i) a bilateral exchange of information and (ii) certain "homogeneity" in the asymmetries of information, and that sets in a neat way the source of conflict and all the relevant incentives of the agents to reveal their information. It can be viewed as a paradigm for gaining at least preliminary insights into the analysis of information sharing in situations of conflict. At a more applied level, the results obtained by analyzing the model in this paper provide insights in a wide range of situations at which two agents who face a common problem decide on information sharing, search activities being the most prominent example.

In terms of generalizations, in a model with more than four possible *paths*, our conclusions will continue to apply as the incentives of the agents to reveal their information would remain unaffected. Such generalizations allow for a broader set of equilibria and, so long as the combination of the pieces of private information is not sufficient to know the actual state, they also obscure the source of conflict thus making the problem less interesting.

The rest of the paper is organized as follows. Section 2 presents the model and describes its relation to the literature. In Section 3 we present and discuss our main results on symmetric information revelation. Section 4 deals with asymmetric information revelation, and Section 5 concludes with a discussion of the results. The proof of Lemma 1 is relegated to the Appendix.

2. The Model

Let Γ denote the *information sharing* game described as follows. There are two agents $i \in \mathcal{I} := \{1, 2\}$, and there is a set of "paths," one of which yields a prize. A *path* is a pair of coordinates $s = (s^1, s^2) \in \mathcal{S}$ and corresponds to a *state of nature*; therefore, $\mathcal{S}$ denotes the *state space* that we consider. Each agent $i \in \mathcal{I}$ starts off by knowing the respective coordinate s^i of the "correct" path, and that constitutes her initial private information. Let $\mathcal{S}^i$ denote the *type space* of agent i which we specify by $\mathcal{S}^i := \{0, 1\}$ for both $i \in \mathcal{I}$. Accordingly, we specify $\mathcal{S} := \mathcal{S}^1 \times \mathcal{S}^2$. Each agent will make the decision of choosing a path from $\mathcal{S}$.

The game is played in three consecutive phases ($t = 0, 1, 2$). In phase 0 a state $s \in \mathcal{S}$ is picked at random according to a probability vector $\pi := ((\pi(s))_{s \in \mathcal{S}}) \in \mathbb{R}^4_{++}$ that specifies the (common) *prior beliefs* of agents, and, accordingly, every agent learns her *type* $s^i \in \mathcal{S}^i$. The two agents are both *senders* and *receivers* of information; in phase 1 each agent $i \in \mathcal{I}$ sends to the other a *message*, $m^i \in M^i \equiv \mathcal{S}^i$, consisting of the name of one of her possible types. In phase 2, after processing the information obtained through messages, each agent $i \in \mathcal{I}$ chooses an *action*, $a^i \in A \equiv \mathcal{S}$, i.e., one of the paths available in $\mathcal{S}$. Messages are sent simultaneously and actions are chosen simultaneously too. The rules of the game Γ are commonly known between the two agents, and perfect recall is assumed, i.e., at each phase, every agent knows what she previously did and what she previously knew.

The utility of each agent is determined by two sources. First, the utility of an agent depends on whether the path she chooses leads to the prize, and on whether she is the only one who "finds" the prize. An agent i who finds the prize obtains either its entire value, which is normalized to 1, if she is the only one who finds it, or some positive amount smaller than its value, $\mathrm{z}^i \in (0, 1)$, if both find it. An agent who "follows" a path that does not lead to the prize receives nothing. Second, we let the utility of an agent depend also upon the combination of messages sent by assuming that an agent i who "declares" a type that differs from her actual one has to pay a penalty $\mathrm{q}^i \geq 0$ to the other.

Let $\mathcal{M} := M^1 \times M^2$ and $\mathcal{A} := A \times A$. A combination of messages, and a combination of actions, are denoted, respectively, by $m \in \mathcal{M}$, and by $a \in \mathcal{A}$. As usual, the profile of the agent other than i will be denoted by the corresponding superscript $-i$.

Let $v^i(a, s)$ and $c^i(m^i, s)$ denote the functions that describe formally the payoff that accrue to agent i, respectively, from the combination of actions, and from her choice over messages. Then the utility of agent i is given by a function $u^i : \mathcal{M} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ such that

$$u^i(m, a, s) = v^i(a, s) - c^i(m^i, s) + c^{-i}(m^{-i}, s),$$

which is specified by

$$v^i(a,s) := \begin{cases} 1 & \text{if } a^i = s \neq a^{-i} \\ \mathrm{z}^i & \text{if } a^i = s = a^{-i} \\ 0 & \text{if } a^i \neq s, \end{cases}$$

and by

$$c^i(m^i,s) := \begin{cases} \mathrm{q}^i & \text{if } m^i \neq s^i \\ 0 & \text{if } m^i = s^i. \end{cases}$$

Thus, each agent wishes to deduce the type of the other from the received message, and mutual gains to information sharing exist since $\mathrm{z}^i > 0$ for all $i \in \mathcal{I}$. Yet both agents have an incentive to mislead the other as $\mathrm{z}^i < 1$ for all $i \in \mathcal{I}$; i.e., the model captures a situation of *conflict of interests*. A source of tension appears in the game from that an agent incurs a penalty by lying. In some economic and/or social contexts, it is natural to think that these costs reflect the existence of an institutional arrangement or social code that punishes agents who transmit false information or "manipulate."

We set $\mathrm{z} := (\mathrm{z}^1, \mathrm{z}^2)$ and $\mathrm{q} := (\mathrm{q}^1, \mathrm{q}^2)$.

A *behavior strategy* for agent i consists of a pair (σ^i, α^i), where $\sigma^i : \mathcal{S}^i \times M^i \to [0,1]$ is a function such that $\sum_{m^i} \sigma^i(s^i, m^i) = 1$ for all $s^i \in \mathcal{S}^i$ with the interpretation that $\sigma^i(s^i, m^i)$ is the probability that i sends message m^i if she is of type s^i. Also, $\alpha^i : \mathcal{S}^i \times \mathcal{M} \times A \to [0,1]$ is a function such that $\sum_{a^i} \alpha^i(s^i, m, a^i) = 1$ for all $(s^i, m) \in \mathcal{S}^i \times \mathcal{M}$ with the interpretation that $\alpha^i(s^i, m, a^i)$ is the probability that i chooses action a^i if she is of type s^i upon the message combination m.

Beliefs over states of agent i at phase 0 are formally given by a function $p^i : \mathcal{S} \to [0,1]$ where $p^i(s)$ denotes the probability that agent i assigns to $s = (s^i, s^{-i})$ being the true state given that she is of type s^i. Also, a belief function for agent i at phase 1 is a function $\mu^i : \mathcal{S} \times M^{-i} \to [0,1]$ where $\mu^i(s, m^{-i})$ denotes the probability that i assigns to $s = (s^i, s^{-i})$ being the actual state given that she is of type s^i and receives message m^{-i}.

Let $\sigma := (\sigma^1, \sigma^2)$, $\alpha := (\alpha^1, \alpha^2)$, $p := (p^1, p^2)$, and $\mu := (\mu^1, \mu^2)$.

For $s^i \in \mathcal{S}^i$ define $r(s^i) := \sum_{\widetilde{s}^{-i}} \pi((s^i, \widetilde{s}^{-i}))$, the prior *marginal* probability that agent i be of type s^i. We assume that a belief function at phase 0 must be consistent with the "strategy" of Nature and therefore, for every $i \in \mathcal{I}$, we specify $p^i(s) := \pi(s)/r(s^i)$ for all $s = (s^i, s^{-i}) \in \mathcal{S}$.

