The Relevance of Envy Freeness as Fairness Criterion

Dorothea K Herreiner
Department of Economics
Loyola Marymount University[1]

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This paper evaluates how relevant envy freeness (Foley 1967) is as an empirical concept of fairness. Several versions of an indivisible-good fair division problem are evaluated in a survey questionnaire. Participants had to determine the fairest allocations of the objects among individuals with different preferences. Each problem features two allocations that are identical in all aspects but envy freeness. Across all treatments and versions of the problem, the envy free allocation is chosen 3.5 times as frequently as the allocation with envy. However, as is shown, only some these choices are based on a conscious use of the criterion of envy freeness. The relevance of other criteria for the choice of the envy free allocation is evaluated.

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[1] University Hall, 1 LMU Drive, Suite 4200, Los Angeles, CA 90045-2659; phone: +1-310-338-2815, fax: +1-310-338-1950, e-mail: dherreiner@lmu.edu, www: http://myweb.lmu.edu/dherreiner.

1. Introduction

The recent experimental literature[2] has suggested and tested various fairness criteria in decision problems, in particular inequality aversion (the smaller the differences between payoffs, the better the allocation). Most of these criteria are based on the assumption that individuals compare what they themselves receive to what others receive. Frequently, an unspecified notion of envy is invoked to explain such fairness criteria, which has not been linked to the fairness criteria discussed in the established axiomatic and theoretical literature[3].

Foley (1967) introduced[4] the criterion of *envy-freeness*, which, in combination with (Pareto) efficiency, is often considered the definition of fairness. An allocation is called *envy-free* if every person is *envy-free* meaning that nobody prefers the bundle someone else receives to his or her own bundle. Such an envy-free allocation has several appealing theoretical features, among them is the fact that no interpersonal utility comparisons are required to find an envy-free allocation. The question investigated here is whether envy-freeness is also an empirically relevant criterion that is (knowingly or unknowingly) employed by individuals when determining fair allocations.

This paper is based on earlier work with Clemens Puppe on the role of envy-freeness in free-form bargaining experiments, see Herreiner/Puppe (2004a, b). Our results there show that other criteria, like inequality aversion, dominate envy-freeness. We also present survey-based (and other) evidence for the relevance of envy-freeness for cases in which no other criteria discriminate between different allocations. Here, I follow up on that evidence by further investigating different variations of a fair division problem where several indivisible objects have to be allocated to different individuals in the fairest manner. The relevant feature of that example is that there are two allocations that are indistinguishable except for the fact that one is envy-free whereas the other is not. The relevance of envy-freeness can thus be evaluated without interference by any other criteria.

The recent experimental literature does not allow for an analysis of envy-freeness because allocations are usually expressed only in monetary terms. In that case, inequality aversion and envy-freeness are indistinguishable in an ordinal sense, because everybody prefers more money to less money. It is impossible to tell whether an unequal allocation is rejected because one person has more money (or a different bundle) than another (envy), or because two individuals have different payoffs (inequality aversion). To test the empirical relevance of envy-freeness requires that individuals have different preference rankings as is the case in this paper.

2. Survey

The survey was conducted with undergraduate students at Loyola Marymount University during March and April 2006. The participants were part of a subject pool in the Economics and Psychology Departments that requires students to participate in research activities for credit. Five

[2] See among others Fehr/Schmidt (1999), Bolton/Ockenfels (2000), Charness/Rabin (2002), Engelmann/Strobel (2004), Bereby-Meyer/Niederle (2003), and also earlier work by for instance Güth/Schmittberger/Schwarze (1982) and Kirchsteiger (1994). Bar-Hillel/Yaari (1984, 1993) investigates how different aspects, for instance need or taste, influence the choice of allocations in division problems. Their work is also one of very few where individuals have different preference rankings over bundles like the example investigated in this project here and in my earlier joint work with Clemens Puppe, such as Herreiner/Puppe (2004a, b). The specific numeric examples used in Bar-Hillel/Yaari (1984), though, do not allow for an evaluation of envy-freeness as fairness criterion.

