Capacity Choice under Uncertainty: The Impact of Market Structure*

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Abstract

We analyze a market game where firms choose capacities under uncertainty about future market conditions and make output choices after uncertainty has unraveled. We show existence and uniqueness of equilibrium under imperfect competition and provide an intuitive characterization of equilibrium investment. We show that investment in oligopoly, in the first and second best solution can be unambiguously ranked, in particular investment is highest in the First Best solution and lowest under imperfect competition. We finally demonstrate that intervention of a social planer only at the production stage leads to strategic uncertainty at the investment stage and moreover decreases total investment below the level obtained under imperfect competition.

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1 Introduction

In this article we investigate the nature of equilibrium outcomes in oligopolistic markets where firms make capacity choices under uncertainty about future market conditions and decide on output after the state of nature has unraveled. The fact that in many industries where non storable goods are produced, capacity is a long run decision, whereas production may be adjusted short–run is a natural motivation for our approach.[1] Consider, for example, the electricity sector or the High Tech industry, where production has to take place just in time, but capacities have to be installed well in advance. In those markets firms usually face considerable demand and cost uncertainty when choosing their capacities. This may be due to uncertainty about the economic trend, about the success of a new product, about future weather conditions, or fuel prices, to give just a few examples.

As we will show, the consideration of uncertainty about future market conditions reveals incentive problems that cannot be addressed in a model with deterministic demand and cost functions. To see this, consider a modification of the game described above where at the second stage firms are regulated to marginal cost pricing whenever unconstrained (and market clearing prices obtain if capacities are binding). It is obvious that in both, the original and the modified game, if future market conditions were perfectly known, capacity choices would equal the (one shot) Cournot quantities and firms would always operate at full capacity. Thus, intervention at the production stage would be ineffective, since firms could exercise their market power already at the investment stage. If capacities are chosen under uncertainty, however, firms will inevitably be unconstrained if demand turns out to be low. Then, an intervention at the production stage has an impact on the investment decision. In our particular example, investment incentives would be lower in the modified game since being capacity constrained is the more attractive the lower unconstrained profits are. The above illustrates that for markets with considerable demand fluctuations a thorough analysis of regulatory interventions cannot be conducted without modeling the investment stage explicitly and accounting for the uncertainty firms are facing. Our research aims to provide the appropriate tools to tackle those issues.

In this paper we develop a rather general and manageable framework to analyze in-

[1] Moreover, our model covers a wide range of scenarios like investment prior to production on many successive markets, that may be of interest for applied theoretical or empirical work. We comment on those issues in the conclusion.

vestment and production choices in an imperfectly competitive environment. We thereby close a gap in the literature between studies that consider investment incentives in perfectly competitive and monopolistic markets[2], respectively (which are covered by our model as the extreme cases). We show that under standard regularity conditions on demand and cost the Cournot two stage market game (where firms invest in capacity under uncertainty about future market conditions and produce when uncertainty has unraveled) always has a unique equilibrium. Equilibrium investment in the *Cournot outcome* can be characterized by an intuitive first order condition that implies that marginal revenue generated by an additional unit of capacity equals marginal investment cost.

In order to asses the impact of strategic behavior on investment incentives and welfare, we also consider the First Best and a Second Best scenario: the *First Best solution* specifies welfare maximizing capacities and production schedules, while the *Second Best solution* specifies welfare maximizing capacity choices given that firms engage in Cournot competition at the production stage. We show that total capacity in the First Best is higher than total capacity in the Second Best solution, which still exceeds equilibrium investment in the Cournot two stage market game. Our results confirm the common perception that in oligopolistic markets there is clearly a role for investment enhancing mechanisms (like capacity obligations or capacity markets).

The second main objective of our work is to provide a framework that allows to shed light on the impact of regulatory intervention *only at the production stage* on investment incentives and welfare. In order to elaborate on this issue, we consider capacity choices by strategic firms that anticipate optimal regulation at the production stage given the capacities chosen.We provide an intuitive characterization of investment in any symmetric equilibrium. However, existence (but not even uniqueness) of equilibrium can only be shown for the case of constant marginal production cost. Moreover, in any symmetric equilibrium of the game with optimal regulation at the production stage, total investment is even lower than in the Cournot market game. Our results have two important implications: First, intervention only at the production stage gives rise to multiple, and possibly asymmetric

[2]The literature on peak–load pricing provides a characterization of investment in those cases. However, the approach used does not allow to analyze the strategic interaction of firms. See, for example, Crew and Kleindorfer (1986) or Crew, Fernando, and Kleindorfer (1994) for an overview. Another related direction of research analyzes *production* decisions under demand uncertainty in perfectly competitive and monopolistic markets. See Dreze and Gabszewicz (1967), Dreze and Sheshinski (1976), Leland (1972), or Sandmo (1971).

equilibria of the game, and thereby generates strategic uncertainty for the firms. Second, it is not even clear that such an intervention is welfare enhancing since it decreases total capacity in the industry. We conclude that interventions only at the production stage have to be carefully reconsidered in markets with highly fluctuating demand.[3]

In the economic literature, capacity choice has been extensively analyzed prior to price competition. This literature was initiated by the seminal article of Kreps and Scheinkman (1983), and has been generalized and extended by many authors, among others by Osborne and Pitchik (1986). Reynolds and Wilson (2000) use the latter to analyze capacity choice prior to Bertrand competition when demand is uncertain. Their analysis reveals that symmetric pure strategy equilibria (in capacities) do not exist unless cost of investment is so high that firms want to be constrained in any demand scenario.

Gabszewicz and Poddar (1997) analyze investment under demand uncertainty prior to quantity competition in a framework where both, demand and marginal cost are linear, and compare it to equilibrium production given the expected demand (which they call the "Certainty Equivalent Game").[4] Our analysis in section 3 includes their model as a special case. However, in terms of generality and technical tractability our approach goes far beyond the one of Gabszewicz and Poddar.[5]

Other papers that investigate investment incentives prior to imperfectly competitive markets were mainly motivated by the liberalization of the electricity sector, where investment incentives have recently become a central issue in the policy debate [see, for example, Murphy and Smeers (2003)]. As a response to the common perception of too low investment incentives, various mechanisms have been proposed to raise investments [see e. g. Cramton

[3]Note that our analysis abstracts from many problems that additionally have to be considered when judging the welfare effects of a particular regulatory policy. Still we provide a framework that can be used in order to explicitly analyze different (more realistic) scenarios at stage two. Examples are the analysis of price cap regulation in Zoettl (2005) or of forward markets prior to spot market competition in Grimm and Zoettl (2005).

[4]In order to relate the results of Gabszewicz and Poddar to ours, in appendix B we analyze a more general version of their "Certainty Equivalent Game".

[5]Our primary goal was to provide a tool to analyze different forms of market organization on investment incentives. This cannot be achieved by the model of Gabszewicz and Poddar, since their discrete approach to model demand uncertainty does not allow to show uniqueness of equilibrium, to analyze an arbitrary number of firms, to use more general demand and cost functions, and finally does not yield intuitive characterizations of equilibria.

and Stoft (2005), or Bushnell (2005) for an overview]. These approaches are well in line with our result that investment is generally too low prior to imperfectly competitive markets. The current policy debate on electricity markets also provides motivation for our second scenario. There is a huge literature that asks whether the firms abuse market power in the spot market and — if so — whether regulatory intervention is desirable.[6] Our results point out that investment incentives may be strongly affected by such an intervention and thus, the welfare effect may be unclear.

Our paper is organized as follows: In section 2 we state the model. In section 3 we show existence and uniqueness of equilibrium for the Cournot two stage market game. Then, in section 4, we characterize the first best solution. Section 5 is devoted to intervention of a planer at only one of the two stages: We characterize the socially optimal capacity levels given that firms compete à la Cournot at the production stage (second best solution) in section 5.1. In section 5.2, we analyze the incentives to invest in case the constrained social optimum is implemented at the production stage. Section 6 contains our main result, an unambiguous ranking of total investment in all scenarios mentioned above. Section 7 concludes.

2 The Model

We analyze a two stage market game where firms have to choose production capacities under demand and cost uncertainty, and make output choices after market conditions unraveled. Uncertainty is represented by a parameter Θ that is distributed on the domain $[\underline{\theta}, \overline{\theta}]$ according to c.d.f. $F(\theta)$ with the corresponding density $f(\theta) = F_\theta(\theta)$.[7] We denote by $q = (q_1, \ldots, q_n)$ the vector of outputs of the n firms, and by $Q = \sum_{i=1}^n q_i$ total quantity produced in the market. Market demand in scenario $\theta \in [\underline{\theta}, \overline{\theta}]$ is given by $P(\cdot, \theta)$. Moreover, all firms have the same cost function in scenario θ, which we denote by $C(\cdot, \theta)$.[8] We make the following regularity assumptions:

[6] See, for example, Wolfram (1999) or Joskow and Kahn (2002).

[7] Throughout the paper we denote the derivative of a function $g(x, y)$ with respect to an argument z, $z = x, y$, by $g_z(x, y)$, the second derivative with respect to that argument by $g_{zz}(x, y)$, and the cross derivative by $g_{xy}(x, y)$.

[8] Note that P and C may depend on independent random events. Then, $F(\cdot)$ approximates the distribution over all potential states of nature that may result from the two random draws.

Assumption 1 *(i) Market demand in scenario θ has a finite satiation point $\overline{Q}(\theta)$, i. e. $P(Q,\theta) = 0$ for all $Q \geq \overline{Q}(\theta)$. Moreover, for each θ there exists a prohibitive price $\overline{P}(\theta)$, such that $P(0,\theta) \leq \overline{P}(\theta)$.*

(ii) $P(Q,\theta)$ is twice continuously differentiable in Q with $P(Q,\theta) > 0$ and $P_q(Q,\theta) < 0$ for all $Q \in [0, \overline{Q}(\theta))$ and $\theta \in (\underline{\theta}, \overline{\theta}]$.

(iii) $C(q_i,\theta)$ is twice continuously differentiable in q_i with $C_q(q_i,\theta) \geq 0$ and $C_{qq}(q_i,\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

(iv) $P(Q,\theta)$ satisfies $P_q(Q,\theta) + P_{qq}(Q,\theta)q_i < 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$ and all $q_i \in [0, \overline{Q} - Q_{-i}]$.[9]

(v) Both, $P(Q,\theta)$ and $C(q_i,\theta)$ are differentiable in θ with $P(0,\underline{\theta}) = C(0,\underline{\theta}) = 0$ and $P_\theta(Q,\theta) - C_{q\theta}(q_i) > 0$.

(vi) $P(Q,\theta)q_i - C(q_i,\theta)$ is (differentiable) strict supermodular in q_i and θ, i. e. $\frac{d^2[P(Q,\theta)q_i - C(q_i,\theta)]}{dq_i d\theta} > 0$ for all i, θ, and q_{-i}.