For a strategy combination (σ, α), and a belief function μ^i, let $U^i(\sigma, \alpha, \mu^i; s^i, m)$ denote the expected utility of i conditioned on being at *decision node* (s^i, m). Analogously, for (σ, α), and a belief function p^i, let $U^i(\sigma, \alpha, p^i; s^i)$ denote the expected utility of i conditioned on being at *decision node* s^i. We have

$$U^i(\sigma, \alpha, \mu^i; s^i, m) := \sum_{s^{-i}} \mu^i(s, m^{-i}) \sum_{a^i} \sum_{a^{-i}} \alpha^i(s^i, m, a^i) \alpha^{-i}(s^{-i}, m, a^{-i}) u^i(m, a, s), \tag{2.1}$$

and

$$U^i(\sigma, \alpha, p^i; s^i) := \sum_{s^{-i}} p^i(s) \sum_{m^i} \sum_{m^{-i}} \sigma^i(s^i, m^i) \sigma^{-i}(s^{-i}, m^{-i}) U^i(\sigma, \alpha, \mu^i; s^i, m). \tag{2.2}$$

The equilibrium concept that we shall employ is that of sequential equilibrium due to Kreps and Wilson [10], which is a Nash equilibrium in the behavior strategies such that the strategy of every agent, for the beliefs that she uses, is a best response to the strategy of the other, and such that those beliefs are consistent in the sense that (i) they are derived from strategies using Bayes' rule, and (ii) they are robust to slight perturbations of strategies that induce that the agents choose completely mixed strategies.

Proposition 1. *The behavior strategies (σ^*, α^*) and the system of beliefs μ^* constitute a sequential equilibrium of Γ if for every $i \in \mathcal{I}$:*

(i) for every $s^i \in \mathcal{S}^i$;

$$\sigma^{i^*} \in \arg\max_{\sigma^i} U^i(\sigma^i, \sigma^{-i^*}, \alpha^*, p^i; s^i), \tag{2.3}$$

(ii) for every $(s^i, m) \in \mathcal{S}^i \times \mathcal{M}$;

$$\alpha^{i^*} \in \arg\max_{\alpha^i} U^i(\sigma^*, \alpha^i, \alpha^{-i^*}, \mu^{i^*}; s^i, m), \tag{2.4}$$

(iii) for every $((s^i, s^{-i}), m^{-i}) \in \mathcal{S} \times M^{-i}$ such that $\sum_{\widetilde{s}^{-i}} \sigma^{-i*}(\widetilde{s}^{-i}, m^{-i})\pi((s^i, \widetilde{s}^{-i})) > 0$;

$$\mu^{i*}((s^i, s^{-i}), m^{-i}) = \sigma^{-i*}(s^{-i}, m^{-i})\pi((s^i, s^{-i})) \Big/ \sum_{\widetilde{s}^{-i}} \sigma^{-i*}(\widetilde{s}^{-i}, m^{-i})\pi((s^i, \widetilde{s}^{-i})). \tag{2.5}$$

Proof. It suffices to prove that strategies and beliefs are consistent in the sense of sequential equilibrium. To do so, consider a $(\sigma^*, \alpha^*, \mu^*)$ that satisfies (2.3)-(2.5), and a sequence $\{\sigma_n\}$ such that, for every $i \in \mathcal{I}$, (a) $\sigma^i_n(s^i, m^i) > 0$ for all $(s^i, m^i) \in \mathcal{S}^i \times M^i$, and (b) $\{\sigma^i_n\} \to \sigma^{i*}$. Let $\{\mu_n\}$ be a sequence specified, for every $i \in \mathcal{I}$, by

$$\mu^i{}_n((s^i, s^{-i}), m^{-i}) := \sigma^{-i}_n(s^{-i}, m^{-i})\pi((s^i, s^{-i})) \Big/ \sum_{\widetilde{s}^{-i}} \sigma^{-i}_n(\widetilde{s}^{-i}, m^{-i})\pi((s^i, \widetilde{s}^{-i}))$$

for all $((s^i, s^{-i}), m^{-i}) \in \mathcal{S} \times M^{-i}$, which is well defined since $\sum_{\widetilde{s}^{-i}} \sigma^{-i}_n(\widetilde{s}^{-i}, m^{-i})\pi((s^i, \widetilde{s}^{-i})) > 0$ for all $m^{-i} \in \mathcal{M}^{-i}$ and for every $i \in \mathcal{I}$, given (a) above. From (b) we obtain that $\{\mu_n\} \to \mu^*$, and this completes the proof. ∎

Remark 1. We note that a strategy combination (σ, α) and a system of beliefs μ that do not satisfy the sequential rationality conditions in (2.3) and (2.4) fails to be a sequential equilibrium of Γ.[3]

Definition 1. Let (σ^*, α^*) be the strategy combination of a sequential equilibrium of Γ. We say that agent i *reveals completely* at equilibrium if $\sigma^{i^*}(s^i, s^i) = 1$ for all $s^i \in \mathcal{S}^i$. We say that agent i *pools* at equilibrium if there is a message $\widehat{m}^i \in M^i$ such that $\sigma^{i^*}(s^i, \widehat{m}^i) = 1$ for all $s^i \in \mathcal{S}^i$. We say that agent i *reveals partially* at equilibrium if there is a type $\widehat{s}^i \in \mathcal{S}^i$ such that (i) $\sigma^{i^*}(\widehat{s}^i, \widehat{s}^i) = 1$, and (ii) $\sigma^{i^*}(\widetilde{s}^i, \widehat{s}^i) = \lambda \in (0, 1)$ for the type $\{\widetilde{s}^i\} := \mathcal{S}^i \setminus \{\widehat{s}^i\}$.

Our model departs from a substantial part of the literature on strategic information provision (see, for example, Crawford and Sobel [5], and Green and Stokey [7]) in that lying is costly, and in that the agents who decide over actions are both senders and receivers of information. It also departs from the *burned money* models (see, for example,

[3]More precisely, conditions (2.3) and (2.4) must be necessarily satisfied by a perfect Bayesian Nash equilibrium of Γ.

Austen-Smith and Banks [2]) principally in the nature of its signaling costs. Burned money models have costs which are independent of the private information of the agents, but which are also *endogenously* chosen by the sender so that the receiver can potentially infer some information about the type of her opponent by observing such costs, which allows for equilibria more informative than in a pure cheap talk framework.

There is a certain parallel between our approach and that followed by Okuno-Fujiwara, Postlewaite, and Suzumura [13] as they too deal with strategic information sharing at an *interim* stage. Their emphasis is in providing sufficient conditions on the information and game structure under which fully revelation corresponds to a sequential equilibrium. Important differences with our model are in assuming that the set of types of each agent is an ordered one, and that only "truthful" messages are taken into account to revise beliefs. More recently, Koessler [9] considers a model of information sharing at an *interim* stage for a class of Bayesian games played at a subsequent stage; he assumes that agents only reveal "truthful" information which is the crucial difference between his approach and ours.

Our model is also related to that of Kartik [8], who studies a sender-receiver game with a one-way flow of information and costs of *misreporting* which depend on the private information of the sender. He assumes that, below a given bound, costs are *endogenously* chosen by the sender and obtains, using a refinement criterion due to Bernheim and Severinov [3], insights similar to ours regarding the existence of a positive relation between information revelation and the amount of the costs.

3. Symmetric Information Revelation

This section analyzes the existence of equilibria in our model in which the agents reveal in a symmetric way, and characterizes them. We provide necessary and sufficient conditions on z and q for the existence of equilibria in which both agents pool and of equilibria in which both agents reveal completely. It is shown that sufficiently high costs of lying are required for complete revelation of information,[4] and that equilibria in which the agents transmit no information exist for low costs. Also, we characterize the existence of a robust class of symmetric equilibria in which both agents reveal partially for a set of intermediate costs under which neither symmetric non-informative equilibria nor symmetric totally revealing equilibria exist.