[3] See Brams/Taylor (1996), Thomson (2003), and Moulin (2003) for an overview.

[4] See also Tinbergen (1953).

different treatments of the survey where administered (on different colored paper: pink, green, white, yellow, and orange) with 6 different situations of a fair division problem with 5 indivisible object and 3 individuals. The first three treatments presented the same 4 situations in the same order, but in different permutations of the problem (see appendix for details). The last two treatments presented situation 2 and 3 from the other treatments and two additional situations. Both of these last two treatments contained the same four situations in the same order, but in different permutations of the problem. The following six situations were considered:

Situation 1

	A	**B**	**C**	**D**	**E**
I	40	2	3	25	30
II	14	26	8	26	26
III	10	26	26	12	26

#
269
(282)

Allocation	Payoffs	PO	EF	U	%
A,BD,CE	40,52,52	x	x	-4	37
A,DE,BC	40,52,52	x		-4	15

Situation 2

	A	**B**	**C**	**D**	**E**
I	48	1	2	23	26
II	18	24	10	24	24
III	12	24	24	16	24

#
369
(386)

Allocation	Payoffs	PO	EF	U	%
A,BD,CE	48,48,48	x	x	-2	56
A,DE,BC	48,48,48	x		-2	16

Situation 3

	A	**B**	**C**	**D**	**E**
I	44	4	6	22	24
II	20	22	14	22	22
III	16	22	22	18	22

#
369
(386)

Allocation	Payoffs	PO	EF	U	%
A,BD,CE	44,44,44	x	x	-2	59
A,DE,BC	44,44,44	x		-2	16

Situation 4

	A	**B**	**C**	**D**	**E**
I	48	1	2	21	28
II	20	21	17	21	21
III	18	21	21	19	21

#
272
(282)

Allocation	Payoffs	PO	EF	U	%
A,BD,CE	48,42,42	x	x	-7	47
A,DE,BC	48,42,42	x		-7	14

Situation 5

	A	**B**	**C**	**D**	**E**
I	40	5	10	23	22
II	14	26	8	26	26
III	10	26	26	12	26

#
97
(104)

Allocation	Payoffs	PO	EF	U	%
A,BD,CE	40,52,52	x	x	x	52
A,DE,BC	40,52,52	x		x	12

Situation 6

	A	**B**	**C**	**D**	**E**

#

Allocation	Payoffs	PO	EF	U	%

I	44	4	5	24	23
II	12	24	16	24	24
III	10	24	24	18	24

97
(104)

A,BD,CE	44,48,48	x	x	x	52
A,DE,BC	44,48,48	x		x	13

The number of valid observations for each situation are indicated under "#" and the number in brackets is the total number of times this situation was presented. The tables to the right indicate the two focal allocations, the frequency of their choice, and their basic characteristics (PO = Pareto Optimality, EF = Envy Freeness, U = Utilitarian, i.e. the difference to the maximal payoff sum).

Each participant was given a letter upon arrival as identifier and a one-page instruction explaining the survey (see appendix). The instructions explained the basic set-up and presentation of the problem with an example. Participants were instructed to identify the fairest allocation in the position as an independent arbiter and then to explain their choice. Having read the instructions, participants then collected the first answer sheet with the four situations. Each situation was presented in a table analogous to the one shown above (depending on the treatment in different permutations of the version shown here), followed by a two-row table where participants had to indicate the allocation they choose (the object given to I, to II, and to III), and several lines for an explanation. To keep situations as separate as possible, two problems were presented on the front page, two on the back and on each side one problem was upside down. Participants were also instructed to consider the problems as independent of each other and to work on them subsequently.

Once the first sheet was completed, students returned that sheet, got the instructions for the second sheet (see appendix) and the second sheet. On the second sheet the same four problems were presented in the same order and layout. Instead of a table to indicate their allocations, the second sheet had a two-row table where candidates could choose one of the two allocations shown above. Participants were told that the three individuals faced with the allocation problem had identified these two allocations as the relevant ones. They now asked the independent arbiter to pick one of the two.

