

Games of Status and Discriminatory Contracts ^{*}

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Abstract

Following recent empirical evidence which indicates the importance of rank for the determination of workers' wellbeing, this paper introduces status seeking preferences in the form of rank-dependent utility functions into a moral hazard framework with one firm and multiple workers, but no correlation in production. Workers' concern for the rank of their wage in the firm's wage distribution may induce the firm to offer discriminatory wage contracts when its aim is to induce all workers to expend effort. Crucial factor for the determination of the profile of optimal wage contracts is the individual worker's valuation of being in front relative to being in the same wage position than another worker.

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‘...to understand what makes workers satisfied it is necessary to look at the distribution of wages inside a workplace. We show that rank matters to people. They care about where their remuneration lies within the hierarchy of rewards in their office or factory. They want, in itself, to be high up the pay ordering.’

Brown et al. (2003, p. 30)

1 Introduction

There is a wealth of experimental and empirical work that attests to the fact that workers in an organisation care about their position among peers. One recent example is Brown et al. (2003) which provides empirical and experimental evidence on the importance of *rank*.¹ Yet many of the theoretical results in the literature on optimal incentives in organisations assume completely self interested workers. The question of what happens to the nature of optimal contracts when agents can be *other-regarding* has only recently started getting attention.²

Itoh (2004) examines moral hazard in incentive contracts when workers have other-regarding preferences, but the analysis is restricted to interdependent and symmetric contracts only. Neilson and Stowe (2004) studies optimal linear contracts for workers with other-regarding preferences. Focusing on independent contracts, they investigate the circumstances under which other-regarding preferences lead workers to expend more effort than they would otherwise and under which circumstances other-regarding preferences lead to lower piece rates. Englmaier and Wambach (2002) presents a model where a worker has other-regarding preferences in the sense that he is inequity averse with respect to the principal. Grund and Sliwka (2002) and Demougin and Fluet (2003) study tournaments when workers are inequity averse. Demougin and Fluet (2003) provide a framework where workers’ choices of effort levels are observable but outcome is uncertain. The effects of other-regarding preferences on the reward systems when there are either group or individual bonus schemes are studied. Biel (2004) studies the incentives of workers with other-regarding preferences to cooperate in a 2×2 normal-form game where output is deterministic and perfectly informative about workers and the productivity of the individual workers is related.

All this literature on other-regarding preferences and optimal contracts has focused on the two agent case, thus abstracting away from the form of the utility function when there are more workers and more wages to compare. They all follow Fehr

¹Brown et al. (2003) also provide a comprehensive list of further empirical and experimental literature in this area.

²There are some early exceptions like e.g. Frank (1984) who argues that the presence of heterogeneous status seeking individuals will lead to wage compression. However he used the framework of perfectly competitive markets and not contract theory.

and Schmidt (1999) in modelling other-regarding preferences. The utility functions considered in some of this literature allow for both *inequality aversion* and status seeking (e.g Itoh (2004), Neilson and Stowe (2004)).

Part of the problem in analysing situations where workers are other regarding is the fear that the specific way in which individuals care about others might be crucial for the results. We investigate in particular, workers who are *status-seeking*, in the sense that they care about their *rank*³ in the reference group. Our focus is on the question of optimal wage contracts in a simple setting of moral hazard with identical status-seeking agents. For simplicity, we follow the literature in making the assumption that status derives only from one dimension, i.e. the order of realised wages although we are aware (as pointed out by Shubik (1971)) that status is often multi-dimensional.

There are two distinguishing features in our modelling approach. First, in contrast to the above mentioned studies we allow the firm to offer asymmetric contracts. A priori if the firm is interested to exploit incentives from status to reduce its wage cost then not allowing the firm to use asymmetric contracts seems artificially restrictive. Second, differently from the existing literature on other-regarding preferences we do not follow Fehr and Schmidt (1999) in allowing wage levels and wage differences to matter to workers. We only allow ordinal differences to matter. In this we are motivated by two considerations: first there is empirical evidence (Brown et al. (2003)) that employees really care about rank and not about the deviation from a certain reference level, and second, that we would like these preferences to generalise to the case of more than two workers⁴. Indeed, making preferences depend on the whole vector of wages (rather than the order) when there are many co-workers in the reference group requires a lot of information on the part of the worker and seems to require strong assumptions about the specific way in which the wages of co-workers enters the utility function.

Shubik (1971) on the other hand had in mind a much simpler notion of status games. In the conversion from a two player game to a *game of status* the set of outcomes reduces to essentially three: Win, Lose or Draw. The natural extension of this to many players suggests that what matters is the number of people below, above or at the same rank. Dubey and Geanakoplos (2004) introduce exactly such a utility function for status seeking students. In other words, rank seems to us to be a more robust way to generalise how wage differences matter in the sense that it is an ordinal measure of status and does not require very precise information on the wage distribution⁵. Motivated both by our interest in exploring the role of

³Rank is based on the order of realised wages only.

⁴We conjecture that our results generalise for more than 2 workers if there is sufficient richness in the distribution of the stochastic shocks.

⁵It might be argued of course that small changes in wage distribution lead to discontinuous changes in rank, but we could easily add some perceptions of changes in rank so that wages would have to change by a significant amount for rank to change. This point is also addressed in the

rank as an indicator of status and the simple and general way in which Dubey and Geanakoplos (2004) allow status to matter through ranks, we use a modification of their model of status seeking.

Our main contribution is to show that when agents are conscious about their rank, then under certain conditions the firm finds it worthwhile to use *discriminatory contracts*. This is a surprising result: the firm offers different wage contracts to agents who are ex-ante identical!

We also find, consistent with the literature, that when agents are status-seeking, there is *wage compression*. Finally, we find that looking for an optimal contract in this framework involves two steps: designing the game of status that maximises incentives (which then implies a given order of wages between the two workers) and finding the wage levels that satisfy the participation and incentive constraints.

Winter (2004) shows that in an environment of complementarities in production and unobservable efforts, optimal mechanisms may be fully *discriminating*, i.e. they require unequal treatment of equals. The driving force in his story is the coordination problems between multiple agents. The more general point of his model is that when peer effects are important we should observe more “hierarchy” in organisations when this hierarchy is not related to different job descriptions. Agents in his model are complementary in production and generate externalities on each other in the sense that the profitability of their own effort level is increasing in the effort of others.

There is no reason why such externalities are generated only when workers are related in production. This paper considers the role of status seeking agents in an environment where effort is not always observable. Status seeking has similar properties as complementarities in production: i.e. the effort that agents put in imposes an externality on other agents. If other agents put in effort then the expected gain from putting in effort for an individual worker increases.

2 The Model

In this section we build a model of moral hazard with multiple agents who are status seeking. The model is standard, apart from the utility functions of workers.

2.1 Workers

There are $n = 2$ identical workers $i = 1, 2$ who have the choice between two different elements of the set of Effort Levels $\epsilon_i = \{e^L, e^H\}$ where $e^H > e^L$. A worker’s cost of

Conclusion.

a certain chosen effort level is assumed to be equal to $c_i(e_i)$ assumed to be increasing and convex.

The workers possess other-regarding preferences. Therefore Worker i 's utility is a function of the cost of providing a specific effort level as well as of the firm's realised wage distribution $\mathbf{w} = [w_1, w_2]$:

$$U_i(\mathbf{w}, e_i) = U(\mathbf{w}) - c(e_i).$$

More specifically it is assumed that Worker i 's utility obtained from a realised wage distribution \mathbf{w} depends on the magnitude of the wage he receives as a consequence of \mathbf{w} and additionally on his rank in the firm's wage distribution $r_i(\mathbf{w})$, i.e.

$$U(\mathbf{w}) = w_i + \beta \rho(r_i(\mathbf{w})), \quad \text{with } \rho'(\cdot) > 0,$$

where $\beta \geq 0$.

Given n and any realised wage distribution \mathbf{w} , *Worker i 's rank* is defined as

$$r_i(\mathbf{w}) \equiv \frac{\#j + \gamma \#k}{n - 1} \tag{1}$$

where $\#j$ is the number of workers that receive a strictly lower wage than Worker i , and $\#k$ is the number of workers that receive the same wage as Worker i , excluding worker i . Furthermore γ is a scalar between zero and one, $\gamma \in [0; 1]$. Note, Worker i 's Rank is not differentiable in any of the workers' wages. This is a modification of the status model in Dubey and Geanakoplos (2004): they take rank to be the number of people below minus the number above. Thus $\gamma = 0$ in their model⁶. We will call such preferences *rank dependent*.