The situation we want to analyze is captured by the following two stage game:

At stage one firms simultaneously build up capacities $x = (x_1, \ldots, x_n) \in [0, \overline{Q}(\overline{\theta})]$. Capacity choices are observed by all firms. Cost of investment $K(x_i)$ is the same for all firms and satisfies

Assumption 2 (Investment Cost) *Investment cost $K(x_i)$ is twice continuously differentiable, with $K_x(x_i) \geq 0$ and $K_{xx}(x_i) \geq 0$.*

Throughout the paper we consider only the interesting cases where it holds that

$$K(0) < \int_{\underline{\theta}}^{\overline{\theta}} [P(0,\theta) - C(0,\theta)] dF(\theta). \tag{1}$$

That is, we assume that the consumers' expected willingness to pay for the "first unit" of capacity is always higher than the cost of the first unit of investment. Note that if the condition does not hold, no firm invests in capacity.

At stage two, facing the capacity constraints inherited from stage one, firms simultaneously choose outputs at the spot market. Since demand uncertainty unravels prior to the

[9]Throughout the paper q_{-i} denotes the quantities produced by the firms other than i, and $Q_{-i} = \sum_{j \neq i} q_j$.

output decision, produced quantities depend on the realized demand scenario. We denote individual quantities produced in demand scenario θ by $q(\theta) = (q_1(\theta), \ldots, q_n(\theta))$, and the aggregate quantity by $Q(\theta) = \sum_{i=1}^{n} q_i(\theta)$.

Finally, we state firm i's stage one expected profit from operating if capacities are given by x and firms plan to choose feasible[10] production schedules $q^F(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.

$$\pi_i\left(x, q^F\right) = \int_{\underline{\theta}}^{\overline{\theta}} \left[P\left(Q^F(\theta), \theta\right) q_i^F(\theta) - C\left(q_i^F(\theta), \theta\right)\right] dF(\theta) - K(x_i). \tag{2}$$

3 Imperfect Competition

In this section we analyze the two stage market game where at stage one firms simultaneously invest in capacity under uncertainty about future market conditions and at stage two, when uncertainty has unraveled, decide on production. We call this game the *Cournot market game* and refer to the equilibrium investments and quantities as the *Cournot outcome.*

In this section we show existence and uniqueness of equilibrium of the Cournot market game and provide and intuitive characterization of equilibrium investment. In the following — using backward induction — we proceed in two steps: we first analyze the equilibria at stage two for all possible investment levels and then characterize equilibrium capacity choices.

Production Stage In the first step we characterize equilibrium outputs of the capacity constrained Cournot games at each $\theta \in [\underline{\theta}, \overline{\theta}]$ given investment choices x. Note that in order to analyze all possible continuation games we have to consider also asymmetric investments. In order to simplify the exposition we will order the firms according to their investment levels, i. e. $x_1 \leq x_2 \leq \cdots \leq x_n$, throughout the paper.

An equilibrium of the capacity constrained Cournot game at stage two in scenario θ given x, $q^C(x, \theta)$, satisfies simultaneously for all firms

$$q_i^C(x, \theta) \in \arg\max_{\mathrm{q}} \left\{P(\mathrm{q} + q_{-i}^C, \theta))\mathrm{q} - C(\mathrm{q}, \theta)\right\} \qquad \text{s.t.} \quad 0 \leq \mathrm{q} \leq x_i. \tag{3}$$

Note that due to assumption 1, part (v), all firms are unconstrained for values of θ close to $\underline{\theta}$. By assumption 1 parts (ii) to (iv), the unconstrained Cournot equilibrium [which we

[10]That is, $0 \leq q_i^F(\theta) \leq x_i$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, $i = 1, \ldots, n$.

denote by $\tilde{q}^{C0}(\theta)$] is unique and symmetric for each $\theta \in [\underline{\theta}, \overline{\theta}]$.[11] From (3) it follows that $\tilde{q}_i^{C0}(\theta)$ is implicitly determined by the first order condition

$$P(n\tilde{q}_i^{C0}, \theta) + P_q(n\tilde{q}_i^{C0}, \theta)\tilde{q}_i^{C0} = C_q(\tilde{q}_i^{C0}, \theta).$$

Now as θ increases, at some critical value that we denote by $\theta^{C1}(x)$, firm 1 (the one with the lowest capacity) becomes constrained. The critical demand scenario is implicitly determined by $x_1 = q_1^{C0}(\theta^{C1})$. If it holds that $x_1 < x_2$, then at $\theta^{C1}(x)$ only firm one becomes constrained. Then, in equilibrium, firm 1 produces at its capacity bound whereas the remaining firms produce their equilibrium output of the Cournot game among $n-1$ firms given the residual demand $P(Q - x_1, \theta)$ [denoted by $\tilde{q}_i^{C1}(x, \theta)$], which solves the first order condition

$$P(x_1 + (n-1)\tilde{q}_i^{C1}, \theta) + P_q(x_1 + (n-1)\tilde{q}_i^{C1}, \theta)\tilde{q}_i^{C1} = C_q(\tilde{q}_i^{C1}, \theta).$$

The capacity constrained Cournot equilibrium in the case where one firm is constrained is a vector $q^{C1}(x, \theta)$, where $q_i^{C1}(x, \theta) = \min\{x_i, \tilde{q}^{C1}(x, \theta)\}$.

As θ increases further, we pass through $n+1$ cases, from case $C0$ (no firm is constrained) to case Cn (all n firms are constrained). Note that two critical values $\theta^{Cm}(x)$ and $\theta^{Cm+1}(x)$ coincide whenever $x_m = x_{m+1}$, and that it holds that $\theta^{Cm}(x) < \theta^{Cm+1}(x)$ (by assumption 1 part (v)) whenever $x_m < x_{m+1}$.

Now we are prepared to characterize the capacity constrained Cournot equilibrium in case Cm where m firms are constrained. In this case, the m firms with the lowest capacities produce at their capacity bound, whereas the $n-m$ unconstrained firms produce

$$\begin{aligned} \tilde{q}_i^{Cm}(x, \theta) \quad = \quad & \left\{ q_i \in \mathbb{R} : P\left(\sum_{i=1}^{m} x_i + (n-m)\,\tilde{q}_i^{Cm}, \theta\right)\right. \\ & \left. + P_q\left(\sum_{i=1}^{m} x_i + (n-m)\,\tilde{q}_i^{Cm}, \theta\right)\tilde{q}_i^{Cm} = C_q\left(\tilde{q}_i^{Cm}, \theta\right)\right\}, \end{aligned} \tag{4}$$

The equilibrium quantities of the capacity constrained Cournot game in case Cm are given by

$$q_i^{Cm}(x, \theta) = \min\{x_i, \tilde{q}_i^{Cm}(x, \theta)\}, \tag{5}$$

[11] See, for example Selten (1970), or Vives (2001), pp. 97/98.

and aggregate production in case Cm is

$$Q^{Cm}(x,\theta) = \sum_{i=1}^{n} q_i^{Cm}(x,\theta). \tag{6}$$

This allows us finally to pin down the profit of firm i in scenario Cm,

$$\pi_i^{Cm}(x,\theta) = \begin{cases} P\left(Q^{Cm},\theta\right) x_i - C\left(x_i,\theta\right) & \text{if } i \le m, \\ \\ P\left(Q^{Cm},\theta\right) \tilde{q}_i^{Cm}(x,\theta) - C\left(\tilde{q}_i^{Cm}(x,\theta),\theta\right) & \text{if } i > m. \end{cases} \tag{7}$$

Note that it holds that $\frac{d\pi_i^{Cm}}{dx_i} > 0$ only if $i \le m$, and $\frac{d\pi_i^{Cm}}{dx_i} = 0$ otherwise, since a firm's capacity expansion only affects production at stage two in case the firm was constrained. Obviously, in this case the derivative must be positive.

Investment Stage Now we are prepared to analyze capacity choices at the investment stage. The results obtained for the production stage enable us to derive a firm i's profit from investing x_i, given that the other firms invest x_{-i} and quantity choices at stage two are given by $q^{Cm}(x,\theta)$ for $\theta \in [\theta^{Cm}(x), \theta^{Cm+1}(x)]$. Recall that when choosing capacities the firms still face demand uncertainty. Thus, a firm's profit from given levels of investments, x, is the integral over equilibrium profits at each θ given x on the domain $[\underline{\theta}, \overline{\theta}]$, taking into account the probability distribution over the demand scenarios. For each θ, firms anticipate equilibrium play at the production stage, which gives rise to one of the $n+1$ types of equilibria, $EQ^{C0}, \ldots, EQ^{Cm}, \ldots, EQ^{Cn}$. Note that, by assumption 1, part (v), any $x > 0$ gives rise to the unconstrained equilibrium if θ is close enough to $\underline{\theta}$. As θ increases, more and more firms become constrained. Thus, a tuple of investment levels that initially gave rise to an EQ^{C0}, then leads to an equilibrium where first one (then two, three, $\ldots$, and finally n) firms are constrained. In order to simplify the exposition we again make use of the definitions $\theta^{C0} \equiv \underline{\theta}$ and $\theta^{Cn+1} \equiv \overline{\theta}$. Then, the profit of firm i is given by[12]

$$\pi_i(x, q^C) = \sum_{m=0}^{m=n} \int_{\theta^{Cm}}^{\theta^{Cm+1}} \pi_i^{Cm}(x,\theta) dF(\theta) - K(x_i). \tag{8}$$

[12]Note that it is never optimal for a firm to be unconstrained at $\overline{\theta}$ and thus, we always obtain $\theta^{Cn} \le \overline{\theta}$.

Note that at each critical value θ^{Cm}, $m = 1, \ldots, n$ it holds that $\pi^{Cm-1}(x, \theta^{Cm}) = \pi^{Cm}(x, \theta^{Cm})$. Thus, $\pi_i(x, q^C)$ is continuous. Differentiating $\pi_i(x, q^C)$ yields[13]

$$\frac{d\pi_i\left(x, q^C\right)}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{Cm}(x)}^{\theta^{Cm+1}(x)} \frac{d\pi_i^{Cm}(x, \theta)}{dx_i} dF(\theta) - K_x(x_i) \tag{9}$$

Note that if all firms invest the same, then it holds that either all firms are constrained, or none, i. e. $\theta^{C1} = \theta^{C2} = \cdots = \theta^{Cn}$. This implies that for symmetric investment the first order condition coincides for all firms. We are able to show the following

Lemma 1 (Cournot (C)) *The Cournot market game has a unique equilibrium which is symmetric. Equilibrium investments $x_i^C = \frac{1}{n} X^C$, $i = 1, \ldots, n$ solve*

$$\int_{\theta^{Cn}(x^C)}^{\overline{\theta}} \left[P\left(X^C, \theta\right) + P_q\left(X^C, \theta\right) \frac{1}{n} X^C - C_q\left(\frac{1}{n} X^C, \theta\right)\right] dF(\theta) = K_x\left(\frac{1}{n} X^C\right). \tag{10}$$

Proof See appendix A.1. □

Let us emphasize two important aspects of our results: First, we could show that *under standard regularity assumptions* the Cournot market game has a *unique* equilibrium. Second, we find that (symmetric) equilibrium investment can be characterized by a rather intuitive condition, (10). The condition simply says that expected marginal revenue generated by an additional unit of capacity must equal marginal cost of investment. When calculating the marginal revenue of capacity, however, one has to take into account that additional capacity affects a firm's revenue only in those states of nature where capacity was binding. Thus, expectation must only be taken with respect to those scenarios in which the firms are capacity constrained, i. e. over the interval $[\theta^{Cn}\left(x^C\right), \overline{\theta}]$, and not over the whole domain of Θ.

