Endogenous heterogeneity in strategic models : symmetry-breaking via strategic substitutes and nonconcavities

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Abstract

This paper is an attempt to develop a unified approach to endogenous heterogeneity by constructing general class of two-player symmetric games that always possess only asymmetric pure-strategy Nash equilibria. These classes of games are characterized in some abstract sense by two general properties: payoff non-concavities and some form of strategic substitutability. We provide a detailed discussion of the relationship of this work with Matsuyama's symmetry breaking framework and with business strategy literature. Our framework generalizes a number of models dealing with two-stage games, with long term investment decisions in the first stage and product market competition in the second stage. We present the main examples that motivate this study to illustrate the generality of our approach.

Keywords: inter-firm heterogeneity, submodular games, business strategy, innovation strategies.

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1 Introduction

One of the most pervasive presumptions in modern economic analysis is the symmetric nature of interacting agents. While often intended solely as a simplifying assumption on *a priori* grounds, this presumption has also permeated economic thinking for a variety of other reasons. When considering noncooperative games, analysts often restrict attention to symmetric equilibrium points even when other asymmetric outcomes exist and may reflect more rationalizable or more pertinent behavior. In mechanism design or policy games, the social planner typically assumes identical treatment of identical agents, although global optimality might dictate otherwise. The design of various forms of joint ventures is also subject to a similar observation.

In most cases, the only justification beyond simplicity is what Schelling (1960) convincingly termed the focal nature of symmetric equilibrium outcomes. Indeed, it is widely recognized that inter-agent heterogeneity is often a critical dimension of several economic and social phenomena. From a positive perspective, heterogeneity is simply a necessary postulate to account for the simple fact that in the real world, one seldom observes identical agents, be it individuals, firms, industries or countries. In a similar vein, even from a normative standpoint, differences across interacting agents often constitute a necessary condition for many important economic activities such as trade or risk-sharing.

Understanding the origins and/or evolution of diversity across economic agents or disparities in economic performance across regions is increasingly perceived as a central goal of economic and social research in a number of different areas (see e.g. Geroski et al. (2003) and Ghemawat (1986)). Macroeconomists attempt to explain the causes of booms and recessions. Development economists wish to understand the forces behind poor and strong economic performances. Labor economists attempt to get a handle on discriminatory treatment of some groups of workers. Business strategists and industrial economists devote a lot of attention to the sources and sustainability of inter-firm heterogeneity within and across industries. Overall, much effort has been expanded with a view to explain "the diversity across space, time and groups" (Matsuyama, 2002).

In view of the diversity of economic research areas involved in this effort, it is not surprising that various conceptual and methodological approaches have been developed in connection with this complex task. While often tailored to a specific area, each of these approaches is broad in explanatory scope and has wide potential applicability. We now briefly review three of these general paradigms that share some relationship to the present paper.

The dominant approach, based on coordination failures, postulates a game with strategic complementarities and multiple Pareto-ranked pure-strategy Nash equilibrium points. Diversity is then synonymous with making different equilibrium selections, with the high-performing entity picking the Pareto-dominant equilibrium and the low-performing entity failing to do so. This argument is thus generally predicated on the presence of two identical and non-interacting economies, each operating under a different equilibrium out of the same equilibrium set. It may also be invoked to explain diversity across time within the same economy, with booms and recessions corresponding to operation under the Pareto dominant and inferior equilibria respectively. This literature includes as key studies Cooper and John (1988), Murphy, Schleifer and Vishny (1989), and is surveyed by Cooper (1999).

The coordination failures approach has been criticized for failing to offer any compelling argument for the diversity in equilibrium selections in the two-economy model or for the regime switch in the one-economy model. Matsuyama (2002) proposes a modification of the former model by creating an interactive link between the two sub-economies and allowing the two players to take two decisions, one in each sub-economy. Under some conditions on the larger game with *a priori* identical players, namely that the players' actions are pairwise strategic complements and each player's actions are substitutes due to a fixed total resource constraint, multiple equilibria arise with the property that the symmetric equilibrium is unstable while the two asymmetric equilibria are stable, both in the sense of Cournot dynamics. Endogenous heterogeneity in this approach is then predicated on the central postulate that only Cournot-stable equilibria are observable outcomes of this complex game, any of which would involve each agent taking different actions in the two *ex ante* identical sub-economies. Matsuyama (2000, 2002, 2004) coined the term "symmetry-breaking" to refer to this heterogeneity-generating process.

The third approach originates in the business strategy literature, and is often presented as part of a general critique of economic theory. With their traditional emphasis on investigating the workings of firms as complex organizations, strategy scholars have been particularly concerned with understanding the sources and dynamics of inter-firm heterogeneity along various functions and characteristics. In the dominant view, as articulated by Nelson and Winter (1982), firms operate in such highly complex and ever-changing environments that they entertain no hope of ever accumulating enough knowledge about their world to view it as a strategic game or formulate a precise game-theoretic strategy to guide their overall behavior. Rather, firms grope for economic performance via a heuristic learning process of trial and error and the continual updating of routines and rules of thumb eschewing optimization. In this evolutionary vision, heterogeneity is simply an inevitable outcome of this groping behavior, with firms ending up with different heuristic strategies and core capabilities to implement them. These "discretionary" differences can then be sustained over extended periods of time due to the presence of barriers to successful imitation generated by the differences in core competencies, and also by forces of path dependence in the evolution of firms' choices. This literature often criticizes economic theory for not adequately accounting for inter-firm differences, other than postulating them either as reflecting variations in initial conditions, or as exogenous consequences of the luck of a draw in stochastic models. This failure is attributed to the fact that economic theorists persist, as part of their excessive reliance on complete rationality, in "taking a firm's choice sets as obvious to it and the best choice similarly clear and obvious" (Nelson, 1991).[1]

[1] An interesting development over the last two decades is a strand of literature straddling the

The present paper constitutes an attempt to contribute to this rich debate along standard lines of argument in applied game theory and industrial organization. Consider a two-player symmetric normal-form game characterized by two key properties: (a) actions form strategic substitutes, and (b) each player's payoff, though continuous, admits a key nonconcavity along the diagonal in action space, which results in a jump of the reaction correspondence across the 45^o line. Such a game always admits pure-strategy Nash equilibrium points due simply to the property of strategic substitutes. Furthermore, due to property (b), no such equilibrium could ever be symmetric. At any of the possibly multiple equilibria, which obviously occur in pairs due to the symmetry of the game, otherwise identical agents will necessarily take different equilibrium actions. While this description exactly fits the main result of the paper, we consider two other related classes of games that always possess asymmetric, but never symmetric, pure-strategy equilibria although they are, strictly speaking, not of strategic substitutes. This suggests that the latter property is not as critical as the diagonal nonconcavity property in generating exclusively asymmetric outcomes.

Since payoffs are continuous in actions in all three classes of games under consideration, these games will typically admit a symmetric mixed-strategy Nash equilibrium (Dasgupta and Maskin, 1986). As this would be the only focal equilibrium in the sense of Schelling (1960), it may reasonably be advanced as a plausible outcome of such a game. Nevertheless, in the actual realization of the equilibrium randomizations, the players will still end up playing different actions with high, if not full, probability. Hence, given a focus on explaining observed heterogeneity, this approach need not rule out mixed strategies *a priori.*

Towards the goal of generating endogenous heterogeneity, this approach is obviously closest in spirit to Matsuyama's symmetry-breaking explanation. By allowing for suitable discontinuities in the players' reaction curves, it dispenses with the need to interconnect two separate games in the somewhat complex

traditional boundaries between industrial organization and business strategy and addressing issues of interest to both fields, making them increasingly related: See Shapiro (1989), Rumelt, Schendel and Teece (1991), Roller and Sinclair-Desgagne (1996).

(and subtle) manner proposed by Matsuyama. More importantly, it also provides a framework that is independent of the controversial argument of outright rejecting Cournot-unstable equilibria. Indeed, even when one ignores the focal nature derived from their symmetry, it is worthwhile to observe that these equilibria cannot be ruled out on account of any of the standard Nash equilibrium refinements, such as normal-form perfection or strategic stability (Kohlberg and Mertens, 1986).[2] Furthermore, in an experimental setting involving a symmetric two-player game with one unstable symmetric equilibrium and a pair of asymmetric equilibria, Cox and Walker (1998) found little support in the data for *any of the three equilibria*. This provocative finding suggests that while a Cournot-unstable equilibrium of a given game may be justifiably regarded as unobservable, it does not thereby follow that some Cournot-stable equilibrium of the same game will necessarily prevail and thus be observable. Rather, the presence of both Cournot-stable and unstable equilibria may engender a high level of indeterminacy, which may critically reduce the predictive power of the game.

Our findings may also be advanced as a rebuttal to the aforementioned criticism from the business strategy literature. Indeed, while sharing their motivation for understanding intra-industry heterogeneity, this approach underlies a general methodology for generating inter-firm differences out of a strategic game with fully rational and completely informed players. The contrast with the evolutionary explanation is rather striking. Instead of discretionary differences that inevitably arise out of the idiosyncratic heuristic response that each firm develops in isolation from other firms as a result of its multi-faceted operation in an extremely complex environment, we uncover strategic differences that arise out of a fully-fledged game-theoretic interaction amongst firms in a simple and completely known environment. We will return to this contrast in the specific context of an R&D game in a subsequent section.

[2] Indeed, they are typically strict Nash equilibria (in the sense that a unilateral deviation will lead to a strict loss for the deviator), and thus would survive any of the well-known refinements.

