A Spatial Interaction Model with Heterogeneous Agents

Virginie Masson	Nicolas Rosenfeld
Department of Economics	Department of Economics
University of Pittsburgh	University of Pittsburgh

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Abstract

We develop an evolutionary model with a neighborhood structure in which two types of individuals coexist: *A-Type* individuals who prefer to coordinate on strategy A and *B-Type* individuals who prefer to coordinate on strategy B. Players meet to play a 2×2 coordination game in which the relevant payoff matrix depends on their types. The selection of a particular decision rule, either imitation or best reply, is conditional on: (i) whether the opponent is a neighbor or a stranger, and (ii) the characteristics of the information they sample. We show that the equilibrium asymptotically selected depends on the distribution of types in the population.

Keywords: Evolutionary Game Theory, Non cooperative games, Stochastic stability, Asymetric information, Networks, Heterogeneity.

1 Introduction

In most models of evolution of conventions populations are assumed to be homogeneous, that is composed by individuals sharing common preferences. However, simple observation of real life examples indicates that the existence of populations of heterogeneous agents abound. Not all individuals vote for the same candidate during an election, or not all individuals choose the same meal on a restaurant menu. Therefore, personal interests can be different, even if individuals face the same strategy space. The present paper develops an evolutionary model incorporating a heterogeneous population and studies its asymptotic properties. More specifically, we consider a population with two types of individuals: *A-Type* and *B-Type*. The payoffs matrices are such that *A-Type* individuals are encouraged to coordinate on strategy A, and *B-Type* individuals to coordinate on strategy B.

Eshel, Samuelson and Shaked [2] considers an evolutionary model where agents can be of two types: altruists or egoists. Players can choose whether or not to contribute to a public good. The benefits of the public good are local: only a certain number of closest neighbors can enjoy the public good while only the provider bears the costs. As time progresses, players can adjust their play by imitating the most successful strategy on average within their neighborhoods. The paper shows how cooperation survives in the long run if altruists are close enough to enjoy the benefits of their mutual cooperation. Myatt and Wallace [8] also considers a public good game with heterogeneous individuals. In their model, a certain number of contributors are necessary to successfully provide the public good. In each period, an individual is selected to revise his strategy. The individual observes the number of current contributors and decide to contribute only if his *contribution incentive* (the difference between valuation and cost) conditional of being pivotal is positive. By making the revising player to draw both his valuation and cost from some noisy distribution, the paper introduces heterogeneity. The population is thus composed of idiosyncratic agents. The authors study the results of the best reply dynamics when the noise parameter becomes small. Specifically, they analyze the case where the threshold public good is efficiently provided in a homogeneous population and then introduce a single 'bad apple'. The results show that the efficient provision of the public good can always be interrupted in the case of a bad apple with sufficiently low valuation or sufficiently high variance. In this paper, we consider types as exogenously given and do not allow individuals to switch between them. The intuition behind this can be explained as follows: when studying public goods, one may suggest the possibility that some individuals' good will becomes contagious. But when considering personal interests, one can find plethora of examples where an individual's choices are not influenced by others. For example, you prefer blue to pink or you prefer cartoons to science fiction movies, no matter the taste of the population you belong to.

Evolutionary models with boundedly rational agents adopt, in general, either the best reply rule or the imitation rule. Focusing the attention on 2×2 coordination games in which there is coexistence of efficiency and risk-dominance; Kandori, Mailath and Rob [5], Young [11] and Ellison [1] show that best reply dynamics favors the selection of the latter. However, the efficient equilibrium survives, when the dynamics is driven by imitation as in Robson and Vega-Redondo [10] and Josephson and Matros [4]. Matros [7] goes one step further and studies a model where agents can choose between best reply and imitation in order to select their strategies. He finds results consistent with those in Eshel et al [2]. Moreover, he shows the robustness of these results to the introduction of the best reply rule. Masson [6] also combines best reply and imitation. An individual considers his opponent a neighbor if he can observe his opponent's past plays and payoffs, otherwise the opponent is regarded a stranger. The use of a specific decision rule is therefore conditional upon the opponent: when players are neighbors they use best reply; but when they are strangers, they select the imitation rule. In this case, evolutionary forces select the efficient equilibrium.