4 First Best

In order to be able to assess the impact of market power on investment incentives, in this section we characterize the first best solution, that is, welfare optimal capacity levels and

[13]Note that continuity of π_i implies that due to Leibnitz' rule the derivatives of the integration limits cancel out. Moreover π_i^{Cm} only changes in x_i if firm i is constrained in scenario FBm, i. e. $i \leq m$. Thus, the sum does not include the cases where firm i is unconstrained, i. e. $m < i$.

output choices given the number of firms in the market. Again we proceed in two steps: We first characterize the socially optimal production plan at stage two for all possible investment levels and then characterize socially optimal investment at stage one.

We moreover show that if firms do not act strategically, investment and production levels coincide with the first best (socially optimal) solution, again given the number of firms. Later, in section 6, we provide a comparison of investment under the First Best solution and in the Cournot outcome.

Production Stage We start with the characterization of the socially optimal production plan at stage two, given the capacities chosen at stage one, which may differ across firms. Recall that we order the firms according to their investment levels, i. e. $x_1 \leq x_2 \leq \cdots \leq x_n$. In the following we specify, for a given vector of capacities x, the optimal production schedule for any possible demand scenario (that is, for any possible value of θ).[14]

Note that due to assumption 1, part (v), all firms are unconstrained for values of θ close to $\underline{\theta}$. It is straightforward to show that in the welfare optimum, all unconstrained firms produce the same (due to convex cost). Thus, the socially optimal total quantity of each firm if all firms are unconstrained is given by $q_i^{FB0}(\theta) = \{q_i \in \mathbb{R} : P(nq_i, \theta) = C_q(q_i, \theta)\}$.

Now, as θ increases, at some critical value, that we denote by $\theta^{FB1}(x)$, firm 1 (the lowest capacity firm) becomes constrained. The critical demand scenario $\theta^{FB1}(x)$ is implicitly defined by $x_1 = q_1^{FB0}(\theta^{FB1})$. If it holds that $x_1 < x_2$, then at $\theta^{FB1}(x)$ only firm 1 becomes constrained and the socially optimal production plan implies that firm 1 produces at its capacity bound whereas the remaining firms produce the unconstrained optimal quantity given the residual demand $P(Q - x_1, \theta)$, i. e. $\tilde{q}_i^{FB1}(x, \theta) = \{q_i \in \mathbb{R} : P((n-1)q_i + x_1, \theta) = C_q(q_i, \theta)\}$. The optimal production plan in scenario $FB1$ is a vector $q^{FB1}(x, \theta)$, where each element is given by $q_i^{FB1}(x, \theta) = \min\{x_i, \tilde{q}_i^{FB1}(x, \theta)\}$.

As θ increases further and more firms become constrained, we pass through $n+1$ cases, from case $FB0$ (no firm is constrained) to case FBn (all n firms are constrained). Note that two critical values $\theta^{FBm}(x)$ and $\theta^{FBm+1}(x)$ coincide whenever $x_m = x_{m+1}$, and that it holds that $\theta^{FBm}(x) < \theta^{FBm+1}(x)$ (by assumption 1 part (v)) whenever $x_m < x_{m+1}$.

Now we are prepared to characterize the socially optimal production plan and social wel-

[14] With convex cost a characterization of the welfare optimum could probably be given with less mathematical burden. However, we will need the characterization developed here also in section 5.2.

fare generated in case FBm, where m firms are constrained. In this case, the m firms with the lowest capacities produce at their capacity bound, whereas the $n-m$ unconstrained firms produce the unconstrained optimal quantity given the residual demand $P(Q-\sum_{i=1}^m x_i,\theta)$, i. e.

$$\tilde{q}_i^{FBm}(x,\theta)=\left\{q_i\in\mathbb{R}:P\left(\sum_{j=1}^m x_j+(n-m)q_i,\theta\right)=C_q(q_i,\theta)\right\}. \tag{11}$$

We denote the optimal production plan in case FBm by $q^{FBm}(x,\theta)$ where each element is given by

$$q_i^{FBm}(x,\theta)=\min\{x_i,\tilde{q}_i^{FBm}(x,\theta)\}\qquad i=1,\ldots,n. \tag{12}$$

Consequently, the optimal total quantity produced in case FBm is

$$Q^{FBm}(x,\theta)=\sum_{i=1}^n q_i^{FBm}(x,\theta). \tag{13}$$

All this allows us finally to pin down maximal social welfare generated in demand scenario $\theta\in[\theta^{FBm},\theta^{FBm+1}]$ (where, given x, the m lowest capacity firms are constrained) as

$$W^{FBm}(x,\theta)=\int_0^{Q^{FBm}(x,\theta)}P(Q,\theta)\,dQ-\sum_{i=1}^n C\left(q_i^{FBm}(x,\theta),\theta\right). \tag{14}$$

Note that W^{FBm} only depends on x_i if firm i is constrained in scenario m, that is if $i\le m$.

Investment Stage Let us now characterize the welfare maximizing level of investment. Total expected welfare is obtained by integrating over all demand realizations. Since the functional form of the maximal attainable welfare changes as we pass from case FBm to case $FBm+1$, we have to integrate piecewisely. In order to facilitate exposition, we define $\theta^{FB0}=\underline{\theta}$ and $\theta^{FBn+1}=\overline{\theta}$. Then, welfare generated by the choice of capacities x, given the optimal production plan is implemented at stage two is

$$\mathcal{W}(x,q^{FB})=\sum_{m=0}^n\int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)}W^{FBm}(x,\theta)dF(\theta)-\sum_{i=1}^n K(x_i). \tag{15}$$

Note that at each critical value θ^{FBm}, $m=1,\ldots,n$, it holds that $W^{FBm-1}(x,\theta^{FBm})=W^{FBm}(x,\theta^{FBm})$. Thus, $\mathcal{W}(x)$ is continuous. Differentiating $\mathcal{W}(x)$ yields

$$\frac{d\mathcal{W}(x,q^{FB})}{dx_i}=\sum_{m=i}^n\int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)}\frac{dW^{FBm}(x,\theta)}{dx_i}dF(\theta)-K_x(x_i). \tag{16}$$

Obviously, the n first order conditions are simultaneously satisfied for all firms if all firms invest the same. We can show the following

LEMMA 2 (FIRST BEST (FB)) *In the welfare optimum, each firm invests* $x_i^{FB} = \frac{1}{n} X^{FB}$, $i = 1, \ldots, n$, *where* x^{FB} *solves*

$$\int_{\theta^{FBn}(x^{FB})}^{\overline{\theta}} \left[P\left(X^{FB}, \theta\right) - C_q\left(\frac{1}{n} X^{FB}, \theta\right) \right] dF(\theta) = K_x\left(\frac{1}{n} X^{FB}\right). \tag{17}$$

PROOF See appendix A.2. □

We obtain a rather intuitive characterization also of the first best investment level. The condition says that in the welfare optimum capacity should be chosen such that expected marginal social welfare of additional capacity [LHS of (17)] should equal marginal cost of investment [RHS of (17)]. Again it is important to notice that expectation is only taken over those scenarios where the firms are actually constrained *given the scheduled stage two–production*, that is, over the interval $[\theta^{FBn}(x^{FB}), \overline{\theta}]$.

REMARK 1 (NON-STRATEGIC FIRMS) *For each number of firms, n, if firms do not behave strategically (i. e. they act as price takers at stage two and ignore their impact on total capacity at stage one), firms invest and produce optimally from a social welfare point of view.*

PROOF See appendix A.3 □

5 Partial Intervention

This section is thought to shed light on the effects that intervention at *only one of the two stages* has on investment incentives. In the following section (5.1) we consider implementation of the welfare optimal capacity level at stage one given that firms strategically choose their outputs at the production stage (Second Best). In section 5.2 we analyze strategic capacity choices if firms anticipate implementation of the welfare optimal production schedule given the capacities chosen at stage two (Optimal Regulation at the Production stage, ORP).[15] Table 2 relates those scenarios to the scenarios already analyzed in sections 3 and 4.

[15] We abstract from all informational problems by assuming that a social planer implements the welfare optimum at one stage given that firms behave strategically at the other one.

		Objective at the Production Stage	
		Profit	**Welfare**
Objective at the Investment Stage	**Profit**	Cournot X^C	ORP X^{ORP}
	Welfare	Second Best X^{SB}	First Best X^{FB}

Table 1: The four scenarios analyzed.

5.1 Second Best

In order to investigate whether the capacity choices of strategic firms are locally inefficient, in this section we characterize the *socially optimal investment levels given that firms play the capacity constrained Cournot equilibrium at the production stage*. Later, in section 6, we will provide a comparison with capacity levels in the First Best and in the Cournot outcome.

If, at stage two, firms play the capacity constrained Cournot equilibrium[16] $q_i^{Cm}(x, \theta)$, $i = 1, \ldots, n$, aggregate production in case Cm is given by $Q^{Cm}(x, \theta)$ as defined in (6). Consequently, total welfare generated in demand scenario $\theta \in [\theta^{Cm}(x), \theta^{Cm+1}(x)]$ is

$$W^{Cm}(x, \theta) = \int_0^{Q^{Cm}(x,\theta)} P(Q, \theta) dQ - \sum_{i=1}^{n} C(q_i^{Cm}(x, \theta), \theta). \tag{18}$$

Note that $W^{Cm}(x, \theta)$ depends on x_i only if firm i is constrained in case Cm, i. e. if $i \leq m$, or, equivalently, $q_i^{Cm}(x, \theta) = x_i$.

Welfare generated by the choice of capacities x, given that the firms play the capacity constrained Cournot equilibrium at stage two is given by

$$\mathcal{W}(x, q^C) = \sum_{m=0}^{n} \int_{\theta^{Cm}(x)}^{\theta^{Cm+1}(x)} W^{Cm}(x, \theta) dF(\theta) - \sum_{i=1}^{n} K(x_i) \tag{19}$$

Note that at each critical value θ^{Cm}, $m = 1, \ldots, n$, it holds that $W^{Cm-1}(x, \theta^{Cm}) =$

[16] See the characterization provided in section 3.