The present paper may also be motivated in relation to various broad strands of literature in industrial economics dealing in some way with strategic endogenous heterogeneity along lines similar to ours here. The first literature that comes to mind is concerned with product differentiation. In a myriad of two-stage games where each firm chooses a quality level or a horizontal characteristic in the first stage, and then a price for its product in the second stage, endogenous heterogeneity naturally emerges out of the firms' perception that identical choices in the first stage will lead to zero profits in the second stage Bertrand competition due to the resulting homogeneity of the products. See in particular Gabszewics and Thisse (1979), Shaked and Sutton (1982) for vertical product differentiation and Tabuchi and Thisse (1995) for horizontal product differentiation with a non uniform density. The present paper will direcly generalize Motta (1993) and Aoki and Prusa (1996).

The second, extensive literature deals with infinite-horizon industry dynamics allowing for entry and exit. One class of models, exemplified by Jovanovic (1982), postulates perfectly competitive firms for which differences emerge due to *exogenous* idiosyncratic technology shocks. Another class is formed by studies that do generate endogenous heterogeneity in long run dynamics by considering firms that invest in capacity expansion (e.g. Besanko and Doraszelski, 2002) or R&D (e.g. Doraszelski and Satterthwaite, 2004). Simpler two-stage models with similar flavor but without entry and exit also generate endogenous differences amongst competing firms: Reynolds and Wilson (2000) and Maggi (1996) for capacity expansion and Amir and Wooders (1999, 2000) for R&D.

There are several other studies in various areas of industrial organization where endogenous heterogeneity emerges in a strategic setting. Hermalin (1994) deals with a two-stage game where firms' choices of managerial structures take place before market competition. Mills (1996) and Amir (2000) deal with R&D games giving rise to equilibrium outcomes with maximal heterogeneity only, i.e. full R&D by one firm and no R&D by the rival.[3] In public economics Mintz

[3] Another strand of literature, not directly related to our setting, deals with endogenous heterogeneity arising out of hybrid models of joint ventures where firms make a cooperative

and Tulkens (1986) exhibit asymmetric tax rates for identical member states.

As a second motivation, the present paper is an attempt to develop a unifying approach to understanding symmetry-breaking mechanisms in general classes of two-player games, encompassing many of the cited studies. These two-stage models share two key features that are critical for the symmetry-breaking arguments they present. The first is a fundamental nonconcavity in the payoffs, which may be confined to the diagonal in action space or hold globally, and the second is some form of strategic substitutes in first-period actions, possibly of an abstract sort (more on this in Section 4). While there is quite some variation in the precise manner versions of these two features are present and interact across all the models, we will be able to capture most of them in three separate general results, which though quite distinct at first sight, nonetheless bear some definite relationship at an abstract level.

The paper is organized as follows. Section 2 contains the overall set-up. Section 3 provides the results on the exclusive existence of asymmetric equilibria for submodular payoff functions. Section 4 deals with nonsubmodular payoff functions and Section 5 presents the results for games with convex payoffs. Section 6 discusses an extension to ordinal complementarity and substitutability conditions. Each section provides a summary of the relevant applications the results pertain to. The appendix provides a brief overview of the supermodularity notions and results.

2 Setup

This section lays out the general notation for use throughout the paper. The nooncooperative game described below may be a simple one-shot game or it may represent the payoffs of a two-stage game as a function of the first period actions, where the unique second stage pure-strategy equilibrium has been substituted in. In the latter case, which actually covers most of the applications of this

decision in the first stage followed by product market competition in the second stage: See e.g. Salant and Shaffer (1998,1999) and Long and Soubeyran (2001).

paper, we obviously restrict consideration to subgame-perfect equilibria and analyze the resulting one-shot game.

Consider a two-player normal form game Γ given by the tuple (X, Y, F, G). Let X and Y be the action sets of player 1 and 2 respectively, such that $X = Y = [0, c] \subset R$. The maps F and $G : X \times Y \to R$ are the payoff functions of players 1 and 2 respectively and F can be expressed as:

$$F(x,y) = \begin{cases} U(x,y), x \geq y \\ L(x,y), x < y \end{cases} \tag{1}$$

By symmetry of the game Γ, G can be expressed as:

$$G(x,y) = \begin{cases} L(y,x), x \geq y \\ U(y,x), x < y \end{cases} \tag{2}$$

Observe that, somewhat contrary to standard practice, the first argument of F is the action of player 1 while the first argument of G is the action of player 2. It is useful to define the following sets:

$$\Delta_U = \{(x,y) \in R^2 : x \geq y\} \text{ and } \Delta_L = \{(x,y) \in R^2 : x \leq y\}.$$

It will be assumed throughout the paper that U, L, F and G are jointly continuous functions of the two actions. Define the best response correspondences (reaction curves) for players 1 and 2 respectively as $r_1(y) = \arg\max\{F(x,y) : x \in [0,c]\}$ and $r_2\,(x) = \arg\max\{G(y,x) : y \in [0,c]\}$.

As usual, a pure strategy Nash equilibrium (or PSNE for short), $(x^*, y^*) \in [0,c]^2$ is said to be symmetric if $x^* = y^*$, and asymmetric otherwise. It follows from the symmetry of the game that if (x^*, y^*) is a PSNE, (y^*, x^*) is also a PSNE.

Each of the next three sections investigates a separate class of normal-form symmetric games that always possess asymmetric Nash equilibria and no symmetric Nash equilibria. For each of the three classes, we provide a general result establishing both the existence and the inexistence conclusions and an illustration based on previous studies where a special case of the result was derived in a specific setting.

The definitions and main results from the theory of supermodular games used in this paper are reviewed in the appendix in a very simple way, which is sufficient for the purposes of this paper.

3 Endogenous heterogeneity with strategic substitutes

In this section, we consider a two-player symmetric normal-form game characterized by two key properties. The first is that actions form strategic substitutes. This means that an increase in one player's strategy lowers the other player's marginal returns to increasing his own strategy. As a result, players respond optimally to an increase of the opponent's choice with a decrease of their own variable. In other words, the best reply correspondences are downward-sloping and a pure-strategy Nash equilibrium exists (see Vives, 1990 and Milgrom and Roberts, 1990).

The second key property is that each player's payoff, though jointly continuous in the two actions, admits a fundamental nonconcavity along the 45^o line, giving rise to a canyon shape along the diagonal. A key consequence of this feature is that a player would never optimally respond to an action of the rival by playing that same action.

Taken together, these two properties imply that each best reply is a decreasing correspondence with a (downward) jump over the 45^o line.[4] Hence, no PSNE could ever be symmetric. At any of the possibly multiple equilibria, which obviously occur in pairs due to the symmetry of the game, *ex ante* identical agents will necessarily take different equilibrium actions.

While all three results presented in this paper share this same flavor, the main result in terms of the generality of the assumptions and thus of the scope of applicability is this section's.

[4] In this paper, we will say that a function $f : R \rightarrow R$ is increasing (strictly increasing) if $x' > x$ implies $f(x') \geq (>) f(x)$. A correspondence is increasing if its maximal and minimal selections are increasing functions (as in the conclusion of Topkis's Monotonicity Theorem).

3.1 The results

Different subsets of the following assumptions will be needed for our conclusions below. The notation is as laid out in Section 2.[5] A full discussion of the assumptions and results is presented at the end of the section. Most of the proofs can be found in the appendix.

A1 U, L are submodular

A2 $U_1(x,x) > L_1(x,x), \forall x \in (0,c)$

A3 $U_1(0,0) > 0, L_1(c,c) < 0$

A1 says that on either side of the diagonal, but not necessarily globally, each player's marginal returns to increasing his action decrease with the rival's action. A2 holds that each player's payoff, though globally continuous in the two actions, has a kink along the diagonal in the shape of a "valley". The role of A3 is simply to rule out PSNEs at $(0,0)$ or (c,c).

These assumptions form a sufficiently general framework to encompass many of the studies mentioned in Section 1 as illustrated below. Furthermore, all three assumptions are easy to check in a particular model.

Theorem 3.1 *Assume that $A1-A3$ hold. Then the game Γ is of strategic substitutes, has at least one pair of asymmetric PSNEs and no symmetric PSNEs.*

The idea of the proof is that overall submodularity of the payoff function is inherited from the submodularity of its components U and L in the presence of assumption $A2$. We know from Topkis's Monotonicity Theorem that global submodularity of the payoff function implies globally decreasing best replies. Assumptions $A2-A3$ imply that the best replies have a downward jump that crosses over the diagonal. This situation is depicted in figure 1.[6] (We caution the

[5] In addition, throughout the paper, partial derivatives are denoted by a subindex corresponding to the relevant variable, i.e. $U_1(x,y) = \frac{\partial U(x,y)}{\partial x}$ and $U_2(x,y) = \frac{\partial U(x,y)}{\partial y}$.

[6] Notice that unusually x is the variable in the vertical axis. This corresponds to analyzing the game from the point of view of player 1 that chooses x as a response to y. We maintain this convention throughout.

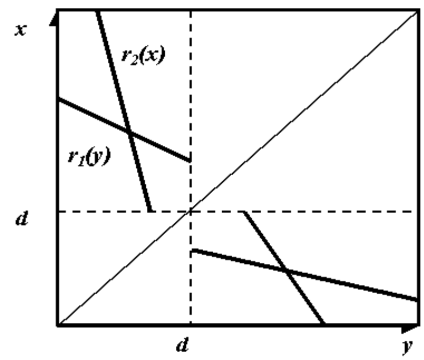

Figure 1: Decreasing reaction curves have a jump along the diagonal, and there is no symmetric equilibrium

reader that the continuity of the reaction curves in each triangle over and below the diagonal is only there for the sake of a clearer figure. It needs not hold under assumptions A1- A3). The first result gives us existence of equilibrium via the strategic substitutes property, and the second precludes symmetric equilibria. The complete proof can be found in the appendix.