Examples: If the realised wage distribution is such that $w_1 > w_2$ then Worker 1's rank is 1, Worker 2's rank is 0. If instead the realised wage distribution is $w_1 = w_2$, then Worker 1's and Worker 2's rank is γ .

Given that the number of workers n is fixed, Worker i 's direct utility from his rank r_i , $\rho(r_i)$, is assumed to be

$$\rho(r_i(\mathbf{w})) \equiv r_i(n - 1)\hat{\rho},$$

where $\hat{\rho} > 0$. This implies that Worker i obtains a direct utility of $\hat{\rho} > 0$ for each worker who receives a realised wage that is strictly smaller than his own wage. Furthermore, for each worker who receives the same wage than Worker i he receives a direct utility of $\gamma\hat{\rho}$. This allows for the fact that there is clear empirical evidence that individuals prefer to have a high rank, but there are no clear guidelines yet from

⁶Shubik (1971) uses a similar utility function for games of status. He also points out the problem in assigning points for handling ties.

the empirical literature for how individuals feel about those who have the same rank as themselves in such a hierarchical framework. For ease of analysis define $\tilde{\rho} \equiv \gamma\hat{\rho}$, then Worker i 's direct utility from his rank $\rho(r_i)$ can be expressed as

$$\rho(r_i) = (\#j)\hat{\rho} + (\#k)\tilde{\rho}. \quad (2)$$

Examples: If the realised wage distribution is such that $w_1 > w_2$ then Worker 1's direct utility from his rank is $\rho(1) = \hat{\rho}$, Worker 2's direct utility from his rank is $\rho(0) = 0$. If instead the realised wage distribution is $w_1 = w_2$, then Worker 1's and Worker 2's direct utility from their rank is $\rho(\gamma) = \tilde{\rho}$.

Assume for simplicity that $c_i(e_i) = \bar{c}e_i^l$ where $l = L, H$. Furthermore the workers' choices of effort level are simplified to either exerting effort ($e^H \equiv 1$) or not ($e^L \equiv 0$). We assume that a worker who rejects the firm's offered contract and therefore does not enter the employment relationship receives a fixed income of zero. Following Neilson and Stowe (2004, p. 10), the natural assumption is made that if a worker is not in an employment relationship then he does not compare his income to that of other workers, and hence the rank dependent component of the utility function is irrelevant. This implies that Worker i 's *reservation utility*, \bar{U} , is normalised to zero. Finally, the assumption stated below holds throughout the following analysis.

Assumption 1. Worker i 's utility from being uniquely on top of the realised wage distribution is bounded from above such that

$$\hat{\rho} < \frac{\bar{c}}{(n-1)\beta}.$$

This assumption (which may alternatively be stated as a bound on β) is made to rule out situations where the worker derives all his utility from status so that he will be ready to work for next to nothing as long as he has status⁷.

2.2 Technology and Output

There are two different states of nature. Each state of nature is characterised by a different level of output. More specifically the output levels in the two states of nature $m = H, L$ are such that $y^H > y^L$. In the following the different output levels are used to refer to the corresponding state of nature. Furthermore sometimes the adjectives good, and bad are used to refer to the different states of nature. We do not allow any correlation in agents shocks nor any technological link between them. These assumptions are made to focus on the case when there are no externalities between agents except those induced by status.

⁷This seems to be the situation in British universities which seem to take full advantage of their status seeking academics to pay them a pittance.

Assume the technology describing the stochastic relationship between effort and output is such that if $e_i = e^H$ then with probability $p^{Hm} > 0$ output is $y_i = y^m$ where $m = L, H$. However, if Worker i chooses instead effort level e^L then with probability $p^{Lm} > 0$ output is equal to $y_i = y^m$ where $m = L, H$. Thus the magnitude of output y_i depends only on the effort level chosen by Worker i and the assumed technology describing the relationship between e_i and y_i , but not on the effort level chosen by Worker j where $j, i = 1, 2$ with $j \neq i$. Additionally for readability define $\Delta p^m \equiv p^{Hm} - p^{Lm}$ for $m = L, H$. Finally, throughout the remainder of this paper the following assumption is made about the technology describing the stochastic relationship between effort and output.

Assumption 2. The technology is such that the distribution of output if a worker expends effort *first-order stochastically dominates* the distribution of output if a worker expends no effort such that

$$p^{HL} < p^{LL} \quad \text{and} \quad p^{HH} > p^{LH}.$$

2.3 Firm

The firm possesses $n = 2$ identical production facilities $i = 1, 2$, called factories. Each factory employs exactly one worker. It is assumed that Worker 1 is employed at Factory 1 and Worker 2 at Factory 2. This interpretation of the two workers is made to simplify exposition, so that we can refer to the two factories without confusion. It should not be taken too literally. In fact, any two workers in the firm who are not related through production would suffice for our model.

The firm is completely unable to monitor the workers' effort levels. The firm only observes the output level at each of the two factories.

Assuming that the firm is risk-neutral, the firm's choice problem is to choose a profile of wage contracts ω and a profile of effort levels \mathbf{e} which maximise its expected combined profit Π from Factory 1 and 2:

$$\max_{\mathbf{w}(y_1, y_2), \mathbf{e} \in \{(e_1, e_2) | e_i \in \{e^L, e^H\}\}} \sum_i \sum_m [p_i^{lm} (y_i^m - w_i(y_i^m))] \quad (3)$$

where p_i^{lm} denotes the probability of getting the outcome m when effort $e^l, l = L, H$ is exerted. subject to the workers' *individual rationality constraints*

$$E(U_i(\mathbf{w}(y_1, y_2), \mathbf{e})) \geq \bar{U} = 0,$$

the two workers' *incentive constraints*, which are case dependent and therefore are specified from case to case, and the workers' *limited liability constraints*, i.e. for $i = 1, 2$

$$w_i^m \geq 0 \quad (m = L, H), \quad (4)$$

where w_i^m is the wage paid to Worker i when the firm observes output y_i^m as specified by the wage contract.

The above profit-maximisation problem of the firm is equivalent to the following two-stage problem:

- (i) For given effort levels, e_1 and e_2 , the firm minimises its expected wage cost EWC .
- (ii) The firm maximises expected profit Π by comparing the different outcomes of stage (i) with each other.

In the following this two-stage process is applied. In most cases, for given effort levels, the focus is on the minimisation of the firm's expected wage cost EWC . Stage (ii) is normally ignored.

3 Complete Information

As a starting point in accordance with Laffont and Martimort (2002, p. 151) "... assume that the principal and a benevolent court of law can both observe effort. This variable is now verifiable and can thus be included into a contract enforced by the court of law." In the following two cases are distinguished. First, the standard complete information problem with rank-independent preferences ($\beta = 0$) is presented for completeness. Then, secondly, the case with rank-dependent preferences ($\beta > 0$) is studied. The optimal contract when effort is verifiable and hence contractable is described for the case when workers have rank-dependent preferences.

All proofs are in the Appendix.

3.1 The benchmark Problem with risk neutral parties

If Worker i 's ($i = 1, 2$) preferences are rank-independent, or in other words if $\beta = 0$ then the whole economic problem degenerates to the standard complete information problem. For each worker individually the firm has to find a wage contract ω_i which maximises its expected profit or equivalently minimises its expected wage cost, and makes the individual worker exert effort.⁸

In principle we can distinguish between many different feasible sets of contracts. The most general set is the set of dependent and asymmetric contracts, where wages can

⁸Therefore in the remainder of this section the subscript i is omitted.

be conditioned on the outcome in both factories and in addition on the identity of workers.

For the two worker case we have a convenient representation:

	H	L
H	(w_1^{HH}, w_2^{HH})	(w_1^{HL}, w_2^{HL})
L	(w_1^{LH}, w_2^{LH})	(w_1^{LL}, w_2^{LL})

The matrix denotes the four possible states of nature. It is entirely possible therefore to have 8 different wages in the most general case. We call a contract *dependent* if wages depend on shocks in both factories. Thus wages are a vector w_i^m where

$$m \in S_D = \{(H, H), (H, L), (L, H), (L, L)\}.$$

A contract is also *dependent and symmetric* if $w_i^{kl} = w_j^{lk}$ where k represents the random shock to worker i and l to worker j .

A contract is *independent* if $w_i^{kl} = w_i^k$, i.e. the wages of i are independent of the shock to worker j . Thus wages are a vector w_i^m where $m \in S_I = \{(H), (L)\}$. Finally a contract is *independent and symmetric* if $w_i^k = w_j^k$ i.e. the two workers get the same wages in the same situations. If workers are not status seeking and have no externalities on each other, the two problems are clearly separable and there is no use in having dependent or asymmetric contracts. This is what the next proposition, a standard result in this field shows.