Upon completion the second sheet was returned to the organizer along with its instructions, and a third sheet was handed to the students with one more page of instructions (see appendix). The third sheet had the same four problems as the earlier two and had the same layout as sheet one. At the top of the front page of sheet the participants were asked to respond to the question "What do you think about this criterion of envy-freeness? Is it relevant for fairness?" In the instructions to sheet 3 envy freeness was defined and applied to the example situation from the initial instructions. Participants were then instructed to reconsider the problems and to choose the fairest allocation (which could be the same or a different one from the one chosen earlier).

Participants indicated their identifying letter on each sheet. They worked first on sheet 1, then sheet 2, and then sheet 3 w/o having access to another sheet at the same time.

3. Results on Envy Freeness

No matter what exact treatment, the envy free allocation is chosen significantly more frequently than the allocation with envy.

		Payoffs	Util[5]	%EF	% ¬EF	P-Value	#
Sheet 1	S1	40,52,52	-4	30	14	0.004321	134
	S2	48,48,48	-2	51	14	0.000000	182
	S3	44,44,44	-2	48	19	0.000001	183
	S4	48,42,42	-7	39	14	0.000038	135
	S5	40,52,52	X	44	15	0.006270	48
	S6	44,48,48	X	48	10	0.000273	50
Sheet 3	S1	40,52,52	-4	44	16	0.000013	135
	S2	48,48,48	-2	62	18	0.000000	187
	S3	44,44,44	-2	69	14	0.000000	186
	S4	48,42,42	-7	54	14	0.000000	137
	S5	40,52,52	X	59	10	0.000019	49
	S6	44,48,48	X	55	17	0.001468	47
Average				51	15	0.0000	1473

A cursory inspection reveals that equal payoffs lead to the Pareto optimal allocations to be chosen more frequently. This applies in particular to the envy free allocation. A similar relationship holds between the shortfall from the utilitarian allocation and the frequency of choice of the envy free allocation. The closer to the maximal payoff sum the Pareto optimal allocation, the more frequently it is chosen. This again occurs more frequently for the envy free allocation than for the allocation with envy. The relevance of the other criteria is further explored below.

		Payoffs	Util[6]	¬EF ⇒ EF	EF ⇒ ¬EF	P-Value	¬EF ⇒ EF	EF ⇒ ¬EF	P-Value
Sheet 1	S1	40,52,52	-4	8	7	0.5000	32	14	0.0057
	S2	48,48,48	-2	18	20	0.6864	50	12	0.0000
	S3	44,44,44	-2	27	12	0.0119	57	14	0.0000
	S4	48,42,42	-7	11	8	0.3238	36	11	0.0002
	S5	40,52,52	X	5	3	0.3633	15	2	0.0012
	S6	44,48,48	X	3	3	0.6563	10	5	0.1509
Total				72	53	0.0535	200	58	0.0000

Although, overall participants switch more frequently from the allocation with envy to the envy free allocation, this effect is significant, and scantly so, at the aggregate level. However,

[5] "Util" indicates the shortfall from the maximal payoff sum. An "X" indicates that the allocation is the payoff sum-maximizing one.
[6] "Util" indicates the shortfall from the maximal payoff sum. An "X" indicates that the allocation is the payoff sum-maximizing one.

results are much starker when considering switches from sheet 1 to sheet 3 from other allocations to one of the two allocations considered here. Individuals switch much more frequently from other allocations to the envy free one than to the allocation with envy. The next section discusses to what extent these switches are influence by participants' thoughts about envy freeness as a criterion of fairness and by their ability and intention to identify the envy free allocation.

4. Reactions, Evaluations and Applications of Envy Freeness by Participants

A very preliminary analysis of the participants thoughts about envy freeness yields some interesting aspects. The majority of the students (64%) states that envy freeness matters.