Theorem 3.1 does not rule out the existence of multiple pairs of PSNEs. Indeed, the two reaction curves may intersect several times above and below the diagonal. In case of multiple pairs of PSNEs, there will typically be co-existence of pairs of Cournot-stable and pairs of Cournot-unstable PSNEs. Nevertheless, Theorem 3.1 does imply that all of these PSNEs are asymmetric. Hence symmetry-breaking in this context does not rely on the rejection of Cournot-unstable symmetric PSNEs. In the same vein, this type of symmetry-breaking is not at odds with Schelling's (1960) notion of focalness of PSNEs.

The next result adds further restrictions of a general nature on the payoff components of our game that lead to a unique pair of PSNEs, which are then necessarily Cournot-stable.[7] In this case, symmetry-breaking is coupled with

[7] It is worthwhile to point out here that our results indicate an even total number of PSNEs, in apparent conflict with the well-known odd number results. The explanation is that the latter results are based on degree theory and require continuity of the best-response form. Given

more predictive power of the game, although the selection of one PSNE from the pair still remains indeterminate, as is standard in symmetric settings.[8]

Theorem 3.2 *Assume that U and L are twice continuously differentiable and that the following holds:*

$$U_{11}(r(z), z) - U_{12}(r(z), z) \geq 0 \tag{3}$$

$$L_{11}(r(z), z) - L_{12}(r(z), z) \geq 0 \tag{4}$$

then there is exactly one pair of PSNEs.

The next result is devoted to comparing the two asymmetric PSNEs from any given pair from the point of view of the players' welfare. Given any pair of asymmetric PSNEs, it is often of interest to determine circumstances where a given equilibrium secures better payoffs for a player. In other words, under what conditions would each player prefer the PSNE where he is the high or the low-activity player? To this end, we need to impose a condition of monotonicity on the payoff function of the player in question along his opponent's best reply as stated in the following result, which lays out conditions for player 1 (say) to prefer the PSNE where he is the low-activity player.[9]

Theorem 3.3 *Let $x^* > y^*$, so that (x^*, y^*) and (y^*, x^*) are equilibria in Δ_U and Δ_L, respectively. If $A1 - A3$ hold and moreover $U(r_1(y), y)$ and $L(r_1(y), y)$ are increasing in $y \in [0, c]$ then $F(x^*, y^*) \leq F(y^*, x^*)$.*

There is a dual statement giving conditions under which each player would prefer the high-activity equilibrium, given any pair of PSNEs. Being obvious from Theorem 3.3, it is omitted for the sake of brevity.

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our systematic and robust jump across the diagonal, our findings are actually consistent with the odd number result in a generic sense.

[8] As mentioned in the Section 1, assumptions $A1 - A3$ imply that our game admits a *symmetric* mixed-strategy Nash equilibrium. However, in the actual realization of such an equilibrium, the two players will be heterogeneous with high, if not full, probability.

[9] This monotonicity assumption is clearly more general than assuming that each player's payoff is increasing in the rival's action. For an example illustrating this point, see von Stengel (2003).

3.2 Applications

In this section we present examples of economic models that constitute special cases of the general framework developed above. While the assumptions validating Theorem 3.1 might at first appear somewhat special, they are satisfied in several *a priori* unrelated studies that have established endogenous heterogeneity in strategic settings. There are also some studies where asymmetric equilibria are produced via a mechanism similar to our Theorem 3.1, without being a special case in a formal sense. Going over some of these examples illustrates the unifying character of our results and allows us to provide some contextual interpretations of endogenous heterogeneity, or our version of symmetry-breaking.

3.2.1 R&D investment

The first example is based on the model by Amir and Wooders (2000). Two *a priori* identical firms with initial unit cost c are engaged in a two stage game of R&D investment and production. In the first stage, autonomous cost reductions x and y for firms 1 and 2, respectively, are chosen. The novel feature of this study is that spillovers are postulated to flow only from the more R&D active firm to the rival, but not vice versa. The effective (post-spillover) cost reductions X and Y when $x \geq y$ are given by:

$$X = x \text{ and } Y = \begin{cases} x \text{ with probability } \beta \\ y \text{ with probability } 1-\beta \end{cases} \tag{5}$$

Second stage product market competition, be it Cournot or Bertrand, is assumed to have a unique PSNE with equilibrium payoffs given by $\Pi : [0,c]^2 \to R$.[10] $\Pi(x,y)$ is the payoff of the firm whose unit cost is the first argument. $f : [0,c] \to R$ is a known R&D cost schedule. Amir and Wooders (2000) assume the following:

C1 Π and f are twice continuously differentiable

[10] This is a standard assumption in the literature.

C2 Π is strictly submodular and $\Pi_1(x,y) < 0$ and $\Pi_2(x,y) > 0$

C3 $\Pi(x,x) < \Pi(y,y)$ if $x > y$

C4 $|\Pi_1(x,x)| > |\Pi_2(x,x)|, \forall x \in [0,c]$

C5 $f'(x) \geq 0$ and $f(0) \geq 0$

C6 $f\prime(0) < -\beta\Pi_2(c,c) - \Pi 1(c,c)$ and $f\prime(c) > -(1-\beta)\Pi_1(0,0)$

The overall payoff of firm $1, F(x,y)$, defined as in (1), is given by the difference between its second stage profit and first stage R&D cost. The payoff of firm 2 is $G(y,x)$ by symmetry.

$$U(x,y) = \beta\Pi(c-x,c-x) + (1-\beta)\Pi(c-x,c-y) - f(x) \tag{6}$$

$$L(x,y) = \beta\Pi(c-y,c-y) + (1-\beta)\Pi(c-x,c-y) - f(x) \tag{7}$$

We can easily check that assumptions $A1, A2$ and $A3$ indeed hold in order to apply Theorem 3.1. U and L are continuous and differentiable because they result from the sum of continuous and differentiable functions. $U(x,x) = L(x,x), \forall x \in [0,c]$, so F and G are continuous. $A1$ can be checked by using the cross-partial test and the fact that $\Pi(x,y)$ is submodular (Assumption $C1$). Also, Using $C2$, and

$$U_1(x,x) = -[\Pi_1(c-x,c-x) + \beta\Pi_2(c-x,c-x)] - f'(x) \tag{8}$$

$$L_1(x,x) = -(1-\beta)\Pi_1(c-x,c-x) - f'(x) \tag{9}$$

we obtain that $U_1(x,x) > L_1(x,x)$, therefore $A2$ is verified.

From Theorem 3.1, we can conclude that payoff functions for both players are submodular and thus reaction curves are downward sloping and there exists at least one PSNE. Moreover the reaction curves do not intersect the diagonal and by $C6$, $U_1(0,0) > 0$ and $L_1(c,c) < 0$, so there is no symmetric equilibrium in $x \in [0,c]$. The uniqueness of a pair of asymmetric PSNEs is shown by imposing conditions that secure that the reaction curves are contractions, so that Theorem 3.2 can be applied. Similarly, extra conditions are needed to apply Theorem 3.3 and Amir and Wooders conclude that player 1 prefers the equilibrium in Δ_U (for details, see Amir and Wooders, 2000).

This model provides a good opportunity for a typical economic interpretation of strategic endogenous heterogeneity in a context that is of particular interest to business strategy scholars. Indeed that field typically attaches a great deal of importance to the innovation process and to its central role in dynamic competition. The key driving force behind asymmetric equilibrium outcomes here is the one-way nature of the spillover process. A firm will always react by performing either less R&D than its rival knowing that it may free ride on the difference in R&D levels, or, in case the rival's R&D is simply too low, by overtaking it. In this vision, firms will endogenously settle into R&D innovator and imitator roles simply as a reflection of the nature of the R&D spillover process. This critical difference arises as a consequence of strategic thinking in a fully interactive setting: though facing equal and known opportunities in all respects, firms emerge as fundamentally different in all possible equilibria of a robust and general model. In strategic settings, there may simply be no single "best choice" (to paraphrase Nelson, 1991) for all *ex ante* identical firms facing the same available choices, simply because one firm's choice has a direct influence on what becomes best for its rivals.

This difference in one key component of firms' overall strategies will then be a causal factor, through natural complementarity-reinforcing developments, for heterogeneity in other aspects of firms' strategies, including in particular firm size and organization (see Amir and Wooders, 1999 for details). This perspective stands in sharp contrast to the explanation for inter-firm differences characterized by idiosyncratic groping behavior on the part of firms and weak interaction amongst them in a world of high uncertainty and complexity, as often envisioned in the strategy literature.

3.2.2 Provision of information

The second example deals with the provision of information in Bertrand oligopoly, see Ireland (1993). Two *a priori* symmetric firms produce a homogeneous product and play a two stage game. In the first stage, each firm sets the level of its product information and in the second stage they compete in prices. Information regards only the existence of the product. Consumers may obtain costless information about prices of products that they know to exist. The number of consumers is normalized to 1. The variables x and y are the proportions of consumers who know about product 1 and 2 respectively. Each consumer is not willing to pay a unit price higher than 1. Firm 1's sales are given by:

$$Q_1 = \begin{cases} x \text{ if } p_1 < p_2 \\ x - \frac{xy}{2} \text{ if } p_1 = p_2 \\ x - xy \text{ if } p_1 > p_2 \end{cases} \tag{10}$$

For $x = y = 1$ the Bertrand oligopoly has a pure strategy Nash equilibrium at $p_1 = p_2 = 0$. If information is not full (x or y or both are less than 1), no pure strategy Nash equilibrium exists. There exists a mixed strategy Nash equilibrium given by the distribution function $G_i(p_i)$ that has the following form respectively for firm 1 and 2 .

$$G_1(p) = \begin{cases} \frac{1-(1-x)/p}{x} \text{ if } 1 - x \leq p \leq 1 \\ 0 \text{ if } 0 \leq p \leq 1 - x \end{cases} \qquad G_2(p) = \begin{cases} \frac{1-(1-x)/p}{y} \text{ if } 1 - x \leq p \leq 1 \\ 0 \text{ if } 0 \leq p \leq 1 - x \end{cases} \tag{11}$$

The overall payoff for (say) firm 1 in the game, upon substituting the second stage equilibrium payoffs, is given by $F(x, y) = E_p(p_1 Q_1)$ or

$$F(x, y) = \begin{cases} U(x, y) = x(1 - y) \text{ if } x \geq y \\ L(x, y) = x(1 - x), \text{ if } x \leq y \end{cases} \tag{12}$$

It is trivial to show that assumptions $A1$, $A2$ and $A3$ are verified in this example. There exists an equilibrium and this equilibrium cannot be symmetric for $p \in (0, 1)$. Moreover $U_1(0, 0) > 0$ and $L_1(1, 1) < 0$. So we can conclude from Theorem 3.1 that no symmetric equilibria exist for $p \in [0, 1]$.