As a first step to solving the game Γ we assume that rationality is common knowledge and, therefore, strictly dominated strategies will not be considered. Then, given the earlier specification of v^i, equation (2.1) can be rewritten as

$$U^i(\sigma, \alpha, \mu^i; s^i, m) = V^i(\alpha, \mu^i; s^i, m) + H^i(\sigma, \mu^i; s^i, m), \tag{3.1}$$

[4] In a context without exogenous costs, Okuno-Fujiwara, Postlewaite, and Suzumura [13] showed that the conditions that ensure information revelation at equilibrium are quite restrictive. While they require that agents can only reveal true messages, in our model the penalties serve the incentives of the agents to transmit truthful information.

where

$$V^i(\alpha, \mu^i; s^i, m) := \sum_{s^{-i}} \mu^i(s, m^{-i})\alpha^i(s^i, m, s)[1 - (1 - \mathrm{z}^i)\alpha^{-i}(s^{-i}, m, s)] \tag{3.2}$$

is the function that specifies the expected payoff of i at (s^i, m), under beliefs μ^i, due to the choices over actions α, and

$$H^i(\sigma, \mu^i; s^i, m) := \sum_{s^{-i}} \mu^i(s, m^{-i})[-c^i(m^i, s) + c^{-i}(m^{-i}, s)] \tag{3.3}$$

specifies her expected cost at (s^i, m), using beliefs μ^i, as a consequence of the choices over messages σ.

PROPOSITION 2. *There exists a sequential equilibrium of* Γ *such that both agents reveal completely if and only if* $\mathrm{q}^i \geq [1 - \mathrm{z}^i]$ *for all* $i \in \mathcal{I}$.

Proof. The outline of the proof is as follows. Given a strategy combination such that both agents reveal completely, and given a system of beliefs consistent with such strategies in the sense of equation (2.5), if an agent $i \in \mathcal{I}$ deviates to any other choice over messages then the beliefs of $-i$ must be such that, upon hearing the name of a type, she "thinks" that the sender is indeed of that type. Since such a deviation must imply that at least one type of the deviant sends the message that differs from the actual type, then every type of the opponent will optimally choose a path different from the true one at phase 2. So the deviant obtains the entire prize, of value 1, and the increase in v^i amounts to $1 - \mathrm{z}^i$ at every state which occurs with positive probability. Yet she also suffers a loss in utility which equals q^i due to the cost of lying. Thus we obtain that every agent $i \in \mathcal{I}$ finds profitable not to deviate from revealing completely so long as the net benefits from such a deviation, $[1 - \mathrm{z}^i] - \mathrm{q}^i$, are non-positive.

Formally, let $(\sigma^*, \alpha^*, \mu^*)$ be such that α^* satisfies (2.4) for μ^*, μ^* satisfies (2.5) for σ^*, and ${\sigma^i}^*(s^i, s^i) = 1$ for all $s^i \in \mathcal{S}^i$ for every $i \in \mathcal{I}$. For a given agent $i \in \mathcal{I}$ consider an arbitrary deviation $\widetilde{\sigma}^i$ from ${\sigma^i}^*$. Then, we must have that $\widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i) > 0$ for some type $\widehat{s}^i \in \mathcal{S}^i$ and for the message $\{\widetilde{m}^i\} \equiv \{\widetilde{s}^i\} := \mathcal{S}^i \setminus \{\widehat{s}^i\}$. From (2.5) the beliefs of $-i$, upon hearing $\widetilde{m}^i$, when agent i is of type $\widehat{s}^i$ and agent $-i$ is of type s^{-i} so that the actual state is $(\widehat{s}^i, s^{-i})$, are given by

$$\mu^{-i^*}((\widehat{s}^i, s^{-i}), \widetilde{m}^i) = 0\pi((\widehat{s}^i, s^{-i})) \Big/ [0\pi((\widehat{s}^i, s^{-i})) + 1\pi((\widetilde{s}^i, s^{-i}))] = 0,$$

and, therefore, by using equation (2.2), we obtain the expected utility of the deviant when she is type $\widehat{s}^i$,

$$\begin{aligned} U^i(\widetilde{\sigma}^i, \sigma^{-i^*}, \alpha^*, p^i; \widehat{s}^i) &= \sum_{s^{-i}} p^i((\widehat{s}^i, s^{-i}))\widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i) U^i(\widetilde{\sigma}^i, \sigma^{-i^*}, \alpha^*, \mu^{i^*}; \widehat{s}^i, (\widetilde{m}^i, s^{-i})) \\ &+ \sum_{s^{-i}} p^i((\widehat{s}^i, s^{-i})) \left[1 - \widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i)\right] U^i(\widetilde{\sigma}^i, \sigma^{-i^*}, \alpha^*, \mu^{i^*}; \widehat{s}^i, (\widehat{s}^i, s^{-i})) \\ &= \widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i)[1 - \mathrm{q}^i] + \left[1 - \widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i)\right] \mathrm{z}^i, \end{aligned}$$

so that

$$U^i(\sigma^*, \alpha^*, p^i; \widehat{s}^i) - U^i(\widetilde{\sigma}^i, \sigma^{-i^*}, \alpha^*, p^i; \widehat{s}^i) =$$

$$\mathrm{z}^i - \big[\widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i)[1-\mathrm{q}^i] + \big[1-\widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i)\big]\,\mathrm{z}^i\big] = \widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i)\big[\mathrm{q}^i - [1-\mathrm{z}^i]\big] \geq 0$$
$$\Leftrightarrow \mathrm{q}^i \geq [1-\mathrm{z}^i]$$

as $\widetilde{\sigma}^i(\widehat{s}^i, \widetilde{m}^i) > 0$.

The proof follows by noting that $\widehat{s}^i$ was arbitrarily chosen, and that a deviation from σ^{i*} only affects the expected utility of agent i when she is of the type that sends not the true message. ∎

Remark 2. Proposition 2 must be interpreted with due care since, for $i \in \mathcal{I}$, trivially, $[1-\mathrm{z}^i] \to 0$ as $\mathrm{z}^i \to 1$.

We need to set a bit of extra notation. For $\widehat{s}^i \in \mathcal{S}^i$ define $\pi(\widehat{s}^i)^+ := \max_{s\in\mathcal{S}} \{\pi(s) \mid s^i = \widehat{s}^i\}$, $\pi(\widehat{s}^i)^- := \min_{s\in\mathcal{S}} \{\pi(s) \mid s^i = \widehat{s}^i\}$, and $s(\widehat{s}^i)^+ := \arg\max_{s\in\mathcal{S}} \{\pi(s) \mid s^i = \widehat{s}^i\}$, the state that agent i assigns maximum prior probability of occurring if she is of type $\widehat{s}^i$.

Assumption 1. For all $i \in \mathcal{I}$, π and z^i are such that $\mathrm{z}^i\pi(s^i)^+ < \pi(s^i)^-$ for all $s^i \in \mathcal{S}^i$.