$W^{Cm}(x, \theta^{Cm})$. Thus, $\mathcal{W}(x, q^C)$ is continuous. Differentiation yields

$$\frac{d\mathcal{W}(x, q^C)}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{Cm}(x)}^{\theta^{Cm+1}(x)} \frac{dW^{Cm}(x, \theta)}{dx_i} dF(\theta) - K_x(x_i) \tag{20}$$

Obviously, the n first order conditions are simultaneously satisfied for all firms if all firms invest the same. We can indeed show the following

Lemma 3 (Second Best (SB)) *The capacities $x_i^{SB} = \frac{1}{n}X^{SB}$, $i = 1, \ldots, n$, a social planner would like to implement prior to Cournot competition at stage two solve*

$$\int_{\theta^{Cn}(x^{SB})}^{\overline{\theta}} \left[P\left(X^{SB}, \theta\right) - C_q\left(\frac{1}{n}X^{SB}, \theta\right)\right] dF(\theta) = K_x\left(\frac{1}{n}X^{SB}\right) \tag{21}$$

Proof See appendix A.4. □

5.2 Optimal Regulation at the Production Stage

In order to investigate the impact of stage two–intervention on capacity choices in oligopolistic markets, we analyze strategic capacity choices at stage one given that firms anticipate that at stage two the socially optimal solution is implemented (e.g. by a social planer). A comparison of equilibrium investment in this scenario with investments in the First Best, Second Best, and Cournot solution is provided in section 6.

If the competitive outcome is implemented at stage two, outputs coincide with the welfare maximizing quantities characterized in equation (12). Thus, a firm i's stage two–profit in scenario $\theta \in [\theta^{FBm}(x), \theta^{FBm+1}(x)]$ where firms have invested x and m firms turn out to be constrained is given by

$$\pi_i^{FBm}(x, \theta) = \begin{cases} P\left(Q^{FBm}(x, \theta), \theta\right) x_i - C(x_i, \theta) & \text{if} \quad i \leq m, \\ \\ P\left(Q^{FBm}(x, \theta), \theta\right) \tilde{q}_i^{FBm}(x, \theta) - C\left(\tilde{q}_i^{FBm}(\cdot), \theta\right) & \text{if} \quad i > m. \end{cases}$$

The stage one expected profit of firm i is obtained by integrating over all profits associated with each demand realization,

$$\pi_i(x, q^{FB}) = \sum_{m=0}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \pi_i^{FBm}(x, \theta) dF(\theta) - K(x_i). \tag{22}$$

Thus, the first order condition is

$$\frac{d\pi_i\left(x, q^{FB}\right)}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \frac{d\pi_i^{FBm}\left(x, \theta\right)}{dx_i} dF\left(\theta\right) - K_x\left(x_i\right). \tag{23}$$

Again, we immediately see that if investment is symmetric across firms, only the last integral in (23) remains positive. We show the following

Lemma 4 (Optimal Regulation at the Production Stage (ORP))

(i) In any symmetric equilibrium of the game where the competitive outcome is implemented at stage two, firms choose capacities $x_i^{ORP} = \frac{1}{n} X^{ORP}$, $i = 1, \ldots, n$ such that

$$\int_{\theta^{FBn}(x^{ORP})}^{\overline{\theta}} \left[P\left(X^{ORP}, \theta\right) + P_q\left(X^{ORP}, \theta\right) \frac{X^{ORP}}{n} - C_q\left(\frac{X^{ORP}}{n}, \theta\right)\right] dF(\theta) \tag{24}$$
$$= K_x\left(\frac{X^{ORP}}{n}\right).$$

(ii) Suppose that marginal cost $C_q(q, \theta)$ is constant in q. Then , there exists at least one symmetric equilibrium, but there may be more than one. No asymmetric equilibria exist.

(iii) The game always has a unique symmetric (degenerate) equilibrium if $X^C \leq \tilde{Q}^{C0}(\underline{\theta})$, i. e. capacity in the Cournot outcome is lower than the unconstrained Cournot equilibrium production at $\underline{\theta}$. In such an equilibrium firms are constrained at any $\theta \in [\underline{\theta}, \overline{\theta}]$.

Proof See appendix A.5 □

Note that we cannot prove existence and uniqueness of a symmetric equilibrium in the general case while for constant marginal cost existence (but not uniqueness) can be shown. The basic problem is that in neither case the stage one profit is quasiconcave, which makes standard analysis impossible. In the case of linear marginal cost, however, we can exploit recent insights on oligopolistic competition that makes use of lattice theory (Amir and Lambson (2000)). In the general case (i. e. strictly convex production cost), however, the game cannot be reformulated as a supermodular game and thus, even more sophisticated techniques do not help.

Finally let us draw the reader's attention to the degenerate case mentioned in part (iii) of the lemma. There we show that the game with optimal regulation at stage two always has a unique equilibrium in case that *even in the Cournot market game (see section 3) firms always want to be constrained, even at the lowest realization of θ.* In section 6 we will provide further intuition on this special case.

6 Comparison of Investment Levels

In this section compare equilibrium investments in the scenarios analyzed in sections 3 to 5. Moreover, in the discussion of our result we demonstrate how the approach can be used to easily obtain insights on the effect that regulatory intervention or market re–organization have on investment incentives, far beyond the stylized scenarios we analyzed.

THEOREM 1 *Suppose that assumptions 1 and 2 hold.*

(i) For any finite number of firms, n, it holds that

- *Capacity in the Cournot outcome is too low from a social welfare point of view, i. e.* $X_n^{SB} > X_n^C$.
- *Capacity in any symmetric stage two–regulated outcome is lower than in the Cournot outcome, i. e.* $X_n^C \geq X_n^{ORP}$.
- *The first best solution yields the highest capacity level among all scenarios.*

Summarizing, it holds that $X_n^{FB} \geq X_n^{SB} > X_n^C \geq X_n^{ORP}$.

(ii) As the number of firms approaches infinity, investment levels in all scenarios coincide, i. e. $X_\infty^{FB} = X_\infty^{SB} = X_\infty^C = X_\infty^{ORP}$.

PROOF **Part (i)** Consider the first order conditions that implicitly define total capacities in the four scenarios considered, as given in lemmas 1 to 4. Note that (i) $P_q(X,\theta) < 0$, (ii) $\theta^{Cn}(x) > \theta^{FBn}(x)$ for all x[17], and that (iii) at (below, above) the demand realization $\theta^{Cn}(x^C)$ we have that $P_q(X^C,\theta)\frac{X^C}{n} + P(X^C,\theta) - C_q(\frac{1}{n}X^C,\theta) = 0$ ($< 0, > 0$). Thus, the lefthand–sides of the first order conditions can be ordered as follows:

$$
\begin{aligned}
FB: &\quad \int_{\theta^{FBn}(x)}^{\overline{\theta}} \left[P(X,\theta) - C_q\left(\frac{1}{n}X,\theta\right)\right] dF(\theta) \qquad (25)\\
SB: &\quad \geq \int_{\theta^{Cn}(x)}^{\overline{\theta}} \left[P(X,\theta) - C_q\left(\frac{1}{n}X,\theta\right)\right] dF(\theta)\\
C: &\quad > \int_{\theta^{Cn}(x)}^{\overline{\theta}} \left[P_q(X,\theta)\frac{1}{n}X + P(X,\theta) - C_q\left(\frac{1}{n}X,\theta\right)\right] dF(\theta)\\
ORP: &\quad \geq \int_{\theta^{FBn}(x)}^{\overline{\theta}} \left[P_q(X,\theta)\frac{1}{n}X + P(X,\theta) - C_q\left(\frac{1}{n}X,\theta\right)\right] dF(\theta)
\end{aligned}
$$

[17] The latter is due to the fact that firms get already constrained at lower demand realizations if they behave competitively and therefore produce where demand equals marginal cost.

Note that according to lemmas 1 to 4, the total capacities are determined as the values of X where the respective term equals $K_x\left(\frac{1}{n}X^Z\right)$, $Z \in \{FB, SB, C, ORP\}$. Recall that in all cases we get interior solutions and note that the above terms (except for the one that determines X^{ORP}) are decreasing in X, while K_x is increasing in X. This immediately implies $X^{FB} \geq X^{SB} > X^C$.

In order to see why the ranking stated in the theorem also holds for ORP, note that the above term in scenario C is strictly decreasing in X, whereas in scenario ORP it satisfies $LHS(0) > K_x(0)$ (by assumption 2) and $LHS(X) < K_x(X)$ for X high enough (by assumption 1 (i)). Since $K_x(X)$ is increasing in X, this immediately implies that for any equilibrium investment X^{ORP} it holds that $X^C \geq X^{ORP}$.

Part (ii) As n approaches infinity, all first order conditions collapse to $\int_{\underline{\theta}}^{\overline{\theta}}[P(X,\theta) - C_q(0,\theta)]dF(\theta) = K_x(0)$. □

In the following we derive exact conditions under which the weak inequalities from theorem 1 are strict, and hold with equality, respectively. They hold with equality whenever already the capacity choice determines production in any demand scenario $\theta \in [\underline{\theta}, \overline{\theta}]$, that is, if firms are always constrained at the production stage. In particular:

Theorem 2 (Degenerate Cases) *Suppose $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. Then it holds*[18]

(i) $X^C \leq \tilde{Q}^{C0}(\underline{\theta}) \quad \Leftrightarrow X^C = X^{ORP}$,

(ii) $X^{FB} \leq \tilde{Q}^{C0}(\underline{\theta}) \quad \Leftrightarrow X^{FB} = X^{SB}$.

Proof Let x^0 be a vector of equal capacities summing up to X^0. We have $\underline{\theta} \leq \theta^{FBn}(x^0) \leq \theta^{Cn}(x^0)$ for all x^0 and both, $\theta^{FBn}(x^0)$ and $\theta^{Cn}(x^0)$ are increasing in X^0.
(i) If $X^C \leq \tilde{Q}^{C0}(\underline{\theta})$, then $\underline{\theta} = \theta^{Cn}(x^C)$. This implies that $\underline{\theta} = \theta^{FBn}(x^{ORP}) = \theta^{Cn}(x^C)$ (since $X^{ORP} \leq X^C$). Then the first order conditions (10) and (24) collapse since the lower limit of integration is given by $\underline{\theta}$. This proves "$\Rightarrow$". In order to prove "$\Leftarrow$", note that $X^C > \tilde{Q}^{C0}(\underline{\theta})$ implies $\underline{\theta} \leq \theta^{FBn}(x^{ORP}) < \theta^{Cn}(x^C)$.[19] Then the the lower limit of integration in first order conditions (10) and (24) does not coincide which implies $X^{ORP} < X^C$ if $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$.
(ii) The proof works analogously to part (i). □

[18]The assumption $f(\theta) > 0$ is only needed for the "$\Leftarrow$"-direction. "$\Rightarrow$" always holds.
[19]Note that whenever $\underline{\theta} < \theta^{Cn}(x^C)$, then it holds that $\theta^{FBn}(x^{ORP}) < \theta^{Cn}(x^C)$.