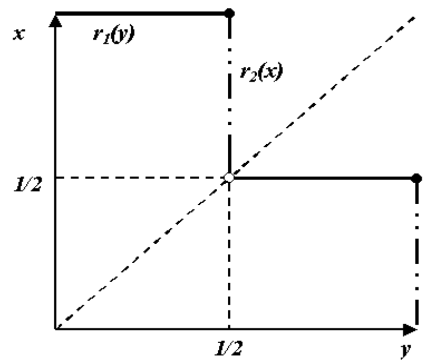

Figure 2: Reaction curves are constant except for a jump down, which precludes symmetric equilibrium.

The number of equilibria may be obtained by finding the explicit form of the reaction curves.

$$r_1(y) = \begin{cases} 1 \text{ if } y \leq 1/2 \\ 1/2 \text{ if } y > 1/2 \end{cases} \qquad r_2(x) = \begin{cases} 1 \text{ if } x \leq 1/2 \\ 1/2 \text{ if } x > 1/2 \end{cases} \tag{13}$$

Figure 2 illustrates that there exists exactly one pair of pure strategy Nash equilibria, namely $(1, \frac{1}{2})$ and $(\frac{1}{2}, 1)$.

We can compare equilibria from the point of view of player 1, using the dual to the Theorem 3.3. The payoff function of player 1 is decreasing along his best replies when $x > y$ and when $x \leq y$ it is constant, i.e. $U_2(r_1(y), y) < 0$ and $L_2(r_1(y), y) = 0$. Therefore we conclude that player 1 prefers the equilibrium where he is more active.

4 Endogenous heterogeneity without monotonic best replies

In the previous section we discussed symmetry breaking via strategic substitutes and nonconcavity of the payoff function. In this section we extend the analysis from the previous section to encompass other forms of strategic interaction.

The main difference is that agents' strategies form (partially) strategic complements. This occurs when a more aggressive strategy from one agent rises the other player's marginal returns to increasing his own strategy. Consequently, an increase of the opponent's choice is responded to by an increase of own choice variable. This property implies that best-replies are increasing in own action. As stated above, strategic complementarities are partial in the sense that they are not observed overall. Reaction curves are piecewise increasing, however due to a symmetry breaking nonconcavity in the payoff function, they possess a jump down across the diagonal. The common aspect to the whole analysis is the canyon shape of the agents' payoff functions along the 45^o line. Once again, a player would never optimally respond to an action of the rival by playing that same action. Whereas in the previous section the submodularity of the payoff function (or alternatively the strategic substitutability) was a global feature, in this section we cannot state global supermodularity (or strategic complementarity). The main consequence is that we cannot guarantee without further assumptions the existence of a PSNE. Nevertheless, when it exists, it will never be symmetric.

When strategies are strategic complements, there is no need to assume the quasiconcavity of the payoff function in order to guarantee the existence of a PSNE. The reason is that existence might be based on the fact that reaction curves are increasing and continuity plays no role. Likewise, when payoff functions are quasiconcave, supermodularity is not crucial for arguing the existence of a PSNE. When player's payoff function is partially quasiconcave and possesses a nonconcavity along the diagonal, we obtain the same type of symmetry-breaking already discussed. In this case, reaction curves are continuous (not necessarily increasing), except for the jump across the diagonal. Existence of PSNE can be assured (as before) via added conditions, however it is clear that no PSNE can be symmetric.

We provide now the main results of this section, first for the case in which strategic complementarities are present and then for the case of partially quasiconcave payoff functions.

4.1 The results

In this section we analyze the case in which the components of the payoff function, U and L, are not submodular. We first consider the case in which U and L are supermodular and then the case where U and L do not have this property, which is replaced by quasiconcavity.

Consider the following assumptions:

B1 U and L are supermodular.

B2 U and L are differentiable and $U_1(x,x) > L_1(x,x), \forall x \in (0,c)$

Define $\overline{r}_1(y) = \arg\max\{U(x,y) : (x,y) \in \Delta_U\}$ and $\underline{r}_1(y) = \arg\max\{L(x,y) : (x,y) \in \Delta_L\}$

B3 $U_2(\overline{r}_1(y), y) < L_2(\underline{r}_1(y), y), \forall y \in Y$

B4 $U_1(0,0) > 0$, $L_1(c,c) < 0$

B5 $U_1(x,0) > 0$, $\forall x < d$ and $L_1(x,c) < 0$, $\forall x > d$, for $d : r_1(d-\varepsilon) > d > r_1(d+\varepsilon)$

$B1$ says that on either side of the diagonal, but not necessarily globally, each player's marginal returns to increasing his action increase with the rival's action. $B2$ holds that each player's payoff, though globally continuous in the two actions, has a valley-like shape along the diagonal. $B3$ is responsible for the uniqueness of the jump in the reaction curves. $B4$ excludes the existence of equilibria in $(0,0)$ and (c,c). Finally, $B5$ restricts the reaction curves to a compact subset of the action space which enables us to prove the existence of an asymmetric PSNE.

These assumptions form a sufficiently general framework to encompass many of the studies mentioned in Section 1 as illustrated below. Furthermore, all assumptions are possible to check in a particular model.

Lemma 4.1 *If $B1 - B4$ hold, then there exists exactly one $d \in [0,c]$ such that $r_1(d-\varepsilon) > d > r_1(d+\varepsilon)$, $\forall \varepsilon > 0$.*

This Lemma guarantees that the reaction curves possess a jump and that this jump is unique. The flavor of the proof is the following: given supermodularity of U and L, we know that reaction curves are increasing in Δ_U and in Δ_L. Assumption $B2$ implies that there is no interior symmetric equilibrium due to the presence of a canyon along the diagonal. Furthermore, assumption $B4$ rules out symmetric equilibrium on the boundary. This is equivalent to saying that the reaction curves do not intersect the diagonal on the whole strategy space. Hence, there must be a jump across the diagonal. Finally assumption $B3$, that entails an idea of monotonicity of maxima along y, guarantees that in case a jump occurs, it is unique. From this Lemma we can conclude that the reaction curves possess a jump across the diagonal in a point d and that once it occurs, the reaction curves never jump back again. The point d is useful to define subsets of the strategy space, which are necessary in the next theorem.

The following theorem implies that no symmetric PSNE exists for a game where assumptions $B1 - B5$ hold and that a PSNE exists. From these premises we can conclude that there are only asymmetric PSNEs.

Theorem 4.1 *Assume that $B1 - B5$ hold, then there exist at least one pair of asymmetric PSNEs and no symmetric one.*

The intuition behind this result is the following: from Lemma 4.1 we have that the reaction curves are partially increasing (as they increase in each side of the diagonal) and possess a unique downward jump at point d. Since reaction curves are not overall increasing we cannot guarantee, without further assumptions, the existence of a PSNE. Introducing assumption $B5$ guarantees that the reaction functions are well defined in $R = \{(x, y) : d \leq x \leq c, 0 \leq y \leq d\}, R \subset \Delta_U$ and $R' = \{(x, y) : 0 \leq x \leq d, d \leq y \leq c\}, R' \subset \Delta_L$, in the sense that they are completely contained in these compact subsets of the strategy space. We obtain, hence, increasing maps in compact sets and Tarski's Fixed Point Theorem can be applied to show that within these sets a PSNE exists.

Note that we can reorder one player 's action set in a nonstandard way to obtain a submodular game. Consider action space of (say) player 1 as $(d, 0] \cup$

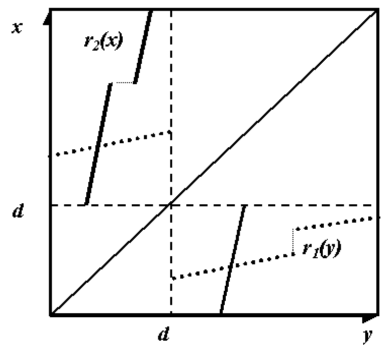

Figure 3: Reaction curves are partially increasing, but posses an unique jump down, which precludes any symmetric equilibrium.

$[c, d)$, ordered from left to right. The other player's action space order remains without change. It can be verified, that the game satisfying $B1 - B5$ then becomes a game of strategic substitutes.

Now consider another property of U and L.

B1' U and L are quasi-concave.

The assumption $B1$ can be replaced by the assumption $B1'$ which guarantees that player's best replies are continuous (not overall) and still the result holds. In other words, the supermodularity of U and L is not necessary (even though often observed in applications) for the existence of only asymmetric PSNE in this framework. In particular, assuming that U and L are quasiconcave implies that the reaction curves are partially continuous (even though not monotone) and thus, as long as the unique jump across the diagonal exists and the reaction curves are completely contained in compact subsets of the domain, we can show the existence of asymmetric PSNE. Since monotonicity cannot be guaranteed anymore Tarski's Fixed Point Theorem must be replaced by Brouwer's Fixed Point Theorem in showing the existence of PSNE.