Consider the modified game obtained from Γ by removing phase 1, and thus also the information exchange and the costs due to the choice over messages so that, after receiving their private information, the agents simply choose one of the available paths before they receive the utility just derived from their decisions over actions. Then Assumption 1 above ensures that for a such a modified game, given the strategy of the opponent, the expected utility of every agent $i \in \mathcal{I}$ at every decision node s^i (which uses only the priors) is higher when she chooses an action different from the one chosen by the opponent. So Assumption 1 describes formally an environment where, the conflict of interests is stronger than that derived from just considering $\mathrm{z}^i < 1$ for all $i \in \mathcal{I}$.[5] This assumption will be useful to specify a combination of choices over actions that be sequentially rational according to the condition in (2.4) when every agent receives no information from her opponent at phase 1 and, therefore, must use her priors to decide her action at phase 2. We note that, since $\mathrm{z}^i \in (0,1)$ for all $i \in \mathcal{I}$, a special case of interest in which Assumption 1 holds is when π is uniformly distributed over states.

Lemma 1. *Suppose that Assumption 1 holds and let σ^* be a combination of choices over messages such that every type of each agent $i \in \mathcal{I}$ sends some given message $\widehat{m}^i \in M^i$. Then, for every type $s^i \in \mathcal{S}^i$ of each agent $i \in \mathcal{I}$, there is a combination of choices over actions $\alpha^*[s^i]$ such that:*
(i) $\alpha^*[s^i]$ satisfies (2.4) under a system of beliefs which satisfies (2.5) for σ^*,
(ii) $V^i(\alpha^*[s^i], p^i; s^i, \widehat{m}) \equiv \max_\alpha V^i(\alpha, p^i; s^i, \widehat{m}) = \pi(s^i)^+/r(s^i)$, and
(iii) $V^i(\alpha^*[s^i], p^i; s^i, \widetilde{m}^i, \widehat{m}^{-i}) \equiv \min_\alpha V^i(\alpha, p^i; s^i, \widetilde{m}^i, \widehat{m}^{-i}) = \mathrm{z}^i\pi(s^i)^+/r(s^i)$ for the message $\{\widetilde{m}^i\} := M^i \setminus \{\widehat{m}^i\}$.

We shall now provide the condition on z and q that completely characterizes the existence of equilibria in which both agents pool. First, we specify some extra notation for describing bounds on penalties. For $i \in \mathcal{I}$, define $\theta^i(p)^- := \min_{s^i\in\mathcal{S}^i} \{\pi(s^i)^+/r(s^i)\}$ and $\theta^i(p)^+ := \max_{s^i\in\mathcal{S}^i} \{\pi(s^i)^+/r(s^i)\}$; clearly, $\theta^i(p)^-$, $\theta^i(p)^+ \in (0,1)$ for any $p \in \mathbb{R}^4_{++}$.

[5] In intuitive terms, Assumption 1 is satisfied when either (i) the shape of π is such that it is not too far from being uniform, or (ii) the shares z are sufficiently small.

PROPOSITION 3. *Under Assumption 1, there exists a sequential equilibrium of* Γ *such that both agents pool if and only if* $0 \leq \mathrm{q}^i \leq [1-\mathrm{z}^i]\theta^i(p)^-$ *for every* $i \in \mathcal{I}$.

Proof. Let (σ^*, μ^*) be such that $\sigma^{i*}(s^i, \widehat{m}^i) = 1$ for all $s^i \in \mathcal{S}^i$ and for some given message $\widehat{m}^i \in M^i$, and for both $i \in \mathcal{I}$, and such that σ^* and μ^* satisfy (2.5).

Consider a given agent $i \in \mathcal{I}$, we have that the type $\{\widetilde{s}^i\} \equiv \{\widetilde{m}^i\} := M^i \backslash \{\widehat{m}^i\}$ is sending a message that does not coincides with that type so that she incurs the penalty q^i. To study how the utility of i changes when she deviates to revealing some information, it suffices to analyze the change in her expected utility at the decision node $\widetilde{s}^i$. Suppose that i deviates from σ^{i*} to a $\tilde{\sigma}^i$ such that $\tilde{\sigma}^i(\widetilde{s}^i, \widetilde{m}^i) > 0$, then her expected utility at $\widetilde{s}^i$ is additively increased by $\tilde{\sigma}^i(\widetilde{s}^i, \widetilde{m}^i)\mathrm{q}^i$. However, i also suffers a loss in her expected utility at $\widetilde{s}^i$ due to the change induced by the deviation on the choice of $-i$ over actions. Given Lemma 1, we know that there is a combination of choices over actions $\alpha^*[\widetilde{s}^i]$ which satisfies (2.4) for the beliefs μ^{i*}, and such that $V^i(\alpha^*[\widetilde{s}^i], p^i; \widetilde{s}^i, \widehat{m}) \equiv \max_\alpha V^i(\alpha, p^i; s^i, \widehat{m}) = \pi(\widetilde{s}^i)^+/r(\widetilde{s}^i)$, and $V^i(\alpha^*[\widetilde{s}^i], p^i; \widetilde{s}^i, \widetilde{m}^i, \widehat{m}^{-i}) \equiv \min_\alpha V^i(\alpha, p^i; s^i, \widetilde{m}^i, \widehat{m}^{-i}) = \mathrm{z}^i\pi(\widetilde{s}^i)^+/r(\widetilde{s}^i)$. Therefore, by using the choice over actions $\alpha^*[\widetilde{s}^i]$ identified in Lemma 1, the additive loss in the expected utility of i from deviating to σ^{i*} amounts to $-\tilde{\sigma}^i(\widetilde{s}^i, \widetilde{m}^i)[1-\mathrm{z}^i]\pi(\widetilde{s}^i)^+/r(\widetilde{s}^i)$ so that the deviation is strictly profitable to i if and only if $\mathrm{q}^i > [1-\mathrm{z}^i]\pi(\widetilde{s}^i)^+/r(\widetilde{s}^i)$, and $(\alpha^*[\widetilde{s}^i], \sigma^*, \mu^*)$ is a sequential equilibrium of Γ if $\mathrm{q}^i \leq [1-\mathrm{z}^i]\pi(\widetilde{s}^i)^+/r(\widetilde{s}^i)$.

The result follows by noting that the strategy choice over actions identified in Lemma 1 implies the maximum loss in utility of the type that deviates from pooling to revealing some information, and that if $\mathrm{q}^i \leq [1-\mathrm{z}^i]\pi(s^i)^+/r(s^i)$ for all $s^i \in \mathcal{S}^i$ and every $i \in \mathcal{I}$, then no type of any agent has strict incentives to deviate from pooling. ∎

The following Lemma will be useful to establish Proposition 4.

LEMMA 2. *For every probability distribution* $\pi \in \mathbb{R}^4_{++}$ *there exists a specification of the state space* $\mathcal{S} := \{(\widehat{s}^1, \widehat{s}^2), (\widehat{s}^1, \widetilde{s}^2), (\widetilde{s}^1, \widehat{s}^2), (\widetilde{s}^1, \widetilde{s}^2)\}$ *satisfying* $s(\widetilde{s}^i)^+ = (\widetilde{s}^i, \widehat{s}^{-i})$ *for both* $i \in \mathcal{I}$.

Proof. The result yields by noting that if it does not hold for some $\widetilde{\pi}$ then, necessarily, the relation $\geq$ fails transitivity on the set $\{\widetilde{\pi}(s) \,|\, s \in \mathcal{S}\}$. ∎

Before stating our results on the existence of symmetric partially revealing equilibria, we introduce a definition that will be useful to specify bounds on penalties. For a type $s^i \in \mathcal{S}^i$ let $\Psi^{s^i} : [0,1] \to \mathbb{R}$ be the function specified by

$$\Psi^{s^i}(\lambda) := (1-\lambda) + \lambda\pi(s^i)^+/r(s^i).$$

The function Ψ^{s^i} is strictly decreasing with respect to λ for every $s^i \in \mathcal{S}^i$ and each $i \in \mathcal{I}$, and it satisfies that, for a given $\widetilde{s}^i \in \mathcal{S}^i$, if $\lambda \to 0$ then $\Psi^{\widetilde{s}^i}(\lambda) \to [1-\mathrm{z}^i]$ while if $\lambda \to 1$ then $\Psi^{\widetilde{s}^i}(\lambda) \to [1-\mathrm{z}^i]\pi(\widetilde{s}^i)^+/r(\widetilde{s}^i)$.