Define p^{Hm} as the probability that Worker i is in the state m given that both workers expend effort. If the wage contracts offered by the firm are independent then $m = L, H$, i.e. there are two states of nature the good state with output level y_i^H and the bad state with output level y_i^L .

If Worker i expends effort, his *participation or individual rationality constraint* is equal to

$$\sum_m p^{Hm} w^m - \bar{c} \geq 0. \quad (5)$$

The *complete information optimal contract* \mathbf{w}^{CI} solves the following problem

$$\min_{\mathbf{w}} \sum_m p^{Hm} w^m \quad \text{subject to} \quad \sum_m p^{Hm} w^m - \bar{c} \geq 0 \text{ and } w^m \geq 0. \quad (6)$$

From (6) it is immediately obvious that there is no unique complete information optimal contract. Instead there is an infinite number of contracts that solve this optimization problem. Any wage contract with $w^m \geq 0$ such that the individual rationality constraint (5) is binding is a complete information optimal contract. The following proposition summarises this standard result of contract theory (see for instance Laffont and Martimort (2002)).

Proposition 1. *Let workers' preferences be rank-independent. Given the assumption of complete information, any wage contract \mathbf{w}^{CI} with $w^m \geq 0$ such that*

$$EWC_i(\mathbf{w}^{CI}) = \bar{c}$$

*is a complete information optimal contract. The **first-best cost** of implementing the high effort level $e^H = 1$ is*

$$C_i^{FB} = EWC_i(\mathbf{w}^{CI}) = \bar{c}.$$

Thus, any wage contract that leads to a minimum wage cost equal to \bar{c} and offers a positive wage in each state of nature is an optimal wage contract. Thus the firm can choose many different contracts, but all lead to the same expected wage cost. This implies for the model with two workers in two different factories that the firm might offer different wage contracts to different workers, but it would not improve the firm's cost structure. The firm's minimum wage cost is always $2\bar{c}$.

Examples: Two obvious examples of complete information optimal contracts, w^{CI} , are the *state-independent* complete-insurance independent wage contract,

$$\bar{\mathbf{w}} = \{\bar{c}, \bar{c}\},$$

and the complete insurance dependent wage contract,

$$\bar{\mathbf{w}} = \{\bar{c}, \bar{c}, \bar{c}, \bar{c}\}.$$

3.2 Rank-Dependence and First-Best

The firm has the choice between choosing independent contracts, or dependent contracts. Moreover, the firm can either offer all workers the same wage contract. or the firm can offer each worker a different wage contract.

The most general contract is the asymmetric dependent one. Thus wages are a vector w_i^m with $m \in S_D = \{(HH), (HL), (LH), (LL)\}$. Let q^m denote the joint probability of event m given that both workers exert effort. Thus e.g. $q^{HH} = (p^{HH})^2$. Similarly w_i^m denotes the wage to worker i when the state is m .

Given that effort is verifiable, the principal's problem is

$$\min_{\mathbf{w}} EWC = \sum_i [q^m w_i^m]$$

subject to the limited liability constraint (4) and the participation constraints

$$\sum_m q^m w_i^m \geq \bar{c} - \beta E(\rho(r_i) | \mathbf{w}), \quad (7)$$

where in the case of $n = 2$ workers

$$E(\rho(r_i)|\mathbf{w}) = q^m \rho(r_i(w_i^m, w_j^m)) \quad (8)$$

It is obvious that with rank-dependent utilities, and for a given type (i.e. symmetric or asymmetric, dependent or independent) of contract the firm's problem can be solved in two steps. Firstly, the firm chooses the structure of \mathbf{w} that minimises $\bar{c} - \beta E(\rho(r_i)|\mathbf{w})$ respectively maximises $E(\rho(r_i)|\mathbf{w})$. Then given the optimal structure of \mathbf{w} , the complete information optimal contract for workers with rank-dependent utilities is chosen such that the participation constraint is binding and the limited liability constraint respected.

In the section below we first examine symmetric and independent contracts as they are easy to characterise. We show too that no symmetric contract even if it is dependent can do better. Finally we provide a characterisation of the optimal Asymmetric independent contract and show that no dependent contract can do better.

3.2.1 Symmetric and Independent Wage Contracts

Let $m \in S_I = \{H, L\}$ denote the two states of nature on which contracts are conditioned (when these are independent). The principal's problem is

$$\min_{\mathbf{w}} EWC = 2 \sum_m p^{Hm} w^m$$

subject to the limited liability constraint (4) and the participation constraint

$$\sum_m p^{Hm} w^m \geq \bar{c} - \beta E(\rho(r_i)|\mathbf{w}), \quad (9)$$

where in the case of $n = 2$ workers

$$\begin{aligned} E(\rho(r_i)|\mathbf{w}) &= p^{HH} [p^{HH} \rho(r_i((w_i^H, w_j^H))) + (1 - p^{HH}) \rho(r_i(w_i^H, w_j^L))] \\ &+ (1 - p^{HH}) [(p^{HH} (\rho(r_i(w_i^L, w_j^H))) + (1 - p^{HH}) (\rho(r_i(w_i^L, w_j^L))))] \end{aligned}$$

Note, given that the firm offers symmetric contracts the two workers have the same participation constraint.

We apply the two-step method for finding the optimal contract. First, the optimal structure of \mathbf{w} is studied. The following two lemmas establish the optimal structure of \mathbf{w} . The first lemma describes the expected rank-utility of wage contracts with a non-egalitarian structure, i.e. wage structures that pay a different wage in each state of nature.

Lemma 1. *Assume all workers exert effort. Given any profile of symmetric and independent wage contracts ω^\neq with a non-egalitarian structure, i.e. $w^H \neq w^L$, the expected rank-utility is the same for all possible ω^\neq and equal to*

$$E(\rho(r_i)|\omega^\neq) = \tilde{\rho} + p^{HH} (1 - p^{HH}) (\hat{\rho} - 2\tilde{\rho}). \quad (10)$$

The reasoning behind Lemma 1 is easy: consider a symmetric independent contract: this imposes the conditions $w_1^{kl} = w_1^k$, $w_2^{lk} = w_2^l$ i.e. the wage for each agent depends only on his own shock. Moreover we require $w_1^k = w_2^k$ so $w_1^H = w_2^H$, and $w_1^L = w_2^L$, by symmetry. Hence we are restricted to considering matrices where the following rank payoff entries are already chosen:

	H	L
H	$(\tilde{\rho}, \tilde{\rho})$	(X, Y)
L	(Y, X)	$(\tilde{\rho}, \tilde{\rho})$

and X, Y denote the ranks corresponding to the choice of $w_H > (\leq) w_L$. In the non-egalitarian contract either we have $w_H > w_L$ or vice-versa and in either case the expected cost is as given by equation (10). If it is an egalitarian contract then $w_H = w_L$ and the total expected cost is given as $\bar{c} - \beta\tilde{\rho}$.

Thus, the expected rank-utility of a profile of symmetric and independent wage contracts with an egalitarian structure, $w^H = w^L$, is

$$E(\rho(r_i)|\omega^\bar{=}) = \tilde{\rho}.$$

Next, Lemma 2 establishes the conditions under which a profile of symmetric and independent wage contracts with a non-egalitarian structure leads to a higher expected rank-utility than a profile of symmetric and independent wage contracts with an egalitarian structure.

Lemma 2. *Assume all workers expend effort and $0 < p^{HH} < 1$. Then,*

$$\frac{1}{2}\hat{\rho} > (\leq)\tilde{\rho}$$

iff

$$E(\rho(r_i)|\omega^\neq) > (\leq)E(\rho(r_i)|\omega^\bar{=}).$$

The following proposition describes the optimal symmetric and independent wage contract in the case of rank-dependent preferences if effort is verifiable.

Proposition 2. Assume $0 < p^{HH} < 1$. (i) let a worker's direct utility from being paid the same wage as another worker be smaller than half the direct utility from receiving a higher realised wage, i.e.

$$\tilde{\rho} < \frac{1}{2}\hat{\rho},$$

then with rank-dependent preferences the complete information optimal symmetric and independent contract possesses a non-egalitarian structure, i.e. $w^H \neq w^L$. Given rank-dependent utilities the **first-best cost** of implementing the high effort level $e^H = 1$ is

$$EWC(\mathbf{w}^{CI}|\omega^\neq) = 2\{\bar{c} - \beta\{\tilde{\rho} + p^{HH}(1 - p^{HH})(\hat{\rho} - 2\tilde{\rho})\}\}. \quad (11)$$

(ii) Let a worker's direct utility from being paid the same wage as another worker be higher than half the direct utility from receiving a higher realised wage, i.e.

$$\tilde{\rho} > \frac{1}{2}\hat{\rho},$$

then with rank-dependent utilities there is a unique complete information optimal symmetric and independent contract which possesses an egalitarian structure,

$$(w^H, w^L) = (\bar{c} - \beta\tilde{\rho}, \bar{c} - \beta\tilde{\rho})$$

Given rank-dependent utilities the **first-best cost** of implementing the high effort level $e^H = 1$ is

$$EWC(\mathbf{w}^{CI}|\omega^\equiv) = 2\{\bar{c} - \beta\tilde{\rho}\}.$$

Proof: This proposition follows directly from Lemma 1 and Lemma 2.