<table>
<tr><th>Genuine Uncertainty</th><th colspan="2">Degenerate Cases</th></tr>
<tr><td>$\tilde{Q}^{C0}(\underline{\theta}) < X^C$</td><td>$X^C \leq \tilde{Q}^{C0}(\underline{\theta}) < X^{FB}$</td><td>$X^{FB} \leq \tilde{Q}^{C0}(\underline{\theta})$</td></tr>
<tr><td>$X^{ORP} < X^C$</td><td colspan="2">$X^{ORP} = X^C$</td></tr>
<tr><td colspan="2">$X^{SB} < X^{FB}$</td><td>$X^{SB} = X^{FB}$</td></tr>
</table>

Table 2: Degenerate Cases and Equivalence of Scenarios.

If condition (i) of theorem 2 holds, in the Cournot market game (section 3) firms want to be constrained at the production stage in any state of nature $\theta \in [\underline{\theta}, \overline{\theta}]$. Since the incentive to be constrained is higher in case of optimal regulation at stage two, the solutions of C and ORP collapse in this case. Moreover, comparison with a result by Reynolds and Wilson (2000) shows that under condition (i) also a game where firms invest prior to Bertrand competition at stage two yields the same capacity as C and ORP.[20]

This result is well known in the absence of uncertainty (when obviously condition (i) is always satisfied). In this case, the equivalence of the Cournot and the Bertrand outcome has already been shown by Kreps and Scheinkman (1983). Our results show that those findings also hold under a weaker condition that basically imposes a restriction on the variance of θ. Obviously, condition (i) describes a degenerate environment where uncertainty does not matter much. Under genuine uncertainty, where firms are unconstrained in at least some states of nature, our analysis demonstrates that in fact market organization at stage two matters a lot.[21]

[20]Reynolds and Wilson show that under condition (i) capacity choice prior to Bertrand competition yields the same outcome as capacity choice in a game where firms cannot adjust their production after uncertainty unraveled. It is easy to show that under condition (i) the latter game yields the same outcome as our Cournot market game (which clearly is not the case if condition (i) does not hold). Reynolds and Wilson fail to recognize, however, their this game does not have a unique equilibrium in case of genuine uncertainty (which is why part (ii) of their theorem is incomplete).

[21]For the Bertrand market game Reynolds and Wilson (2000) show that under genuine uncertainty equilibria with equal capacities of the firms do not exist.

If condition (ii) holds, at the welfare maximizing (First Best) capacity level even strategic firms are constrained in any demand scenario $\theta \in [\underline{\theta}, \overline{\theta}]$ at stage two. Notice that condition (ii) is stronger than condition (i) [since $X^{FB} > X^{C}$, as we have shown in theorem 1]. Consequently, (ii) can only hold in a degenerate environment where uncertainty is not an important issue.

Why the level of uncertainty is not the only decisive factor for a equivalence of X^{FB} and X^{SB} can best be illustrated in case of certain demand. At the production stage, strategic firms play either their Cournot quantity given marginal *production* cost, or their capacity, whichever is lower. This implies that even under certainty the First Best and the Second Best outcome coincide only in those cases where the First Best capacity level is *below* the Cournot quantities at stage two. Thus, condition (ii) requires that marginal capacity cost is sufficiently high compared to marginal production cost and that uncertainty does not matter much. As we have shown in our analysis, however, under genuine uncertainty the First Best solution always implies higher investment than the second best solution, independent of marginal capacity and production cost.

Let us finally draw the reader's attention to the particular structure of all four first order conditions. They all equalize marginal profit or welfare of additional capacity [LHSs of the first order conditions as listed in equation (25)] with marginal cost of capacity [RHS] (see lemmas 1 to 4). Note that the stage one–objective is reflected only in the integrand at the LHS while the stage two–objective enters exclusively into the lower limit of integration. That is, we integrate over *marginal profit* in cases where the firms maximize profits at stage one (C and ORP) and over *marginal welfare* in cases where welfare is the stage one-objective (FB and SB). The game at stage two enters only in form of the lower limit of integration, which is the state of nature from which on firms are constrained given the capacities chosen at stage one (i. e. $\theta^{Cn}(x)$ in the case of Cournot competition at stage two and $\theta^{FBn}(x)$ if the welfare optimum is implemented).

7 Conclusion

In this paper we have provided a general model of strategic investment decisions under uncertainty prior to imperfectly competitive markets. We have shown existence and uniqueness of equilibrium and provided an intuitive characterization of equilibrium investment.

We found that increasing capacity is desirable from a social welfare point of view. We also demonstrated that intervention only at the production stage leads to strategic uncertainty at the investment stage and, moreover, decreases total investment. Thus, in markets with considerable demand fluctuations, (partial) intervention only at the market stage has to be carefully reconsidered.

The particular structure of the first order conditions discussed at the end of the previous section moreover allows several conjectures about the desirability of interventions at either stage one or stage two. First, our model suggests that any stage two–intervention which increases production above the level obtained in the Cournot outcome in every state of the world *reduces investment*. Second, increasing the capacity above the level freely chosen by the firms is desirable from a social welfare point of view whenever firms exercise market power to some extent at the production stage.

While the model provides a solid intuition for how investment incentives and welfare are affected by regulatory intervention, specific market designs under consideration still have to be analyzed carefully in order to obtain reliable policy conclusions. In this respect, our model provides a tractable framework for the analysis of different scenarios at the market stage. The framework captures the stylized fact that at the time when they make their investment decisions firms face considerable uncertainty both about future demand and production cost, and probably also with respect to future regulatory regimes. Let us outline several directions of research that can directly benefit from the analysis done in this paper.

The most obvious application of the model is to modify the game played at the second stage in order to analyze how different market designs or regulatory interventions affect investment incentives and welfare. However, modeling a more complicated strategic context at the production stage usually comes at the cost of loosing some generality (i. e. restriction to linear demand). Grimm and Zoettl (2005) analyze how investment incentives are affected by the introduction of forward markets prior to spot trading and Zoettl (2005) considers price cap regulation at the spot market. Whereas the results of Grimm and Zoettl (2005) confirm the intuition that making the spot market more competitive decreases investments, Zoettl (2005) finds that price caps at stage two may actually increase investment incentives. The reason is that price caps eliminate an important feature of the present model, i. e. prices do not rise in case of insufficient capacity, which crucially affects the incentives.

A second line of research for which the current model serves as a starting point is the analysis of capacity expansion, probably even allowing the choice between different technologies. On the one hand, such a model would allow to analyze the effect of measures like emission permits in electricity markets that affect variable costs of different technologies to different extents. On the other hand it could serve as the theoretical benchmark that allows to estimate market power at the investment stage.

A Proofs

A.1 Proof of Lemma 1

We prove the lemma in two parts. In part I we show existence and in part II uniqueness of the equilibrium. For the proof we first need to establish the following

Property 1 (Monotonicity of θ^{Cm}) *$\frac{d\theta^{Cm}(x)}{dx_i}$ is strictly positive if $i \leq m$, and zero otherwise.*

Proof $\theta^{Cm}(x)$ is the demand realization from which on firm m cannot play its unconstrained output any more. At $\theta^{Cm}(x)$ it holds that $q_i^C(\theta^{Cm}(x)) = \tilde{q}^{Cm}(\theta^{Cm}(x)) = x_m$ for all $i \geq m$ and $q_i^C(\theta^{Cm}(x)) = x_i < x_m$ for all $i < m$. Thus, $\theta^{Cm}(x)$ is implicitly defined by the conditions

$$\begin{aligned} &P\left(\sum_{i=1}^{m} x_i + (n-m)x_m, \theta^{Cm}(x)\right) \\ &+P_q\left(\sum_{i=1}^{m} x_i + (n-m)x_m, \theta^{Cm}(x)\right) x_m - C_q\left(x_m, \theta^{Cm}(x)\right) = 0. \end{aligned}$$

Differentiation with respect to x_i, $i < m$, yields

$$P_q(\cdot) + P_\theta(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} + P_{qq}(\cdot)\,x_m + P_{q\theta}(\cdot)\,x_m\frac{d\theta^{Cm}(x)}{dx_i} - C_{q\theta}(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} = 0,$$

and solving for $\frac{d\theta^{Cm}(x)}{dx_i}$ we obtain

$$\frac{d\theta^{Cm}(x)}{dx_i} = -\frac{P_q(\cdot) + P_{qq}(\cdot)\,x_m}{P_\theta(\cdot) + P_{q\theta}(\cdot)\,x_m - C_{q\theta}(\cdot)} > 0$$

due to assumption 1, parts (iv) and (vi) [note that the expression in the denominator is the cross derivative which was assumed to be positive in part (vi) of assumption 1].

Differentiation with respect to x_i, $i = m$, yields

$$(n-m+2)P_q(\cdot) + P_\theta(\cdot)\frac{d\theta^{Cm}(x)}{dx_i}$$
$$+(n-m+1)P_{qq}(\cdot)x_m + P_{x\theta}(\cdot)x_m\frac{d\theta^{Cm}(x)}{dx_i} - C_{xx}(\cdot) - C_{q\theta}(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} = 0,$$

and solving for $\frac{d\theta^{Cm}(x)}{dx_i}$ we obtain

$$\frac{d\theta^{Cm}(x)}{dx_i} = -\frac{(n-m+2)P_q(\cdot) + (n-m+1)P_{qq}(\cdot)x_m - C_{xx}(\cdot)}{P_\theta(\cdot) + P_{q\theta}(\cdot)x_m - C_{q\theta}(\cdot)} > 0,$$

also due to assumption 1, parts (iv) and (vi). Finally, differentiation with respect to x_i, $i > m$, yields

$$P_\theta(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} + P_{x\theta}(\cdot)x_m\frac{d\theta^{Cm}(x)}{dx_i} - C_{q\theta}(\cdot)\frac{d\theta^{Cm}(x)}{dx_i} = 0,$$

which implies that $\frac{d\theta^{Cm}(x)}{dx_i} = 0$ for $i > m$. □

PartI: Existence of Equilibrium In the following we show that a symmetric equilibrium of the two stage Cournot market game exists, and that equilibrium choices $x_i^C = \frac{1}{n}X^C$, $i = 1, \ldots, n$, are implicitly defined by equation (10). For this purpose it is sufficient to show quasiconcavity of firm i's profit given the other firms invest x_{-i}^C, $\pi_i(x_i, x_{-i}^C)$, which we do in the following.