Among those rejecting envy freeness as a relevant criterion, several students reject the counterfactual notion of envy freeness. Some argue that allowing to consider swapping bundles, would open the door to consider as reference points also getting other, larger bundles, or even everything. Considering other individuals' bundles seems to be no more obvious or justified than considering any arbitrary bundle to compare the actual bundle with.

Quotes
"No. It is a means of having individuals accept a justifiable reason for not getting their choices met, as they might have liked them to be. [... It] neutralizes 'the envy' to a point, but does not eliminate the desire created by envy."

"I think it plays a small factor, but it isn't too good, because you'll always be thinking 'what if I got …' and so there will always be a chance of being envious."

"Envy freeness is relevant for fairness because if nobody cared what they received (hence were not envious), then there would be no point in a distribution process."

5. Relevance of Other Criteria

[Regressions]

6. Conclusion

APPENDIX

Different Treatments and Number of Observations

Version 0 Original, as shown in text.
Version 1 Columns A and B, and columns C and D are exchanged.
Version 2 Columns A and D, and rows I and II are exchanged.
Version 3 Columns A and E, and rows I and III are exchanged.Number of

Sheet	Task	Pink (47)	Green (47)	White (47)	Yellow (26)	Orange (26)
		Situation 1 Version 0	Situation 1 Version 1	Situation 1 Version 0	Situation 5 Version 0	Situation 5 Version 1
1	1	45	44	45	23	25
2	5	47	44	46	26	26
3	9	45	46	44	25	24
		Situation 2 Version 1	Situation 2 Version 2	Situation 2 Version 3	Situation 2 Version 1	Situation 2 Version 2
1	2	45	44	44	24	25
2	6	47	40	45	26	25
3	10	46	45	46	25	25
		Situation 3 Version 2	Situation 3 Version 0	Situation 3 Version 1	Situation 6 Version 2	Situation 6 Version 0
1	3	45	45	44	25	25
2	7	46	42	45	26	25
3	11	45	46	46	23	24
		Situation 4 Version 3	Situation 4 Version 3	Situation 4 Version 2	Situation 3 Version 3	Situation 3 Version 3
1	4	45	46	44	23	26
2	8	46	44	46	26	25
3	12	46	47	44	24	25

Survey Instructions

Today’s activity consists of three parts. Once you are done with one part, please get up and collect the next part from me. If I am talking with another participant, please wait until we are done. Please indicate on the top right of each answer sheet the letter that you were assigned when you arrived for today’s activity.

Today’s activity asks you to consider several situations in which five different objects have to be allocated to three individuals. Assume that these three individuals approached you because they are looking for an independent arbiter who picks a fair distribution of the five objects among them; they are looking for someone who does not benefit from the distribution and therefore takes an impartial decision. Your task is therefore to analyze the situation and then to determine the distribution of the objects that you consider the fairest. You do not know any of the three

individuals and should take as impartial a decision as possible. What matters is what you consider a fair distribution – there is no “right” or “wrong” answer.
You decision is final. The three individuals cannot swap or otherwise trade the objects or money once you have made a decision. Please think carefully about the task at hand. Once you have identified the fairest allocation, write it down and then explain why you chose this distribution, i.e. explain why you consider that distribution the fairest possible.

The objects that need to be distributed are indivisible, i.e. every object has to be given to one person in its entirety. All five objects need to be distributed among the three individuals. The three individuals are called I, II and III. The five objects are called A, B, C, D, and E.

For each situation you are given a payoff table. The entries in that table tell you how much each individual likes each object. Consider, for instance, this case:

Example Situation

	A	B	C	D	E
I	20	20	5	25	30
II	14	26	24	10	26
III	30	7	17	26	20

Here, person I likes getting A or getting B equally much, but prefers getting D or even better E. Person II, for instance, likes getting objects A and D together as much as getting object C, etc. If you decide, for instance, that you want to give A, C and D to person I, B to person II and E to person III, then you indicate this as follows:

Distribution

Individual	I	II	III
Objects	A, C, D	B	E

Please provide an explanation for your choice in the space underneath the distribution on the answer sheet.
This example here is only for illustrational purposes. It does not imply or suggest any judgment about what would constitute a fair allocation in this or any other case.

