Efficiency out of Disorder — Contested Ownership in Incomplete Contracts

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Abstract

This paper studies the role of contested ownership in a situation where two players have to make a person- and asset-specific investment, and when no complete contracts can be written. It compares contested ownership to the various ex ante ownership structures typically discussed in the literature (following the influential work by Grossman, Hart, and Moore). The paper shows that contested ownership mitigates the inefficiency of investments due to the incompleteness of contracts generating an exchange surplus which comes closer to the first-best surplus as compared to any other ex ante distribution of ownership. For example, if the contest is perfectly discriminatory, *each* player makes a transaction-specific investment as if he or she owns the asset.

Keywords: Incomplete Contracts, Contests, Bargaining, Theory of the Firm.

1 Introduction

This paper studies the role of contested ownership in a situation where two players have to make a person- and asset-specific investment in human capital, and when no complete contracts can be written. Following the influential work by Grossman, Hart, and Moore (GHM),[1] ownership over an asset is defined as having the residual control right over this asset which gives the owner the right to exclude the other player from using the asset. The setup is similar to the one used in Hart (1995): At $t = 0$, a team of two players simultaneously choose their respective transaction-specific investment levels. At $t = 1$, production takes place and the team surplus is divided up according to the Nash bargaining solution where the bargaining disagreement points depend on the ownership structure agreed on in an initial (incomplete) contract in period $t = 0$. GHM show that this ownership structure matters because it gives players stronger or weaker incentives to make transaction-specific investments (hold-up problem). For example, they recommend that if one player is "relatively unproductive" then the other player should be the owner of all the assets (integration). Contrary to GHM, this paper suggests that players should *not* sign a contract which distributes residual control rights in period $t = 0$. Instead, players should leave ownership over an asset unspecified (disorder) and they should specify ownership only ex post by a contest in case contract (re)negotiations fail. The paper shows that unspecified but potentially contested ownership may give players a stronger incentive to make transaction-specific investments as compared to *any* possible ex ante distribution of ownership typically discussed following GHM. If the contest is perfectly discriminatory, the result is general and holds for any specification of the skill technology. If the contest is imperfectly discriminatory, then the result is less general but still holds in important cases. For example, if one player is "relatively unproductive" or if skill technologies are sufficiently symmetric contested ownership always produces a higher surplus. In the latter case, this paper complements GHM because for a sufficiently symmetric skill technology, GHM do not produce a clear prediction of who should own the asset. In the former case, the paper contradicts GHM because it suggests that not distributing residual control rights produces a higher surplus than any ex ante distribution of ownership rights.

[1] See the two influential articles Grossman and Hart (1986), and Hart and Moore (1990). For a comprehensive discussion of this approach see the book Hart (1995).

In GHM, ex-ante ownership leads to the bargaining disagreement points where one or the other player has access to an asset (deterministic ownership), or each player has access to an asset with some probability (random ownership). Similar to random ownership, with contested ownership each player has access to the asset with some probability since players win the contest with some probability. But unlike the probability with random ownership, the probability of winning a contest is a strategic variable and it is determined non-cooperatively when players make their asset-specific human capital investment in period $t = 0$. Thus, all what this paper does in comparison to GHM is to change the bargaining disagreement points when bargaining over the team surplus by introducing a contest as an alternative way of ownership determination.

The economic intuition for this result is that asset-specific investments partially serve as a *commitment device* to be a strong contestant in case contract (re)negotiations fail and ownership is determined by a contest. Each players' valuation of the contested asset increases in each players' respective investment, and a higher valuation means that players will spend a higher contest effort, should a contest occur. Although a contest never happens in equilibrium, players are forced to make those investments in order to render their respective threats to contest forcefully credible.

Consider the following situation: There are two electrical engineers supplying their labor and skills to a joint start up company. For example, their objective is to develop and sell an electronic device, where one player engineers the device while the other designs a chip for this device. Both players have access to some equipment, and both players need to make a person- and asset-specific investment in human capital. Assume further that investments and the output are observable by both players but they are not verifiable by outsiders. What organizational structure should the company use? The theory of property rights following GHM suggests to give the player with the more important investment decision the residual control right over the asset. For example, this player should be the company's boss.[2] The paper here makes an alternative suggestion: Players should write no contract at all and leave ownership unresolved (disorder). Then, in case they fail to agree on how to divide up the company's surplus they should resolve ownership

[2]This player can fire the other player which gives him the de facto control right over the asset, even though the asset may be owned by somebody different, such as venture capitalists or a university.

by a contest. For example, both engineers lobby the venture capitalists to fire their opponent, or both hire a lawyer and resolve ownership by seeking the courts, or by another means of conflict resolution assuming that conflict resolution has the properties of a contest. Thus, if some venture capitalists finance the start up company they should insist that neither player is the company's boss with the discretionary power to fire the other team member (thus no player has de facto control rights over the asset). To my knowledge, there is no paper that studies the role of contests in a setting of incomplete contracts.[3]

The remainder of the paper is organized as follows: Section 2 gives a short summary of Hart's (1995) model of optimal ownership structure. Section 3 presents the main result. It analyzes the impact of contested ownership on the levels of transaction-specific investments assuming a perfectly discriminatory contest technology (all-pay auction). Section 4 analyzes how the main result is affected when changing the setup of the basic model: First, it studies the impact of an imperfectly discriminatory contest technology. Then, it discusses the role of the timing of the contest, the assumption that transaction-specific investments are in the form of human and not physical capital, and finally the difference between contested and joint ownership. This section relates the paper to the existing literature. Finally, some further comments conclude the paper.

2 Hart's Model of Optimal Ownership Structure

In this section I will give a short summary of Hart's model of optimal ownership structure developed in his 1995 book. There is a team of two players, labelled i and j, each supplying labor and skills to a production process. Both players are risk-neutral. There is a physical asset such as a machine, labelled A, which makes both team members more productive if they have access to this asset. Contrary to Hart (1995), I assume that there is only one asset.[4] Assume that worker j produces an input which is needed by i to produce the team's output. For example, the two players are both engineers

[3] See Section 4 for the discussion of the literature that is related to this paper.

[4] Having more than one asset will only strengthen the result of the paper because more assets increase the value of the contested object.

developing some high tech device, where j may design the chip for this device, while i is responsible for the engineering of the device using j's chip. The players work jointly on some high tech equipment when doing their job. Both need to invest in their skills which are partially specific to the skills of the other player and to the equipment they use. There are two periods. In period $t = 0$, both players simultaneously choose a transaction-specific investment in skills, e_i and e_j respectively. The players' skill technology has the following properties: Player i's investment e_i increases the revenue, $R(e_i)$, for the team output and player j's investment e_j decreases the cost of producing the team input, $C(e_j)$. It is assumed that $R(0) = R^0$, $R' > 0$, $R'' < 0$, $C(0) = C^0$, $C' < 0$, and $C'' > 0$, where R^0 is small, and C^0 is large. Assume that the marginal cost for both kinds of investments e_i and e_j is constant and equals one. If for some reason, the team fails to come to an agreement on how to divide the team surplus, then there is a market substitute for the traded input in the team. This substitute is exchanged at a price p. That is, player i will demand and player j will supply this input on the market. However, because of transaction specificity market exchange is only an imperfect substitute for team exchange. Similar to Hart (1995), transaction specificity which consists of *person*- and *asset* specificity leads to the following two assumptions regarding the skill technology R and C:

Assumption 1 (Person specificity)**.**

$$
\begin{aligned}
R^{Aj}(e_i) &> R^{A}(e_i) \quad \textit{for all } e_i > 0,\\
R^{Aj\prime}(e_i) &> R^{A\prime}(e_i) \quad \textit{for all } e_i > 0,\\
C^{Ai}(e_j) &< C^{A}(e_j) \quad \textit{for all } e_j > 0,\\
|C^{Ai\prime}(e_j)| &> |C^{A\prime}(e_j)| \quad \textit{for all } e_j > 0,
\end{aligned}
$$

and

Assumption 2 (Asset specificity)**.**

$$
\begin{aligned}
R^{A}(e_i) &> R^{0}(e_i) \quad \textit{for all } e_i > 0,\\
R^{A\prime}(e_i) &> R^{0\prime}(e_i) \quad \textit{for all } e_i > 0,\\
C^{A}(e_j) &< C^{0}(e_j) \quad \textit{for all } e_j > 0,\\
|C^{A\prime}(e_j)| &> |C^{0\prime}(e_j)| \quad \textit{for all } e_j > 0.
\end{aligned}
$$

Superscripts denote a player's access to resources. For example, $R^{Aj}(e_i)$ is the revenue given i's investment e_i in case i has access to the asset A and

j's input. Or $R^0(e_i)$ is i's revenue given his or her investment when he or she has no access to the asset nor to player j's input. The *ex post* surplus equals $R^{Aj} - C^{Ai}$ and it is assumed that there is a range of e_i and e_j for which this surplus is positive. This implies that trading in the team is efficient since the team surplus is strictly larger than $R^A - C^0$ or $R^0 - C^A$ – the ex post surplus obtained when trading in the team fails. Total welfare or the ex ante surplus equals $W = R^{Aj} - C^{Ai} - (e_i + e_j)$. Assumptions 1 and 2 imply that person- and asset specificity not only holds for total values of revenue and cost, but also for the marginal ones. It implies that total and marginal values are positively correlated.[5]

Assumption 2 is slightly stronger than in Hart (1995) since the assumption here uses strict inequalities while Hart uses weak inequalities. That is, Hart allows for the possibility that a technology exhibits no asset specificity (if $R^A = R^0$). However, it is quite reasonable to assume that investments are asset-specific. For example, when discussing the empirical relevance of the model, Moore and Hart (1986, p. 1122) emphasize asset specificity over person specificity as shown in the following quotation: "... some skill or productivity acquisition is asset-specific ... as well as possibly person-specific (the asset specificity may come from the fact that the assets have special characteristics or that workers have sunk costs to locate near them)." Note also that in the empirical examples Hart (1995) discusses, asset specificity receives substantial emphasis as exemplified by the General Motors-Fisher body case. I emphasize the importance of asset specificity because it is crucial for the result of this paper. As can be seen below, a contest enhances the players incentive to make transaction-specific investments only if investments are partially asset-specific.

Finally, it is assumed that

Assumption 3. *R^k, C^k, e_i and e_j, $\forall k \in \{Ai, Aj, A, 0\}$[6] are observable to both players, but are not verifiable by outsiders.*

Assumption 3 implies that no enforceable contract containing these variable can be written. However, the players can write an incomplete contract

[5] A positive correlation between total and marginal values is assumed in Hart and Moore (1990) and Hart (1995), but not in Grossman and Hart (1986) which explains why in the Grossman-Hart model over-investment is possible.

[6] I abuse notation slightly because $k = Aj$ and $k = Ai$ refers only to R and C respectively.

which may give one or the other player the discretionary power to make decisions what ever the circumstances (e.g. ownership).

2.1 First-best Investment Levels

Note that because of transaction specificity it is always optimal for a team to stick together. Thus, in equilibrium player i, j, and the asset A will *always* form a production entity. If players could write an enforceable contract specifying the investment levels, investment levels would be specified as to maximize total welfare W. The first-order conditions determining the first-best investment effort, e_i^* and e_j^*, is given by:

$$R^{Aj'}(e_i) = 1, \text{ and} \tag{1}$$

$$-C^{Ai'}(e_j) = 1. \tag{2}$$

However, because contracts are incomplete, the first-best cannot be achieved. The key insight provided by GHM is that ownership of the asset matters because it affects the investment levels of either player. Here, I focus on three possible ex ante ownership distributions: Either player i or player j owns the asset. One can think of this situation as of one player being the company's boss with the discretionary power to fire the other player. That is, one company member has de-facto control rights over the asset. In the example introduced earlier, some venture capitalist with an interest that the company succeeds in the long run may impose some organizational structure (ownership) in order to maximize the probability of survival of the company. The third form is random ownership, where ownership is determined randomly once investments have been realized. This last form of ownership is mostly used for technical reasons since it allows me to analyze any convex combination of i-ownership and j-ownership. A common property of all three forms of ex ante ownership is that they are exogenously enforced by a third party.

2.2 Player i Ownership

Consider the situation, where i owns the asset A. Assuming the 50:50 Nash bargaining solution, the first-order conditions in this case become:[7]

$$\frac{1}{2}\left(R^{Aj'}(e_i) + R^{A'}(e_i)\right) = 1, \text{ and} \tag{3}$$

[7]See Hart (1995, p. 39) for details of how these conditions are derived.

$$-\frac{1}{2}\left(C^{Ai'}(e_j) + C^{0'}(e_j)\right) = 1. \tag{4}$$

From Assumptions 1 and 2 and the fact that $R''(e_i) < 0$ it follows that investment effort $e_i^{i-\text{ownership}}$ with i-ownership is smaller than first-best investment level e_i^*. The same holds for j's optimal effort, $e_j^{i-\text{ownership}}$, under i-ownership.

2.3 Player j Ownership

If player j owns the asset A then the first-order conditions become:

$$\frac{1}{2}\left(R^{Aj'}(e_i) + R^{0'}(e_i)\right) = 1, \text{ and} \tag{5}$$

$$-\frac{1}{2}\left(C^{Ai'}(e_j) + C^{A'}(e_j)\right) = 1. \tag{6}$$

Again, from Assumptions 1 and 2, and the fact that $R''(e_i) < 0$ it follows that investment effort $e_i^{j-\text{ownership}}$ with j-ownership is smaller than with i-ownership and is smaller than the first-best investment level, e_i^*. In contrast, the investment effort $e_j^{j-\text{ownership}}$ with j-ownership is larger than with i-ownership. This trade-off highlights the *costs and benefits of ownership*. Hart (1995) argues that the ownership structure should prevail for which total welfare created in the team is higher. Thus, if

$$W(e_i^{i-\text{ownership}}, e_j^{i-\text{ownership}}) \geq W(e_i^{j-\text{ownership}}, e_j^{j-\text{ownership}}),$$

then i should own the control rights over the asset. If this condition is not satisfied, player j should own the asset.[8]

2.4 Random Ownership

A third form of ownership is random ownership. As Hart (1995, p. 86) notes, random ownership can increase total welfare if it acts as a smoothing device between i- and j-ownership. With random ownership, player i and j could

[8]The property rights approach following GHM does not present a mechanism which shows that the optimal property rights structure will actually prevail. One may think of an evolutionary mechanism where a less efficient property rights structure dies out. Or one can think of a conflict model – as done in the previous section – where one property rights structure emerges as the equilibrium outcome.

write a contract that ownership will be determined randomly in case they fail to agree on how to divide up the company's surplus. Assume that random ownership – as any of the other ownership structures – is enforced by a third party.

Let ζ be the probability that the random device assigns ownership to player i, and let $1-\zeta$ be the probability that the random device assigns ownership to player j. Then, the first-order conditions become:

$$\frac{1}{2}\left(R^{Aj'}(e_i)+\zeta R^{A'}(e_i)+(1-\zeta)R^{0'}(e_i)\right)=1, \text{ and} \tag{7}$$

$$-\frac{1}{2}\left(C^{Ai'}(e_j)+(1-\zeta)C^{A'}(e_j)+\zeta C^{0'}(e_j)\right)=1. \tag{8}$$

Let ζ^* be the ex ante ownership distribution that maximizes welfare W. Note that if $\zeta^*=1$ then i-ownership is optimal, and if $\zeta^*=0$ then j-ownership is optimal.