Note that $\pi_i(x_i, x_{-i}^C)$ is defined piecewisely. For $x_i < x_i^C$, we have to examine to profit of firm 1 (by convention the lowest capacity firm) given that $x_2 = x_3 = \cdots = x_n$. Since this implies that $\theta^{C2} = \cdots = \theta^{Cn}$ and thus it follows from (8) that

$$\begin{aligned}\pi_1(x_1, x_{-1}^C) &= \int_{\underline{\theta}}^{\theta^{C1}(x)} \pi_1^{C0}(x,\theta)dF(\theta) + \int_{\theta^{C1}(x)}^{\theta^{Cn}(x)} \pi_1^{C1}(x,\theta)dF(\theta) \\ &+ \int_{\theta^{Cn}(x)}^{\overline{\theta}} \pi_i^{Cn}(x,\theta)dF(\theta) - K(x_1)\end{aligned} \tag{26}$$

For $x_i > x_i^C$, the profit of firm i is the profit of the highest capacity firm (firm n according to our convention), given all other firm have invested the same, i. e. $x_1 = \cdots = x_{n-1}$. We get

$$\begin{aligned}\pi_n(x_n, x_{-n}^C) &= \int_{\underline{\theta}}^{\theta^{Cn-1}(x)} \pi_n^{C0}(x,\theta)dF(\theta) + \int_{\theta^{Cn-1}(x)}^{\theta^{Cn}(x)} \pi_n^{Cn-1}(x,\theta)dF(\theta) \\ &+ \int_{\theta^{Cn}(x)}^{\overline{\theta}} \pi_n^{Cn}(x,\theta)dF(\theta) - K(x_1)\end{aligned} \tag{27}$$

(i) The shape of $\pi_i(x_i, x_{-i}^C)$ for $x_i > x_i^C$: The second derivative of the profit function π_n is given by[22]

$$\frac{d^2\pi_n}{(dx_n)^2} = -\frac{d\theta^{Cn}(x)}{dx_n} \underbrace{\left[\frac{d\pi_n^{Cn}(x,\theta^{Cn})}{dx_n}\right]}_{=0 \ (x_n \text{ is opt. at} \theta^{Cn})} f(\theta^{Cn}) + \int_{\theta^{Cn}(x)}^{\overline{\theta}} \underbrace{\frac{d^2\pi_n^{Cn}(x,\theta)}{(dx_n)^2}}_{<0 \text{ by A1 part (iv)}} f(\theta)d\theta < 0. \tag{28}$$

Note that the first term cancels out and the second term is negative by concavity of the spot market profit function (implied by assumption 1, part (iv)). We find that for $x_i > x_i^C$, $\pi_i(x_i, x_{-i}^C)$ is concave, which implies that upwards deviations are not profitable.

(ii) The shape of $\pi_i(x_i, x_{-i}^C)$ for $x_i < x_i^C$: This region is more difficult to analyze since the profit function $\pi_1(x_1, x_{-1}^C)$ is not concave. But we can show quasiconcavity of $\pi_1(x_1, x_{-1}^C)$. For this purpose we need the following properties of marginal profits at stage two for the cases $(C1)$ and (Cn) [that can be derived from equations (7)].

Property 2 [Marginal Profits at Stage Two in Cases $(C1)$ and (Cn)]

(i) $\frac{d\pi_1^{C1}(x,\theta)}{dx_1} \geq 0.$

(ii) $\frac{d\pi_1^{Cn}(x_1',x_{-1},\theta)}{dx_1} \geq \frac{d\pi_1^{C1}(x_1'',x_{-1},\theta)}{dx_1} \geq 0$ *for* $x_1' < x_1''.$

Proof (i) The first part holds due to the fact in case firm 1 is constrained, i. e. $(\theta \geq \theta^{C1})$, firm 1 would like to produce more than x_1 for all demand realizations $\theta \geq \theta^{C1}$, which, however, is not possible due to the capacity constraint.
(ii) The first inequality follows from concavity of the profit functions in the spot markets, which is implied by assumption 1, part (iv). Thus, the first order condition at each spot-market is decreasing in x_1 until $\tilde{q}_i^{C0}$, which immediately yields the first inequality of part (ii). The second inequality is due to the fact that in case all firms are constrained, i. e. $(\theta \in [\theta^{Cn}, \overline{\theta}])$, firm 1 would like to produce more for all demand realizations θ (which is not possible because it is constrained). □

Now we can use property 2 in order to complete the proof of existence (part I). We can show quasiconcavity of $\pi_1(x_1, x_{-1}^C)$ by showing that

$$\frac{d\pi_1(x_1^0, x_{-1}^C)}{dx_1} \geq \frac{d\pi_1(x_1^C, x_{-1}^C)}{dx_1} = 0 \qquad \text{for all} \quad x_1^0 < x_1^C.$$

[22] It is obvious that there is no incentive for any firm to deviate such that it is unconstrained at $\overline{\theta}$. Thus, we only consider the case that all firms are constrained at $\overline{\theta}$.

This holds true, since [compare also equation (9)]

$$
\begin{aligned}
\frac{d\pi_1(x_1^0, x_{-1}^C)}{dx_1} &= \underbrace{\int_{\theta^{C1}(x_1^0,x_{-1}^C)}^{\theta^{Cn}(x_1^0,x_{-1}^C)} \frac{d\pi_1^{C1}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta)}_{>0 \quad \text{by property 2, part (i)}} + \int_{\theta^{Cn}(x_1^0,x_{-1}^C)}^{\overline{\theta}} \frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta) \\
&> \int_{\theta^{Cn}(x_1^0,x_{-1}^C)}^{\overline{\theta}} \frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta) \\
&= \underbrace{\int_{\theta^{Cn}(x_1^0,x_{-1}^C)}^{\theta^{Cn}(x_{-1}^C,x_{-1}^C)} \frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} dF(\theta)}_{\geq 0 \quad \text{by properties 1 and 2, part (ii)}} \\
&\quad + \underbrace{\int_{\theta^{Cn}(x_1^C,x_{-1}^C)}^{\overline{\theta}} \left[\frac{d\pi_1^{Cn}(x_1^0, x_{-1}^C, \theta)}{dx_1} - \frac{d\pi_1^{Cn}(x_1^C, x_{-1}^C, \theta)}{dx_1} \right] dF(\theta)}_{\geq 0 \quad \text{by property 2, part (ii)}} \\
&\quad + \underbrace{\int_{\theta^{Cn}(x_1^C,x_{-1}^C)}^{\overline{\theta}} \frac{d\pi_1^{Cn}(x_1^C, x_{-1}^C, \theta)}{dx_1} dF(\theta)}_{=\frac{d\pi_i(x^C)}{dx_i}=0 \text{ [recall that } \theta^{C1}(x^C)=\theta^{Cn}(x^C)]} \geq 0.
\end{aligned}
$$

To summarize, in part I we have shown that $\pi_i(x_i, x_i^C)$ is quasiconcave. We conclude that the first order condition given in lemma 1 indeed characterizes equilibrium investment in the Cournot market game.

Part II: Uniqueness In this part we show that (i) x^C is the unique symmetric equilibrium and (ii) that there are no asymmetric equilibria.

(i) x^C is the unique symmetric equilibrium. If capacities are equal, i. e. $x_1^0 = x_2^0 = \cdots = x_n^0$, we have

$$\frac{d\pi_i(x^0)}{dx_i} = \int_{\theta^{Cn}(x^0)}^{\overline{\theta}} [P(nx_i^0, \theta) + P_q(nx_i^0, \theta)x_i^0 - C_q(x_i^0, \theta)]f(\theta)d\theta - K_x(x_i^0).$$

Differentiation yields[23]

$$\frac{d^2\pi_i(x^0)}{(dx_i)^2} = \int_{\theta^{Cn}(x^0)}^{\overline{\theta}} \left[(n+1)P_q(nx_i^0, \theta) + nP_{qq}(nx_i^0, \theta)x_i^0 - C_{qq}(x_i^0, \theta)\right] dF(\theta) - K_{xx}(x_i^0) < 0,$$

which is negative due to assumption 1 part (iv). Thus, since $\frac{d\pi_i(x^C)}{dx_i} = 0$ and moreover $\pi_i(x)$ is concave along the symmetry line, no other symmetric equilibrium can exist.

(ii) There cannot exist an asymmetric equilibrium. Any candidate for an asymmetric equilibrium $\hat{x}$ can be ordered such that $\hat{x}_1 \leq \hat{x}_2 \leq \cdots \leq \hat{x}_n$, where at least one inequality

[23]Differentiation works as in (28).

has to hold strictly. This implies $\hat{x}_1 < \hat{x}_n$. The profit of firm n can be obtained by setting $i = n$ in equation (8), and the first derivative is given by

$$\frac{d\pi_n}{dx_n} = \int_{\theta^{Cn}(x)}^{\overline{\theta}} \frac{d\pi_n^{Cn}(x,\theta)}{dx_n} f(\theta) d\theta - K_x(x_n).$$

It is easy to show that firm n's profit function is concave by examination of the second derivative [see equation (28)]. Thus, any asymmetric equilibrium $\hat{x}$, if it exists, must satisfy $\frac{d\pi_n(\hat{x})}{dx_n} = 0$. We now show that whenever it holds that $\frac{d\pi_n(\hat{x})}{dx_n} = 0$, firm 1's profit is increasing in x_1 at $\hat{x}$ (which implies that no asymmetric equilibria exist).

From equation (9) it follows that the first derivative of firm 1's profit function is given by

$$\frac{d\pi_1}{dx_1} = \int_{\theta^{C1}(x)}^{\theta^{C2}(x)} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta) d\theta + \cdots + \int_{\theta^{Cn}(x)}^{\overline{\theta}} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta) d\theta - K_x(x_1).$$

Note that all the integrals in $\frac{d\pi_1}{dx_1}$ are positive since firm 1 is constrained at all demand realizations and therefore would want to increase its production. Thus, we have

$$\frac{d\pi_1}{dx_1} > \int_{\theta^{Cn}(x)}^{\overline{\theta}} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta) d\theta - K_x(x_1),$$

where the RHS are simply the last two terms of $\frac{d\pi_1}{dx_1}$. Note furthermore that $\hat{x}_1 < \hat{x}_n$ also implies that $K_x(\hat{x}_1) < K_x(\hat{x}_n)$ (due to assumption 2) and

$$\frac{d\pi_1(\hat{x})}{dx_1} = P(\hat{x},\theta) + P_q(\hat{x},\theta)\hat{x}_1 - C_q(\hat{x}_1,\theta) < P(\hat{x},\theta) + P_q(\hat{x},\theta)\hat{x}_n - C_q(\hat{x}_n,\theta) = \frac{d\pi_n(\hat{x})}{dx_n}$$

(due to assumption 1, part (iv)). Now we can conclude that

$$\frac{d\pi_1}{dx_1} > \int_{\theta^{Cn}(x)}^{\overline{\theta}} \frac{d\pi_1^{Cn}(x,\theta)}{dx_1} f(\theta) d\theta - K_x(x_1) > \int_{\theta^{Cn}(x)}^{\overline{\theta}} \frac{d\pi_n^{Cn}(x,\theta)}{dx_n} f(\theta) d\theta - K_x(x_n) = 0.$$

The last equality is due to the fact that this part is equivalent to the first order condition of firm n, which is satisfied at $\hat{x}$ by construction. To Summarize, we have shown that $\frac{d\pi_1}{dx_1} > 0$, which implies that there exist no asymmetric equilibria, since at any equilibrium candidate, firm 1 has an incentive to increase its capacity.