In this paper, we study the impact of introducing a heterogeneous population in an environment similar to the one developed in Masson [6]. Time is discrete and in each period, a finite number of individuals are paired to play a 2×2 coordination game. Players can have one of two possible types: an *A-Type* whose members prefer to coordinate on strategy A and a *B-Type* whose members prefer coordination on strategy B. The relevant payoff matrix is now dependent upon the type of players being matched. With two types, there are 3 possible payoff matrices and 3 possible matchings. The payoff matrices depend on the type of each player, whereas the matching refers to the relationship between both players, i.e if each player considers his opponent as neighbor or stranger.

Given this environment, it is shown that the stochastically stable equilibrium depends upon the proportion and the distribution of the two types in the population. More specifically, a population of a given type may be diverted from its preferred equilibrium due to the presence of at least two neighbors from the other type.

The paper is organized as follows. Section 2 presents the formal model introducing the unperturbed and perturbed version of the process. In section 3, the asymptotic behavior of the process is analyzed and our main results are presented. Section 4 concludes.

2 The Model

In this section, we first describe the population structure and then introduce the game and its dynamics. Time is discrete and let $t = 1, 2, ...$ be the time periods. Let $N = \{1, 2, ..., 2n\}$ be the finite number of individuals in the population and let each $i \in N$ be represented by a node in a graph. Each individual is of a specific type: either *A-Type* or *B-Type*. For all i, define N_i as the set of individual $i's$ neighbors. This relationship among the individuals is defined by a set of directed edges $E = \cup_i \{(i,j) : j \in N_i\}$. That is, if an individual $j \in N_i$, then individual i knows individual $j's$ type and has access to the last *m-period* plays and payoffs of individual j. In this case, individual i considers individual j a neighbor. When individual $j \notin N_i$, individual j is considered a stranger by individual i who cannot access to any information about $j's$ type, past plays and payoffs. Formally, let $i \rightarrow j$ means that i has access to $j's$ *m-period* strategy-payoff pairs, and that i knows $j's$ type; i.e. the first player considers the second a neighbor. If no edge exist between two players, they consider themselves as strangers. For simplicity, consider the following notations:[1]

(i) $N_i = \{j : i \rightarrow j\}$; the set of players that individual i considers his neighbors.

(ii) $R_i = \{j : j \rightarrow i\}$; the set of players who consider individual i as their neighbor.

(iii) $I_i = N_i \cap R_i$; the set of players who have a double-sided link with i.

(iv) $S_i = \overline{N_i \cup R_i}$; the set of players who have no link with i.

Before introducing the dynamics of the model, we make some additional assumptions concerning the population structure:

(I) "Symmetric" neighborhoods: $\forall i$, $\exists\, j \neq i$ such that $I_i = \{i, j\}$

(II) Diversity: $Card(S_i) \geq Card(I_i), \forall i$

(III) Connectedness: From every individual in I_i, there exists a directed path to any other individual in the population.

Assumption (I) allows us to partition the population into groups in which any pair of individuals consider each other as neighbors. Assumption (II) ensures that each individual can be matched with a stranger with positive probability and reflects the fact that individuals usually know a small

[1] For the rest of the study, we assume that an individual has access to his own last $m-period$ strategy-payoff pairs, i.e individual i considers himself a neighbor

proportion of the whole population. Assumption (III) imposes an overlapping structure to the neighborhoods in the spirit of Ellison [1] and Eshel et al [2].

The dynamics of the game depend on the type of the players and on the relationship between the players. In each period, individuals are randomly matched to play a 2×2 coordination game. Each individual decides which decision rule to use according to the relationship with his opponent and the type of his neighbors: individual i uses the best reply rule whenever his opponent is one of his neighbors. But when individual $i's$ opponent is a stranger, individual i may either use the imitation rule or the best reply rule, depending on the type of his neighbors. Individual i only imitates neighbors of the same type.

The type of players in each match determines the relevant payoff matrix they face. We assume that both types have the same strategy set $X_i = \{A, B\}$, and that players are myopically rational.

We define the payoff matrices as follows. For *A-Type* individuals, the payoff matrix is represented by:

	A	B
A	a	b
B	c	d

Table 1

For *B-Type* individuals, the payoff matrix is represented by:

	A	B
A	d	c
B	b	a

Table 2

Regarding the payoffs, we assume $a > c > d > b \geqslant 0$ and $a - c < d - b$. Given these payoffs, we see that matches of players with the same type display tension between risk-dominance and efficiency. *A-Type* individuals obtain the highest payoff if they coordinate on the efficient equilibrium represented by (A, A) in the first table, while *B-Type* individuals obtain the highest payoff if they agree on the efficient equilibrium represented by (B, B) in Table 2. When matching occurs between individuals of different types, Nash equilibrium is reached only if one of the individual deviates from his preferred strategy. Efficiency in this latter case could only be reached if both types deviate. Intuitively, the fact that an individual deviates from its preferred strategy can be seen as willingness to compromise. If both individuals compromise, Pareto efficiency is reached. However, if only one of them yields, he bears the cost of the compromise. Moreover, the worst outcome occurs when both individuals refuse to sacrifice one's own.