3 Optimal Skill Investments with Contested Ownership

The key insight provided in this paper is that there is no need to ex ante specify ownership of an asset. Person- and asset-specificity imply that in equilibrium players, their skills, and the asset form *one* production unit. It follows naturally that ownership may become an issue only if things start to go wrong – as exemplified by off-the equilibrium path scenarios of what happens if players fail to reach a bargaining agreement. Transaction specificity has the consequence that there is no need to assign ownership *from a technological point of view* (see Section 4.2 for a further discussion of this issue). One such scenario is that ownership is decided ex post, and parties seek the courts, or another form of conflict resolution. I will assume that conflict resolution involves costly resources (time and money), whereby each parties success of winning the conflict will depend not only on the own resources spent but also on the resources spent by the opponent. Then, assigning ownership has the characteristics of a contest which happens only after transaction-specific investments have been incurred.

3.1 The Basic Model

Consider the following contest. Player i's net benefit of keeping the asset as compared to separating without the asset is $V_i = R^A(e_i) - R^0(e_i)$, while j's net benefit equals $V_j = C^0(e_j) - C^A(e_j)$. Note that $V_i(0) = V_j(0) = 0$, and by Assumption 2, $V_i(e_i), V_j(e_j) > 0 \ \forall e_i, e_j > 0$. Thus, the value of the contested object for player i and j is V_i and V_j respectively.

Assume some contest technology, $\mu(f_i, f_j)$, where μ denotes player i's probability of winning the contest which depends on the contest efforts f_i and f_j of player i and j respectively. Player j's probability of success equals $1-\mu(f_i, f_j)$. The player who wins the contest, owns the asset A. Assume that the marginal cost of contest effort equals one. This contest situation is non-standard because the value of the contested object V_i and V_j is endogenous.[9] The value of owning the asset depends on each player's transaction-specific investment in human capital. Denote by e_i^{**} and e_j^{**} i's and j's optimal investment level if ownership is contested. Now it becomes clear that without asset specificity (Assumption 2), $V_i = V_j = 0$ which means that a contest cannot affect the players' level of transaction-specific investments.

The setup of the game is identical to the setup described in Hart (1995) with the only difference that the status quo points or disagreement points of the Nash bargaining solution are determined by the contest just described. Thus, the timing of the game is as follows: At $t = 0$, players simultaneously choose there respective investment levels e_i^{**} and e_j^{**}. At $t = 1$, the surplus created in the team is divided up according to the Nash bargaining solution, where the players disagreement points are determined by the contest characterized by $(f^*, \mu(f^*))$.

With a contest, player i's and player j's ex post payoff is as follows when using the 50:50 Nash bargaining solution to divide up the team's surplus:

$$\pi_i = \mu(R^A - p) + (1-\mu)(R^0 - p) - f_i^* + 1/2S, \quad \text{and} \tag{9}$$

$$\pi_j = \mu(p - C^0) + (1-\mu)(p - C^A) - f_j^* + 1/2S, \tag{10}$$

where p denotes the price of the market substitute, and S denotes the net ex post surplus (net of each players' value of the disagreement point) created in the team which equals

$$S = R^{Aj} - C^{Ai} - \mu(R^A - C^0) - (1-\mu)(R^0 - C^A) + f_i^* + f_j^*.$$

[9]Depending on the skill technology, players' equilibrium valuation of the contested object may be different. For an analysis of contests with asymmetric valuations see Nti (1999).

Note that $\pi_i + \pi_j = R^{Aj} - C^{Ai}$. Furthermore, note that f_i^* and f_j^* are part of the surplus S which implies that the contest has a cost to each player – even though a contest never happens in equilibrium – which comes from the fact that it reduces each players disagreement payoff and increases S which is shared equally. Differentiating (9) with respect to e_i yields the first-order condition for player i.

$$\frac{1}{2}[R^{Aj'} + \mu R^{A'} + (1-\mu)R^{0'} + \left(\frac{\partial \mu}{\partial V_i}(V_i + V_j) + \frac{\partial f_j^*}{\partial V_i} - \frac{\partial f_i^*}{\partial V_i}\right) V_i'] = 1. \quad (11)$$

Differentiating (10) with respect to e_j yields the first-order condition in case of contested ownership for player j.

$$-\frac{1}{2}\left[C^{Ai'} + (1-\mu)C^{A'} + \mu C^{0'} + \left(\frac{\partial \mu}{\partial V_j}(V_i + V_j) + \frac{\partial f_j}{\partial V_j} - \frac{\partial f_i}{\partial V_j}\right) V_j'\right] = 1. \quad (12)$$

The last term of (11) and (12) comes from the fact that the probability of success, $\mu(V_i(e_i), V_j(e_j))$, is a strategic variable and depends on e_i. For example, if ownership were to be decided on a purely random manner then $\partial\mu/\partial e_i = \partial f_i^*/\partial e_i = \partial f_j^i/\partial e_i = 0$, and $\mu = \zeta$ with $\zeta = 1$ when i owns the asset or $\zeta = 0$ if j owns the asset. In these cases, the first-order conditions of the previous section are obtained. The magnitude and behavior of the last term in (11) and (12) will depend on the specific form of the contest technology $\mu(f_i, f_j)$. The first result of the paper uses a contest technology which is *perfectly discriminatory.* This form of contest is well studied, and it is also called an "all-pay auction." (Hillman and Riley 1989; Baye, Kovenock and de Vries 1996)

Definition 1. *The contest technology $\mu(f_i, f_j)$ is perfectly discriminatory if and only if*

$$\mu(f_i, f_j) = \begin{cases} 1 & \text{if } f_i > f_j \\ 0 & \text{if } f_i < f_j \\ \frac{1}{2} & \text{if } f_i = f_j. \end{cases}$$

The first result of the paper is stated in the following proposition:

Proposition 1. *Under assumptions 1 - 3, and if ownership is decided by a perfectly discriminatory contest then the game has a unique Nash equilibrium with the property that*

$$W(e_i^*, e_j^*) > W(e_i^{**} = e_i^{i-ownership}, e_j^{**} = e_j^{j-ownership}) > W(e_i^{\zeta}, e_j^{1-\zeta}),$$

$\forall \zeta \in [0, 1]$.

This result is quite remarkable because it says that if ownership is decided ex post by a perfectly discriminating contest then *each* player behaves as if he or she owns the asset. Thus, contested ownership removes the trade-off associated with an ex ante ownership structure as pointed out by GHM since both invest as if they own.

Proof. First, note that a perfectly discriminatory contest has no equilibrium in pure strategies. In order to see this assume that $V_i > V_j$. Then i could spend an effort of $V_j + \epsilon$ and win the contest for sure. But given this strategy, j's best-response is to bid zero. But given j's response, i's best-response is to reduce his or her effort to ϵ, in which case j's best-response is to change the effort to just above ϵ, which then in turn ... an so on and forth.

However, this contest has an *unique* equilibrium in mixed strategies which is characterized by the following proposition which is proven in Hillman and Riley (1989, p. 24):

Proposition 2 (Proposition 2, Hillman & Riley (1989))**.** *With perfect discrimination and two agents whose gross valuations are v_1 and v_2 ($v_2 \leq v_1$), agent 1 always enters the contest while agent 2 enters with probability v_2/v_1. Conditional upon entry each agent spends according to a uniform mixed strategy over the interval $[0, v_2]$.*

Applying this result to the context here, player i's and j's expected contest effort equals

$$Ef_i^* = \begin{cases} \frac{V_j}{2} & \text{if } V_i \geq V_j \\ \frac{V_i^2}{2V_j} & \text{if } V_i < V_j. \end{cases}$$

and

$$Ef_j^* = \begin{cases} \frac{V_i}{2} & \text{if } V_j \geq V_i \\ \frac{V_j^2}{2V_i} & \text{if } V_j < V_i. \end{cases}$$

Note that if player i has a higher valuation than player j, his or her average contest effort does not depend on his or her own valuation. Given the equilibrium strategy defined in Proposition 2, player i's expected probability of winning the contest equals

$$E\mu = \begin{cases} 1 - \frac{V_j}{2V_i} & \text{if } V_i \geq V_j \\ \frac{V_i}{2V_j} & \text{if } V_i < V_j. \end{cases}$$

Given this contest technology, the last term of i's first-order condition (11) reduces to

$$\left(\frac{V_j}{V_i}\frac{(V_i+V_j)}{2V_i}-\frac{V_j^2}{2V_i^2}+0\right)V_i' = (1-\mu)V_i' \quad \text{if } V_i > V_j, \text{ and} \tag{13}$$

$$\left(\frac{V_i+V_j}{2V_j}+\frac{1}{2}-\frac{V_i}{V_j}\right)V_i' = (1-\mu)V_i' \quad \text{if } V_i \leq V_j. \tag{14}$$

This implies that player i's first-order condition (11) reduces to (3), the first-order condition of player i obtained under i-ownership. Thus, $e_i^{**} = e_i^{i-\text{ownership}}$.