PROPOSITION 4. *There exists a sequential equilibrium of* Γ *such that both agents* $i \in \mathcal{I}$ *reveal partially by choosing, for some given type* $\widehat{s}^i \in \mathcal{S}^i$, *(i)* $\sigma^{i*}(\widehat{s}^i, \widehat{m}^i) = 1$ *where* $\widehat{m}^i \equiv \widehat{s}^i$, *and (ii)* $\sigma^{i*}(\widetilde{s}^i, \widehat{m}^i) = \lambda \in (0,1)$ *for the type* $\{\widetilde{s}^i\} := \mathcal{S}^i \setminus \{\widehat{s}^i\}$, *and choose symmetrically over actions, if and only if* $\mathrm{q}^i \geq [1-\mathrm{z}^i]\Psi^{\widehat{s}^i}(\lambda)$ *and* $\mathrm{q}^i \equiv [1-\mathrm{z}^i]\Psi^{\widetilde{s}^i}(\lambda)$ *for all* $i \in \mathcal{I}$.

Proof. Given Lemma 2, fix a type $\widehat{s}^i \in \mathcal{S}^i$ for each agent $i \in \mathcal{I}$, and set $\{\widetilde{s}^i\} := \mathcal{S}^i \setminus \{\widehat{s}^i\}$ in a way such that $s(\widetilde{s}^i)^+ = (\widetilde{s}^i, \widehat{s}^{-i})$ for both $i \in \mathcal{I}$. Also, set $\widehat{m}^i \equiv \widehat{s}^i$ and $\widetilde{m}^i \equiv \widetilde{s}^i$ for every $i \in \mathcal{I}$.

Now consider a given agent $i \in \mathcal{I}$, and let $(\sigma^*, \alpha^*, \mu^*)$ be such that $\sigma^{-i^*}(\widehat{s}^{-i}, \widehat{m}^{-i}) = 1$, $\sigma^{-i^*}(\widetilde{s}^{-i}, \widehat{m}^{-i}) = \lambda \in (0,1)$, and such that μ^* and σ^* satisfy (2.5). Then, using (2.2) combined with (3.1), upon substituting each $H^i(\sigma^*, \mu^{i^*}; s^i, m)$ according to (3.3), we obtain that, for each $s^i \in \mathcal{S}^i$ and for $\{\breve{s}^i\} := \mathcal{S}^i \setminus \{s^i\}$,

$$\begin{aligned}
U^i(\sigma^*, \alpha^*, p^i; s^i) = \sigma^{i^*}(s^i, \breve{s}^i)\Big[& p^i((s^i, \widehat{s}^{-i}))[V^i(\alpha^*, \mu^{i^*}; s^i, (\breve{s}^i, \widehat{m}^{-i})) - \mathrm{q}^i] \\
&+ p^i((s^i, \widetilde{s}^{-i}))\Big[\lambda[V^i(\alpha^*, \mu^{i^*}; s^i, (\breve{s}^i, \widehat{m}^{-i})) - \mathrm{q}^i + \mathrm{q}^{-i}] + \\
&\qquad (1-\lambda)[V^i(\alpha^*, \mu^{i^*}; s^i, (\breve{s}^i, \widetilde{m}^{-i})) - \mathrm{q}^i]\Big]\Big] \\
+ \sigma^{i^*}(s^i, s^i)\Big[& p^i((s^i, \widehat{s}^{-i}))V^i(\alpha^*, \mu^{i^*}; s^i, (s^i, \widehat{m}^{-i})) \\
&+ p^i((s^i, \widetilde{s}^{-i}))\Big[\lambda[V^i(\alpha^*, \mu^{i^*}; s^i, (s^i, \widehat{m}^{-i})) + \mathrm{q}^{-i}] + \\
&\qquad (1-\lambda)V^i(\alpha^*, \mu^{i^*}; s^i, (s^i, \widetilde{m}^{-i}))\Big]\Big].
\end{aligned} \tag{3.4}$$

We exploit the indifference condition for type $\widetilde{s}^i$ to randomize between sending messages $\widehat{m}^i$ and $\widetilde{m}^i$ and, therefore, we obtain from (3.4) that

$$\begin{aligned}
q^i \equiv & \left[p^i((\widetilde{s}^i, \widehat{s}^{-i})) + \lambda p^i(\widetilde{s})\right]\left[V^i(\alpha^*, \mu^{i^*}; \widetilde{s}^i, \widehat{m}) - V^i(\alpha^*, \mu^{i^*}; \widetilde{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}))\right] + \\
& (1-\lambda)p^i(\widetilde{s})\left[V^i(\alpha^*, \mu^{i^*}; \widetilde{s}^i, (\widehat{m}^i, \widetilde{m}^{-i})) - V^i(\alpha^*, \mu^{i^*}; \widetilde{s}^i, \widetilde{m})\right]
\end{aligned} \tag{3.5}$$

is a necessary and sufficient condition for $\sigma^{i^*}(\widetilde{s}^i, \widehat{m}^i) = \lambda \in (0,1)$ in equilibrium. As for type $\widehat{s}^i$, from (3.4) it follows that

$$\begin{aligned}
q^i \geq & \left[p^i(\widehat{s}) + \lambda p^i((\widehat{s}^i, \widetilde{s}^{-i}))\right]\left[V^i(\alpha^*, \mu^{i^*}; \widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i})) - V^i(\alpha^*, \mu^{i^*}; \widehat{s}^i, \widehat{m})\right] + \\
& (1-\lambda)p^i((\widehat{s}^i, \widetilde{s}^{-i}))\left[V^i(\alpha^*, \mu^{i^*}; \widehat{s}^i, \widetilde{m}) - V^i(\alpha^*, \mu^{i^*}; \widehat{s}^i, (\widehat{m}^i, \widetilde{m}^{-i}))\right]
\end{aligned} \tag{3.6}$$

is a necessary and sufficient condition for $\sigma^{i^*}(\widehat{s}^i, \widehat{m}^i) = 1$ in equilibrium.

We turn to analyze the expected payoffs of the agents due to a symmetric choice over actions in equilibrium. First, note that, since μ^{i^*} and σ^{-i^*} satisfy (2.5) for every $i \in \mathcal{I}$, it follows that, for all $s^i \in \mathcal{S}^i$,

$$\mu^{i^*}((s^i, \widetilde{s}^{-i}), \widetilde{m}^{-i}) = (1-\lambda)\pi((s^i, \widetilde{s}^{-i}))\Big/[(1-\lambda)\pi((s^i, \widetilde{s}^{-i})) + 0\pi((s^i, \widehat{s}^{-i}))] = 1.$$

Second, for the received message $\widehat{m}^{-i}$, we have, for all $s^i \in \mathcal{S}^i$, and every $i \in \mathcal{I}$,

$$\mu^{i^*}((s^i, \widehat{s}^{-i}), \widehat{m}^{-i}) = \pi((s^i, \widehat{s}^{-i}))\Big/[\pi((s^i, \widehat{s}^{-i})) + \lambda\pi((s^i, \widetilde{s}^{-i}))].$$