Let the vector $x^t = \left(x_1^t, ...x_{2n}^t\right)$ be the play at time t, where x_i^t is player $i's$ strategy in that period. The history at time t is given by the vector $h^t = (x^1, ..., x^t)$. Let integers m and s be such that $m \geqslant 4$ and $1 < s \leqslant \frac{m}{2}$. When individual i faces a neighbor, he randomly selects s past plays from the history of length m of his current opponent, and plays best reply against this sample as in Young[11]. In the case where individual i is matched with a stranger, he randomly draws s strategy-payoff pairs generated previously by some of his neighbors. If some of these s strategy-payoff pairs are drawn from neighbors with the same type as individual i, individual i imitates the strategy that gave the highest average payoff among these particular strategy-payoff pairs only. If none of the strategy-payoff pairs drawn belong to an individual of the same type as i, individual i only considers the strategies used, and plays best reply against the whole sample, in an *A-Type* versus *B-Type* game. If an individual has more than one optimal strategy, we assume that he chooses among them with equal probability.

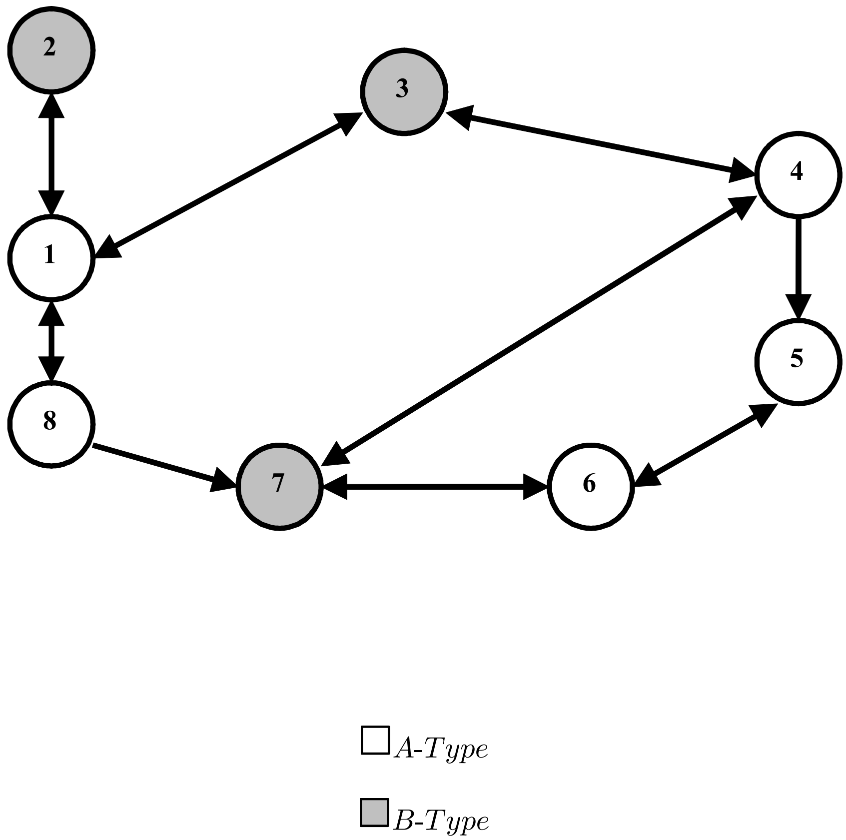
In period $t+1$, the relevant history of play corresponds to the most recent m periods. Denote the finite history of length m by $h = (x^{t-m+1}, ..., x^{t})$. From this state the process just described will move to a new state $h' = (x^{t-m+2}, ..., x^{t+1})$ in period $t+1$. For each $x_j \in X$, let $p_i(x_j \mid h)$ be the probability that individual i chooses strategy x_j in state h. Given our assumptions, $p_i(x_j \mid h)$ is independent of time and $p_i(x_j \mid h) > 0$ if and only if: x_j is a best reply of individual i when he faces a neighbor; x_j is the best reply to the sample drawn by individual i from some of his neighbors' most recent m strategy-payoff pairs, when individual i faces a stranger and when none of his neighbors is of the same type as him; or x_j is the strategy that gave the highest average payoff in the sample drawn among his neighbors who share the same type, and individual i faces a stranger.