□

Three remarks can be made here. First notice that if workers are risk averse, non-egalitarian wage contracts may not be desirable even if the conditions of Proposition (2) are satisfied.

Second, we could ask whether the presence of status seeking agents causes *wage compression*. Wage Compression in this setting occurs when in the optimal (symmetric independent) contract, the wage difference $w_i^H - w_i^L$ is lower with rank dependent preferences than in the benchmark case.

Such a result is shown e.g. in Neilson and Stowe (2004) for whom the key driving force that causes wage compression is *behindness aversion* i.e. *changes* in payoff matter more to the worker when he is behind than when he is ahead of co-workers.

In our model, we interpret behindness aversion as the following: Starting from a position of equal wages, w , being ahead by an amount $x > 0$, generates a utility of $\hat{\rho} - \tilde{\rho}$, while being behind by an amount $x > 0$ generates a disutility of $\tilde{\rho}$. Thus a worker is behindness averse iff $\frac{\hat{\rho}}{2} < \tilde{\rho}$.

Comparing the optimal contract with rank dependent preferences and the optimal contract in the benchmark case, we do have a wage compression result that holds *independently* of behindness aversion. Intuitively if the firm can design contracts such that both agents get a positive utility from status then the expected wages that have to be paid out can be reduced commensurately.

Third, we might well ask if we could lower the expected costs of the principal even further with dependent and asymmetric contracts.

First we claim that with dependent *symmetric* contracts we can do no better than Proposition (2). To see this note that symmetry imposes the condition that $w_1^{HH} = w_2^{HH}, w_1^{LL} = w_2^{LL}$ and $w_1^{HL} = w_2^{HL}, w_1^{LH} = w_2^{LH}$. This gives us the following rank matrix which is the same as the matrix with symmetric and independent contracts⁹.

	H	L
H	$(\tilde{\rho}, \tilde{\rho})$	(X, Y)
L	(Y, X)	$(\tilde{\rho}, \tilde{\rho})$

Thus dependence by itself does not buy us any extra degrees of freedom.

Can we do better with Asymmetric dependent contracts? We show that while Asymmetry is crucial, the dependent contract is redundant. We can do no better with a dependent contract than with an independent asymmetric one. The way we do this is to first consider the optimal Asymmetric independent contract and show that this achieves the lowest possible cost, i.e. it is not possible to achieve costs lower than this level. The implication is that no dependent contract can do strictly better. This is shown in the next subsection.

3.3 Independent Asymmetric Contracts

With independent contracts we can have four different wages: $w_1^H, w_1^L, w_2^H, w_2^L$. Note that either $w_i = w_j$ in which case both get a rank utility of $\tilde{\rho}$, or $w_i > w_j$ in which case one worker gets $\hat{\rho}$ and the other worker gets zero. Thus if we maximise the incentives from status for one player by giving him a higher rank, the other player's

⁹Hence, the minimum feasible expected wage cost cannot be lower than that with the symmetric independent contracts.

incentives are adversely affected for that combination of shocks. The expected wage cost of the firm, EWC , depend on both participation constraints such that

$$EWC \geq 2\bar{c} - \beta (E(\rho(r_1)|\mathbf{w}) + E(\rho(r_2)|\mathbf{w})).$$

Note, the minimum expected wage cost possible for the firm, EWC^{min} are equal to $2\bar{c} - \beta\hat{\rho}$ if $(1/2)\hat{\rho} > \tilde{\rho}$ and equal to $2(\bar{c} - \beta\tilde{\rho})$ if $(1/2)\hat{\rho} < \tilde{\rho}$.

The following theorem establishes that the assumption of independent contracts allows the firm to formulate a profile of wage contracts such that it has to pay minimum expected wage cost as long as the individual wage contracts are allowed to be asymmetric.

Theorem 1. *Assume all workers expend effort. Then with complete information for all $0 < p^{HH} < 1$, there exists a profile of (optimal) independent wage contracts that guarantees the firm minimum wage cost EWC^{min} as the **first-best cost**.*

From Theorem 1 it follows that under certain conditions the optimal contract is asymmetric.

Corollary 1. *Assume $0 < p^{HH} < 1$, let a worker's direct utility from being paid the same wage as another worker be smaller than half the direct utility from receiving a higher realised wage, i.e.*

$$\tilde{\rho} < \frac{1}{2}\hat{\rho},$$

then there exists a profile of asymmetric and independent contracts $\tilde{\omega}$ which does strictly better than any symmetric contract, where

$$\tilde{\omega}_1 = \left\{ \frac{\bar{c} - \beta p^{HH} \hat{\rho}}{p^{HH}}, 0 \right\},$$

$$\tilde{\omega}_2 = \{ \bar{c} - \beta (1 - p^{HH}) \hat{\rho}, \bar{c} - \beta (1 - p^{HH}) \hat{\rho} \}.$$

whenever $p^{HH} \leq \bar{p}^{HH} = \frac{\sqrt{5}-1}{2}$

and

$$\tilde{\omega}_1 = \left\{ 0, \frac{\bar{c} - \beta (1 - p^{HH}) \hat{\rho}}{1 - p^{HH}} \right\},$$

$$\tilde{\omega}_2 = \{ \bar{c} - \beta p^{HH} \hat{\rho}, \bar{c} - \beta p^{HH} \hat{\rho} \}.$$

whenever $p^{HH} \geq \underline{p}^{HH} = \frac{3-\sqrt{5}}{2}$.

The following remark follows from the above theorem.

Remark: With complete information such that effort is verifiable independent contracts allow the firm to achieve whatever it can achieve with dependent asymmetric contracts as long as it has the ability to choose independent asymmetric contracts.

Thus, in the case of observable effort we find that under some conditions the optimal contract is asymmetric and independent. What can we say when effort is unobservable? This is what the next section addresses.

4 Moral Hazard with Risk Neutral parties

Assume the firm's aim is to minimise its expected wage cost and to induce both workers to expend effort, $e_1 = e_2 = 1$. If the firm is unable to observe its workers' actions, i.e. their choice of effort, directly, then the firm can offer only a contract that is based on the observable and therefore verifiable output levels of the two factories. However again the firm has the choice between dependent or independent contracts, and symmetric or asymmetric contracts.

The remainder of this section is structured as follows. First, the standard moral-hazard problem is presented. As in the complete information case when there is no rank-dependence in utilities, then the two factories problems are separable and so independent symmetric contracts will do as well as the most general contracts. So in this section we only consider independent and symmetric contracts.

We then study the moral-hazard problem in an environment of rank-dependent preferences. Here, first, we focus on symmetric contracts, but later we show that in some circumstances the firm can improve its situation by offering asymmetric independent contracts. We also show that symmetric dependent contracts can do no better than independent contracts.

4.1 The Benchmark Moral-Hazard Problem

If Worker i 's ($i = 1, 2$) utility is rank-independent, or in other words if $\beta = 0$, then the whole economic problem degenerates to the standard moral-hazard problem. For each of its factories, the firm's strategy is to find a wage contract ω_i which maximises its expected profit or equivalently minimises its expected wage cost, and makes the individual worker exert effort at Factory i .¹⁰

Let $m \in S_I$. With incomplete information, i.e. if Worker i 's effort level is not

¹⁰Therefore in the remainder of this section the subscript i is omitted.

verifiable, the problem of the firm is to find a wage contract \mathbf{w} that minimises

$$\min_{\mathbf{w}} EWC = \sum_m p^{Hm} w^m,$$

where EWC represents the expected wage cost at Factory i , subject to the limited liability constraint (4), and Worker i 's *incentive constraint*¹¹:

$$\Delta p^H w^H + \Delta p^L w^L \geq \bar{c}$$

or using the fact that $\Delta p^L = -\Delta p^H$

$$\Delta p^H [w^H - w^L] \geq \bar{c}. \quad (12)$$

The following proposition describes the optimal contract for the standard moral-hazard problem. In its main part it repeats Proposition 1 of Itoh (2004).