Using this, together with the fact that $\lambda < 1$ and $\pi((\widetilde{s}^i, \widehat{s}^{-i})) \geq \pi(\widetilde{s})$, it follows that

$$\alpha^{i^*}(s^i, (m^i, \widetilde{m}^{-i}), (s^i, \widetilde{s}^{-i})) = 1, \quad \text{and} \quad \alpha^{i^*}(s^i, (m^i, \widehat{m}^{-i}), (s^i, \widehat{s}^{-i})) = 1,$$

for all $m^i \in M^i$, all $s^i \in \mathcal{S}^i$, and for each $i \in \mathcal{I}$, specifies the unique symmetric combination of choices over actions that satisfies equation (2.4) for μ^*. Thus, by using (3.2), we obtain, for every $s^i \in \mathcal{S}^i$, and every $i \in \mathcal{I}$, under the α^* specified above,

$$V^i(\alpha^*, \mu^{i*}; s^i, (\breve{s}^i, \widehat{m}^{-i})) = \pi((s^i, \widehat{s}^{-i})) \Big/ [\pi((s^i, \widehat{s}^{-i})) + \lambda\pi((s^i, \widetilde{s}^{-i}))],$$

$$V^i(\alpha^*, \mu^{i*}; s^i, (s^i, \widehat{m}^{-i})) = \left[\pi((s^i, \widehat{s}^{-i})) \Big/ [\pi((s^i, \widehat{s}^{-i})) + \lambda\pi((s^i, \widetilde{s}^{-i}))]\right] \mathrm{z}^i,$$

$$V^i(\alpha^*, \mu^{i*}; s^i, (\breve{s}^i, \widetilde{m}^{-i})) = 1, \quad \text{and} \quad V^i(\alpha^*, \mu^{i*}; s^i, (s^i, \widetilde{m}^{-i})) = \mathrm{z}^i,$$

so that, by doing the algebra, the conditions in (3.5) and in (3.6) become, respectively, $\mathrm{q}^i \equiv [1 - \mathrm{z}^i]\Psi^{\widetilde{s}^i}(\lambda)$ and $\mathrm{q}^i \geq [1 - \mathrm{z}^i]\Psi^{\widehat{s}^i}(\lambda)$ as required. ∎

Remark 3. From the specification of Ψ^{s^i}, the requirement $\mathrm{q}^i \geq [1 - \mathrm{z}^i]\Psi^{\widehat{s}^i}(\lambda)$ in the statement of Proposition 4 can be replaced by $\theta^i(p)^+ = \pi(\widetilde{s}^i)^+ / r(\widetilde{s}^i)$.

Remark 4. Proposition 4 establishes the existence of a class of equilibria in which the agents behave symmetrically with respect to both the choice over messages and the choice over actions whenever $[1-\mathrm{z}^i]\theta^i(p)^+ < \mathrm{q}^i < [1-\mathrm{z}^i]$, i.e., a region for q^i under which Proposition 2 together with Proposition 3 have established the non-existence of symmetric pooling and of symmetric completely revealing equilibria. Further, we see that the informativeness of such equilibria is increasing with respect to q.

Remark 5. Proposition 4 provides conditions on q and z that characterize completely the existence of a particular class of equilibria of Γ under which both agents reveal partially in a symmetric way. Nonetheless, since signaling games exhibit typically multiple equilibria and Γ is a bilateral signaling game, there is every reason to think that it may have other classes of equilibria such that both agents reveal partially.

4. Asymmetric Information Revelation

This section establishes the existence of equilibria in which the agents reveal their private information in a asymmetric way. It is shown that there exists a class of equilibria of Γ in which an agent reveals completely for costs above a bound that is positively related to the amount of information transmitted by her opponent. While we cannot offer a complete exercise of comparative statics as a multiplicity of equilibria is to be expected for Γ, the results we obtain enable us to answer a few questions on comparative statics given the non-existence results shown in Proposition 2 and in Proposition 3.

The following assumption will be useful to strengthen the results obtained in this section.

Assumption 2. There exists a given type $\widehat{s}^i \in \mathcal{S}^i$ of some agent $i \in \mathcal{I}$ such that $s(s^{-i})^+ = (\widehat{s}^i, s^{-i})$ for all $s^{-i} \in \mathcal{S}^{-i}$.

Assumption 2 says that priors are such that there is a type of an agent which is assigned maximum prior probability by her opponent, independently of the initial private information of the latter. It enables us to construct a particular choice over actions that be part of an equilibrium when an agent faces an opponent who transmits her no information. We note that Assumption 2 is trivially satisfied if π is uniformly distributed over states.

Proposition 5. *Suppose that (i)* $\mathrm{q}^i \equiv [1-\mathrm{z}^i]$ *for some given agent* $i \in \mathcal{I}$*, and that (ii)* $\mathrm{q}^{-i} \geq [1-\mathrm{z}^{-i}]\Psi^{s^{-i}}(\lambda)$ *for all* $s^{-i} \in \mathcal{S}^{-i}$*, and all* $\lambda \in [0, \widehat{\lambda}]$ *where* $\widehat{\lambda} > 0$ *is sufficiently small. Then there exists a sequential equilibrium of* Γ *such that* i *chooses (1)* $\sigma^{i*}(\widehat{s}^i, \widehat{m}^i) = 1$*, for a given type* $\widehat{s}^i \in \mathcal{S}^i$ *and for* $\widehat{m}^i \equiv \widehat{s}^i$*, and (2)* $\sigma^{i*}(\widetilde{s}^i, \widehat{m}^i) = \lambda \in [0, \widehat{\lambda}]$ *for the type* $\{\widetilde{s}^i\} := \mathcal{S}^i \setminus \{\widehat{s}^i\}$*, and* $-i$ *reveals completely.*
Further, if Assumption 2 is satisfied for the type $\widehat{s}^i$ *then the result above holds for all* $\lambda \in [0, 1]$.

Proof. Let $(\sigma^*, \alpha^*, \mu^*)$ be such that $\sigma^{1*}(0, 0) = 1$, $\sigma^{1*}(1, 0) = \lambda \in [0, \widehat{\lambda}]$ for (a) $\widehat{\lambda} > 0$ sufficiently small, as hypothesized without loss of generality in the proposition, such that $\sigma^{2*}(s^2, s^2) = 1$ for all $s^2 \in \mathcal{S}^2$, and such that and μ^* and σ^* satisfy (2.5).

From this it follows that $\mu^{1*}((s^1, s^2), m^2) = 1$ for every $s^1 \in \mathcal{S}^1$, and for all $s^2 \in \mathcal{S}^2$ and $m^2 \in M^2$ such that $m^2 \equiv s^2$, so that (b) $\alpha^{1*}(s^1, (m^1, s^2), (s^1, s^2)) = 1$, for all $(s^1, m^1) \in \mathcal{S}^1 \times M^1$ and all $s^2 \in \mathcal{S}^2$, is obtained by imposing (2.4) to α^*. Also, it follows that, for all $s^2 \in \mathcal{S}^2$, (c) $\mu^{2*}((0, s^2), 0) = \pi((0, s^2))/[\pi((0, s^2)) + \lambda\pi((1, s^2))]$, and (d) $\mu^{2*}((1, s^2), 1) = (1-\lambda)\pi((1, s^2))/[(1-\lambda)\pi((1, s^2)) + 0\pi((0, s^2))] = 1$. Therefore, by combining (a), (b), and (c) above, we obtain that if (2.4) is satisfied then $\alpha^{2*}(s^2, (0, s^2), (0, s^2)) = 1$ for all $s^2 \in \mathcal{S}^2$. In addition, by using (d) above, we know that $\alpha^{2*}(s^2, (1, s^2), (1, s^2)) = 1$, for all $s^2 \in \mathcal{S}^2$, must hold necessarily when (2.4) is satisfied. This completes a specification of α^* that satisfies the sequential rationality condition in (2.4) for the beliefs μ^* obtained above. Furthermore, we note that if Assumption 2 is satisfied for the type $\widehat{s}^i$ then that specification of α^* continue to satisfy the condition in (2.4) under μ^* for any $\lambda \in [0, 1]$.