Similarly for player j, the first-order condition (12) reduces to (6), the first-order condition of player j obtained under j-ownership, implying that $e_j^{**} = e_j^{j-\text{ownership}}$. Thus, player j also invests in his or her skills as if he or she owns the asset A.

Since each players' transaction-specific investment level is equal or larger as compared to any ex ante ownership structure but strictly below the first-best investment levels e_i^* and e_j^*, $W(e_i^*, e_j^*) > W(e_i^{**} = e_i^{i-\text{ownership}}, e_j^{**} = e_j^{j-\text{ownership}}) > W(e_i^{\zeta}, e_j^{1-\zeta})$, $\forall \zeta \in [0,1]$ as claimed. $\square$

The economic intuition for this result is that increasing V_i produces a benefit since it increases the expected probability of winning the contest. If $V_i > V_j$, to increase V_i has no other cost than the indirect cost coming from the fact that increasing V_i decreases j's expected contest effort which reduces the ex post net surplus, S, which is divided equally. If $V_i < V_j$ then increasing V_i has a direct cost which comes from the fact that it increases i's expected contest effort which increases S. However, in this case there is an additional benefit associated with increasing V_i which comes from the fact that increasing V_i increases j expected contest effort which increases S. This discussion shows that contested ownership provides strong incentives to incur transaction-specific investments due the fact that benefits dominate the costs associated with these investments.

With a perfectly discriminatory contest technology *each* player behaves as if he or she owns the asset. The implication of this result is that if conflict resolution has the property of a perfectly discriminatory contest then players should *not* write a contract which distributes ownership of the asset among the two players in period $t = 0$. This result is contrary to the recommendation made by GHM. The next section analyzes whether this recommendation can be maintained if, for example, players use an *imperfectly discriminatory* contest technology.

4 Robustness of the Basic Model

In this section I analyze how the main result in Section 3 is affected by changes of the exact setup of of the basic model. First, I analyze the optimal investment levels when the contest technology is *imperfectly* discriminatory. Second, I analyze the role of the timing of the contest. Third, I discuss the relevance of the assumption that investments consist in human capital investments rather than physical capital investments for the contest outcome. And finally, I discuss briefly the difference between contested and joint ownership.

4.1 Contest Technology and Optimal Investment Levels

One important economic variable which characterizes different types of contests is given by the "effectiveness" of a contest technology (Skaperdas 1992). Assume that the difference between the players' optimal contest effort is $\epsilon > 0$ but small. With a perfectly discriminatory contest technology, the player with the slightly lower contest effort looses the contest with probability one. In this sense, the contest technology is highly effective. On the other hand, with a less effective contest technology, the player with a slightly lower contest effort still has a positive probability of winning the contest. Thus, the winning probability is not entirely determined by contest efforts but has also a random component. In this case, the contest technology is imperfectly discriminatory.

One way to model the different degrees of effectiveness is to use the Tullock contest success function:

$$\mu(f_i, f_j) = \frac{f_i^r}{f_i^r + f_j^r},$$

where $r \geq 0$. The effectiveness of the contest success function increases in r. For example, if $r = 0$, then the contest success is purely random. Players' contest efforts have no impact on the outcome. In contrast, if $r = \infty$, then a small difference in contest efforts makes the players with the higher effort the winner with probability one. In this case, the contest technology is perfectly discriminatory as in Section 3.

Consider an imperfectly discriminating contest technology, where $r < \infty$. This contest technology is only well understood to the point where the contest has a unique equilibrium in pure strategies.[10] For example, if $V_i = V_j$, then the contest has a unique pure strategy Nash equilibrium if and only if $r \leq 2$. If the players' valuations differ then r needs to be smaller to assure an equilibrium in pure strategies. If $r \leq 1$ then the contest has a unique pure strategy equilibrium no matter the relative value of V_i and V_j. To simplify the analysis, I assume that $r = 1$. In Appendix A, I solve the model without this simplifying assumption.

Lemma 1. *Under assumptions 1 - 3 and if ownership is decided with an imperfectly discriminating contest technology with $r = 1$ then $e_i^{**} \in [\underline{e}_i, \bar{e}_i)$ and $e_j^{**} \in [\underline{e}_j, \bar{e}_j)$. $\underline{e}_i$ and $\bar{e}_i$ are implicitly defined by $\frac{1}{2}(R^{Aj'} + \frac{3}{4}R^{A'} + \frac{1}{4}R^{0'}) = 1$ and $\frac{1}{2}(R^{Aj'} + 2R^{A'} - 1R^{0'}) = 1$ respectively. $\underline{e}_j$ and $\bar{e}_j$ are implicitly defined by $-\frac{1}{2}(C^{Ai'} + \frac{3}{4}C^{A'} + \frac{1}{4}C^{0'}) = 1$ and $-\frac{1}{2}(C^{Ai'} + 2C^{A'} - 1C^{0'}) = 1$ respectively.*

Proof. The game is solved backwards. Given the investment efforts e_i^{**} and e_j^{**} player $k \in \{i, j\}$ chooses his or her contest effort f_k to maximize

$$\max_{f_k} \left(\frac{f_k}{f_i + f_j} V_k - f_k \right) \qquad \text{s.t.} \quad \left(\frac{f_k}{f_i + f_j} V_k - f_k \right) \geq 0.$$

This problem has a closed form solution which equals $f_i^* = \frac{Vi^2 V_j}{(V_i+V_j)^2}$ and $f_j^* = \frac{Vj^2 V_i}{(V_i+V_j)^2}$, and player i's probability of successfully winning ownership equals $\mu = \frac{V_i}{V_i+V_j}$. It follows that the probability of success and the optimal contest efforts depend on e_i and e_j.

The contest outcome affects the first-order conditions (11) and (12). Using the solution to the contest to calculate the partial derivatives of the last term in (11) and (12) and simplifying yields player i's first-order condition

$$\frac{1}{2}\left[R^{Aj'} + R^{0'} + X_i(R^{A'} - R^{0'})\right] = 1, \tag{15}$$

where $X_i = \frac{2+\theta^2(3+\theta)}{(1+\theta)^3}$ and $\theta = V_i/V_j$. Player j's first-order condition is now

$$-\frac{1}{2}\left[C^{Ai'} + C^{A'} + X_j(C^{0'} - C^{A'})\right] = 1, \tag{16}$$

[10] See Baye, Kovenock, and De Vries (1994) for an analysis of a contest for which $r > 2$. They are able to specify a symmetric mixed strategy equilibrium only for the case where the strategy space is discrete.

where $X_j = \frac{(3-\theta)\theta^2}{(1+\theta)^3}$. Note that conditions (15) and (16) both depend on e_i and e_j.

The game is solved by finding a Nash equilibrium where the equilibrium investment levels e_i^{**} and e_j^{**} solve $e_i^{**} = B_i(e_j^{**})$ and $e_j^{**} = B_j(e_i^{**})$. B_i is player i's and B_j is player j's best-response function. They are implicitly defined by the first-order conditions (15) and (16).

