Discriminatory pricing schemes in ascending auctions with anonymous bidders

N. Immorlica, A. Karlin, M. Mahdian, and K. Talwar

April 6, 2006

Abstract

A single auctioneer is offering identical copies of an indivisible good with fixed production cost and zero marginal cost to bidders with unit demand. Equivalently, one can imagine the auctioneer is offering an excludable public good of fixed cost. We suppose the auctioneer is restricted to sell the goods through the use of an ascending auction and would like to maximize his profit. If the bidders are distinguishable and the auctioneer has complete information about their types, then he can clearly extract the full surplus of the market with a discriminatory pricing scheme which charges each bidder according to his valuation. We study how much surplus can be extracted when bidders are *ex-ante identical*. It is natural to wonder whether the auctioneer can again employ discriminatory pricing to increase his profit beyond that of the simple uniform pricing scheme. We show that even in the case of interdependent values (i.e., values drawn according to an arbitrary symmetric joint distribution), no ascending auction extracts more than a constant times the revenue of the uniform pricing scheme.

1 Introduction

Price discrimination occurs when identical goods or services are sold to buyers at different prices. Such pricing policies are quite prevalent in many markets. In the airline industry, ticket prices vary with length of stay; in the movie industry, cinemas offer discounts to students and senior citizens; and in nightclubs, the cover charge is waived for women. Economic theory suggests one explanation for this phenomenon by noting that differential pricing can raise the revenue of a monopolistic seller (see, e.g., [6]).

Under full information, discriminatory pricing permits a seller to charge each buyer his valuation, thereby extracting the full surplus of the market. With incomplete information, surplus extraction is still possible when buyers are distinguishable. However, when buyers are ex-ante identical (i.e., their values are drawn from a symmetric joint distribution), the situation is more complex. When buyers have independent and identically distributed values, a sequence of work [1, 2, 4, 5] shows that for a general class of cost functions (including the fixed cost case described here-in), the maximum profit is extracted by a uniform price. However, in the case of interdependent values, one might imagine that a seller could use the value dependencies to deduce optimal prices for the buyers, and indeed, Cremer and McLean [3] prove that full surplus extraction is possible when values are interdependent for a range of priors, including some where the buyers are ex-ante identical.

We are interested in whether price discrimination can increase the revenue of an auctioneer who wishes to sell goods through the use of an ascending auction to anonymous (i.e. ex-ante identical) bidders. The restriction to the class of ascending auctions is especially interesting as many auctions in practice are implemented in an ascending manner, including the FCC auctions and most online auctions like eBay, eBid, and Yahoo! Auctions. The assumption of anonymous bidders is motivated by applications where it is particularly difficult or prohibitively costly to verify the identity of a bidder and thus segment the market, as is the case in many online auctions.

We consider an environment in which a monopolistic auctioneer wishes to sell identical copies of an indivisible good of fixed production cost and zero marginal cost to a finite number of bidders with unit demand (alternatively, one can imagine the auctioneer has an excludable public good of fixed cost). The bidders' values for the good are private information, and they are drawn according to a (commonly known) symmetric joint distribution. The main result of this paper is that no ascending auction can extract significantly more revenue than the uniform pricing scheme.

2 Model and Pricing schemes

A monopolistic auctioneer wishes to sell identical goods to a set of n bidders. These goods have a fixed production cost and zero marginal cost. Each bidder i has a private value v_i for a single unit of the good. We assume bidders are *ex-ante identical*, i.e. the bidders' valuations for these goods are drawn from a symmetric joint distribution F on $[0, \infty]^n$ which is common knownledge. For ease of notation, we label the bidders' values in decreasing order, i.e. $v_i \geq v_{i+1}$ for $1 \leq i \leq n$, and let π be the permutation such that the value of bidder i is $v_{\pi(i)}$. Our goal is to design an ascending auction that extracts maximum revenue in the dominant strategy equilibrium. In an *ascending auction*, the auctioneer specifyies a price for each bidder and raises it during the course of the auction. The procedure for raising prices and the termination condition are parameters of the auction design.

One obvious candidate for a revenue-maximizing ascending auction is the *uniform pricing scheme*. This scheme computes the maximum expected revenue $R = \max_p E_F[|\{v_i : v_i > p\}| \cdot p]$ and then charges all bidders the price p which maximizes the above expression, thereby achieving a revenue of R in expectation. However, when values are interdependent, the uniform pricing scheme does not necessarily maximize revenue. Consider, for example, a setting in which there is one high bidder with value 1 and $(n-1)$ low bidders with value $1/n$. The optimum uniform pricing scheme achieves a revenue of 1. However, an alternate auction offers each bidder sequentially a price of 1 until someone accepts and thereafter offers a price of $1/n$. In expectation, the high bidder will receive the $n/2$'th offer. The preceding $n/2$ bidders pay zero, the proceeding $n/2$ bidders pay $1/n$, and the high bidder pays 1. Thus the expected revenue of the auction is $3/2$ which is one-and-a-half times the revenue of the uniform pricing scheme. In fact, the example can be made more extreme by considering the joint distribution F which is simply a random permutation of the set of values $\{1/i : 1 \leq i \leq n\}$. For this distribution, a uniform price of v_i achieves a revenue of 1 for any $i \in N$. However, a generalization of the above auction described below actually exceeds the revenue of the uniform pricing scheme on this input by a factor of $\pi^2/6 \approx 1.64$ in expectation.

Auction 1

1. Initialization: Let $V \leftarrow \{1, 1/2, \ldots, 1/n\}$ be the set of possible values for bidders still under consideration.

2. For each bidder successively, slowly increase his price to the maximum value v in V. If he leaves the auction when the price is $1/i$, set $V = V - \{1/i\}$. Otherwise set $V = V - \{v\}$.

3 Maximum revenue of an ascending auction

Our main result states that the uniform pricing scheme extracts nearly as much revenue as any ascending auction and is thus, in some sense, the best ascending auction. More precisely, we prove the following theorem. Let $A(\mathbf{v}_\pi)$ denote the revenue of auction A when bidder i has value $v_{\pi(i)}$.

Theorem 1 *For all symmetric joint distributions F and for all ascending auctions A,*

$$\mathrm{E}_F[A(\mathbf{v}_\pi)] \leq \alpha \cdot \max_p E_F[|\{v_i : v_i > p\}| \cdot p].$$

for some constant $\alpha \in (\pi^2/6, 10]$.

The idea of the proof is to first show that the distribution described in the last section is the worst-case distribution. We then bound the revenue of any auction on that distribution by revenue of a semi-omniscient auction which considers bidders sequentially.

References

[1] J. Bergin and L. Zhou. Optimal uniform pricing procedures. manuscript.

[2] J. Bulow and J. Roberts. The simple economics of optimal auctions. *Journal of Political Economy*, 97:1060–1090, 1990.

[3] J. Cremer and R.P. McLean. Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent. *Econometrica*, 53(2):345–361, 1985.

[4] M. Harris and A. Raviv. A theory of monopoly pricing schemes with demand uncertainty. *American Economic Review*, 71:347–365, 1981.

[5] R. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6:58–73, 1981.

[6] H.R. Varian. Differential pricing and efficiency. *First Monday*, 1996. http://www.firstmonday.dk/issues/issue2/different/.