Building a Reputation Under Frequent Decisions*

Eduardo Faingold†

March 31, 2005

Extended Abstract

This paper studies reputation phenomena in repeated games in which monitoring is imperfect and players observe noisy public signals of each other's moves. As in Fudenberg and Levine (1992), the games considered in this paper are played by a single long-run player and by a sequence of short-run players, each of whom plays only once. I depart from the standard literature and consider a model of repeated games in which players take actions frequently. In particular, I investigate the extent to which reputation effects are affected by the possibility that players react quickly to new information.

As it is well known (e.g. Abreu, Milgrom and Pearce (1991)), the possibility that players make frequent moves renders the signals poorly informative. For instance, consider a dynamic game played in continuous time such that: (i) players take actions at times $t = 0, \Delta, 2\Delta, \ldots$, where $\Delta > 0$ is the *period length*; (ii) a player's moves are unobservable to her opponents; (iii) the public signals are the sample paths of a Poisson process whose intensity, λ, is jointly controlled by the players; and (iv) the long-run player discounts payoffs at an exponential rate $r > 0$. This dynamic game can be formally viewed as a standard discrete-time repeated game in which the long-run player's discount factor and the monitoring structure depend on Δ. More precisely, in the formulation of this game as a discrete-time repeated game, the long-run player's discount factor is $\delta = e^{-r\Delta}$, and the *monitoring structure*, defined as the probability distribution over public signals, corresponds to the law of a Poisson random variable with parameter $\lambda\Delta$. Pick two action profiles a and a' and consider the signal informativeness $\mathcal{I}^{\Delta}$, as

*I am very grateful to George J. Mailath for invaluable advice and guidance throughout this project. This document was prepared for submission to the 16th International Conference on Game Theory (Stony Brook, July 2005).

†Department of Economics, University of Pennsylvania. E-mail: faingold@econ.upenn.edu

measured by the distance between the probability distribution over signals induced by profiles a and a'. Clearly, one has

$$\mathcal{I}^{\Delta} = |\lambda(a) - \lambda(a')|\,\Delta + o(\Delta) \to 0$$

This, in turn, adversely affects the ability of the long-run player to develop a favorable reputation, as it might take many periods for the short-run players to learn about the long-run player's behavior.

Notwithstanding, I show herein that in a broad class of games with frequent actions reputation effects impose intuitive bounds on the set of Nash equilibrium payoffs of a patient long-run player. Specifically, for games with a monitoring structure that possesses a well-behaved continuous-time limit (such as in the example above), I derive reputation bounds akin to Fudenberg and Levine's (1992) and show that these bounds hold *uniformly* over all games with sufficiently short periods of inaction. The main result of the paper, stated rather informally, is:

Suppose that as $\Delta \to 0$ the monitoring converges to a "well-behaved" continuous-time stochastic process, such as drift-controlled Brownian motion or intensity-controlled Poisson process. Then, for every prior probability p over the long-run player's types and for every $\varepsilon > 0$ there exist $r_0 > 0$ such that for all $0 < r < r_0$ and all $\Delta > 0$, every Nash equilibrium of the game with prior p, discount rate r and period length Δ yields the long-run player a payoff no smaller than $\pi^* - \varepsilon$, where π^* is the long-run player's Stackelberg payoff.

The strength of this result is the uniformity in Δ. Clearly, for a fixed period length Δ, the results of Fudenberg and Levine (1992) can be applied and one has a weaker form of the above result, with r_0 possibly depending on Δ. This leaves open the possibility that $r_0 \to 0$ as $\Delta \to 0$, which would mean that reputation effects would not have a bite in games with frequent actions. I show this not to be the case, as long as the monitoring has a nicely behaved continuous-time limit.

The proof of this result involves the study of the continuous-time limit game and it is based on a merging argument (see Sorin (1999)) in continuous time, the formulation of which employs methods of optimal nonlinear filtering of random processes.