4.1.1 i's Best-Response Function

Let $\tilde{e}_i(e_j)$ a function that assigns to each value of e_j a value of e_i such that $\theta = 1$. Let $\tilde{e}_j(e_i) = \tilde{e}_i^{-}(e_i)$. Using the implicit function theorem, $\partial\tilde{e}_i/\partial e_j = \frac{V_i V_j'}{V_j V_i'} > 0$. Note that if skill technologies are identical for the two players, then $\partial\tilde{e}_i/\partial e_j = 1$ (see Figure 1). $\tilde{e}_i(0) = 0$ since $V_i(0) = V_j(0) = 0$.

Note that $\lim_{\theta\to 0} X_i = 2$, $\lim_{\theta\to 1} X_i = 3/4$, and $\lim_{\theta\to\infty} X_i = 1$. X_i has its global minimum at $\theta = 1$. Thus, $\partial X_i/\partial\theta = \frac{6(\theta-1)}{(1+\theta)^4} \lesseqqgtr 0$ if $\theta \lesseqqgtr 1$. Denote by $\underline{e}_i$ i's optimal investment level if $\theta = 1$. Thus, $\underline{e}_i$ solves

$$\frac{1}{2}[R^{Aj'} + \frac{3}{4}R^{A'} + \frac{1}{4}R'] = 1. \tag{17}$$

Denote by $\bar{e}_i$ i's optimal investment level if θ approaches zero. Thus, $\bar{e}_i$ solves

$$\frac{1}{2}[R^{Aj'} + 2R^{A'} - 1R'] = 1. \tag{18}$$

Finally note that $B_i = e_i^{i\text{-ownership}}$, if θ approaches infinity. It follows that $\underline{e}_i < e_i^{i-\text{ownership}} < \bar{e}_i$.

Now I claim that $B_i(e_j) \in [\underline{e}_i, \bar{e}_i)$, $\forall e_j \geq 0$. Using the implicit function theorem, $\partial B_i/\partial e_j = \frac{\lambda V_i V_j'}{\varphi + \lambda V_j V_i'}$, where $\lambda = 6V_j(V_i - V_j)V_i'$ and $\varphi = (V_i + V_j)(3V_iV_j^2(R^{0''} + R^{Aj''}) + V_i^3(R^{Aj''} + R^{A''}) + 3V_i^2V_j(R^{Aj''} + R^{A''}) + V_j^3(R^{Aj''} + 2R^{A''} - R^{0''})) < 0$. If $V_j = 0$, then $\partial B_i/\partial e_j = 0$ and $B_i = e_i^{i-\text{ownership}}$. If $V_j = \epsilon > 0$, where ϵ is small, then $\partial B_i/\partial e_j < 0$ since $\lambda > 0$ and $|\varphi| > \lambda$. If $V_j = V_i$ then $B_i = \underline{e}_i$ since $\theta = 1$, and $\partial B_i/\partial e_j = 0 < \partial\tilde{e}_i/\partial e_j$. This implies that for a marginal increase in e_j when $V_i = V_j$, V_j will become larger than V_i. Then, $\partial B_i/\partial e_j > 0$ and it is always the case that $\partial B_i/\partial e_j < \partial\tilde{e}_i/\partial e_j$ as e_j grows since $\lambda < 0$. This implies that θ goes toward zero and B_i approaches $\bar{e}_i$. Thus, $B_i \in [\underline{e}_i, \bar{e}_i)$ as claimed.

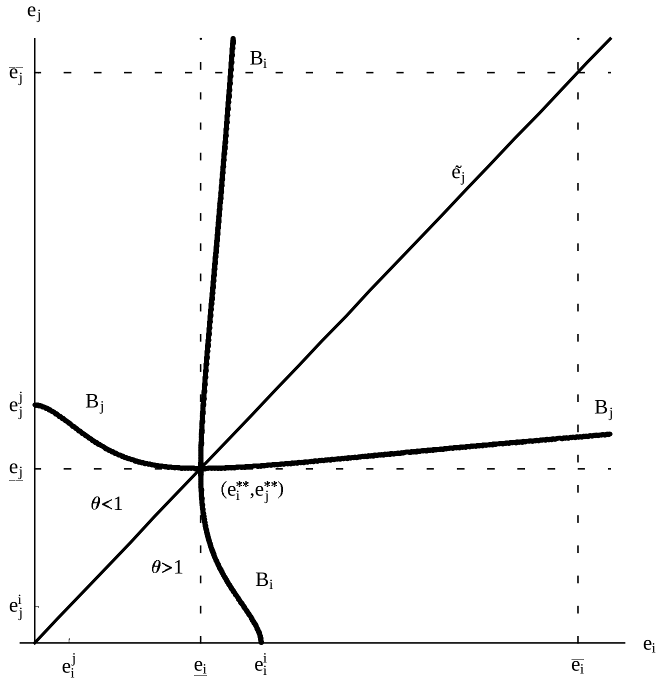

Figure 1: Best-Response Functions — Identical Players

4.1.2 j's Best-Response Function

Player j's best-response function can be derived in exactly the same way than i's best-response function. Again note that $\lim_{\theta\to 0} X_j = 0$, $\lim_{\theta\to 1} X_j = 1/4$, and $\lim_{\theta\to\infty} X_j = -1$. X_j has a global maximum at $\theta = 1$. Thus, $\partial X_j/\partial\theta = \frac{6(1-\theta)\theta}{(1+\theta)^4} \lesseqqgtr 0$ if $\theta \gtreqqless 1$. By the same reasoning as above it can be shown that $B_j \in [\underline{e}_j, \bar{e}_j)$, where $\underline{e}_j$ and $\bar{e}_i$ are implicitly defined by

$$-\frac{1}{2}[C^{Ai'} + \frac{3}{4}C^{A'} + \frac{1}{4}C'] = 1, \text{ and} \tag{19}$$

$$-\frac{1}{2}[C^{Ai'} + 2C^{A'} - 1C'] = 1. \tag{20}$$

As with B_i, B_j will cross $\tilde{e}_j(e_i)$ only once. B_j slopes downward till this crossing point, and then slopes upwards approaching $\bar{e}_j$.

Note that with $\theta = 1$ investment incentives due to the contest are weakest. Both, e_i and e_j increase as θ becomes larger or smaller than one.