Now, for agent 1, given the α^* specified above, we know, by using (2.2) combined with (3.1), upon substituting each $H^1(\sigma, \mu^{1*}; s^1, m)$ according to (3.3), that $\sigma^{1*}(0, 0) = 1$ satisfies the sequential rationality condition in (2.3) if

$$\mathrm{q}^1 \geq \sum_{s^2} p^1((0, s^2))\Big[V^1(\alpha^*, \mu^{1*}; 0, (1, s^2)) - V^1(\alpha^*, \mu^{1*}; 0, (0, s^2))\Big] = [1-\mathrm{z}^1]. \tag{4.1}$$

In addition, for the α^* specified above, $\sigma^{1*}(1, 0) = \lambda \in [0, 1]$ is compatible with (2.3) if

$$\mathrm{q}^1 \equiv \sum_{s^2} p^1((1, s^2))\Big[V^1(\alpha^*, \mu^{1*}; 1, (0, s^2)) - V^1(\alpha^*, \mu^{1*}; 1, (1, s^2))\Big] = [1-\mathrm{z}^1]. \tag{4.2}$$

Obviously, (4.1) and (4.2) are simultaneously satisfied if and only if $\mathrm{q}^1 \equiv [1-\mathrm{z}^1]$.

As for agent 2, given the α^* specified above, by using (2.2), (3.1) and (3.3), it follows that $\sigma^{2*}(s^2, s^2) = 1$ for all $s^2 \in \mathcal{S}^2$ satisfies condition (2.3) if, for all $s^2 \in \mathcal{S}^2$ and for $\{\breve{s}^2\} := \mathcal{S}^2 \setminus \{s^2\}$,

$$\begin{aligned}
\mathrm{q}^2 \geq\ & \Big[p^2((0, s^2)) + \lambda p^2((1, s^2))\Big]\Big[V^2(\alpha^*, \mu^{2*}; s^2, (0, \breve{s}^2)) - V^2(\alpha^*, \mu^{2*}; s^2, (0, s^2))\Big] \\
& + (1-\lambda)p^2((1, s^2))\Big[V^2(\alpha^*, \mu^{2*}; s^2, (1, \breve{s}^2)) - V^2(\alpha^*, \mu^{2*}; s^2, (1, s^2))\Big] \\
=\ & \Big[(1-\lambda) + \lambda\frac{\pi((0, s^2))}{r(s^2)}\Big][1-\mathrm{z}^2] \\
=\ & [1-\mathrm{z}^2]\Psi^{s^2}(\lambda).
\end{aligned}$$

The result follows since, under the required hypotheses, we have specified without loss of generality an equilibrium as described in the statement of Proposition 5. ∎

Remark 6. If $\mathrm{q}^i \equiv [1-\mathrm{z}^i]$ for some given agent $i \in \mathcal{I}$ then Proposition 5 combined with the result in Proposition 2 show the existence of a class of equilibria in which agent $-i$ reveals completely for penalties that decrease as agent i reveals less amount of information. Further, we see that there are equilibria in which agent i does not reveal totally and agent $-i$ reveals completely for penalties lower than $[1-\mathrm{z}^{-i}]$ which would be incompatible with $-i$'s best response if it were the case that i reveals totally, as shown in Proposition 2. Also, Proposition 5 shows the existence of equilibria in which agent i pools, provided that she receives some information from $-i$, for penalties higher than $[1-\mathrm{z}^i]\theta^i(p)^-$ which cannot correspond to her best response if it were the case that the choice of $-i$ be totally uninformative, as shown in Proposition 3.

Remark 7. It is natural to address the question of what costs ensure *ex-ante* Pareto optimality in our model. However, since for a given q uniqueness of equilibria of Γ is not guaranteed, such an exercise needs further qualification of our investigation (e.g., by invoking equilibria selection criteria using, possibly, a refinement of sequential equilibrium).

5. Discussion

This paper attempts to study the nature of the incentives to share private information in situations of conflict under incomplete information. We wanted to work with a model in which the agents (i) were both senders and receivers of information, (ii) were able to condition their information sharing decisions on their *realized* private information, and (iii) may choose not to reveal the truth. Our results suggest that there is a positive relation between information revelation and the amount of the costs of lying, and that symmetric fully revelation requires one to impose sufficiently "high" transfers.[6] Also, we have shown that, for intermediate income transfers or costs, there are equilibria in which the agents wish to communicate symmetrically in a partial manner. Another interesting conclusion suggested by our model is that information disclosure has a collective bilateral structure.[7] This is due to that the information that every agent reveals affects the beliefs of the other and thus the computation of expected utility that they use to decide the amount of information that they provide.

The model we have proposed is special in several respects. For example, we have assumed very specific information structures and payoffs. Clearly a detailed analysis of information structures and of the nature of the payoffs present in relevant economic and/or social encounters is required. Also, a worthwhile extension of our investigation should attempt to select among the equilibria of Γ, which would allow for both a more detailed comparative statics exercise and for an appropriate analysis of *ex-ante* Pareto optimality. We believe that all these are subjects for future research.

[6] In a sender-receiver game with a one-way flow of information, Crawford and Sobel [5] argue that fully revelation is not to be expected when the interests of the agents do not coincide.

[7] Aumann and Hart [1] have noted the importance of considering bilateral "conversations" in their work on information revelation in cheap talk games.

6. Appendix

This appendix is devoted to the proof of Lemma 1.

Proof of Lemma 1. Let σ^* be such such that for each $i \in \mathcal{I}$ there is a given message $\widehat{m}^i \in M^i$ such that $\sigma^{i*}(s^i, \widehat{m}^i) = 1$ for all $s^i \in \mathcal{S}^i$. First, we note that if μ^* is a system of beliefs consistent with σ^* as required by equation (2.5) then, necessarily, $\mu^{i*}(s, \widehat{m}^{-i}) = p^i(s)$ for all $s \in \mathcal{S}$, for each $i \in \mathcal{I}$.

Now, fix a type $\widehat{s}^i$ of a given agent $i \in \mathcal{I}$, and let $s(\widehat{s}^i)^+ =: (\widehat{s}^i, \widehat{s}^{-i})$ for some type $\widehat{s}^{-i} \in \mathcal{S}^{-i}$. Set $\{\widetilde{s}^i\} := \mathcal{S}^i \setminus \{\widehat{s}^i\}$ for every $i \in \mathcal{I}$. We begin specifying $\alpha^*[\widehat{s}^i]$ by imposing:

(a) $\alpha^{i*}[\widehat{s}^i](\widehat{s}^i, \widehat{m}, s(\widehat{s}^i)^+) = 1$,
(b) $\alpha^{-i*}[\widehat{s}^i](\widehat{s}^{-i}, \widehat{m}, s(\widehat{s}^i)^+) = 0$,
(c) $\alpha^{-i*}[\widehat{s}^i](\widetilde{s}^{-i}, \widehat{m}, (\widehat{s}^i, \widetilde{s}^{-i})) = 1$, and
(d) $\alpha^{i*}[\widehat{s}^i](\widetilde{s}^i, \widehat{m}, (\widetilde{s}^i, \widetilde{s}^{-i})) = 1$.