In the first part of today’s activity you will have to make a decision in four different situations which are independent of each other. This means, that what you decided for an earlier situation is irrelevant for a later situation. Please work on the situations in the order in which they are presented, 1 through 4; finish analyzing and deciding one situation before you move on to the next.

Any questions? Please raise your hand.

Please think carefully, write legibly, and provide detailed explanations.
Please do not talk to the other participants.

[Sheet 1 with the four problems; two on the front, two on the back; on each side one problem was right side up, the other was presented upside down; the problem was presented in table format as throughout the paper, followed by several lines for the explanation]

In this part you will reconsider the situations you encountered earlier. In the same situations as before, the three individuals now approach you with two possible distributions they are considering. They ask you to pick the one that you consider the fairest. Indicate your choice by ticking off either Distribution 1 or Distribution 2. Provide a short explanation for your choice. Your decisions in this part are independent of your earlier decisions. Do not feel bound by the distributions you suggested or discarded earlier. Enter this part with a fresh mind and consider the two distributions that are presented to you for each situation. As before, the different situations are independent of each other – what you pick in one case has not relevance or relation to what you pick in another situation.

[Sheet 2 with the same four problems, now offering two choices for each problem and 3 lines for an explanation under each problem; otherwise same layout as Sheet 1]

One criterion to determine a distribution of objects among several individuals is called envy-freeness. The idea of envy-freeness is that a distribution of objects is going to be acceptable to all individuals if they are not envious of each other. Specifically, this means that nobody should prefer what someone else gets to what that person him/herself gets.

Let's consider the example situation introduced at the very beginning.

Example Situation

	A	**B**	**C**	**D**	**E**
I	20	20	5	25	30
II	14	26	24	10	26
III	30	7	17	26	20

Let's assume the chosen distribution is as suggested in the introduction:

Distribution

Individual	**I**	**II**	**III**
Objects	A, C, D	B	E

Would such a distribution be envy-free?
Person I gets objects A, C, and D, whereas person II only gets B. Person II receives a value of 26 from object B, but would be getting a value of 14+24+10=48 if person II managed to get person I's bundle of A, C, and D. Person II would clearly be better off with person I's objects. Person II is therefore envious of person I. The distribution is not envy-free because there is one person, person II, who is envious.
As a matter of exercise, let's see whether anyone else is envious. Person III gets E and therefore a value of 20. If person III got person II's object B, then that would be worth only 7. Person III is therefore not envious of person II. However, person III is envious of person I – the combination

of A, C, and D is worth 30+17+26=73 to person III, which clearly is a lot more than the 20 person III gets from object E.
Person I is not envious of either of the other. Objects A, C, and D are worth 20+5+25=50 for person I, whereas getting B with a value of 20 or E with a value of 30 is worse.
Obviously, there may be many other reasons why someone could consider the suggested distribution of giving A, C, and D to I, B to II, and E to III as fair or unfair.

What do you think about this criterion of envy-freeness? Is it relevant for fairness? Briefly explain your point of view on the answer sheet.

Have another look at the four situations considered before. If you found yourself again in the position of an independent arbiter where the three individuals approach you with the request of identifying the fairest distribution for them as in part one – what would you choose in each situation now? You are no longer limited to only two choices as in part 2. Feel free to pick the same choices as earlier or to change your mind. Whatever you pick, it should be the distribution YOU consider the fairest. Please explain your choice briefly.

[Sheet 3, same problems as the two previous sheets; the front page starts with the question “What do you think about this criterion of envy-freeness? Is it relevant for fairness?” and some lines for an answer, followed by the first problem; otherwise, the layout is the same as Sheet 1]