Lemma 4.2 *If $B1'$ and $B2 - B4$ hold, then there exists exactly one $d \in [0, c]$ such that $\forall \varepsilon > 0$: $r_1(d - \varepsilon) > d > r_1(d + \varepsilon)$,*

Theorem 4.2 *Assume that $B1'$ and $B2 - B5$ hold then there exist at least one pair of asymmetric PSNEs and no symmetric one.*

The proofs of these results follow the same reasoning as the proofs of the precedent ones, however, existence of PSNE is now guaranteed through the application of Brouwer's Fixed Point Theorem (which is suitable given that payoff functions are quasiconcave).

Uniqueness of a pair of equilibria can be shown if reaction functions are contractions by using Banach's Fixed Point Theorem as in the previous section.

A comparison of equilibria when payoff functions are partially supermodular (or quasiconcave) can be done in the same spirit of Theorem 3.3. Instead of assuming $A1 - A4$ we assume $B1 - B5$ ($B1' - B5$). In the proof, the second inequality follows now from the fact that $x^* \geq d$ and Assumption $B3$.[11]

Adopting the results of Echenique (2004), wherein the order on the action spaces is not exogenously given would enlarge the scope of supermodular games. In particular, since our game here has at least two PSNEs, one can always find a partial order such that it becomes a supermodular game (Echenique, 2004, Theorem 5).

4.2 Applications: quality investment

We illustrate the results of this section with two papers dealing with quality investment problems. The first paper we analyze is Aoki and Prusa (1996). In this paper, two identical firms produce products differentiated by quality, in a two-stage setting. In the first stage firms 1 and 2 decide the level of quality investment $x \in [0, c]$ and $y \in [0, c]$ respectively, and in the second stage they simultaneously announce prices.[12]

Consumers are diversified in their willingness to pay for quality. Production cost is assumed to be 0 and firm 1 (firm 2) incurs a cost of quality investment

[11] It is easy to see that Assumption $B3$ means that for $z > d$, $U(r_1(z), z) < L(r_1(z), z)$.

[12] Aoki and Prusa (1996) consider unlimited quality investments. We impose the upper limit c, arbitrarily big, such that the strategy spaces are compact.

$f(x) = kx^2$ $\left[f(y) = ky^2\right]$, $k > 0$. A detailed study of the second stage equilibrium of this game can be found in Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). The subgame perfect equilibrium of the whole game can be obtained by backward induction and first stage overall payoffs for firm 1 are given as follows:

$$F(x,y) = \begin{cases} U(x,y) = \frac{4x^2(x-y)}{(4x-y)^2} - kx^2 \text{ if } x \geq y \\ L(x,y) = \frac{yx(y-x)}{(4y-x)^2} - kx^2 \text{ if } x < y \end{cases} \tag{14}$$

Now we can check whether assumptions of Theorem 4.1 are fulfilled. Given that U and L are differentiable we can check supermodularity (Assumption $B1$) by recurring to Topkis's Characterization Theorem. Consider that $U_1(x,x) = \frac{4}{9} - 2x$ and $L_1(x,x) = -\frac{1}{9} - 2x$, so $U_1(x,x) > L_1(x,x)$ which verifies $B2$. To check $B3$ we compute:

$$U_2(\overline{r}_1(y), y) = -4\overline{r}_1^2(y)\frac{2\overline{r}_1(y)+y}{(4\overline{r}_1(y)-y)^3} \quad L_2(\underline{r}_1(y), y) = -\underline{r}_1^2(y)\frac{2y+\underline{r}_1(y)}{\left(-4y+\underline{r}_1(y)\right)^3} \tag{15}$$

Since $\overline{r}_1(y) > y > \underline{r}_1(y)$ the condition $B3$ holds. We know, hence, that the reaction curves are upward sloping except for a downward jump at a point d. Since $U(0,0)$ is not defined we cannot check the first part of $B4$ directly, we must compute $\lim_{x\to 0} U_1(x,x)$. We obtain $\lim_{x\to 0} U_1(x,x) = \frac{4}{9} > 0$ which means that increasing quality investment at point $(0,0)$ is profitable for both firms. Point $(0,0)$ is thus ruled out as a pure strategy Nash equilibrium of the game. As for the second part of $B4$, it is easily verified by the fact that $L_1(c,c) = -\left(\frac{1}{9} + 2kc\right) < 0$. From Lemma 4.1 we have that if there is an equilibrium, it cannot be symmetric in $[0,c]^2$.

To apply Theorem 4.1 we must also check, whether $B5$ is verified. To this end, we must find the point d where the reaction curve has a jump. If the reaction curve is given implicitly, there is no algorithm to find this point d, however it is possible to find it through numerical methods. In the case of the model by Aoki and Prusa (1996), the point d is equal to $\frac{1}{12k}$. It is possible to compute that $U_1(x,0) = \frac{1}{4} - 2kx \geq 0$ for $x > d$ and that for sufficiently big c, $L_1(x,c) = \lim_{y\to\infty} L_1(x,y) = -2kx + \frac{1}{16} < 0$ for $x > d$. So the assumptions of Theorem 4.1 are satisfied and we can conclude that there exist only asymmetric equilibria in the game. Furthermore, reaction curves in this model are contractions and so a unique pair of equilibria exists.

The second paper that can be analyzed under our framework is Motta (1993). He develops two versions of a vertical product differentiation model, one with fixed and the other with variable cost of investment in quality. In each of them, he compares price versus quality competition. The model with fixed cost and price competition can illustrate the results of this section. Considering the same notation as in Aoki and Prusa (1996), the payoff function is defined as follows:

$$F(x,y) = \begin{cases} \frac{4v^2x^2(x-y)}{(4x-y)^2} - \frac{x^2}{2} \text{ if } x \geq y \\ \frac{xyv^2(y-x)}{(4y-x)^2} - \frac{x^2}{2} \text{ if } x \leq y \end{cases} \tag{16}$$

Where v is the upper limit of the set of consumer's taste parameters. This model is analogous to Aoki and Prusa (1996) if we let $v = 1$ and $k = \frac{1}{2}$. All the results follow directly from the above discussion.

5 Convex payoffs

Certain features of the production technologies or consumer preferences may lead to a situation where payoff functions are convex. In particular, the presence of highly convex demand functions or strongly concave costs translating intensely decreasing elasticity of demand or decreasing marginal costs might have this effect. This property of the payoff functions implies that agents prefer corner solutions. Moreover certain additional conditions on the payoffs might generate asymmetric equilibria and even rule out the symmetric ones. In this section we analyze a class of games in which players have convex payoff functions and only asymmetric PSNEs arise.

Let $S = [0, c]$ be the strategy space of a player in a two-player game, Γ. Consider $F : S \times S \to R$ as the payoff function of player 1. $G(y, x) = F(x, y)$ is the payoff of player 2 because the game is *a priori* symmetric. Then we can apply the following theorem to conclude about the properties of the equilibria of Γ.

Theorem 5.1 *Assume that the following assumptions hold:*

1. *F is strictly quasi-convex in own strategy*
2. *$F(c,0) > F(0,0)$ and $F(0,c) > F(c,c)$*

Then, the game has no symmetric equilibrium and it has exactly one pair of asymmetric equilibria given by $(0,c)$ and $(c,0)$.

Proof. From the definition of strict quasi-convexity, we know that:

$$F(\lambda z_1 + (1-\lambda)\, z_2) < \max\{F(z_1), F(z_2)\}, \forall z_1, z_2 \in S \times S$$

Let $z_1 = (0,y)$ and $z_2 = (c,y)$, then we know that any $x \in (0,c)$ yields a lower payoff than $x = 0$ or $x = c$, $\forall y \in [0,c]$. This means that $\forall y \in [0,c]$, $r_1(y) = 0$ or $r_1(y) = c$. Analogously, for player 2 we have the same result, due to symmetry.

Finally, from Assumption 2 we have that $(c,0)$ and $(0,c)$ are the only PSNEs of the game. ■

Figure 4 illustrates the results of this section. Notice that we depicted reaction curves which are not continuous, specifically, they possess an odd number of jumps. This is a direct consequence of the Assumptions 1 and 2 that imply either that the reaction curves are continuous, or that, if they jump, they must jump an odd number of times.

5.1 Applications

This theorem generalizes the results of Amir (2000) and Mills and Smith (1996), whose models are usually presented within the literature about endogenous heterogeneity of firms.[13] In these papers it is considered a two-stage duopoly game. In the first stage, firms make long term investment decisions that affect the production costs. In the second stage, firms compete *à la* Cournot. Both firms face

[13] Another example where the game can be reduced to a two person normal form game is presented in Boyer and Moreaux (1997). Conditions for the non-existence of symmetric equilibria are the same, even if the payoff function is not convex.

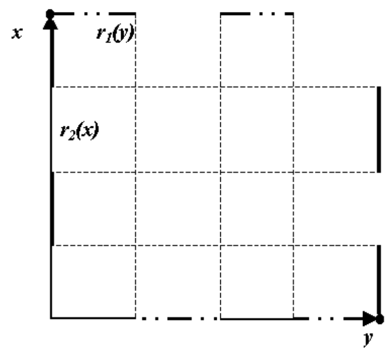

Figure 4: Quasi-convexity of the payoffs implies that players prefer corner solutions.

a linear demand function and for some parameterization of the cost functions, profits are convex in own quantity. In both papers it is then concluded that if some conditions on the parameters hold, asymmetric equilibria might arise. It is easily shown that the conditions presented in the papers can be deduced from the assumptions of Theorem 5.1.

6 Extensions

Most of the results of the previous sections depend on some form of monotonicity of the reaction curves. We used the cardinal notions of complementarity and substitutability to obtain this property mainly due to its convenient characterization through the cross partial derivatives of the payoff functions. Supermodularity and submodularity are cardinal notions that are not preserved by monotone transformations of the objective function. The usefulness of ordinal properties under which comparative statics results are invariant is clear. Milgrom and Shannon (1994) proved that the result of Topkis holds when single crossing property substitutes supermodularity of the objective function. In this section we show that our results can be generalized to the ordinal definitions of complementarity.