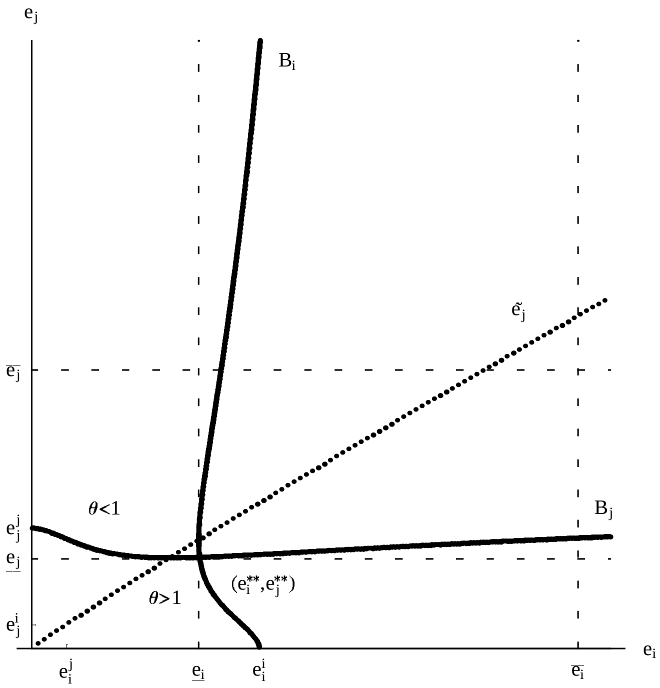

Figure 2: Best-Response Functions — Non-identical Players

4.1.3 Equilibrium

The best-response functions B_i and B_j will have to intersect in the rectangle $(\underline{e}_i, \underline{e}_j), (\bar{e}_i, \underline{e}_j), (\bar{e}_i, \bar{e}_j), (\underline{e}_i, \bar{e}_j))$. They will intersect only once because $\partial B_i^- / \partial e_i$ and $\partial B_j / \partial e_i$ have opposite signs except in one case. If in the equilibrium $\theta = 1$, then B_i^- and B_j touch each other at $(\underline{e}_i, \underline{e}_j)$, and their slopes have the same sign. However, B_j approaches the point $(\underline{e}_i, \underline{e}_j)$ from above, while B_i^- approaches this point from below. The best-response function cannot intersect another time because $\partial B_i^- / \partial e_i > \partial \tilde{e}_j / \partial e_i > \partial B_j / \partial e_i$ (see Figure 1 for this case, graphing the best-response functions of a numerical example.) In contrast, if in equilibrium $\theta \neq 1$ then the slopes of B_i^- and B_j have opposite signs in the rectangle $(\underline{e}_i, \underline{e}_j), (\bar{e}_i, \underline{e}_j), (\bar{e}_i, \bar{e}_j), (\underline{e}_i, \bar{e}_j))$ (see Figure 2 for this case, graphing the best-response functions for a numerical example in case of players with a non-identical skill technology.) Thus, the best-response functions cut each other exactly once in the rectangle $(\underline{e}_i, \underline{e}_j), (\bar{e}_i, \underline{e}_j), (\bar{e}_i, \bar{e}_j), (\underline{e}_i, \bar{e}_j))$.

□

Lemma 1 implies that equilibrium investment levels are at least $e_i^{**} = \underline{e}_i$ and $e_j^{**} = \underline{e}_j$. The following result relates the investment levels under contested ownership to the ones under ex ante ownership.

First, consider the case where skill technologies between player i and j are roughly identical (as depicted in Figure 1).

Definition 2. *Player i's and player j's skill technologies are approximately identical if* $|C^{k'}(e)| \approx R^{k'}(e)$, $\forall k \in \{Ai, Aj, A, 0\}, \forall e \geq 0$.

Second, consider the case where skill technologies between i and j are different. Hart (1995) considers this situation when defining the skill technology of one player as being "relatively unproductive".

Definition 3. *Player i's investment is relatively unproductive if* R^k *can be replaced by* $\phi R^k(e_i) + (1-\phi)e_i$, $\forall k \in \{Aj, A, 0\}$, *where* $\phi > 0$ *is small. An equivalent definition applies to the case when player j is relatively unproductive.*

Hart (1995, p. 45) shows in his proposition 2 that if one player is "relatively unproductive", then the other player should own all the assets. In the context here, if i's investment is relatively unproductive, player j should own the asset. However, when considering contested ownership, this recommendation has to be revised.

Proposition 3. *Assume an imperfectly discriminating conflict technology with* $r = 1$.

i) If skill technologies are approximately identical then $W(e_i^*, e_j^*) > W(e_i^{**}, e_j^{**}) > W(e_i^{\zeta}, e_j^{1-\zeta})$, $\forall \zeta \in [0, 1]$.

ii) If one player is relatively unproductive then $W(e_i^*, e_j^*) > W(e_i^{**}, e_j^{**}) > W(e_i^{\zeta}, e_j^{1-\zeta})$, $\forall \zeta \in [0, 1]$, *provided that* ϕ *is small enough.*

Proof. Statement i): The fact that players have approximately identical skill technologies implies that $e_i^{**} \approx \underline{e}_i$ and $e_j^{**} \approx \underline{e}_j$. If $\zeta^* \in [0.25, 0.75]$, then contested ownership clearly generates a surplus which is larger than the surplus generated under any ex ante ownership. Without a loss of generality assume that $\zeta^* = 1$. By comparing the first-order conditions (3) and (17) for player i, and (4) and (19) for player j, it follows that $\Delta e_i \equiv e_i^{i-\text{ownership}} - e_i^{**} > 0$ and $\Delta e_j \equiv e_j^{**} - e_j^{i-\text{ownership}} > 0$ when changing from i-ownership to contested ownership. Furthermore, $\Delta e_i < \Delta e_j$. The fact that $\Delta e_i < \Delta e_j$ implies that total welfare W must increase when going from i-ownership to contested ownership since $R^{Aj'} \approx |C^{Ai'}|$ by assumption.

Statement ii): Assume that player i's investment is relatively unproductive. Then player i's first-order condition in (15) can be rewritten as:

$$\frac{1}{2}\left[\phi R^{Aj'} + (1-\phi) + \phi R' + (1-\phi) + X_i(\phi)\phi(R^{A'} - R')\right] = 1,$$

which, simplifies to

$$\frac{1}{2}\left[R^{Aj'} + R' + X_i(\phi)(R^{A'} - R')\right] = 1.$$

Thus, player i's first-order condition is affected by ϕ through X_i only. V_i now equals $\phi(R^A - R)$. Note that $\lim_{\phi\to 0} V_i = 0$ which implies that $X_i \to 2$ and $X_j \to 0$ as ϕ goes to zero. Furthermore, $\bar{e}_i \to e_i^{i-\text{ownership}}$ as $\phi \to 0$. Thus, the first-order conditions (15) and (16) reduce to (3) and (6) respectively. Thus, players choose $e_i^{**} \approx e_i^{i-\text{ownership}}$ and $e_j^{**} \approx e_j^{j-\text{ownership}}$ provided ϕ is small enough. □

In Appendix A, I show that Proposition 3 holds for any contest technology which is imperfectly discriminatory (i.e. $r \in (0,2]$). The weaker the effectiveness of the contest technology, the broader the range of skill technologies for which contested ownership does *not* increase welfare. Thus, it will be always the case that contested ownership enhances efficiency for sufficiently asymmetric or symmetric skill technologies. If a player is relatively unproductive, or if the players' skill technology is sufficiently symmetric then again players should *not* write a contract that distributes residual control rights in period $t = 0$ also when the contest technology is imperfectly discriminatory.

Figure 1 uses the numerical example where $R^{Aj} = 2(1+e_i)^{0.9}$, $R^A = 2(1+e_i)^{0.5}$, $R^0 = 2(1+e_i)^{0.2}$, $C^{Ai} = 3 - 2(1+e_j)^{0.9}$, $C^A = 3 - 2(1+e_j)^{0.5}$, and $C^0 = 3 - 2(1+e_j)^{0.2}$. These function satisfy the assumptions regarding R and C specified earlier. In this example, the marginal values are identical for both players with produces the symmetry in Figure 1. Calculating the ex post surplus for the various forms of ex ante ownership reveals that in this case $i-$ownership or $j-$ownership both are an optimal ex ante ownership structure producing an ex post surplus of \$8.11. Contested ownership, however, produces a larger ex post surplus which equals \$10.12. This has to be the case as proven in Proposition 3 since the skill technologies of the two players are identical. Thus, in the case of approximately symmetric skill technologies the paper here complements GHM since their approach does not make a clear prediction of who should own the asset. Note that this is true no matter the contest technology (see Appendix A). In contrast, Figure 2 plots an example where players have non-identical skill technologies. The numerical example is as follows: Player i is as in Figure 1, and player j's skill technology is $C^{Ai} = 3 - 3(1+e_j)^{0.58}$, $C^A = 3 - 3(1+e_j)^{0.5}$, and $C^0 = 3 - 3(1+e_j)^{0.2}$. In this example, i-ownership is the optimal ownership structure since i-ownership produces an ex post surplus of \$8.69 whereas contested ownership produces a lower ex post surplus which equals \$8.59. As one can see in Figure 2, in equilibrium $\theta \approx 1$. As pointed out earlier, in this case the incentives coming from a contest are the weakest. Contested ownership does not increase welfare because the increase in e_j due to the change from ex ante to contested ownership fails to offset the decrease in e_i due to this change. However, if increasing the asymmetry by for example setting $\phi = 0.5$ for player j – i.e. making player j relatively unproductive – contested ownership generates with \$8.01 a larger surplus than i-ownership which produces a surplus of \$7.58. Thus, for the intermediate cases as depicted in Figure 2, considering contested ownership does not alter the conclusions of GHM. However, if the contest technology becomes sufficiently asymmetric, contested ownership becomes again an important alternative to any ex ante ownership distribution.