Proposition 3. *For all $m = L, H$ and risk neutral parties the unique optimal contract solving the standard moral-hazard problem is*

$$\mathbf{w}^S = \{w^H, w^L\} = \left\{ \frac{\bar{c}}{\Delta p^H}, 0 \right\},$$

and the expected cost of implementing effort is

$$EWC_i(\mathbf{w}^S) = \frac{p^{HH} \bar{c}}{\Delta p^H}$$

in Factory i . The firm's overall expected cost of implementing effort is $2EWC_i(\mathbf{w}^S)$.

4.2 Rank-Dependence and Moral Hazard

If effort is no longer observable and verifiable, and workers possess rank-dependent preferences then the firm's choice problem changes. Like in the standard moral-hazard problem the firm has to base its contracts on the observable variable output. However, with rank-dependent preferences, when offering a wage contract to one worker the firm has to take into account the likely effects of this contract on the other worker.

The most general type of contract that can be offered is (as before) the Asymmetric Dependent one.

The firm's problem is to find contracts that will be conditioned on workers identity and the shocks in both factories to induce both workers to exert effort while minimising its expected wage cost:

¹¹Remember the individual rationality constraint is implied by the limited liability constraint and the incentive constraint.

$$\min_{\omega} EWC = \sum_i \{p^{HH}[p^{HH}w_i^{HH} + (1-p^{HH})w_i^{HL}] + (1-p^{HH})[p^{HH}w_i^{LH} + (1-p^{HH})w_i^{LL}]\}$$

subject to the limited liability constraints, which follow from (4),

$$w_i^m \geq 0.$$

with $m \in S_D$, and the incentive constraints ($i, j = 1, 2$ with $j \neq i$)

$$\Delta p^H [\{p^{HH}w_i^{HH} + (1-p^{HH})w_i^{HL}\} - \{p^{HH}w_i^{LH} + (1-p^{HH})w_i^{LL}\}] \geq \bar{c} - \beta E(\Delta\rho_i), \quad (13)$$

where $E(\Delta\rho_i)$ is Worker i 's expected gain or loss in expected rank-utility from expending effort. Let $E(\rho_i^H) = \{p^{HH}(\rho(r_i(w_i^{HH}, w_j^{HH}))) + (1-p^{HH})(\rho(r_i(w_i^{HL}, w_j^{HL})))\}$. This is the expected rank payoff when the worker i is in the good state. and $E(\rho_i^L) = \{p^{HH}(\rho(r_i(w_i^{LH}, w_j^{LH}))) + (1-p^{HH})(\rho(r_i(w_i^{LL}, w_j^{LL})))\}$. This is the expected rank payoff when the worker i is in the bad state.

Then,

$$E(\Delta\rho_i) \equiv E(\rho(r_i)|e^H) - E(\rho(r_i)|e^L) = \Delta p^H [E(\rho_i^H) - E(\rho_i^L)],$$

Note the individual rationality constraints can be ignored in this framework because they are implied by the limited liability constraints and the incentive constraints.¹²

Define the *incentives from status* as follows:

$$\begin{aligned} I_i(\mathbf{w}) &= E(\Delta\rho(r_i)|\mathbf{w}) \\ &= (\Delta p^H)[p^{HH}\{\rho(r_i(w_i^{HH}, w_j^{HH})) - \rho(r_i(w_i^{LH}, w_j^{LH}))\} \\ &\quad + (1-p^{HH})\{\rho(r_i(w_i^{HL}, w_j^{HL})) - \rho(r_i(w_i^{LL}, w_j^{LL}))\}] \end{aligned}$$

Note that $\rho(r_i(w_i^m, w_j^m)) = \hat{\rho}$ if $w_i^m > w_j^m$, $\rho(r_i(w_i^m, w_j^m)) = \tilde{\rho}$ if $w_i^m = w_j^m$, and $\rho(r_i(w_i^m, w_j^m)) = 0$ if $w_i^m < w_j^m$, for any $m \in S_D$.

Obviously it is easier to characterise the optimal independent contract. We believe that going to dependent contracts cannot help the firm to do any better than with independent contracts. It is easy to see that this is true for *symmetric contracts* as the following proposition shows:

Proposition 4. *Suppose that the cost minimising optimal Symmetric Dependent (SD) contract is given by w_i^{m*} for $i = 1, 2$ and $m \in S_D$. The principal can achieve the same expected cost by a Symmetric Independent (SI) contract with wages w_i^H, w_i^L , $i = 1, 2$, where $w_i^H = p^{HH}(w_i^{HH*}) + (1-p^{HH})w_i^{HL*}$ and $w_i^L = p^{HH}(w_i^{LH*}) + (1-p^{HH})(w_i^{LL*})$.*

¹²For the sketch of a proof see Laffont and Martimort (2002, p. 164).

For Asymmetric contracts, the proof would require us to first derive the optimal rank payoff matrix. Given this we could construct a corresponding independent asymmetric contract in much the same way as we constructed the SI one. We conjecture that dependent contracts cannot do better than independent contracts and we present below a heuristic argument to find the rank payoff matrix corresponding to the optimal Asymmetric Dependent contract:

The firm has to satisfy the incentive constraints of both workers. In order to maximise each worker's incentive from status, the firm realises the strictly competitive nature of the game of status¹³. Thus, if the status incentives improve for one worker they must decrease for the other worker and that constraint will become binding. Thus we need to choose the order of wages that achieves the $\max_i \min_j (I_i(\mathbf{w}))$ by appropriate choice of the pairwise orders w_i^m, w_j^m . If we follow this procedure and find the orders that maximise the minimum incentives for each worker, we can find the optimal rank payoff matrix. Then we simply follow the same construction as that in proposition (4) above.

Henceforth we focus on Independent contracts.

4.3 Independent Contracts

Thus the firm's problem is now to find independent contracts that induce both workers to exert effort and minimise its expected wage cost, i.e.

$$\min_{w_i^m} EWC = \sum_i \{p^{HH} w_i^H + (1 - p^{HH}) w_i^L\}$$

subject to the limited liability constraints, which follow from (4),

$$w_i^m \geq 0.$$

and the incentive constraints ($i, j = 1, 2$ with $j \neq i$)

$$\Delta p^H [w_i^H - w_i^L] \geq \bar{c} - \beta E(\Delta \rho_i), \quad (14)$$

where $E(\Delta \rho_i)$ is Worker i 's expected gain or loss in expected rank-utility from expending effort. For independent contracts this is

$$E(\Delta \rho_i) \equiv E(\rho(r_i)|e^H) - E(\rho(r_i)|e^L) = \Delta p^H E(\rho_i^H) + \Delta p^L E(\rho_i^L),$$

where $E(\rho_i^m)$ represents Worker i 's expected rank-utility conditional on receiving wage w_i^m , in short expected conditional rank-utility.

¹³Notice that the game of status is (conceptually) a zero sum game. Our payoffs from rank are not consistent with this, but we could choose the payoff from being behind as $-\hat{\rho}$ and the payoffs from being equal as being 0.

With only two states of nature $\Delta p^L = -\Delta p^H$ and hence

$$E(\Delta \rho_i) = \Delta p^H \{E(\rho_i^H) - E(\rho_i^L)\}$$

where

$$\begin{aligned} E(\rho_i^H) - E(\rho_i^L) &= [p^{HH} \rho_i(w_i^H, w_j^H) + (1 - p^{HH}) \rho_i(w_i^H, w_j^L)] \\ &\quad - [p^{HH} \rho_i(w_i^L, w_j^H) + (1 - p^{HH}) \rho_i(w_i^L, w_j^L)] \\ &= [p^{HH} (\rho_i(w_i^H, w_j^H) - \rho_i(w_i^L, w_j^H))] \\ &\quad + (1 - p^{HH}) [(\rho_i(w_i^H, w_j^L) - \rho_i(w_i^L, w_j^L))] \end{aligned}$$

Note the individual rationality constraints can be ignored in this framework because they are implied by the limited liability constraints and the incentive constraints.¹⁴

There are now two possibilities for the firm to approach this problem. The firm can either offer symmetric independent contracts or asymmetric independent contracts. With symmetric independent contracts the wage paid out to the workers depends only on the observed output of the respective factory the worker is working for, but both workers receive the same wage contract. With asymmetric independent wage contracts the wage paid out to the workers depends on the observed output of the respective factory the worker is working for, and additionally each worker receives a different wage contract.

We first look for the optimal symmetric independent contract in the next section.