If h' is a successor of h and x is the right-most element of h', the transition probability becomes

$$P^{m,s}(h' \mid h) = \prod_{i=1}^{2n} p_i(x_i \mid h).$$

If h is not a successor of h', $P^{m,s}(h' \mid h) = 0$. Hence, we have defined a Markov process $P^{m,s}$ on the finite space of histories $Z = X^m$ (where $X^m = \prod_{i=1}^{2n} X_i$). We refer to $P^{m,s}$ as selection play with memory m and sample size s.

Let us now consider a population that has the following struture (Example 1):



One can verify that this population structure satisfies assumptions (I) to (III):

$N_1 = \{1, 2, 3, 8\}$	$R_1 = \{1, 2, 3, 8\}$	$I_1 = \{1, 2, 3, 8\}$	$card(S_1) = 4$	$card(I_1) = 4$
$N_2 = \{1, 2\}$	$R_2 = \{1, 2\}$	$I_2 = \{1, 2\}$	$card(S_2) = 6$	$card(I_2) = 2$
$N_3 = \{1, 3, 4\}$	$R_3 = \{1, 3, 4\}$	$I_3 = \{1, 3, 4\}$	$card(S_3) = 5$	$card(I_3) = 3$
$N_4 = \{3, 4, 5, 7\}$	$R_4 = \{1, 3, 7\}$	$I_4 = \{3, 7\}$	$card(S_4) = 4$	$card(I_4) = 2$
$N_5 = \{5, 6\}$	$R_5 = \{4, 5, 6\}$	$I_5 = \{5, 6\}$	$card(S_5) = 5$	$card(I_5) = 2$
$N_6 = \{5, 6, 7\}$	$R_6 = \{5, 6, 7\}$	$I_6 = \{5, 6, 7\}$	$card(S_6) = 5$	$card(I_6) = 3$
$N_7 = \{4, 6, 7\}$	$R_7 = \{4, 6, 7, 8\}$	$I_7 = \{4, 6, 7\}$	$card(S_7) = 4$	$card(I_7) = 3$
$N_8 = \{1, 7, 8\}$	$R_8 = \{1, 8\}$	$I_8 = \{1, 8\}$	$card(S_8) = 5$	$card(I_8) = 1$

The payoffs matrices are as follow:

	A	B
A	10, 10	0, 9
B	9, 0	7, 7

A-Type versus *A-Type*

	A	B
A	7, 7	9, 0
B	0, 9	10, 10

B-Type versus *B-Type*

	A	B
A	10, 7	0, 0
B	9, 9	7, 10

A-Type versus *B-Type*

Assume that individuals are paired at random in such a way that individual 7 faces individual 8. That means that individual 8 plays the game according to the best reply rule whereas individual 7 selects his decision rule depending on the type of the neighbors from whom he samples.

Let $m = 8$ and $s = 3$. The state h could for example be as follow:

	Ind 1	Ind 2	Ind 3	Ind 4	Ind 5	Ind 6	Ind 7	Ind 8
	A-Type	*B-Type*	*B-Type*	*A-Type*	*A-Type*	*A-Type*	*B-Type*	*A-Type*
t	$(A, 10)$	$(A, 7)$	$(B, 0)$	$(A, 0)$	$(A, 10)$	$(A, 10)$	$(A, 3)$	$(B, 3)$
$t-1$	$(A, 0)$	$(B, 10)$	$(B, 0)$	$(A, 10)$	$(A, 10)$	$(B, 9)$	$(A, 9)$	$(B, 10)$
$t-2$	$(B, 7)$	$(B, 10)$	$(B, 7)$	$(B, 9)$	$(A, 0)$	$(A, 10)$	$(A, 7)$	$(B, 10)$
$t-3$	$(A, 10)$	$(B, 10)$	$(B, 10)$	$(A, 10)$	$(B, 7)$	$(A, 10)$	$(B, 10)$	$(A, 10)$
$t-4$	$(B, 3)$	$(A, 0)$	$(A, 3)$	$(A, 10)$	$(B, 9)$	$(A, 0)$	$(A, 9)$	$(A, 10)$
$t-5$	$(A, 10)$	$(A, 3)$	$(A, 7)$	$(B, 7)$	$(A, 0)$	$(B, 3)$	$(B, 0)$	$(B, 7)$
$t-6$	$(B, 9)$	$(B, 0)$	$(B, 10)$	$(A, 0)$	$(A, 10)$	$(A, 10)$	$(B, 10)$	$(A, 0)$
$t-7$	$(B, 7)$	$(B, 10)$	$(B, 10)$	$(A, 10)$	$(B, 9)$	$(A, 10)$	$(B, 10)$	$(A, 0)$

