Cheap Talk on the Circle

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Abstract

In this paper we modify the 'cheap-talk' model of Crawford and Sobel 1982 by taking the state space to correspond to a circle (instead of a line). It is shown that in such a setup the relationship between the 'bias' (i.e., the parameter capturing the divergence of interests between sender and receiver), and the 'informativeness' of equilibria, is reversed from what it is in the original Crawford and Sobel story: Now, a higher bias can be associated with more informative equilibria, rather than the other way around. We also attempt to characterize more generally the equilibria of this modified model.

1 Introduction

Crawford and Sobel 1982 showed that informative equilibria existed in sender-receiver games with costless signalling, even when the interests of the sender and the receiver did not fully coincide. Moreover, they showed that as the divergence of interests between sender and receiver increased, there was a sense in which the equilibria became less informative. This paper focuses on this last feature of the Crawford and Sobel analysis. Whereas the Crawford and Sobel story has been extended in many directions, as far as I know, this particular aspect of their work has not been explored further -perhaps because this conclusion accords so well with our (a priori) intuition on the problem. However, in this paper we show that this conclusion relies crucially on a not very 'intuitive' auxiliary assumption of the Crawford and Sobel model (certainly one they did not motivate explicitly), namely, taking the state space to correspond (essentially) to the real line. If one takes instead the state space to correspond to the circle, there is a sense in which a greater divergence of interests between sender and receiver results in more informative equilibria. In fact, one can show that, in a limiting sense, the largest 'bias' (the divergence of interests in the circle is naturally bounded) leads to the most informative equilibrium concievable in this class of models. To witness, one where, for each pair of distinct states, a distinct message is sent in equilibrium. Actually, one can go even further, and show that, as the bias falls, the most informative equilibrium attainable (again, in a limiting sense) becomes less and less informative. Moreover, the equilibria we construct are qualitatively very different from the equilibria of Crawford and Sobel: All involve mixed responses only[1]. In contrast, in the Crawford and Sobel model, mixed responses are excluded by the strict concavity of the receiver's objective (mind that we posit the same objective for both the sender and the receiver as Crawford and Sobel do; the circular form of the state space, however, introduces non-concavities into the agents' choice problems).

We do not think that these results in any way invalidate the Crawford and Sobel results, as it is not obvious whether a circle state space is more appropiate than a real line one (as usual the answer will presumably depend on the specific context). But they do point to the need to reflect more on this aspect of cheap talk models.

2 The Model

The game is played by a Sender (S) and a Receiver (R). S directly observes a state of nature ω in a set ('the state space'). We take the set to be the circle of unit circumference, with each state corresponding to a number between 0 and 1. The numbers 0 and 1 will describe one and the same state (I will refer to this state as 'the origin'), while numbers strictly between 0 and 1 will describe distinct states. R does not directly observe the state of nature, but takes it to be distributed uniformly on . R's payoff is a function of both the state of nature, and an action a R chooses from a set A ('the action space'). The action space A is also taken to be the circle of unit circumference, with each number strictly between 0 and 1 representing a different action, and both 0 and 1 designating the same action.

[1] Actually, it is easy to show that in this setup there are no equilibria with only pure responses.

S's payoff function is given by the von Neumann-Morgenstern utility,

$$U^S(a,\omega) = -\min\left((\omega - a)^2, (1 - |\omega - a|)^2\right)$$

R's payoff function is given by the von Neumann-Morgenstern utility,

$$U^R(a,\omega) = -\min\left(([\omega + b]_1 - a)^2, (1 - |[\omega + b]_1 - a|)^2\right)$$

where b is a scalar parameter in $\left[-\frac{1}{2}, \frac{1}{2}\right]$ meant to capture the divergence of interests between S and R ('the bias'), and $[\omega + b]_1$ denotes the sum in the brackets taken modulo 1[2].

These utilities imply that S's 'ideal action', i.e., the action that maximizes S's utility given the realized state, is exactly that state, while R's ideal action corresponds to the realized state shifted clockwise by the amount of the bias b. More generally, given a realized state ω', S will prefer actions closer to it (modulo 1), while R will prefer actions closer to $\omega' + b$ (modulo 1). The following diagram illustrates:

This interpretation of the posited utilities should make it clear why R's bias is assumed not to exceed $\frac{1}{2}$ in absolute terms: Any larger bias would be equivalent to a smaller bias of the opposite sign. Further, note the linearity of the payoff functions, which will play an important role in what follows.

Apart from the shape of the state space (the unit circle rather than the unit interval), the game played by S and R coincides with the cheap signalling game of CS: After observing the state of nature, S sends a message m from a set of possible messages M ('the message space' which is taken to coincide with the state space). After observing the message sent by S (but not the state of nature), R chooses an action based on the beliefs induced by the observed message. As in CS, the message sent does not directly enter either players' utility ('talk is cheap').

The solution concept we use is (weak) Perfect Bayesian Equilibrium (for a definition, see Mas-Colell et al. ?). In order to formally define an equilibrium, we introduce the following objects:

[2]For any $\omega \in$, and any number $a \in [0,1]$,

$$[a + \omega]_1 = \begin{cases} \omega + a & if\ \omega + a \leq 1 \\ \omega + a - 1 & else \end{cases}$$

A *message rule* is a mapping from the space of states into the space of messages,

$$\mu : \quad \to M$$

It describes a strategy for the sender. Note that we allow only for pure actions by the sender. This is without loss of generality[3].

An *action rule* is a mapping from the space of messages into the set of probability distributions over the space of actions,

$$\alpha : M \to \Delta A$$

It describes a strategy for the receiver. We write $\alpha\left(a|m\right)$ for the probability that action $a \in A$ is taken when message m is received.

Definition 1 *An equilibrium is made up of a message rule* σ, *and an action rule* α, *such that*

1) for each $\omega \in \quad$,

$$\mu\left(\omega\right) \in \arg\max_{m} \int_{A} U^{S}\left(a,\omega\right)\alpha\left(a|m\right)da$$

2) for each $m \in M$, and for each action a' in the support of $\alpha\left(m\right)$,

$$a' \in \arg\max_{a} \int U^{R}\left(a,\omega\right) f\left(\omega|m\right)d\omega$$

where

$$f\left(\omega|m\right) = \begin{cases} 1 \;\; if \;\; \omega \in \mu^{-1}\left(m\right) \\ \\ 0 \;\; else \end{cases}$$

[3] There is no loss in generality in doing this, as.... . Of course, with our formulation, out-of-equilibrium beliefs must be specified. Assume, for example, that beliefs do not change when an out of equilibrium message is observed.