4.3.1 Symmetric Independent Contracts

The focus of this part is on symmetric independent contracts. With symmetric contracts each worker receives the same contract, i.e. in the following it is assumed

$$\mathbf{w}_1 = \mathbf{w}_2 = (w^H, w^L).$$

The firm's problem is to find the symmetric contracts that induces both workers to expend effort and minimises its expected wage cost, i.e.

$$\min_w EWC = 2 \{p^{HH} w^H + (1 - p^{HH}) w^L\}$$

subject to the incentive constraint, which follow from (14) after some rearranging,

$$[w^H - w^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} (\tilde{\rho} - \rho^L(w^H)) + (1 - p^{HH}) (\rho^H(w^L) - \tilde{\rho})\}. \quad (15)$$

and the limited liability constraints, which follows from (4),

$$w^H, w^L \geq 0.$$

¹⁴For the sketch of a proof see Laffont and Martimort (2002, p. 164).

We now need to find the order of wages w^H, w^L which maximises the incentives of both workers. The rank payoff matrix has less flexibility now as symmetry imposes the conditions that $w_1^k = w_2^k$ for $k = H, L$. The rank payoff matrix is as below with the only choice being whether to choose $w^H \geq w^L$ or vice versa.

	H	L
H	$\tilde{\rho}, \tilde{\rho}$	X, Y
L	Y, X	$\tilde{\rho}, \tilde{\rho}$

with $X = \hat{\rho}, Y = 0$ if $w^H > w^L$ and $X = 0, Y = \hat{\rho}$ if $w^H < w^L$. Clearly incentives are maximised if $w^H > w^L$. This is what the next proposition shows.

The following proposition characterises the optimal symmetric independent contract that minimises the firm's expected wage cost.

Proposition 5. *The unique optimal symmetric independent wage contract that minimises the firm's expected wage cost is*

$$\mathbf{w}^{S^*} \equiv (w^H, w^L)^{S^*} = \left(\frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} \tilde{\rho} + (1 - p^{HH}) (\hat{\rho} - \tilde{\rho})\}, 0 \right).$$

Given the optimal symmetric wage contract, \mathbf{w}^{S^*} , the firm's expected wage cost are

$$EWC^S = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} \tilde{\rho} + (1 - p^{HH}) (\hat{\rho} - \tilde{\rho})\} \right\}. \quad (16)$$

As with the complete information case, we see a wage compression in this setting as well: the high wage is smaller than in the benchmark case without rank dependence.

The question now arises: is this the best contract possible or as in the complete information case does there exist an asymmetric contract that does strictly better than the optimal symmetric contract? The next section analyses this question.

4.3.2 Asymmetric Contracts

In the previous section the unique optimal symmetric independent contract has been described. The aim of this part is to answer the question whether the firm can improve its outcome by offering asymmetric contracts. In other words is there a profile of asymmetric contracts which can make the firm better off than the unique optimal symmetric contract described in Proposition 5.

Define an order on wages $w_1^H, w_1^L, w_2^H, w_2^L$ as an *wage structure*. Observe that by Assumption 1 we must always have $w_i^H > w_i^L$. Hence there are only a limited

number of such structures that are feasible. Surprisingly, we find that all but one of the feasible wage structures that are implied by asymmetric contracts lead to higher wage costs for the firm than the symmetric optimal contract. The next Lemma characterises all such wage structures implied by asymmetric contracts.

Lemma 3. *There is no profile of asymmetric independent contracts ω^{AS} such that either $\mathbf{w}_i^{AS} \gg \mathbf{w}_j^{AS}$ or $\mathbf{w}_i^{AS} \geq \mathbf{w}_j^{AS}$ or $w_i^H > w_j^H > w_j^L > w_i^L$ ($i, j = 1, 2$ with $i \neq j$) which induces both workers to exert effort and makes the firm better off than with the optimal symmetric independent contract, i.e. $EWC(\omega^{AS}) \geq EWS^S$ for all ω^{AS} .*

Given this negative result, we are left with only one possible wage structure that could possibly be used to construct an asymmetric contract that is better than the optimal symmetric contract. This wage structure is $w_i^H > w_j^H > w_i^L > w_j^L$ ($i, j = 1, 2$ with $i \neq j$).

The following proposition describes the minimum expected wage cost with an asymmetric contract with the wage structure given above.

Proposition 6. *There exists a profile of asymmetric independent wage contracts ω^{AS*} with a asymmetric implied structure that is strictly increasing such that ($i, j = 1, 2$ with $j \neq i$)*

$$w_i^H > w_j^H > w_i^L > w_j^L,$$

which induces all workers to expend effort and has expected wages costs $EWC(\omega^{AS})$ given as:*

If $p^{HH} \leq \frac{1}{2}$:

$$EWC(\omega^{AS*}) = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \frac{\hat{\rho}}{2} \right\} + \epsilon. \quad (17)$$

If $p^{HH} > \frac{1}{2}$:

$$EWC \geq 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{1 - p^{HH}\} \hat{\rho} \right\} + \epsilon \quad (18)$$

where $\epsilon > 0$ but equal to the smallest monetary unit.

The following theorem establishes the conditions under which a profile of asymmetric contracts does strictly better than the optimal symmetric contract.

Theorem 2. Let $p^{HH} < 1/2$. Then if Worker i 's direct utility from being paid the same wage as another worker is higher than half the direct utility from receiving a higher realised wage, i.e.

$$\frac{1}{2}\hat{\rho} < \tilde{\rho},$$

then there exists an $\bar{\epsilon} \equiv \beta p^{HH} [1 - 2p^{HH}] [2\tilde{\rho} - \hat{\rho}]$, such that if $\epsilon < \bar{\epsilon}$, then

$$EWC(\omega^{AS*}) < EWC(\omega^{S*}).$$

The intuition behind this result is as follows. Starting from symmetric contracts can the firm do better by switching to an asymmetric contract? With the optimal asymmetric contract one worker has always higher expected status. For ease of exposition we assume this is Worker 1. However what is crucial for the firm is not the expected status of a worker, but the incentives from status. Depending on the parameters it can be either Worker 1 or Worker 2 whose incentives from status increase after the switch to an asymmetric contract. If it is Worker 1 who gains in terms of incentives at the expense of Worker 2 then the firm cannot profit from a switch to an asymmetric contract, because it is the wages of Worker 2 that provide a lower bound for the wages of Worker 1. However note that this lower bound increases due to the switch. Thus, it is only in the case where Worker 2 gains in terms of incentives from status that the firm can benefit from switching to an asymmetric contract. Theorem 2 provides the conditions under which it is indeed Worker 2 who gains in incentives from status while Worker 1 gains in terms of the expected status.

Therefore under some circumstances it might be profitable for the firm to switch to the asymmetric contract.

5 Concluding Remarks

Much of contract theory has focused on agents who are self interested. What happens to the theory of optimal contracts when this assumption is relaxed? In particular, what happens when workers care about their *rank*? We investigate this question in a simple model with two status-seeking agents. The model we use is the standard moral hazard model used in the literature but with status seeking agents. We show that then the problem of finding the optimal contract involves (1) Maximising the incentives from status given a particular distribution of the stochastic shocks to the two workers: here we are essentially designing a game of status Shubik (1971) to maximise the incentives of both workers. (2) Look for the levels of wages that minimise expected wage costs and satisfy the Incentive Constraints.

Our main results are illustrated in our simple two agent model. With some standard assumptions we show that Asymmetric contracts or discriminatory contracts

where the two agents are offered different contracts even though they are ex-ante identical, dominate symmetric contracts in the case of observable effort. In the case of unobservable effort, we find circumstances under which asymmetric contracts do better. Moreover, no symmetric contract *even if it is dependent* can replicate the discriminatory one. This result relies on the assumption that the rank payoff from being ahead is higher by a critical amount than the rank payoff from being at the same level.

How robust is this result to different specifications of rank payoffs? Intuitively, the result relies on the fact that there are enough states of nature that any given agent can be both ahead in some states and behind or equal in others¹⁵. Moreover since the high state is more likely with high effort we need to design the rank payoffs so that each agent has a chance of being ahead in the high state (for him) and being behind in the low state (for him). Thus, putting in effort increases the chances of being ahead and reduces the chances of being behind. This effect can only be reinforced if we make the rank payoff from being ahead higher and the rank payoff from being behind lower¹⁶. The only crucial factor is the relation of the rank payoff from being equal compared to that of being ahead or behind.

Our result relies on being able to pay one worker just a little bit more than the other (the ϵ in our story). A criticism that might be levelled at us is that people perceive rank differences only when there are noticeable or significant differences in wages (see e.g. Shubik (1971)). We have two answers to this. First, we refer to anecdotal evidence from A. Oswald who cites the story of Professor X who refused a job in a top university because he was paid a wage \$ 10 below that of the (then) highest paid professor. In other words, the satisfaction of being top ranked comes from the fact that this hierarchy in wages is common knowledge. This is what we assume in our model (contracts have to be common knowledge to both workers). Second, what we suggest (like Winter (2004)) is that there may be benefits to introducing an artificial hierarchy between workers even when the job is ex-ante identical – again it is the common knowledge about status that is important rather than the actual wage differences. Indeed Baron (1988) suggests that reference actors are people who are not too different from oneself in terms of pay. Pay differences matter more when they are across people in the same job title.