A.2 Proof of Lemma 2

Part I: Existence Note that on a compact set any continuous function has at least one global maximum. The result applies to our setup since $\mathcal{W}(x, q^{FB})$ is continuous and $x \in [0, \overline{Q}(\overline{\theta})]$. Now it remains to show that the optimal investment levels cannot be asymmetric (see part II) and that the symmetric solution as characterized in lemma 2 is unique (see part III).

Part II: Symmetry We first show that the optimal capacity choices cannot be asymmetric across firms. We start at the first order condition (16), which at the optimal solution has to hold simultaneously for all firms. It can be rewritten as follows:

$$\frac{d\mathcal{W}(x,q^{FB})}{dx_i} = \sum_{m=i}^{n}\int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \frac{dW^{FBm}(x,\theta)}{dx_i}dF(\theta) - K_x(x_i) \tag{29}$$

Let us first define the relevant *industry marginal cost function*,, given the capacities chosen by the firms, x. Note that in any of the cases FBm, $m = 1, \ldots, n$, the unconstrained firms produce the same in the socially optimal solution. Thus, in case $FB0$, the relevant industry marginal cost is given by $C_q(\frac{Q}{n}, \theta)$. Increasing Q leads, at some point, to a situation where $x_1 = \frac{Q}{n}$. A further increase of Q then has to be produced by firms 2 to n, and thus, from here on industry marginal cost is given by $C(\frac{Q-x_1}{n-1}, \theta)$. Continuation of this argument yields a general formulation of the industry marginal cost function as follows:

$$C_x^I(Q,\theta|x) = \begin{cases} C_q\left(\frac{Q}{n},\theta\right) & \text{if } \ Q \in [0, nx_1), \\ & \\ C_q\left(\frac{Q-\sum_{i=1}^{m}x_i}{n-m},\theta\right) & \text{if } \ Q \in [\sum_{i=1}^{m-1}x_i + (n-m+1)\,x_m, \\ & \qquad \sum_{i=1}^{m}x_i + (n-m)\,x_{m+1}) \\ & \\ \infty & \text{if } \ Q \in [\sum_{i=1}^{n}x_i, \infty) \end{cases}$$

Now we can rewrite maximal social welfare generated in case FBm (given by (14)) as follows:

$$\begin{aligned} W^{FBm}(x,\theta) &= \int_0^{nx_1}\left[P(Q,\theta) - C_q\left(\frac{Q}{n},\theta\right)\right]dQ \\ &\quad + \sum_{k=1}^{m-1}\int_{\sum_{i=1}^{k-1}x_i+(n-k+1)x_k}^{\sum_{i=1}^{k}x_i+(n-k)x_{k+1}}\left[P(Q,\theta) - C_q\left(\frac{Q-\sum_{i=1}^{k}x_i}{n-k},\theta\right)\right]dQ \\ &\quad + \int_{\sum_{i=1}^{m-1}x_i+(n-m+1)x_m}^{Q^{FBm}}\left[P(Q,\theta) - C_q\left(\frac{Q-\sum_{i=1}^{m}x_i}{n-m},\theta\right)\right]dQ \end{aligned}$$

Now we can compute the derivatives that we need in order to analyze the first order conditions given by (29). First note that $\frac{dW^{FBm}}{dx_i} = 0$ whenever $i > m$, i. e. firm i is not constrained in case FBm. Thus, for the highets capacity firm, firm n, we get that only $\frac{dW^{FBn}}{dx_i} \neq 0$, that is

$$\frac{dW^{FBn}}{dx_n} = P(X,\theta) - C_q(x_n,\theta). \tag{30}$$

Thus, according to (29) it must hold for the highest capacity firm that

$$\int_{\theta^{FBn}(x)}^{\overline{\theta}} [P(X,\theta) - C_q(x_n,\theta)]\, dF(\theta) - K_x(x_n) = 0. \tag{31}$$

Now suppose that one of the inequalities in $x_{\leq} x_2 \leq \cdots \leq x_n$ is strict, such that firm 1 has invested strictly less than firm n. It holds that

$$\begin{aligned}
\frac{dW^{FBm}(x,\theta)}{dx_1} &= \sum_{k=1}^{m-1} \int_{\sum_{i=1}^{k-1} x_i+(n-k+1)x_k}^{\sum_{i=1}^{k} x_i+(n-k)x_{k+1}} \frac{1}{n-k} - C_{qq}\left(\frac{Q-\sum_{i=1}^{k} x_i}{n-k},\theta\right) dQ \\
&\quad + \int_{\sum_{i=1}^{m-1} x_i+(n-m+1)x_m}^{Q^{FBm}} \frac{1}{n-m} C_{qq}\left(\frac{Q-\sum_{i=1}^{m} x_i}{n-m},\theta\right) dQ \\
&\quad + \frac{dQ^{FBm}}{dx_1}\left[P\left(Q^{FBm},\theta\right) - C_q\left(q^{FBm},\theta\right)\right] \\
&= \underbrace{C_q(q^{FBm},\theta) - C_q(x_1,\theta)}_{>0} + \underbrace{\frac{dQ^{FBm}}{dx_1}}_{>0} \underbrace{\left[P\left(Q^{FBm},\theta\right) - C_q\left(q^{FBm},\theta\right)\right]}_{\geq 0}
\end{aligned}$$

Now consider the first order condition of firm 1:

$$\begin{aligned}
\frac{d\mathcal{W}(x,q^{FB})}{dx_1} &= \sum_{m=i}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \frac{dW^{FBm}(x,\theta)}{dx_1} dF(\theta) - K_x(x_1) \\
&> \int_{\theta^{FBn}(x)}^{\overline{\theta}} \frac{dW^{FBn}(x,\theta)}{dx_1} dF(\theta) - K_x(x_1) \\
&= \int_{\theta^{FBn}(x)}^{\overline{\theta}} \left(C_q(q^{FBn},\theta) - C_q(x_1,\theta) + \frac{dQ^{FBn}}{dx_1}\left[P\left(Q^{FBn},\theta\right) - C_q\left(x_n,\theta\right)\right]\right) dF(\theta) - K_x(x_1) \\
&> \int_{\theta^{FBn}(x)}^{\overline{\theta}} \left[P\left(Q^{FBn},\theta\right) - C_q\left(x_n,\theta\right)\right] dF(\theta) - K_x(x_n) = \frac{d\mathcal{W}(x,q^{FB})}{dx_n} \equiv 0
\end{aligned}$$

Consequently, it cannot be that in the social optimum firm 1 invests less than firms n.

Part III: Uniqueness We now show that there can be no other symmetric equilibrium than x^{FB}. If capacities are equal, i. e. $x_1^0 = x_2^0 = \cdots = x_n^0$, we have

$$\frac{d\mathcal{W}(x^0,q^{FB})}{dx_i} = \int_{\theta^{FBn}(x^0)}^{\overline{\theta}} [P(nx_i^0,\theta) - C_q(x_i^0,\theta)]dF(\theta) - K_x(x_i^0).$$

Differentiation yields

$$\begin{aligned}
\frac{d^2\mathcal{W}(x^0,q^{FB})}{(dx_i)^2} &= \int_{\theta^{FBn}(x^0)}^{\overline{\theta}} [nP_q(nx_i^0,\theta) - C_{qq}(x_i^0,\theta)]dF(\theta) \\
&\quad - \frac{d\theta^{FBn}(x^0)}{dx_i}[P(nx_i^0,\theta^{Cn}(x^0)) - C_q(x_i^0,\theta^{Cn}(x^0))]f(\theta^{Cn}(x^0)) \\
&\quad - K_{xx}(x_i^0).
\end{aligned}$$

Note that the second term is equal to zero, since x^0 is the unconstrained first best solution at demand realization $\theta^{Cn}(x^0)$. Thus, we are left with the first term, i. e.

$$\frac{d^2\mathcal{W}(x^0, q^{FB})}{(dx_i)^2} \quad = \quad \int_{\theta^{FBn}(x^0)}^{\overline{\theta}} [nP_q(nx_i^0, \theta) - C_{qq}(x_i^0, \theta)]dF(\theta) - K_{xx}(x_i^0) < 0$$

Since $\frac{d\mathcal{W}(x^{FB}, q^{FB})}{dx_i} = 0$, it follows that $\frac{d\mathcal{W}(x^0, q^{FB})}{dx_i} > (<) 0$ for $x_i^0 < (>) x_i^{FB}$. Thus, no other symmetric optimal solution can exist and X^{FB} is the unique welfare maximizing investment level.