In order to understand the economic intuition it is helpful to analyze the last term of player i's first-order condition (11). By using the contest outcome derived earlier, $\frac{\partial\mu}{\partial V_i}(V_i + V_j)$ is positive and can be interpreted as the marginal benefit of increasing V_i. Increasing V_i increases the probability of winning. On the other hand, $-\frac{\partial f_i^*}{\partial V_i}$ is negative, which can be interpreted as the marginal cost of increasing V_i. Increasing V_i increases f_i^* which is part of the surplus divided among the two players. Note that the marginal benefit outweighs the marginal cost. This is true for both players, which can be interpreted as the commitment effect of a contest. However, there is also an asymmetry which comes from the fact that $\frac{\partial f_j^*}{\partial V_i} = \frac{V_j^2(V_j - V_i)}{(V_i+V_j)^3}$ is positive if $V_i < V_j$ and it is negative if $V_i > V_j$. Thus, the relatively unproductive player i has an extra incentive to invest in V_i because this increases the contest effort of the opponent j which benefits i since the contest effort is part of the surplus to be divided equally. Thus, under contested ownership a relatively unproductive player has strong incentives to make transaction-specific investments. Regarding the investment decision of the productive player it is not surprising that he or she invests as if the owner of the asset since this player is "de facto" the owner of the asset.

With Proposition 3 the question arises of how important is contested ownership as an alternative form of ownership structure. Clearly, if skill technologies are approximately identical, or if one player's skill technology is relatively unproductive then contested ownership produces a higher surplus. For all the other cases one has to get a sense of which contest technology comes closest to modelling situations of conflict resolution in the real world. Or in other words, how much luck is involved when courts or other authorities resolve ownership disputes. If one believes that conflict resolution is competitive and only little affected by luck then r is large. In this case, contested ownership is important for all possible cases of skill technology. On the other hand, if one believes that conflict outcomes are mainly determined by luck and only little by efforts, then contested ownership is only important in the cases of sufficiently symmetric or asymmetric skill technologies. However, note that a conflict technology with an $r = 1$ or $r = 2$ involves a large portion of luck. For example, if $r = 1$ then the player who exerts 80% of the other player's contest effort has still a chance of roughly 45% of winning the contest. If $r = 2$, then this chance reduces to roughly 40%.

4.2 Timing of the Contest

How does the main result change if contest efforts have to be incurred before players start to bargain. In the setting here, players have not actually to incur these efforts, but they have to be able to make credible threats of incurring them if needed. In a related paper, Anbarci, Skaperdas, and Syropoulos (2001) compare various bargaining solutions, where bargaining – as in the context of the paper here – takes place in the *shadow of conflict*. Their result differs to the one presented here: In their paper the shadow of conflict always creates an inefficiency. The reason is that in their setup, players have to make an irreversible investment in conflict efforts (guns) before they start to bargain. These investments increase a player's disagreement payoff, but reduce the output (butter) a player can produce. Players fight for or bargain over a distribution of land, and then produce on the obtained land. They find that bargaining solutions that put less weight on the disagreement point Pareto-dominate other solutions. Similar to the paper here, conflict does not occur in equilibrium, but it is still optimal for players to invest in guns. This discussion shows that if contest efforts have to be incurred before bargaining takes place then the results presented here may change.

In the model here, a potential conflict does not involve such irreversible and wasteful investments for a particular reason. Anbarci et al.'s (2001) context is different: In their setting, it is a *technological requirement* that (contested) land is divided up before production can take place. Thus, players divide up the asset (by a contest or by bargaining), and then produce. In the context studied here, the distribution of ownership over the asset is *not* a technological requirement. On the contrary, transaction specificity produces the requirement that both players must have access to the asset. In equilibrium, the players, their skills, and the asset must always form one production entity. Thus, the situation is reversed: First, players produce together and then they bargain (over the surplus, and not over ownership of the asset). It, therefore, is not necessary that contest efforts have to be incurred before bargaining takes place. Critical, however, is that players can make credible threats to incur those efforts if needed. This discussion, however, shows that if the setting were changed so that contest efforts would have to be incurred in period $t = 0$, then the conclusions of this paper may change.

4.3 Human Capital vs. Physical Capital Investments

The assumption that transaction-specific investments are made in skills and human capital is a crucial assumption in the GHM framework.[11] This property affects the contest outcome in this paper. In another related paper, Konrad (2002) analyzes a situation in which one player (the incumbent) decides of how much to invest in a productive process, when the output generated by this process is contested by another player. Konrad analyzes the impact of different forms of asymmetries in the contest technology on this investment decision. Unlike here, in his setup a higher investment necessarily increases the rival's contest effort, since it increases the value of the contested object. Investment disincentives are small (large) if the incumbent has a large (small) contest advantage. This setup differs to the setup analyzed here because in Konrad's model the investment increases the value of the contested prize for *both* players. Here, in contrast, a player's asset-specific investment does only affect the valuation of this same player. The investment consists of an investment in human skills which is not appropriable by others by the very nature of this investment. This means that an extra investment may increase or decrease the rivals's contest effort, depending on whether his or her valuation is higher than this player's valuation. This property explains why the

[11] See the discussion in Hart (1995, p.68), Hart and Moore (1990, Appendix B).

incentives to make transaction-specific investments increase with an increasing asymmetry in the players' skill technology. The result ii) in Proposition 3 hinges critically on the fact that investments are done in human capital rather than physical capital.

4.4 Joint Ownership, Public Goods, and Contested Ownership

Contested ownership is different from *joint* ownership. Joint ownership means that player i and j own the asset together as an *ex ante agreement* that access to the asset is possible only when *both* agree with it. According to Hart (1995,p. 48), the implication of this agreement is "... that if negotiations break down neither [player] has access to [the asset] independently (since any asset usage must be agreed by both)." The value of the bargaining disagreement points in this case is determined by neither player having access to the asset leading to the investment levels $e_i^{j-\text{ownership}}$ and $e_j^{i-\text{ownership}}$. Given this scenario, joint ownership is never optimal. As Hart (1995, p. 48, footnote 23) notes, this argument assumes that an asset cannot be used by two people independently. This is a critical assumption which explains that simply declaring the asset A as a *public good* does not enhance total welfare. However, if an asset can be used independently by each player, then declaring the asset as a public good will generate the same level of welfare than contested ownership where the contest technology is perfectly discriminatory. In case negotiations break down, both players have continued access to the asset which means that the disagreement points equal $R^A - p$ and $p - C^A$ for player i and j respectively. Then, each player invests as if he or she owns the asset. This argument, however, does not consider other inefficiencies that may exist as a result of A being a public good. If neither player owns the asset, players may be negligent when using the asset so that its value depreciates faster than when the asset is privately owned. Clearly, contested ownership will not produce this inefficiency since both players may end up being the owner of the asset. In this sense, contested ownership is like private ex ante ownership.

Contested ownership, obviously, is not the same as joint ownership because it will be never the case that an asset cannot be used by either player as in joint ownership. Thus, in this sense contested ownership is equivalent to i- or j-ownership, but ownership is simply determined ex post should a contest occur.

5 Conclusions

Hart (1995,p. 86) admits when discussing his property rights model that his analysis is not complete because he derives the optimality of one property rights structure in his model relative to a limited set of possible property rights structures.[12] For example, contested ownership is not considered as a possibility. Hart, however, is confident that the main message will remain as a result of a complete analysis, namely "... that the allocation of scarce ownership rights – that is, the allocation of residual rights of control – matters when contracts are incomplete." This paper shows that this statement has to be revised when including contested ownership into the analysis. If the contest is perfectly discriminatory, *not* allocating scarce ownership rights enhances welfare, no matter the characteristics of the skill technology. If the contest is imperfectly discriminatory, then the result is not as general but *not* allocating residual control rights is still optimal in important cases such as when one player is relatively unproductive or if both players have a similar skill technology. Furthermore, the paper shows that the range of skill technologies for which contested ownership is optimal increases in the effectiveness of the contest technology.