Let the sample of individual 8 be the plays of individual 7 at time $t-1$, $t-3$ and $t-6$, represented by the sequence of strategies A, B and B. So individual 8 expected payoff is $\frac{10}{3}$ if he plays strategy A, and $\frac{23}{3}$ if he plays strategy B. This implies that individual 8 will play B next period. Now assume that individual 7 samples what individual 4 did at time t, and what individual 6 did at time $t-1$ and $t-4$. Individual 7 then observed the following strategy payoff-pairs $\{(A,0),(B,9),(A,0)\}$. Because individual 7 does not have the same type as individuals 4 and 6, he plays best reply against the sequence of strategies A, B and A, in the *A-Type* versus *B-Type* game. This sequence of plays gives to individual 7 an expected payoff of $\frac{23}{3}$ for playing A and $\frac{10}{3}$ for playing B. Therefore, the successor of h has in its first row (corresponding to period $(t+1)$) the strategy B in the eighth position and A in the seventh position. Suppose now that during the same period individuals 4 and 5 were also matched together. Individual 4 samples from individual 5's most recent plays, giving him a frequency of plays as follow: A, A, A. Since individuals 4 and 5 are of the same type, the best reply to this sample for individual 4 is A. Let the sample drawn by individual 5 be the strategy-payoff pairs of individual 6 at time $t-5$, $t-6$ and $t-7$. Individual 5 then observes the following: $\{(B,3),(A,10),(A,10)\}$. Since both individuals are of the same type, individual 5 plays by imitating the most successful strategy on average, which is A. Therefore, the successor of h has in its first row the strategy A in the fourth and fifth positions.

According to Young [11], we also define the perturbed version of the process. In each period and for each player i, there is a small probability $\varepsilon > 0$ that an individual disregards the decision rule selection described above and just chooses his strategy at random from X_i. It is assumed that the event that individual i experiments is independent of the event that another individual j experiments. Experimentations or mistakes are also independent across periods. The introduction of the experimentation probability makes the process ergodic: now $P^{m,s,\varepsilon}(h' \mid h) > 0$ for all $h, h' \in Z$. We denote the perturbed process as $P^{m,s,\varepsilon}$ and refer to it as the selection play with memory m, sample size s and experimentation probability ε.

3 Asymptotic behavior: what happens in the long run?

Following Young [11], we define a recurrent class as a set of states such that there is positive probability of reaching any state in the class from any state outside, but zero probability of moving outside the class from any state within the class. If the set of states is a singleton, the recurrent class is called an absorbing state. We denote by h_x a state of the form $h = (x, ..., x)$ where x can either be A or B, and refer to h_x as a *convention*. Finally, let $\gamma = \frac{n_B}{n_A+n_B}$ be the proportion of individuals of type B in the population where n_i, $i = A, B$ is the number of individuals of type i such that $n_A + n_B = 2n$.

For $\gamma = 0$, Masson [6] showed that the unperturbed process we described in the last section converges to a convention from any initial state. Then, when the process is perturbed by incorporating a small probability of mistake, she also showed that, under certain conditions on the sample size, the process selects the Pareto efficient state h_A, that is, if $s \in \left(\frac{2}{Int(\frac{a-c}{a-b-c+d})}, \frac{m}{2} \right)$ [2] the stochastically stable state is the efficient convention.[3]

We begin the study of the asymptotic behavior by examining the unperturbed process $P^{m,s}$. We show next that, when the population is constituted of individuals of both types, no matter their proportions, none of the conventions are stochastically stable. The following theorem shows that the unperturbed version of the process converges to a convention independently of the initial state.

Theorem 1 *For $s \leq \frac{m}{2}$ a set of states is a recurrent class if and only if it is a convention.*

Proof. It is obvious that a convention is a recurrent class of the unperturbed process $P^{m,s}$: if the state is a convention, every sample includes only the same strategy, so no matter which decision rule is used, each individual in the population plays the same strategy forever.

[2] Where $Int(x)$ is x if x is an integer, and the integer part of $x + 1$ if x is not an integer.
[3] Conversely, it can also be shown that process selects the efficient convention (B, B) when $\gamma = 1$.