A.3 Proof of Remark 1

If firms act as price takers at stage two, outputs coincide with the welfare maximizing quantities characterized in equation (12). Thus, a firm i's stage two–profit in scenario θ if firms have invested x and m firms turn out to be constrained in the welfare optimum is given by

$$\pi_i^{FBm}(x, \theta) = \begin{cases} P\left(Q^{FBm}(x, \theta), \theta\right) x_i - C(x_i, \theta) & \text{if} \quad i \leq m, \\ \\ P\left(Q^{FBm}(x, \theta), \theta\right) \tilde{q}_i^{FBm}(x, \theta) - C\left(\tilde{q}_i^{FBm}(\cdot), \theta\right) & \text{if} \quad i > m. \end{cases}$$

The stage one expected profit of firm i is obtained by integrating over all profits associated with each demand realization,

$$\pi_i(x, q^{FB}) = \sum_{m=0}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \pi_i^{FBm}(x, \theta) dF(\theta) - K(x_i) \tag{32}$$

Analogously to equation (16), the first order condition of firm i's maximization problem is given by[24]

$$\frac{d\pi_i(x, q^{FB})}{dx_i} = \sum_{m=i}^{n} \int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)} \left[P(Q^{FBm}, \theta) - C_q(x_i, \theta)\right] dF(\theta) - K_x(x_i),$$

Note that the investment levels cannot be asymmetric by the following argument: Suppose

[24]Note that we assume that firms ignore their impact on X since they behave perfectly competitive. In this case, concavity of (32) is easy to establish.

that firm n invests strictly more than firm 1. The first order condition of firm 1 is given by

$$\begin{aligned}\frac{d\pi_1(x,q^{FB})}{dx_1} &= \sum_{m=1}^{n}\int_{\theta^{FBm}(x)}^{\theta^{FBm+1}(x)}\left[P(Q^{FBm},\theta)-C_q(x_1,\theta)\right]dF(\theta)-K_x(x_1)\\ &> \int_{\theta^{FBn}(x)}^{\overline{\theta}}\left[P(X,\theta)-C_q(x_1,\theta)\right]dF(\theta)-K_x(x_1)\\ &> \int_{\theta^{FBn}(x)}^{\overline{\theta}}\left[P(X,\theta)-C_q(x_n,\theta)\right]dF(\theta)-K_x(x_n)\equiv 0.\end{aligned}$$

Thus, firm 1 would like to increase its investment whenever it is lower than firm n's. Consequently, we have a unique solution, which must be the same for each firm. Let $x^n=(\frac{1}{n}X^N,\ldots,\frac{1}{n}X^N)$ denote the capacities invested by the n non–strategic firms. Since all firms face the same first order condition, we obtain a symmetric solution characterized by

$$\int_{\theta^{FBn}(x^N)}^{\overline{\theta}}\left[P\left(X^N,\theta\right)-C_q\left(\frac{1}{n}X^N,\theta\right)\right]dF(\theta)=K_x\left(\frac{1}{n}X^N\right). \tag{33}$$

Comparison with condition (17) implies that the investment level if firms do not behave strategically coincides with the welfare optimal investment level (as characterized in lemma 2), i. e. $X^{FB}=X^N$.

A.4 Proof of Lemma 3

The structure of the proof is equivalent to the proof of lemma 2. The welfare function in the case of Cournot competition at stage has exactly the same structure as welfare if the social optimum is implemented at stage two. As in the proof of lemma 2, the derivative of the welfare in a scenario Cm can be pinned down by using industry marginal cost. The only difference is that in the analysis, the Cournot equilibrium quantities q^{Cm}, Q^{Cm} of the unconstrained players have to be substituted for the socially optimal quantities of the unconstrained players, q^{FBm}, Q^{FBm}, that have been used in the analysis of the welfare optimum. We get

$$\frac{dW^{Cm}(x,\theta)}{dx_1} = \underbrace{C_q(q^{Cm},\theta)-C_q(x_1,\theta)}_{>0}+\underbrace{\frac{dQ^{Cm}}{dx_1}}_{>0}\underbrace{\left[P\left(Q^{Cm},\theta\right)-C_q\left(q^{Cm},\theta\right)\right]}_{>0}.$$

Note that in case of Cournot competition at stage two the last term is strictly positive. The remainder of the proof is equivalent to the proof of lemma 2.

A.5 Proof of Lemma 4

(i) First note that $\frac{d\pi_i}{dx_i} > 0$ at $X = 0$ (by equation (1)), that $\frac{d\pi_i}{dx_i} < 0$ for some finite value of X (by assumption 1 part (i)), and that $\frac{d\pi_i}{dx_i}$ is continuous. Thus, a corner solution is not possible, and we have at least one point where (24) is satisfied and $\frac{d\pi_i}{dx_i}$ is decreasing. Note, however, that this does not assure existence. In fact, in the scenario considered here a firm's stage one profit is not even quasiconcave, and it is not possible to reformulate the game as a supermodular game.

(ii) First note that in the case of constant marginal costs it is, independently of the capacity choices firms made at stage one, always true that either all firms are constrained at $p = C_q(\cdot, \theta)$, or none of them. Thus, it holds that $\theta^{FB1}(x) = \cdots = \theta^{FBn}(x)$.

In order to prove the second part of the lemma we apply theorem 2.1 of Amir and Lambson (2000), p. 239. They show that the standard Cournot oligopoly game has at least one symmetric equilibrium and no asymmetric equilibria whenever demand $P(\cdot)$ is continuously differentiable and decreasing, cost $C(\cdot)$ is twice continuously differentiable and nondecreasing and, moreover, the cross partial derivative $\frac{d\pi(X,q)}{dX_{-i}dX} > 0$, where X denotes total capacity and X_{-i} capacity chosen by the firms other than i. In order to see that the results of Amir and Lambson apply to our setup, note that our game is equivalent to a game where firms choose output given the expected demand and cost function. Note that if the first best outcome occurs whenever capacity is sufficient, it follows that expected inverse demand is given by

$$EP(X) = \int_{\underline{\theta}}^{\theta^{FBn}(x)} P\left(Q^{FB0}(\theta), \theta\right) dF(\theta) + \int_{\theta^{FBn}(x)}^{\overline{\theta}} P(X, \theta)\, dF(\theta), \tag{34}$$

and expected cost is given by

$$EC(x_i) = \int_{\underline{\theta}}^{\theta^{FBn}(x)} C\left(q_i^{FB0}, \theta\right) dF(\theta) + \int_{\theta^{FBn}(x)}^{\overline{\theta}} C(x_i, \theta)\, dF(\theta) + K(x_i), \tag{35}$$

Note that $EP(X)$ is strictly decreasing in X and $EC(x_i)$ is strictly increasing in x_i, but they do not satisfy assumption 1, part (iv), which is why existence and uniqueness are not implied by standard (textbook) analysis.[25] However, Amir and Lambson's assumptions[26] are satisfied, since the cross partial derivative

$$\begin{aligned} \frac{d\pi^2(X, q^C)}{dX_{-i}dX} &= -\frac{d\theta^{FBn}(x)}{dX} \underbrace{\left[-P(X, \theta^{FBn}(x)) + C_q(X - X_{-i}, \theta^{FBn}(x))\right]}_{=0 \text{ at } \theta^{FBn}(x)} f(\theta^{FBn}(x)) \\ &\quad + \int_{\theta^{FBn}(X)}^{\overline{\theta}} \underbrace{\left[-P_q(X, \theta) + C_{qq}(X - X_{-i}, \theta)\right]}_{>0} f(\theta) d\theta \end{aligned}$$

is positive. This guarantees that we have at least one symmetric equilibrium and no asymmetric equilibria in case of constant marginal cost.

B The "Certainty Equivalent Game"

As already mentioned, our analysis of imperfect competition in section 3 covers a contribution by Gabszewicz and Poddar (1997) who analyzed the imperfect competition scenario with linear demand and deterministic and constant marginal cost for a discrete distribution over demand realizations. In order to relate our results more closely to theirs, in the following we consider the game to which they compare capacities chosen in the imperfect competition scenario (lemma 3). In this hypothetical game, which they call the "*Certainty Equivalent Game*", firms are assumed to choose production given the *expected* demand. However, rather than throwing away what they cannot (or do not want to) sell in low demand scenarios, they have to sell the quantity they chose at any price (in particular also at negative prices).[27] Since the demand function we defined does not allow for neg-

[25] In fact, the expected profit function is not even quasiconcave, as it is easily seen by inspecting its second derivative. Those observations point to an error in the article of Reynolds and Wilson (2000). They make almost the same assumptions on demand as we do, but are more restrictive regarding cost (i. e. $C_q(x_i) = 0$ and $K(x_i) = kx_i$). They state (p.126 of the article) that $E[x_i P(x_i + x_{-i}, \theta) - kx_i]$ (in our notation) is strictly concave and differentiable in x_i and therefore has a unique solution. Since $E[x_i P(x_i + x_{-i}, \theta) - kx_i]$ is exactly the profit given by equation (32) for $C_q(x_i) = 0$ and $K(x_i) = kx_i$, our analysis in section 5.2 shows that this is not true.

[26] The assumptions are: $P(\cdot)$ is continuously differentiable with $P_q(\cdot) < 0$, $C(\cdot)$ is twice continuously differentiable and nondecreasing, and $P_q(X) - C_{qq}(x_i) < 0$.

[27] This can also be implemented by intervention at stage two, namely if the regulator prohibits withholding of capacity at any demand scenario. In the formulation of Gabszewicz and Poddar demand is linear and becomes negative for capacities higher than demand at price zero. This corresponds to the assumption of considerable destruction cost in case of excess capacity, which does not seem plausible in most cases.

ative prices we define an extended demand function $\hat{P}(Q,\theta)$ that coincides with $P(Q,\theta)$ for all $Q \leq \overline{Q}(\theta)$ and that may become negative for $Q > \overline{Q}(\theta)$. $\hat{P}$ is assumend to satisfy assumption 1, parts (ii) to (vi) for all $Q, q_i \in [0,\infty)$.[28]

In our terminology the requirement that capacity is always sold, whatever the price is, implies that firms are never "unconstrained". Thus, in order to determine the stage one profit we do not need to integrate piecewisely but we simply get[29]

$$\pi_i(x,x) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\hat{P}(X,\theta)x_i - C(x_i,\theta)\right] dF(\theta) - K(x_i). \tag{36}$$

Differentiation yields the first order condition as stated in the following[30]

Property 3 (The Certainty Equivalent Game) *The "Certainty Equivalent Game" has a unique equilibrium which is symmetric. Equilibrium investments $x_i^{CE} = \frac{1}{n}X^{CE}$, $i = 1,\dots,n$ solve*

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\hat{P}\left(X^{CE},\theta\right) + \hat{P}_q\left(X^{CE},\theta\right)\frac{X^{CE}}{n} - C_q\left(\frac{X^{CE}}{n},\theta\right)\right) dF(\theta) = K_x\left(\frac{1}{n}X^{CE}\right). \tag{37}$$

We get that

$$\begin{aligned} CE: &\quad \int_{\underline{\theta}}^{\overline{\theta}} \left[P_q(X,\theta)\frac{1}{n}X + P(X,\theta) - C_q\left(\frac{1}{n}X,\theta\right)\right] dF(\theta) \\ ORP: &\quad < \int_{\theta^{FBn}(x)}^{\overline{\theta}} \left[P_q(X,\theta)\frac{1}{n}X + P(X,\theta) - C_q\left(\frac{1}{n}X,\theta\right)\right] dF(\theta) \end{aligned}$$

which, analogously to the proof of theorem 1 allows us to show that $X^{CE} < X^{ORP}$.

C References

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[28]Note that while Gabzewicz and Poddar consider linear demand, in our model the slope of the demand function may considerably change as prices become negative. This allows to model the more realistic scenario of some moderate destruction cost.

[29]Since in GP cost is deterministic and cost is linear, equation (36) would simplify to $(E[\theta] - X)x_i - cx_i - kx_i$, compare p. 139 in GP.

[30]Existence of the symmetric equilibrium is straightforward, since π_i^{CE} is concave. Uniqueness follows immediately by separating (36) into expected demand $E_\theta[\hat{P}]$ and expected cost $E_\theta[C]$ and showing that $E_\theta[\hat{P}]$ and $E_\theta[C]$ satisfy standard assumptions.

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