Let F be defined as in (1) and consider the following assumptions:

$A1\prime$ U and L have dual single crossing property

$A4$ $U_2(\overline{r}_1(y), y) < L_2(\underline{r}_1(y), y), \forall y \in [0, c]$

The Assumption $A1'$ is alternative to Assumption $A1$ defined in Section 3 and provides the ordinal condition for the reaction curves to be decreasing. Assumption $A4$ expresses monotonicity of U and L with respect to the opponent's action along the best replies.

We now show the main result of this section in the following steps: first we conclude about decreasing best-replies; then we observe that there is a downward jump in the reaction function at point d and that this jump is unique. Finally we use Tarski's Fixed Point Theorem to conclude of the existence of equilibria in the subsets of the strategy space defined using point d.

Finally we treat the setting of Section 4, extending its results to the ordinal definitions of complementarity.

Theorem 6.1 *If $A1', A2, A3$ and $A4$ hold, then the game Γ has at least one pair of asymmetric PSNEs and no symmetric one.*

The idea of the proof is that within Δ_U and Δ_L the dual single crossing property of the payoff function F holds from assumption $A1'$. As Milgrom and Shannon (1994) showed, the dual single crossing property allows us to draw the same conclusions as the submodularity of the payoff function in terms of monotonicity of the reaction curves. Furthermore the dual single crossing property has the advantage of being more general and preserved by monotonic transformations. Then assumption $A2$ implies that there exists a jump down in the reaction curves at a certain point d and $A4$ implies that this jump is unique. With point d we define compact subsets of the strategy space where reaction curves are decreasing and thus we can guarantee the existence of a PSNE by Tarski's Fixed Point Theorem.

Some results from Section 4 can also be extended into an ordinal version.

Let F be defined as in (1) and consider the following assumption:

$B1''$ U and L have the single crossing property

Theorem 6.2 *If the assumptions $B1''$ and $B2 - B5$ hold, then there exist at least one pair of asymmetric PSNEs and no symmetric one.*

From Milgrom and Shannon's Theorem Assumption $B1''$ implies that reaction curves are partially increasing in Δ_U and Δ_L. From this fact and as long as they are well defined in a compact subset of the strategy space, we can obtain existence of PSNEs. The valley-shape of the profit precludes symmetric equilibria in the same spirit as Theorem 4.1.

7 Conclusion

Our theorems assert that, under specific conditions, heterogeneity in agent's behavior might arise even when they are *a priori* identical. This paper constitutes, hence, a contribution to the discussion about the sources of diversity across economic agents and disparities in economic performances. While previous literature stands on arguments related to multiplicity of equilibria and strategic complementarities (Cooper, 1999) or on strategic substitutability and stability of equilibria (Matsuyama, 2002), our approach stands on the existence of a fundamental nonconcavity of the payoff function and on some form of strategic substitutability. It is, thus, similar in spirit to Matsuyama's work. However, we show that endogenous heterogeneity does not rely on the idea that only stable equilibria are observable as in Matsuyama. With respect to Cooper's approach, where agents can still choose symmetrically, our results guarantee that symmetric equilibria can never arise in a two player setting. Even though we have, in our model some form of strategic substitutability (notice that when talking about two-player games strategic substitutability can be converted into complementarity through a simple inversion of one agent's strategy space), the critical assumption for the inexistence of symmetric equilibria is the nonconcavity of the payoffs. In fact, we show that strategic substitutability can be replaced by partial quasiconcavity and still the results follow.

An alternative explanation for endogenous heterogeneity can be found in business strategy literature. Our results can also be considered as a response to its critique as asymmetries arise here in a completely deterministic and rational setup. We should thus expect that heterogeneity generated in such a framework can be the origin of long-term diversity.

8 Appendix

8.1 Summary of supermodular/submodular games

We give an overview of the main definitions and results in the theory of supermodular games that are used in the paper, in a simplified setting that is sufficient for our purposes. Details may be found in Topkis (1978).[14]

Let I_1 and I_2 be compact real intervals and $F : I_1 \times I_2 \rightarrow R$. F is (strictly) supermodular if $\forall x_1, x_2 \in I_1, x_2 > x_1$ and $\forall y_1, y_2 \in I_2, y_2 > y_1$ we have $F(x_2, y_2) - F(x_2, y_1)(>) \geq F(x_1, y_2) - F(x_1, y_1)$. F is (strictly) submodular if $-F$ is (strictly) supermodular.

Theorem 8.1 (Topkis's Characterization Theorem) *Let F be twice continuously differentiable. Then*

(i) $F_{12} = \frac{\partial^2 F}{\partial x \partial y} \geq 0$ [≤ 0] for all $x, y \Leftrightarrow F$ is supermodular [submodular].

(ii) $F_{12} = \frac{\partial^2 F}{\partial x \partial y} > 0$ [< 0] for all $x, y \Rightarrow F$ is strictly supermodular [submodular].

The supermodularity property is not preserved by monotonic transformations of the function F. An alternative notion (ordinal) is the single crossing property defined as follows: F has the single crossing property [the dual single crossing property] in (x, y) if $\forall x_1, x_2 \in I_1, x_2 > x_1$ and $\forall y_1, y_2 \in I_2, y_2 > y_1$ we have

$$F(x_1, y_2) - F(x_1, y_1) \geq 0 \,[\leq 0] \Rightarrow F(x_2, y_2) - F(x_2, y_1) \geq 0 \,[\leq 0] \,.$$

The single crossing property does not have a correspondent differential characterization and thus it is often more difficult to check. Now we present the main monotonicity theorems.

Theorem 8.2 (Topkis's Monotonicity Theorem) *If F is continuous in y and (strictly) supermodular [submodular] in (x, y), then* $\arg\max_{y \in I_2} F(x, y)$ *has (all of its) maximal and minimal selections increasing [decreasing] in $x \in I_1$.*

[14] Other aspects of the theory may be found in Topkis (1979), Milgrom and Roberts (1990) and Vives (1990).

Theorem 8.3 (Milgrom and Shannon) *The conclusion of Topkis's Monotonicity Theorem continues to hold when supermodularity [submodularity] is replaced by the [dual] single crossing property.*

We can introduce now the notion of supermodular game and of its properties. A two player game is supermodular (submodular) if both payoff functions are continuous, supermodular (submodular) and both action spaces are compact real intervals. [15] The fixed point theorems associated with this framework are due to Tarski (1955).

Theorem 8.4 (Tarski's Fixed Point Theorem) *Let $f : I_1 \times I_2 \to I_1 \times I_2$ be an increasing function, then f has a fixed point.*

Theorem 8.5 *A two player supermodular (submodular) game has a pure strategy Nash equilibrium.*

In general, this theory dispenses with assumptions of concavity or differentiability of payoff functions, making it an extremely general framework to study the properties of equilibria.

8.2 Proofs of Section 3

The proof of Theorem 3.1 is organized as follows: we begin with proving four preliminary lemmas, and then present the main proof in two steps: first we show existence of PSNE and afterwards that all PSNEs must be asymmetric.

The first lemma states that for a small enough square of points on the diagonal we have submodularity.

Lemma 8.1 *Consider the following points as depicted in figure 5. If $A1-A2$ hold, then for small enough $\alpha > 0 : U(x, x-\alpha) - U(x-\alpha, x-\alpha) \geq L(x, x) - L(x-\alpha, x), \forall x \in [0, c]$.*

[15] Compactness is not necessary, it is required in order to use a simplified version of Tarski's Fixed Point Theorem, without referring to lattices.

Proof. (Lemma 8.1) Take any point (x, x) on the diagonal, belonging to the domain of F. For $\alpha > 0$ small enough

$$U_1(x, x) \simeq U(x + \alpha, x) - U(x, x) \text{ and } L_1(x, x) \simeq (x, x) - L(x - \alpha, x). \quad (17)$$

Hence, from $A2$:

$$U(x + \alpha, x) - U(x, x) > L(x, x) - L(x - \alpha, x). \quad (18)$$

From $A1$ we know that

$$U(x + \alpha, x - \alpha) - U(x, x - \alpha) \geq U(x + \alpha, x) - U(x, x). \quad (19)$$

Take $\widehat{\varepsilon} > 0$ such that

$$U(x + \alpha, x) - U(x, x) \geq L(x, x) - L(x - \alpha, x) + \widehat{\varepsilon}. \quad (20)$$

From continuity of $U(x, y)$ in x (for fixed $y = x - \alpha$), it follows that

$$\forall 0 < \varepsilon \leq \frac{\widehat{\varepsilon}}{2}, \ \exists \delta > 0 \text{ such that}$$
$$|x - (x - \alpha)| = \alpha \leq \delta \text{ and } |U(x, x - \alpha) - U(x - \alpha, x - \alpha)| \leq \varepsilon \leq \frac{\widehat{\varepsilon}}{2}.$$

Moreover,

$$\forall 0 < \varepsilon \leq \frac{\widehat{\varepsilon}}{2}, \ \exists \delta > 0 \text{ such that}$$
$$|x + \alpha - x| = \alpha \leq \delta \text{ and } |U(x + \alpha, x - \alpha) - U(x, x - \alpha)| \leq \varepsilon \leq \frac{\widehat{\varepsilon}}{2}.$$

This allows us to establish that:

$$(U(x + \alpha, x - \alpha) - U(x, x - \alpha)) - (U(x, x - \alpha) - U(x - \alpha, x - \alpha)) \leq 2\varepsilon \leq \widehat{\varepsilon}. \quad (21)$$

Rewriting (21),

$$U(x + \alpha, x - \alpha) - U(x, x - \alpha) \leq U(x, x - \alpha) - U(x - \alpha, x - \alpha) + \widehat{\varepsilon}. \quad (22)$$

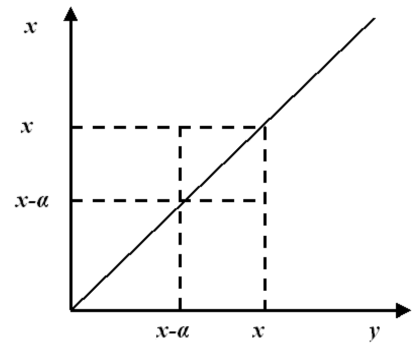

Figure 5: Square of the length α.