Is it better to have status seeking agents as far as the principal is concerned? We might argue that the extra utility that agents get from rank might cause the total expected wages paid out to be *lower* than in the case of agents who are not status seeking (this is the *wage compression* result that is discussed by many authors in this area (e.g. Frank (1984), Neilson and Stowe (2004))). Let us compare symmetric

¹⁵In this respect our result parallels the observation in Dubey and Geanakoplos (2004) that to maximise incentives to students from working some randomness must be introduced in the payoffs from rank. However in our context the randomness is given as part of the moral hazard set up and we can only play with the ranks.

¹⁶We have to take some care to ensure that participation constraints are satisfied.

contracts with and without status seeking agents. It turns out that the wage compression result holds in our model both with observable effort and with unobservable effort. This is quite intuitive in that when agents get some utility from rank (in the cases when the state is different across workers), they need to be paid less to exert effort.

We conclude this paper with some ideas for extensions of this work. One obvious extension is to investigate the case of status seeking agents who are not identical (adverse selection). Another interesting question is that of information about wage scales. Why do we observe e.g. that many organisations give broad information about wages (i.e. the bands within which the wages for a given job title lie) but not detailed information (e.g. which employee is getting how much? Is this a case of making the reference group endogenous? How are these bands chosen? We hope to tackle these questions in future work.

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A Appendix: Proof of Lemma 1

Rename the two states of nature in an arbitrary way, i.e. $m = A, B$. Let p^m be the probability that Worker i is in state m given that he expends effort. Take the wage contract \mathbf{w}^{AB} with the non-egalitarian structure $w^A > w^B$. Let ω^{AB} be the profile of symmetric contracts that offers each worker \mathbf{w}^{AB} . Then Worker i 's expected rank utility is

$$E(\rho(r_i)|\omega^{ABC}) = \{Pr(w_i > w_j|\omega^{AB}) \hat{\rho} + Pr(w_i = w_j|\omega^{AB}) \tilde{\rho}\},$$

where $Pr(w_i > w_j|\omega^{ABC})$ is the probability that Worker i 's wage is higher than Worker j 's given ω^{AB} . $Pr(w_i = w_j|\omega^{AB})$ is then the probability that Worker i 's wage is equal to Worker j 's wage given ω^{ABC} . Note,

$$Pr(w_i > w_j|\omega^{AB}) = p^A p^B$$

and

$$Pr(w_i = w_j|\omega^{AB}) = (p^A)^2 + (p^B)^2.$$

Worker i 's expected rank utility is

$$\begin{aligned} EWC(\rho(r_i)|\omega^{AB}) &= p^A p^B \hat{\rho} + [(p^A)^2 + (p^B)^2] \tilde{\rho} \\ &= p^A p^B \hat{\rho} - 2p^A p^B \tilde{\rho} + [(p^A)^2 + 2p^A p^B + (p^B)^2] \tilde{\rho} \\ &= p^A p^B (\hat{\rho} - 2\tilde{\rho}) + \tilde{\rho}. \end{aligned}$$

Given that the two states of nature have been renamed in an arbitrary way and the fact that $p^A + p^B = 1$ this completes the proof of Lemma 1.

B Appendix: Proof of Lemma 2

Note, $E(\rho(r_i)|\omega^=) = \tilde{\rho}$ and from Lemma 1 it is known that $E(\rho(r_i)|\omega^\neq) = \tilde{\rho} + p^{HH} (1 - p^{HH}) (\hat{\rho} - 2\tilde{\rho})$.

Then $E(\rho(r_i)|\omega^\neq) > (\leq) E(\rho(r_i)|\omega^=)$ iff

$$\begin{aligned} \tilde{\rho} + p^{HH} (1 - p^{HH}) (\hat{\rho} - 2\tilde{\rho}) &> (\leq) \tilde{\rho} \\ \frac{1}{2}\hat{\rho} &> (\leq) \tilde{\rho}. \end{aligned}$$

C Appendix: Proof of Theorem 1

Proof: It is sufficient to show Proposition 1 holds when $(1/2)\hat{\rho} > \tilde{\rho}$, since when $(1/2)\hat{\rho} < \tilde{\rho}$, Proposition 2 already shows the result (the symmetric independent contract is sufficient).

Consider the profile of asymmetric and independent wage contracts, $\tilde{\omega}$ with

$$\tilde{w}_1^H \equiv \frac{\bar{c} - \beta p^{HH} \hat{\rho}}{p^{HH}}.$$

$$\tilde{w}_1^L = 0, \text{ and } \tilde{w}_2^H = \tilde{w}_2^L = \tilde{w}_2 \equiv \bar{c} - \beta (1 - p^{HH}) \hat{\rho}.$$

$$\text{whenever } p^{HH} \leq \bar{p}^{HH} = \frac{\sqrt{5}-1}{2}$$

and

$$\tilde{w}_1^H = 0, \text{ and } \tilde{w}_2^H = \tilde{w}_2^L = \tilde{w}_2 \equiv \bar{c} - \beta p^{HH} \hat{\rho}.$$

$$\tilde{w}_1^L \equiv \frac{\bar{c} - \beta (1 - p^{HH}) \hat{\rho}}{(1 - p^{HH})}.$$

$$\text{whenever } p^{HH} \geq \underline{p}^{HH} = \frac{3-\sqrt{5}}{2}.$$

We show that this is the optimal asymmetric contract that minimises expected wage costs and moreover that it leads to EWC^{min17} .

Let $p^{HH} \leq \bar{p}^{HH} \frac{\sqrt{5}-1}{2}$. Observe that given the profile $\tilde{\omega}$, $EWC = EWC^{min}$ if the order of wages satisfies:

$$\begin{aligned} \tilde{w}_1^H &> \tilde{w}_2^H \\ \tilde{w}_2^L &> \tilde{w}_1^L. \end{aligned}$$

(since then worker 1's expected rank utility is $E(\rho(r_1)|\tilde{\omega}^A) = p^{HH} \hat{\rho}$. Worker 2's expected rank utility is $E(\rho(r_1)|\tilde{\omega}^A) = (1 - p^{HH}) \hat{\rho}$).

It is sufficient to show that the profile $\tilde{\omega}$ is such that both workers participation constraints are satisfied and the order (19) is satisfied.

Worker 1's participation constraint is

$$p^{HH} w_1^H + (1 - p^{HH}) w_1^L \geq \bar{c} - \beta p^{HH} \hat{\rho},$$

¹⁷The contract is not unique when $\underline{p}^{HH} \leq p^{HH} \leq \bar{p}^{HH}$. Any of the two wage vectors can be used in this case.

and Worker 2's participation constraint is

$$p^{HH} w_2^H + (1 - p^{HH}) w_2^L \geq \bar{c} - \beta (1 - p^{HH}) \hat{\rho}.$$

It is obvious that with the profile \tilde{w} both constraints are satisfied and that $\tilde{w}_2^L = \tilde{w}_2 > \tilde{w}_1^L$. Thus it is sufficient to check that $\tilde{w}_1^H > \tilde{w}_2$. This is the case if

$$\frac{(p^{HH})^2}{(1 - p^{HH})} \leq 1 \quad \text{or} \quad p^{HH} \leq \frac{\sqrt{5} - 1}{2}.$$

Now consider the case when $p^{HH} \geq \underline{p}^{HH}$. Observe that given the profile \tilde{w} , $EWC = EWC^{min}$ if the order of wages satisfies:

$$\begin{aligned} \tilde{w}_1^L &> \tilde{w}_2^H \\ \tilde{w}_2^L &> \tilde{w}_1^H \end{aligned}$$

(since if this is the case then Worker 1's expected rank utility is $E(\rho(r_1)|\tilde{\omega}^A) = (1 - p^{HH}) \hat{\rho}$. Worker 2's expected rank utility is $E(\rho(r_1)|\tilde{\omega}^A) = p^{HH} \hat{\rho}$.)

It is sufficient to show that the profile \tilde{w} is such that both workers participation constraints are satisfied and the order (19) is satisfied.