Fix s and m, such that $s \leq \frac{m}{2}$ and $m > 2$. Let $h(t) = (x^{t-m+1}, ..., x^t)$ be an arbitrary state in period $t \geqslant m$. Consider I_i for a fixed i. There exists two cases: I_i is only composed by the same type of individuals or I_i contains individuals from both types. Let us first assume we are in the first case, and let's assume without loss of generality that individuals in I_i are *A-Type*. Given assumption (II) about the structure of the population, there is a positive probability that every player in I_i meets a stranger from period $t+1$ to $t+s$ inclusive, and that they all draw the sample composed of the most recent strategy-payoff pairs of player i.[4] From period $t+s+1$ to $t+m$ inclusive, there is a positive probability that every member of I_i meets a stranger again and sample from $i's$ most recent plays. At the end of period $t+m$, every individual in I_i has the same history of plays.[5] In the case where I_i is composed with both types of individuals, let's consider $I_i(A)$ the group of *A-Type* individuals who belong to I_i and $I_i(B)$ the group of *B-Type* individuals who also belong to I_i. Without loss of generality let's consider only individuals from $I_i(A)$. Using the above argument, we can show that there is a positive probability that at the end of period $t+m$, all individuals from $I_i(A)$ face the same history of plays. Considering now the whole neighborhood I_i and given assumption (II) about the structure of the population, there is a positive probability that every player in I_i meets a stranger from period $t+1$ to $t+s$ inclusive and that they all draw the sample composed of the most recent strategy-payoff pairs from one of $I_i(A)$ individuals. This will lead all individuals from I_i to play the same strategy the next period. This can be seen as follow: if individuals from $I_i(A)$ face an history of plays composed only by A [resp. B], individuals from $I_i(B)$ use strategy A [resp. B] for best response. By assumptions (I) and (III), some individuals $j \in I_i$ have their own R_j that contains individuals who do not belong to I_i, such that there exists some $k \in R_j$ satisfying $I_i \neq I_k$. Let $O(I_i)$ be the set of these players. According to assumption (II), there

[4]Note that individual i also samples from himself.

[5]The strategy that is common to all individuals in I_i depends on i's type. If i is $A-type$ (resp. $B-type$), each individual from I_i faces an history of plays of A (resp. B). Note also that payoffs may differ across individuals.

exists a positive probability that each individual from each I_j, $j \in O(I_i)$, and also each individual from $I_i \backslash O(I_i)$ meets a stranger for the next m periods. There is also a positive probability that individuals in I_i sample from individuals within I_i and that individuals in each I_j sample from j for m periods. Then, m periods later, all individuals from I_i and all individuals in each I_j for all $j \in O(I_i)$ face the same history of play. Applying the same reasoning to those players $k \in I_j$ that have their own R_k including individuals l who do not belong to either to I_i or any of the $I_j s$, we have shown that a convention has been reached. It follows that the only recurrent classes of the unperturbed process are conventions. ■

Consider now the perturbed version of the process $P^{m,s,\varepsilon}$ and let focus on the limiting distribution of this process when the probability of experimention tends to zero. By arguments similar to Young [11], the perturbed process $P^{m,s,\varepsilon}$ is a regular Markov chain and hence it has a unique stationary distribution μ^ε that satisfies $\mu^\varepsilon P^{m,s,\varepsilon} = \mu^\varepsilon$. Moreover, Young [11] proves that $\lim_{\varepsilon \to 0} \mu^\varepsilon = \mu^0$ exists and that μ^0 is a stationary distribution of the unperturbed process $P^{m,s}$. From Foster and Young [3] and Young [11], we say that a state $h \in H$ is stochastically stable relative to the process $P^{m,s}$ if $\lim_{\varepsilon \to 0} \mu^\varepsilon(h) > 0$. Let h' be a successor of h and let x be the right-most element of h'. A mistake in the transition $h \to h'$ is a component x_i of x that either is not a best reply to a sample drawn by an individual who used the best reply rule or it does not have the highest average payoff in the sample drawn by an imitator. Denote by $r(h, h')$ the resistance for any two states h and h' as being the total number of mistakes involved in the transition $h \to h'$ if h' is a successor of h; otherwise $r(h, h') = \infty$. In the 2×2 symmetric coordination game with heterogeneous types described in section 2, we can apply this procedure as follows. This game has only two absorbing states: the A-convention and the B-convention. Consider a directed graph with one vertex for each absorbing state. The directed edge $h_A \to h_B$ has weigh $r(h_A, h_B)$ and corresponds to the resistance of the tree rooted at h_A. The stochastic potential ρ of the absorbing state h_A is the minimum resistance of the tree rooted at h_A. Similar concepts apply to the tree rooted at h_B. The following theorem states which conventions can be observed asymptotically when the probability of mistake in the perturbed process vanishes.