Finally, summarizing, we have:

$$L(x,x) - L(x-\alpha,x) + \widehat{\varepsilon} \leq$$
$$\leq U(x+\alpha,x) - U(x,x) \leq$$
$$\leq U(x+\alpha,x-\alpha) - U(x,x-\alpha) \leq$$
$$\leq U(x,x-\alpha) - U(x-\alpha,x-\alpha) + \widehat{\varepsilon}$$

where, the first inequality comes from (20), the second one from (19) and the last one from (22). So we obtain,

$$U(x,x-\alpha) - U(x-\alpha,x-\alpha) + \widehat{\varepsilon} \geq (x,x) - L(x-\alpha,x) + \widehat{\varepsilon}. \qquad (23)$$

Subtracting $\widehat{\varepsilon}$ from both sides ends the proof. ■

The next lemma extends the property of submodularity of F form the small square of length α to any square with two vertices on the diagonal.

Lemma 8.2 *If Lemma 8.1 holds, then for any square with points in the diagonal, such as depicted in figure 6, we have,*

$$F(z,x) - F(z,z) \geq F(x,x) - F(x,z)\,.$$

Proof. (Lemma 8.2) Consider the square formed by the four points defined in the lemma. Divide this square into rectangles such that their height is equal to

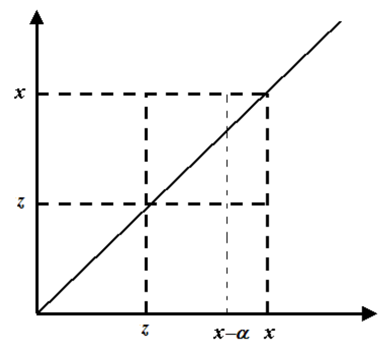

Figure 6: Partition of the square whose vertices coincide with the diagonal.

the original height of the square and its length is not bigger than α, as defined in Lemma 8.1. We will now show that for points in the vertices of such rectangles the thesis holds and *a fortiori* it is possible to obtain the conclusion for the whole square. Figure 6 illustrates the proof.

Let $x - z = k\alpha$, where $k \in R$, and $\alpha > 0$ is small enough. Now consider the rectangle defined by the following points $(x, x), (x, x - \alpha)$, (z, x) and $(z, x - \alpha)$. From Lemma 8.1 we know that

$$F(x, x - \alpha) - F(x - \alpha, x - \alpha) \geq F(x, x) - F(x - \alpha, x)$$

Also, from $A1$ we know that

$$F(x - \alpha, x - \alpha) - F(z, x - \alpha) \geq F(x - \alpha, x) - F(z, x)$$

Adding these two inequalities we obtain that

$$F(x, x - \alpha) - F(z, x - \alpha) \geq F(x, x) - F(z, x) \tag{24}$$

Repeating the procedure, consider the rectangle defined by: $(x, x - \alpha)$, $(x, x - 2\alpha)$, $(z, x - \alpha)$, $(z, x - 2\alpha)$. From $A1$ we know that:

$$F(x, x - 2\alpha) - F(x - \alpha, x - 2\alpha) \geq F(x, x - \alpha) - F(x - \alpha, x - \alpha)$$

and also

$$F(x - 2\alpha, x - 2\alpha) - F(z, x - 2\alpha) \geq F(x - 2\alpha, x - \alpha) - F(z, x - \alpha).$$

Using Lemma 8.1 we know that

$$F(x-\alpha, x-2\alpha) - F(x-2\alpha, x-2\alpha) \geq F(x-\alpha, x-\alpha) - F(x-2\alpha, x-\alpha)$$

Adding the three inequalities we obtain:

$$F(x, x-2\alpha) - F(z, x-2\alpha) \geq F(x, x-\alpha) - F(z, x-\alpha)$$

From (24) we obtain

$$F(x, x-2\alpha) - F(z, x-2\alpha) \geq F(x, x) - F(z, x)$$

We can repeat this argument k times until getting a rectangle whose length is not bigger than α. Once again we apply assumption $A1$ and Lemma 8.1 to show that submodularity holds for this rectangle as well and we can conclude that,

$$F(x, z) - F(z, z) \geq F(x, x) - F(x, z).$$

Hence submodularity is satisfied for any square with vertices coinciding with the diagonal. ■

The following lemma establishes that the analysis of submodularity of any rectangle formed by four points of the domain $[0, c]^2$ can be reduced to the analysis of submodularity for points placed in such a way that they form a square with vertices coinciding with the diagonal.

Lemma 8.3 *If $A1$ and $A2$ hold, then F and G are submodular on $[0, c]^2$.*

Proof. (Lemma 8.3) Due to the kink along the diagonal, one cannot invoke Topkis's simple cross-partial test (Topkis's Characterization Theorem in the Appendix) to verify submodularity of F and G. Instead, we use definition of submodularity for any configuration of four points in the domain, constituting a rectangle. If the rectangle is completely contained in either Δ_U or Δ_L, the submodularity condition follows from $A1$. Every other situation can be reduced by adding subrectangles, each of which lying fully in either Δ_U or Δ_L, to the situation depicted in figure 5 as we now show, say for F. Consider the case of

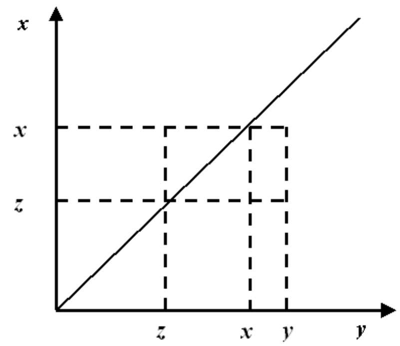

Figure 7: If F satisfies submodularity on the square on the diagonal, this implies it satisfies submodularity on the rectangle.

figure 7 with the four points (x, z), (z, z), (x, y) , (z, y) as shown. With $z < x < y$, we know from $A1$ that, since $F = U$ on Δ_U,

$$F(x, y) - F(x, x) \geq F(z, y) - F(z, x).$$

From Lemma 8.2, submodularity holds for the configuration of the square (x, x), (z, x), (z, z), (z, x), hence we have

$$F(x, x) - F(x, z) \geq F(z, x) - F(z, z).$$

Adding the two inequalities yields

$$F(x, y) - F(x, z) \geq F(z, y) - F(z, z),$$

which is just the definition of submodularity for the original points (x, z), (z, z), (x, y) and (z, y).

It can be shown via analogous steps that the submodularity of F for any other configuration of points can be reduced to showing submodularity for squares with two vertices on the diagonal. The details are left out. ■

The next result allows us to conclude that the two reaction curves always admit a discontinuity that skips over the diagonal, a key step for our endogenous heterogeneity result.

Lemma 8.4 *Given $A1 - A3$, there exists exactly one point $d \in (0, c)$, such that $r_i(d - \varepsilon) > d > r_i(d + \varepsilon), i = 1, 2.$*

Proof. (Lemma 8.4) From Topkis's Monotonicity Theorem and Lemma 8.3 , all the selections from the best reply correspondences are downward sloping. Hence, both $r_i(d - \varepsilon)$ and $r_i(d + \varepsilon)$ exist for any selection of r_i and are independent of the selection.

From assumption $A3$, we know that $(0, 0) \notin Graph\ r_i$ and $(c, c) \notin Graph\ r_i$ (i.e. r_i does not go through $(0, 0)$ or (c, c)). These two properties imply that r_i cannot be identically 0 or c.

We next show that the reaction correspondence r_1 (say) cannot ever cross the 45^o line at an interior point, i.e. in $(0, c)$. The generalized first order condition for a maximum of F (say) to occur at a point (x, x) with $x \in (0, c)$, which applies even in the absence of differentiability, is that $U_1(x, x) \leq L_1(x, x)$. Assumption $A2$ rules out this case . Hence no $x \in (0, c)$ can ever be a best reply to itself, meaning that the reaction curves do not cross the 45^o line at any interior point.

Since r_1 starts strictly above 0 (for $y = 0$) and ends strictly below c (for $y = c$), the above properties of r_1 imply that there exists exactly one $d \in (0, c)$ such that $r_1(d - \varepsilon) > d > r_1(d + \varepsilon)$. In words, there must exist a jump in the best reply function past the diagonal as in figure 1. ■

Using the Lemmas 8.3 and 8.4 we can now prove Theorem 3.1.

Proof. (Theorem 3.1) From Lemma 8.3 we have overall submodularity of the payoff function. This guarantees that a PSNE exists.

Consider now the behavior of the reaction curves in the area Δ_U. The same conclusion follows for Δ_L by symmetry. Define the following restricted reaction curves: $r_1|_{\Delta_U}(y) : [0, d] \to [d, c]$ and $r_2|_{\Delta_U}(x) : [d, c] \to [0, d]$ both decreasing as implied by Lemma 8.3. Define the mapping $B : [d, c] \to [d, c]$, $B(x) = r_1|_{\Delta_U} \circ r_2|_{\Delta_U}(x)$, which is increasing given that each of $r_1|_{\Delta_U}$ and $r_2|_{\Delta_U}$ is decreasing. From Tarski's Fixed Point Theorem, we know that there exists $\bar{x}$ such that $B(\bar{x}) = r_1|_{\Delta_U} \circ r_2|_{\Delta_U}(\bar{x})$, therefore $(\bar{x}, r_2(\bar{x})|_{\Delta_U})$ is a PSNE. From Lemma 8.4, there is no symmetric PSNE in $[0, c]$. Hence, there must exist at least one pair of asymmetric PSNEs. ■

Theorem 3.2 rules out the existence of multiple pairs of asymmetric equilibria.