Transaction specificity is the driving force of this result because it implies that the distribution of ownership is not a necessity *from a technological point of view*. Transaction specificity is a technological characteristic which glues the players together so that they achieve a better outcome without ex ante agreements of who owns what. It follows naturally that ownership may become an issue only when things start to go wrong. In this case, player will use courts or other means of conflict resolution. The paper shows that if conflict resolutions has the property of a contest then the distribution of ownership may not be a necessity *from an economic point of view* either.

The importance of contests in a situation of incomplete contracts with transaction-specific investments depends on how much luck is involved when resolving property rights disputes. If the outcome of such disputes depends to a large extent on the efforts exerted by the parties involved in this dispute, then contested ownership enhances welfare no matter the properties of the

[12] In this sense, the analysis in this paper is not complete either. I simply add one other form of possibly many other forms of ownership structures.

skill technology. In this case the result of this paper is general: Disorder increases efficiency.

References

Anbarci, Nejat; Skaperdas, Stergios; Syropoulos, Constantinos (2002). "Comparing Bargaining Solutions in the Shadow of Conflict: How Norms against Threats Can Have Real Effects," *Journal of Economic Theory* **106, (1)**, 1–16.

Baye, Michael R.; Kovenock, Dan; de Vries, Casper G. (1994). "The Solution to the Tullock Rent-Seeking Game when R>2: Mixed-strategy Equilibria and Mean Dissipation Rates," *Public Choice* **81**, 363–380.

Baye, Michael R.; Kovenock, Dan; de Vries, Casper G. (1996). "The All-Pay Auction with Complete Information," *Economic Theory* **8, (2)**, 291–305.

Grossman, Sanford; Hart, Oliver D. (1986). "The Cost and Benefits of Ownership: A Theory of Vertical and Leteral Integration," *Journal of Political Economy* **94**, 691–719.

Hart, Oliver D. (1995). *Companys, Contracts, and Financial Structure.* Oxford: Clarendon Press.

Hart, Oliver D.; Moore, John (1990). "Property Rights and the Nature of the Company," *Journal of Political Economy* **98,(6)**, 1119–1158.

Hillman, Arye L.; Riley, John G. (1989). "Politically Contestable Rents and Transfers," *Economics and Politics* **1, (1)**, 17–39.

Konrad, Kai A. (2002). "Investment in the Absence of Property Rights; the Role of Incumbency Advantages," *European Economic Review* **46**, 1521–1537.

Nti, Kofi O. (1999). "Rent-seeking with Asymmetric Valuations," *Public Choice* **98**, 415–430.

Skaperdas, Stergios (1992). "Cooperation, Conflict, and Power in the Absence of Property Rights," *American Economic Review* **82, (4)**, 720–739.

Appendix A

Consider a contest with $\mu(f_i, f_j) = \frac{f_i^r}{f_i^r + f_j^r}$. This contest has an equilibrium in pure strategies given the investment levels (e_i^{**}, e_j^{**}) if

$$\frac{1}{(r-1)^{1/r}} \geq \theta(e_i^{**}, e_j^{**}) \geq (r-1)^{1/r}. \tag{21}$$

I shall assume that the skill technology has the property such that a unique equilibrium in pure strategies of the contest is assured. For example, if $r = 2$ then it has to be the case that in equilibrium $\theta(e_i^{**}, e_j^{**}) = 1$.[13]

Given the investment efforts e_i^{**} and e_j^{**} player $k \in \{i, j\}$ chooses his or her contest effort f_k to maximize

$$\max_{f_k} \left(\frac{f_k^r}{f_i^r + f_j^r} V_k - f_k \right) \qquad \text{s.t.} \quad \left(\frac{f_k^r}{f_i^r + f_j^r} V_k - f_k \right) \geq 0.$$

The contest outcome has a closed form solution where $\mu = \frac{V_i(e_i)^r}{V_i(e_i)^r + V_j(e_j)^r}$, $f_i^* = \frac{rV_i^{r+1}V_j^r}{(V_i^r+V_j^r)^2}$, and $f_j = \frac{rV_j^{r+1}V_i^r}{(V_i^r+V_j^r)^2}$.

The first-order conditions for i and j are equal to (15) and (16) with

$$X_i = \frac{\theta^{r-1}(r^2(\theta-1)(\theta^r-1) + r(1+\theta^r) + (1+\theta^r)^2)}{(1+\theta^r)^3}, \text{ and}$$

$$X_j = \frac{-\theta^r(r^2(\theta-1)(\theta^r-1) + r\theta(1+\theta^r) - (1+\theta^r)^2)}{(1+\theta^r)^3}.$$

The game is solved by finding the equilibrium investment levels levels e_i^{**} and e_j^{**} that solve $e_i^{**} = B_i(e_j^{**})$ and $e_j^{**} = B_j(e_i^{**})$. B_i is player i's and B_j is player j's best-response function.

Note that if $r = 0$, then $X_i = X_j = 1/2$. Then, the first-order conditions are identical to the one with random ownership with $\zeta = 0.5$. Otherwise, the best-response function behave similarly than in the case with $r = 1$. First, $\lim_{\theta\to\infty} X_i = 1$ if $r \leq 1$, and $\lim_{\theta\to 0} X_j = 0$. Second, $\lim_{\theta\to 1} X_i = \frac{2+r}{4}$ and $\lim_{\theta\to 1} X_j = \frac{2-r}{4}$. Third, if $r < 1$ both function increase without bound because $\lim_{\theta\to 0} X_i = \infty$ and $\lim_{\theta\to\infty} X_j = -\infty$. Note that if $r > 1$, then extreme values of θ are ruled out by condition (21).

These observations imply that $B_i(0) = e_i^{i-\text{ownership}}$ and $B_j(0) = e_j^{j-\text{ownership}}$. Then, B_i and B_j slope downwards till they reach $\tilde{e}_i$ and $\tilde{e}_j$ respectively, at which point $\theta = 1$. Again denote by $\underline{e}_i$ and $\underline{e}_j$ i's and j's optimal skill level if $\theta = 1$. Thus, $\underline{e}_i$ solves

$$\frac{1}{2}[R^{Aj\prime} + \frac{2+r}{4}R^{A\prime} + \frac{2-r}{4}R'] = c, \tag{22}$$

[13]In this case B_i and B_j consist of one point only, since I am not able to solve the contest for values of e_i and e_j for which $\theta \neq 1$.

and $\underline{e}_j$ solves

$$-\frac{1}{2}[C^{Ai'} + \frac{2+r}{4}C^{A'} + \frac{2-r}{4}C'] = c. \tag{23}$$

Note that $\underline{e}_i$ and $\underline{e}_j$ increase in r.

Both best-response functions have their minimum at $\underline{e}_i$ and $\underline{e}_j$ respectively. By inspecting (22) and (23) one can see that contested ownership always generates a larger surplus than any ex ante ownership structure if skill technologies are approximately identical (see Proposition 3, Part i)). If $r = 2$, then the model has the same outcome than the one with a perfectly discriminatory contest technology. Then, each player makes an investment as if he or she owns the asset.

In case $r \leq 1$ contested ownership always generates a larger ex post surplus than any ex ante ownership structure when one player is relatively unproductive (see Proposition 3, Part ii)). For example, if i is relatively unproductive, $V_i \to 0$ as $\phi \to 0$ which implies that $X_j \to 0$ and $X_i \to \infty$. Furthermore, as $\phi \to 0$ $\bar{e}_i \to e_i^{i-\text{ownership}}$. These observation imply that each player invests as if he or she owns the asset when one player is relatively unproductive for any $r \in (0, 1]$. Note that I cannot solve the case for $r > 1$ since in this case the contest has no equilibrium in pure strategies if one player is relatively unproductive.