Theorem 2 *The stochastically stable states of $P^{m,s,\varepsilon}$ are the conventions with minimum stochastic potential.*

Proof. This follows from Theorem 1 and Theorem 4 in Young [11]. ■

We can now present the main results of this paper.

Theorem 3 *If there exists at least one I_i that contains at most one A-Type [resp. B-Type] individual, the stochastically stable equilibrium is the convention h_B [resp. h_A].*

Proof. Let us first consider the case where we move from h_B to h_A. The whole population plays B. Consider I_i, where i is an *A-type* individual. In this case, whether this *A-type* individual is matched with a stranger or a neighbor, he plays using the best reply rule. Let $Int(x)$ be x if x is an integer, and the integer part of $x+1$ if x is not an integer. In period t, there exists a positive probability that this *A-type* individual is matched with one of his *B-type* neighbors for $Int(\frac{d-b}{a-c-b+d})s$ periods, and that the *B-type* individual makes the mistake of playing A instead of B during these $Int(\frac{d-b}{a-c-b+d})s$ periods. Then, there is a positive probability that these two individuals are matched together again for s periods, and that the *A-type* individual always samples the same plays from this *B-type* individual and therefore plays A. Given assumption (II), there is a positive probability that all individuals in I_i face a stranger for the next m periods and sample from the *A-type* individual most recent plays. Since these individuals are *B-type*, their best response to strategy A in a *A-type* versus *B-type* game is to play A also. At the end of these m periods, all individuals in I_i face a history of plays composed uniquely by the strategy A. By assumptions (I) and (III), some individuals k from I_i have their own R_k that contains individuals who do not belong to I_i, such that there exists some $l \in R_k$ satisfying $I_i \neq I_k$.With positive probability, all individuals from I_k face strangers in period $t+m+1$ and sample the s most recent strategy-payoff pairs from the individual who also belongs to I_i. Applying the same reasoning as the one used for I_i, we can see that after m periods, all individuals who belong to I_k have the same history of play. Considering now individuals from I_k that connect to other neighborhoods other than I_i and I_k, and applying the same argument, we can see that h_A is reached by only $Int(\frac{d-b}{a-c-b+d})s$ mistakes.

Let now consider the case where the system moves in the opposite direction, i.e $h_A \longrightarrow h_B$. Consider I_i as above, where i is an *A-type* individual. There is a positive probability that the *A-type* individual is matched with one of his neighbors, who makes the mistake of playing B instead of A for $Int(\frac{a-c}{a-c-b+d})s$ periods. Then, there is a positive probability that these two individuals are matched again for m periods and that the *A-type* individual plays best response against the same sample, drawn from the *B-type* individual, that contains $Int(\frac{a-c}{a-c-b+d})s$ times the strategy B. Using assumption (II) on the structure of the population, there is a positive probability that all individuals from I_i meet strangers for the next m periods. Following the same reasoning as the one developed for the case where we move from h_B to h_A, we can see that h_B has been reached only by $Int(\frac{a-c}{a-c-b+d})s$ mistakes. Therefore, the stochastically stable equilibrium is the convention h_B. ■

Theorem 3 says that the presence of an individual of one type in a neighborhood belonging to a population of the other type is not enough to move the equilibrium in his favor. The same result holds even when there are more than one such individual, as long as they are located in separated neighborhoods. Theorem 4 states the conditions under which agents from a minority type can influence the selection of their preferred equilibrium. In particular, the minimum requirement is to have a couple of individuals from the minority type leaving together in one neighborhood. However, as shown below, their success is limited to the imposition of their preferred equilibrium only half of the time.