Proof. (Theorem 3.2) Once again we concentrate on the area Δ_U. Conclusions follow for the area Δ_L by symmetry. Whenever r_1 $[r_2]$ is interior, first order condition $U_1(r_1(y), y) = 0$ $[L_1(r_2(x), x) = 0]$, together with the Implicit Function Theorem and the assumptions (3) and (4), implies that r_1 $[r_2]$ is differentiable in Δ_U and that $r_1'(y) = -\frac{U_{12}(r_1(y),y)}{U_{11}(r_1(y),y)} \geq -1$, also $r_2'(x) = -\frac{L_{12}(r_2(x),x)}{L_{11}(r_2(x),x)} \geq -1$. Hence, $r_1(y)|_{\Delta_U}$ and $r_2(x)|_{\Delta_U}$ are contractions. Using Banach's Fixed Point Theorem we can conclude that there exists exactly one pure strategy Nash equilibrium in Δ_U.[16] In the same way there exists exactly one pure strategy Nash equilibrium in Δ_L. Concluding, we have exactly one pair of pure strategy Nash equilibrium. ■

Finally we provide a proof of Theorem 3.3.

Proof. (Theorem 3.3) Since $x^* > d > y^*$, we have $F(x^*, y^*) = U(x^*, y^*)$ and $F(y^*, x^*) = L(y^*, x^*)$. Also $U(r_1(d), d) = L(r_1(d), d)$ if d denotes the unique point of jump of reaction curve between Δ_U and Δ_L, as defined in Lemma 8.4. Then

$$\begin{aligned} F(x^*, y^*) &= U(x^*, y^*) = \\ &= U(r_1(y^*), y^*) \leq U(r_1(d), d) = \\ &= L(r_1(d), d) \leq L(r_1(x^*), x^*) = L(y^*, x^*) = F(y^*, x^*) \end{aligned}$$

where both inequalities follow from the monotonicity of $U(r_1(y), y)$ and $L(r_1(y), y)$ in y. ■

8.3 Proofs of Section 4

First we prove Lemma 4.1 and Lemma 4.2 since these proofs are similar. We then move to proving Theorems 4.1 and 4.2.

[16] (Banach's Fixed Point Theorem): Let $S \subset \mathcal{R}^n$ be closed and $f : S \to S$ be a contraction mapping, then there exists $x \in S : f(x) = x$.

Proof. (Lemma 4.1) We consider the area Δ_U. From $B1$ and Topkis's Monotonicity Theorem, we know that the reaction curves are increasing. The generalized first order condition for a maximum to occur in the point (x, x) in the absence of differentiability of F (and G, by symmetry) is that $U_1\left(x, x\right) \leq L_1\left(x, x\right)$. Assumption $B2$ rules out this possibility so we have that no y (nor x), belonging to $(0, c)$, can be best reply to itself, meaning that the reaction curves do not cross the 45^o line. Assumption $B4$ excludes that 0 can be a best reply to 0 and that c can be a best reply to c. Hence, there must exist a $d \in [0, c]$, such that $r_1(d-\varepsilon) > d > r_1(d+\varepsilon)$.

To exclude the possibility of another jump we use assumption $B3$. Consider $\underline{r}_1(y)$ and $\bar{r}_1(y)$ defined as in Section 4. Denote $W(y) = L(\underline{r}_1(y), y)$ and $V(y) = U(\bar{r}_1(y), y)$. From the Envelope Theorem, $\frac{\partial W(y)}{y} = L_2(\underline{r}_1(y), y)$, and $\frac{\partial V(y)}{y} = U_2(\bar{r}_1(y), y)$. Hence, when $B3$ holds, we know that W increases in y quicker then V does. It means that when the overall reaction curve jumps down along the diagonal, it never jumps up again. ■

Proof. (Lemma 4.2) If $B1'$ holds reaction curves are continuous in Δ_U and in Δ_L. $B2$ and $B4$ rules out the possibility that F (and G, by symmetry) has a maximum in a point $[x, x]$, that secures that the reaction curve must have a jump down in a point $d \in [0, c]$. As in the proof of Lemma 4.1, $B3$ rules out other possible upward jumps between Δ_L and Δ_U. ■

We may now show that only asymmetric pure strategy Nash equilibria exist.

Proof. (Theorem 4.1) Consider $R \subset \Delta_U$ as defined in Section 4. Define as before the restricted reaction curves as $r_1(y)|_{\Delta_U}$ and $r_2(x)|_{\Delta_U}$. From assumption $B5$, the best reply of player 1 to $y = 0$ cannot be less than d as U is decreasing in x for $y = 0$ when $x < d$. For $y > 0$, and given assumption $B1$ (supermodularity), Topkis's Monotonicity Theorem allows us to conclude that the best reply of 1 is increasing. Hence $r_1(y)|_{\Delta_U} \in R$. Seemingly the best reply of player 2 for $x = c$ cannot exceed d as L is decreasing in y when $x = c$. Also by Topkis's, the reaction curve is an increasing map. Therefore $r_2(x)|_{\Delta_U} \in R$. Consider the mapping, $B : R \to R$ such that $B(x, y) = (r_1(y), r_2(x))$. $B(x, y)$ is an increasing correspondence, given that both its components are increasing. R is a compact set and hence we may use Tarski's Fixed Point Theorem to conclude that there exists a pair $(\bar{x}, \bar{y})$ such that $B(\bar{x}, \bar{y}) = (r_1(\bar{y}), r_2(\bar{x}))$. $(\bar{x}, \bar{y})$ is a pure strategy Nash equilibrium.

Finally, by Lemma 4.1 we know that no equilibrium can be symmetric. ■

Proof. (Theorem 4.2) Define R, $r_1(y)|_{\Delta_U}$ and $r_2(x)|_{\Delta_U}$ as before. From assumption $B5$, $r_1(y)|_{\Delta_U} \in R$ and $r_2(x)|_{\Delta_U} \in R$ and from assumption $B1'$ they are continuous. Consider the mapping $B : R \to R$ such that $B(x, y) = (r_1(y), r_2(x))$. B is a continuous correspondence, given that both its components are continuous, R is a compact set and hence we may use Brouwer's Fixed Point Theorem to conclude that there exists a pair $(\bar{x}, \bar{y})$ such that $B(\bar{x}, \bar{y}) = (r_1(\bar{y}), r_2(\bar{x}))$. $(\bar{x}, \bar{y})$ is a pure strategy Nash equilibrium.

By Lemma 4.2 we know that no equilibrium can be symmetric. ■

8.4 Proofs of Section 6

To prove the theorem we first formulate a useful lemma.

Lemma 8.5 *If $A1\prime$, $A2$ and $A4$ hold, then there exist exactly one point $d \in [0, c]$ such that $r_1(d - \varepsilon) > d > r_1(d + \varepsilon), \varepsilon > 0$.*

Proof. (Lemma 8.5) Consider Δ_U. From $A1'$ and Topkis's Monotonicity Theorem we know that reaction curve is decreasing in this area. The generalized first order condition for a maximum to occur in (x, x) is (in the absence of differentiability in this point) $U_1(x, x) \leq L_1(x, x)$. Assumption $A2$ rules out this possibility, thus no $x \in (0, c)$ can be a best response to itself. Moreover, neither $(0, 0)$ nor (c, c) can be an equilibrium, since from $A3$ follows, that for player 1 it is always profitable to deviate from any of these points. Hence, the reaction curve does not cross the 45^o line, and there must exist a point, call it $d \in [0, c]$ such that $r_1(d - \varepsilon) > d > r_1(d + \varepsilon)$.

Now, we prove uniqueness of this point. Consider $\underline{r}_1(y)$ and $\bar{r}_1(y)$ defined as in Section 4. Denote $W(y) = L(\underline{r}_1(y), y)$ and $V(y) = U(\bar{r}_1(y), y)$. From the Envelope Theorem, $\frac{\partial W(y)}{y} = L_2(\underline{r}_1(y), y)$, and $\frac{\partial V(y)}{y} = U_2(\bar{r}_1(y), y)$. Hence, if

$A3$ holds, we know that W increases in y quicker then V does. It means that when the overall reaction curve jumps down along the diagonal, it never jumps up again. $\blacksquare$

Proof. (Theorem 6.1) Consider restricted reaction curves $r_1|_{\Delta_U}(y)$ and $r_2|_{\Delta_U}(x)$, both decreasing as implied by $A1'$ and Topkis's Monotonicity Theorem. From Lemma 8.5 and the monotonicity it follows that $c \geq r_1|_{\Delta_U}(0) \geq r_1|_{\Delta_U}(d) > d$ and $0 < r_2|_{\Delta_U}(0) \leq r_2|_{\Delta_U}(d) \leq d$, therefore $r_1|_{\Delta_U}(y) : [0,d] \to [d,c]$ and $r_2|_{\Delta_U}(x) : [d,c] \to [0,d]$ are well defined. Define the mapping $B : [d,c] \to [d,c]$, $B(x) = r_1|_{\Delta_U} \circ r_2|_{\Delta_U}(x)$, which is increasing given that each of $r_1|_{\Delta_U}$ and $r_2|_{\Delta_U}$ is decreasing. From Tarski's Fixed Point Theorem, we know that there exists $\bar{x}$ such that $B(\bar{x}) = r_1|_{\Delta_U} \circ r_2|_{\Delta_U}(\bar{x})$, therefore $(\bar{x}, r_2(\bar{x})|_{\Delta_U})$ is a PSNE.

From Lemma 8.5, there is no symmetric PSNE in $[0,c]$. Hence, there must exist at least one pair of PSNEs. $\blacksquare$

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