

## Solutions comparing in multistage cooperative games

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The comparison problem of TU-cooperative game values is introduced and treated. The idea of the proposed method is based on multicriteria methodology and ASPID technique. The Shapley value and the nucleolus are compared under different information about excess preferences.

*Keywords:* Cooperative game; multicriteria assessment; time-consistent dominance.

### 1. Introduction

Joint implementation projects demand to develop distribution principles to allocate the profit of a cooperation. Cooperative games with transferable utility (TU-games) are the mathematical models of such conflicts. Cooperative theory takes into account only income of cooperation but does not consider various coalitional attitudes towards extra payoffs due to common actions. In the problem of extra payoff allocation analyst should pay attention to desires and ambitions of individual players and coalitions to propose realizability of a solution. Cooperative game theory treats many optimality concepts (core, Shapley value, nucleolus, etc.) It is possible to choose an appropriate solution by a number of axioms (Moulin (1991)). One of the possible ways to choose an appropriate solution is realized in the concept of weighed nucleolus (Derks and Haller (1999)). Authors assign significance expressed in the weight coefficients to the excesses. The values of the game received for various sets of the weight coefficients possess different properties. In this case there can be a question on comparison of varied weighed nucleolus. It is obvious, that the choice of comprehensible value of the game depends on the attitude of community of the

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players to the magnitude of the values of the excesses. In this paper we try to realize the idea of such attitudes with reference to several known values of the cooperative games in order to compare the corresponding solutions. Using of multicriteria assessment to compare a number of alternatives is not new, however, it is rarely met in game-theoretical literature. The methodology of such a comparison can be constructed on the basis of the approach stated in Hovanov (1996). Due to the given methodology, we reduce the comparison problem to a problem multicriteria evaluation. We illustrate an opportunity of such an approach in an example with three person game.

### References

- Moulin, Herve (1991). "Axioms of cooperative decision making," Cambridge University Press.
- Derks, Jean and Haller, Hans (1996). "Weighted nucleoli," *The International Journal of Game Theory* No. 28, 173-187.
- Hovanov, Nikolay (1996). "Analysis and synthesis of indices under informational deficit," Saint-Petersburg University Press. (Russian)