Theorem 4 *If there exists at least one I_i that contains at least two A-type individuals and at least one I_j that contains at least two B-type individuals, the limiting distribution puts equal probability on both conventions h_A and h_B.*

Proof. Let us first consider the case $h_B \to h_A$. Consider I_i where there is at least two A-*type* individuals. There exists a positive probability that in period t, two of these A-*type* individuals face each other and simultaneously make the mistake of playing A instead of B, yielding the highest possible payoff of a. There is also a positive probability that these two individuals face strangers for the next s periods, and sample from each other. Considering assumption (II), with positive probability, every individual in I_i can face a stranger from period $t+s+1$ to $t+s+m$ and sample from the s most recent strategy-payoff pairs of one of these two A-*type* individuals. This means that at the end of period $t+s+m$, all individuals in I_i face a history composed uniquely by As. This comes from the fact that the only samples drawn by individuals in I_i are constituted by strategy A. So if we consider an A-*type* individual in I_i, he imitates the only available strategy, that is A. And if we consider a B-*type* individual in I_i, he plays a best response to A in an A-*type* versus B-*type* game, which is A. Using assumptions (I) and (III), some individuals k from I_i have their own R_k that contains individuals who do not belong to I_i, such that there exists some $l \in R_k$ satisfying $I_i \neq I_k$.By assumption (II), there is a positive probability that all individuals from I_k face strangers for the next m periods and sample the s most recent strategy-payoff pairs from one of the individuals, possibly the only one, who also belong to I_i. Applying the same reasoning as the one for I_i to all I_k while using assumptions (II) and (III), we can see that after m periods, all individuals who belong to I_k have the same history of plays. Considering now individuals from I_k that connect to other neighborhoods other than I_i and I_k, and applying the same argument, we can see that h_A is reached by only 2 mistakes.

Consider now the case where we go from h_A to h_B. Everybody plays A. Consider I_i where there is at least two *B-type* individuals. There exists a positive probability that in period t, two of these *B-type* individuals face each other and simultaneously make the mistake of playing B instead of A, yielding the highest possible payoff of d. Following the same argument as the one developed for the case $h_B \rightarrow h_A$, one can see that the convention h_B has been reached in only two mistakes, letting thus the selection of a convention undetermined. ■

The previous theorems show that the selection of a convention is dependent on the distribution of types in the population. In particular, if there is only one individual of a given type per 'symmetric' neighborhood, the system will converge to the convention that is preferred by the type which is the majority. In the more general case where we can find at least one neighborhood that contains two individuals of one type, and another neighborhood, possibly the same, that contains two individuals of the other type, the system spends half of the time in each equilibria.

4 Concluding Remarks

In this paper, it has been shown that heterogeneity of types among individuals may lead to both conventions to exist in the limit. The minimum condition required for this result is the presence of two neighbors from the minority type. In the terminology of Myatt and Wallace [8], the introduction of a 'bad apple' – in our case, an agent of the other type – is not enough to move the system to the new agent's preferred equilibrium. On the other hand, a minority of agents can still be capable of influencing the equilibrium selection, no matter their proportion in the whole population, as long as two of them belong to the same neighborhood.

It would be interesting to extend this line of research by allowing individuals to create and to severe links. In this context, we may see the emergence of seggregated neighborhoods, in the spirit of Schelling [9].

References

[1] ELLISON G. (1993): "Learning, Local Interaction, and Coordination", Econometrica v61(5), 1047-1071.

[2] ESHEL I., L. SAMUELSON and A. SHAKED (1998): "Altruists, Egoists, and Hooligans in a Local Interaction Model", The American Economic Review v88(1), 157-179.

[3] FOSTER D. and P. H. YOUNG (1990): "Stochastic Evolutionary Game Dynamics", Theoretical Population Biology 38, 219-232.

[4] JOSEPHSON J. and A. MATROS (2004): "Stochastic Imitation in Finite Games", Games and Economic Behavior 49, 244-259.

[5] KANDORI M., G. MAILATH and R. ROB (1993): "Learning, Mutation, and Long Run Equilibria in Games", Econometrica v61(1), 29-56.

[6] MASSON V. (2004): "Neighbors Versus Strangers in a Spatial Interaction Model", Working Paper, University of Pittsburgh.

[7] MATROS A. (2004): "Virtuous Versus Spiteful Behavior in a Public Good Game", Working Paper, University of Pittsburgh.

[8] MYATT D. and C. Wallace (2004): "The Evolution of Collective Action", Discussion Paper Series, Department of Economics, University of Oxford.

[9] SCHELLING, T. (1971): "Dynamic Models of Segregation", Journal of Mathematical Sociology 1, 143-186.

[10] ROBSON A. and F. VEGA-REDONDO (1996): "Efficient Equilibrium Selection in Evolutionary Games with Random Matching", Journal of Economic Theory 70, 65-92.

[11] YOUNG H. P. (1993): "The Evolution of Conventions", Econometrica v61(1), 57-84.