Then, using (3.1) and (3.2) together with the specification of $\alpha^*[\widehat{s}^i]$ given in (b) and (c) above, we obtain

$$U^i(\sigma^*,\alpha^i, \alpha^{-i*}[\widehat{s}^i], p^i; \widehat{s}^i, \widehat{m}) = H^i(\sigma^*, p^i; \widehat{s}^i, \widehat{m}) + \frac{1}{r(\widehat{s}^i)}\left[\left[\pi(\widehat{s}^i)^+ - \pi((\widehat{s}^i, \widetilde{s}^{-i}))\mathrm{z}^i\right] \alpha^i(\widehat{s}^i, \widehat{m}, s(\widehat{s}^i)^+) + \pi((\widehat{s}^i, \widetilde{s}^{-i}))\mathrm{z}^i\right], \tag{6.1}$$

an expression which is maximized by choosing $\alpha^{i*}[\widehat{s}^i](\widehat{s}^i, \widehat{m}, s(\widehat{s}^i)^+) = 1$ since $\pi(\widehat{s}^i)^+ \geq \pi((\widehat{s}^i, \widetilde{s}^{-i}))$ and $\mathrm{z}^i < 1$, as stated in (a) above. Analogously, consider agent $-i$. From the specification of $\alpha^*[\widehat{s}^i]$ in (a) and (d) above, we have

$$U^{-i}(\sigma^*,\alpha^{-i}, \alpha^{i*}[\widehat{s}^i], p^{-i}; \widehat{s}^{-i}, \widehat{m}) = H^{-i}(\sigma^*, p^{-i}; \widehat{s}^{-i}, \widehat{m}) + \frac{1}{r(\widehat{s}^{-i})}\left[\left[\pi(\widehat{s}^i)^+\mathrm{z}^{-i} - \pi((\widetilde{s}^i, \widehat{s}^{-i}))\right] \alpha^{-i}(\widehat{s}^{-i}, \widehat{m}, s(\widehat{s}^i)^+) + \pi(\widehat{s}^i)^+\mathrm{z}^{-i}\right],$$

so that the sequential rationality condition in (2.4) is satisfied at node $(\widehat{s}^{-i}, \widehat{m})$ by choosing $\alpha^{-i*}[\widehat{s}^i](\widehat{s}^{-i}, \widehat{m}, s(\widehat{s}^i)^+) = 0$, as stated in (b) above, since, from Assumption 1, we know that $\pi(\widehat{s}^i)^+\mathrm{z}^{-i} - \pi((\widetilde{s}^i, \widehat{s}^{-i})) < 0$. Using Assumption 1, it can be shown in a completely analogous manner that, for the message combination $\widehat{m}$, the combination of choices over actions specified by (a)-(d) above also satisfies (2.4) for the types $\widetilde{s}^i$ and $\widetilde{s}^{-i}$. Furthermore, from (6.1) it follows, by taking $\alpha^{i*}[\widehat{s}^i](\widehat{s}^i, \widehat{m}, s(\widehat{s}^i)^+) = 1$, that $V^i(\alpha^*[\widehat{s}^i], p^i; \widehat{s}^i, \widehat{m}) = \pi(\widehat{s}^i)^+/r(\widehat{s}^i) \geq V^i(\alpha, p^i; \widehat{s}^i, \widehat{m})$ for all α, as stated in Lemma 1, (ii).

We specify $\alpha^*[\widehat{s}^i]$ for the message combination $(\widetilde{m}^i, \widehat{m}^{-i})$, where $\{\widetilde{m}^i\} := M^i \setminus \{\widehat{m}^i\}$ by:

(e) $\alpha^{-i*}[\widehat{s}^i](\widehat{s}^{-i}, (\widetilde{m}^i, \widehat{m}^{-i}), s(\widehat{s}^i)^+) = 1$,
(f) $\alpha^{-i*}[\widehat{s}^i](\widetilde{s}^{-i}, (\widetilde{m}^i, \widehat{m}^{-i}), (\widehat{s}^i, \widetilde{s}^{-i})) = 1$, and
(g) $\alpha^{i*}[\widehat{s}^i](\widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}), s(\widehat{s}^i)^+) = 1$.

First, note that if μ^* and σ^* satisfy (2.5) then $\mu^{-i*}(\cdot, \widetilde{m}^i)$ can be determined arbitrarily since $\sum_{s^i} \sigma^{i*}(s^i, \widetilde{m}^i)\pi((s^i, s^{-i})) = 0$ for every $s^{-i} \in \mathcal{S}^{-i}$. Thus, we set $\mu^{-i*}((\widehat{s}^i, s^{-i}), \widetilde{m}^i) = 1$ for every $s^{-i} \in \mathcal{S}^{-i}$, it follows from this that if $\alpha^*[\widehat{s}^i]$ is as specified in (e) and (f) above then agent $-i$ chooses a best response to any $\alpha^{i*}[\widehat{s}^i]$ at both $(\widehat{s}^{-i}, (\widetilde{m}^i, \widehat{m}^{-i}))$ and $(\widetilde{s}^{-i}, (\widetilde{m}^i, \widehat{m}^{-i}))$, as required by (2.4). Second, from the specification of $\alpha^*[\widehat{s}^i]$ in (e) and (f) above we obtain

$$U^i(\sigma^*, \alpha^i, \alpha^{-i*}[\widehat{s}^i], p^i; \widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i})) = H^i(\sigma^*, p^i; \widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i})) + \frac{1}{r(\widehat{s}^i)}\left[\mathrm{z}^i \left[\pi(\widehat{s}^i)^+ - \pi((\widehat{s}^i, \widetilde{s}^{-i}))\right] \alpha^i(\widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}), s(\widehat{s}^i)^+) + \pi((\widehat{s}^i, \widetilde{s}^{-i}))\mathrm{z}^i \right], \tag{6.2}$$

so that, since $\pi(\widehat{s}^i)^+ \geq \pi((\widehat{s}^i, \widetilde{s}^{-i}))$, it follows that (2.4) is also satisfied at $(\widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}))$ if $\alpha^{i*}[\widehat{s}^i](\widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}), s(\widehat{s}^i)^+) = 1$, as stated in (g) above. Furthermore, from (6.2) it follows that $V^i(\alpha^*[\widehat{s}^i], p^i; \widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i})) = \mathrm{z}^i\pi(\widehat{s}^i)^+/r(\widehat{s}^i) \leq V^i(\alpha, p^i; \widehat{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}))$ for all α, as given in the statement of Lemma 1, (iii).

The specification of $\alpha^*[\widehat{s}^i]$ is completed by choosing $\alpha^{i*}[\widehat{s}^i](\widetilde{s}^i, (\widetilde{m}^i, \widehat{m}^{-i}), s(\widetilde{s}^i)^+) = 1$, i.e., by inducing $\widetilde{s}^i$ to follow the path that she assigns maximum prior probability of being the "correct" one. Also, the choice over actions of $-i$, given a message combination $(\widehat{m}^i, \widetilde{m}^{-i})$ where $\{\widetilde{m}^{-i}\} := M^{-i} \setminus \{\widehat{m}^{-i}\}$, can be specified in a symmetric manner to those of i so that all the arguments above continue to work for $-i$. Finally, we note that for the message combination $\widetilde{m}$, the beliefs of both agents can be arbitrarily determined, thereby, by setting appropriately the choices over actions along with those beliefs, the sequential rationality condition in equation (2.4) is satisfied at every node $(s^i, \widetilde{m})$, for every $s^i \in \mathcal{S}^i$, and every $i \in \mathcal{I}$. This completes the proof. ∎

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