Worker 1's participation constraint is

$$p^{HH} w_1^H + (1 - p^{HH}) w_1^L \geq \bar{c} - \beta (1 - p^{HH}) \hat{\rho},$$

and Worker 2's participation constraint is

$$p^{HH} w_2^H + (1 - p^{HH}) w_2^L \geq \bar{c} - \beta p^{HH} \hat{\rho}.$$

It is obvious that with the wages \tilde{w} specified above, these two constraints are satisfied. It is also obvious that $\tilde{w}_2 > \tilde{w}_1^H$. It remains to check that $\tilde{w}_1^L > \tilde{w}_2$.

From Assumption 1 it follows that this condition is fulfilled whenever

$$\frac{(1 - p^{HH})^2}{p^{HH}} \leq 1 \quad \text{or} \quad p^{HH} \geq \frac{3 - \sqrt{5}}{2}.$$

This completes the proof of the proposition. □

D Appendix: Proof of Proposition 3

Proof: The Lagrange Function representing the standard moral-hazard problem is

$$L(\mathbf{w}, \lambda) = p^{HH} w^H + (1 - p^{HH}) w^L + \lambda [\bar{c} - \Delta p^H [w^H - w^L]]. \quad (19)$$

Minimising (19) leads to the following Kuhn-Tucker conditions:

$$p^{HH} - \lambda \Delta p^H \geq 0 \quad \text{and} \quad w^H \geq 0 \quad \text{and} \quad w^H [p^{HH} - \lambda \Delta p^H] = 0$$

$$(1 - p^{HH}) + \lambda \Delta p^H \geq 0 \quad \text{and} \quad w^L \geq 0 \quad \text{and} \quad w^L [(1 - p^{HH}) + \lambda \Delta p^H] = 0$$

and

$$[\bar{c} - \Delta p^H [w^H - w^L]] \leq 0 \quad \text{and} \quad \lambda \geq 0 \quad \text{and} \quad \lambda [\bar{c} - \Delta p^H [w^H - w^L]] = 0.$$

These Kuhn-Tucker conditions imply that the unique solution is equal to $w^L = 0$, $w^H = \frac{\bar{c}}{\Delta p^H}$, and $\lambda = \frac{p^{HH}}{\Delta p^H}$. The results of Prop 3 follow immediately.

□

E Appendix: Proof of Proposition 4

Proof:

Step 1: The rank payoff matrix corresponding to the optimal SD contract is:

	H	L
H	$\tilde{\rho}, \tilde{\rho}$	$\hat{\rho}, 0$
L	$0, \hat{\rho}$	$\tilde{\rho}, \tilde{\rho}$

To see this note that symmetry imposes the following structure on the rank payoff matrix:

	H	L
H	$\tilde{\rho}, \tilde{\rho}$	X, Y
L	Y, X	$\tilde{\rho}, \tilde{\rho}$

Clearly, choosing $X \leq Y$ is suboptimal because we can increase *both* I_1 , and I_2 by choosing $X > Y$. This completes the proof of the Claim. This implies that $w_1^{HL*} > w_2^{HL*}$ and $w_2^{LH*} > w_1^{LH*}$.

Step 2: Observe that in a symmetric optimal SD contract $w_1^{HH*} = w_2^{HH*}$, $w_1^{HL*} = w_2^{HL*}$, $w_1^{LH*} = w_2^{LH*}$ and $w_1^{LL*} = w_2^{LL*}$. This implies that $w_1^H = w_2^H$ and $w_1^L = w_2^L$.

Step 3: Now we show that we can find an SI contract that can achieve the same *EWC* as the optimal SD contract. Given the construction of the independent contract, we have from Step 2 that $w_1^H = w_2^H$ and $w_1^L = w_2^L$. Moreover since $w_1^{HL*} > w_2^{HL*}$ and $w_2^{LH*} > w_1^{LH*}$, we have $w_1^H > w_2^L$ and $w_2^H > w_1^L$. Thus we get the same rank payoff matrix as the one corresponding to the optimal symmetric SD contract.

It remains to show that the constructed w_1^H, w_2^H and w_1^L, w_2^L satisfy the incentive constraints. Note that the incentive constraints are given by (13). The RHS of the incentive constraints is the same between the SD and the SI contract (since the rank payoffs are the same). But this means that by construction the incentive constraints are satisfied (since the LHS is also the same) and total costs are the same.

□

F Appendix: Proof of Proposition 5

Proof: The firm's expected wage cost can be rewritten as

$$EWC = 2 \{w^L + p^{HH} [w^H - w^L]\}.$$

Using the Workers' limited liability constraints and incentive constraints, the following condition must hold:

$$w^L + p^{HH} [w^H - w^L] \geq p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} (\tilde{\rho} - \rho(w^L, w^H)) + (1 - p^{HH}) (\rho(w^H, w^L) - \tilde{\rho})\} \right\}.$$

The *RHS* of this inequality is minimised iff $w^H > w^L$. The above condition then becomes

$$w^L + p^{HH} [w^H - w^L] \geq p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} (\tilde{\rho} - 0) + (1 - p^{HH}) (\hat{\rho} - \tilde{\rho})\} \right\}.$$

The *LHS* and hence the firm's expected wage cost are minimised but fulfill the above condition iff $w^L = 0$ and

$$w^H = \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} (\tilde{\rho} - 0) + (1 - p^{HH}) (\hat{\rho} - \tilde{\rho})\}.$$

In other words, the optimal symmetric wage contract is determined by choosing the two state-dependent wages such that the workers' incentive constraints are binding and then setting w^L equal to zero.

□

G Appendix: Proof of Lemma 3

Proof: This proposition deals with four different cases of asymmetric contracts:

Case (i) $w_i^H, w_i^L > w_j^H, w_j^L$: Worker i 's incentive constraint is

$$[w_i^H - w_i^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH}(\bar{\rho} - \tilde{\rho}) + (1 - p^{HH})(\bar{\rho} - \tilde{\rho})\},$$

or simplified

$$[w_i^H - w_i^L] \geq \frac{\bar{c}}{\Delta p^H}.$$

Similarly Worker j 's incentive constraint is

$$[w_j^H - w_j^L] \geq \frac{\bar{c}}{\Delta p^H}.$$

Note, the firm's expected wage cost is

$$\begin{aligned} EWC &\equiv \{p^{HH} w_1^H + (1 - p^{HH}) w_1^L\} + \{p^{HH} w_2^H + (1 - p^{HH}) w_2^L\} \\ &= w_1^L + p^{HH} [w_1^H - w_1^L] + w_2^L + p^{HH} [w_2^H - w_2^L]. \end{aligned}$$

The expressions for the two workers' incentive constraints imply that the following condition holds for the firm's expected wage cost:

$$EWC \geq 2p^{HH} \frac{\bar{c}}{\Delta p^H}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm.

Case (ii) $w_i^H > w_j^H > w_i^L = w_j^L$: Worker i 's incentive constraint is

$$[w_i^H - w_i^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} \hat{\rho} + (1 - p^{HH})(\hat{\rho} - \tilde{\rho})\}.$$

Similarly, Worker j 's incentive constraint is

$$[w_j^H - w_j^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{(1 - p^{HH})(\hat{\rho} - \tilde{\rho})\}.$$

Worker i 's incentive constraint cannot be binding, but Worker j 's is. The wage w_i^H is determined by the fact that $w_i^H > w_j^H$. Hence, for the firm's expected wage cost the following inequality holds:

$$EWC > 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{(1 - p^{HH})(\hat{\rho} - \tilde{\rho})\} \right\}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm.

Case (iii) $w_i^H = w_j^H > w_i^L > w_j^L$: Worker i 's incentive constraint is

$$[w_i^H - w_i^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} \tilde{\rho}\}.$$

Similarly, Worker j 's incentive constraint is

$$[w_j^H - w_j^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} \tilde{\rho} + (1 - p^{HH}) \hat{\rho}\}.$$

Worker j 's incentive constraint cannot be binding, but Worker i 's is. The wage w^H is determined by the fact that $w_i^L > w_j^L$ and Worker i 's binding incentive constraint. Hence, for the firm's expected wage cost the following inequality holds:

$$EWC > 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{p^{HH} \tilde{\rho}\} \right\}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm.

Case (iv) $w_i^H > w_j^H > w_j^L > w_i^L$: Worker i 's incentive constraint is

$$[w_i^H - w_i^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta \{\hat{\rho}\}.$$

Similarly, Worker j 's incentive constraint is

$$[w_j^H - w_j^L] \geq \frac{\bar{c}}{\Delta p^H}.$$

Worker i 's incentive constraint cannot be binding, but Worker j 's is. The wage w_i^H is determined by the fact that $w_j^L > w_i^L = 0$ and Worker j 's binding incentive constraint. Hence, for the firm's expected wage cost the following inequality holds:

$$EWC > 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} \right\}.$$

But this inequality establishes that deviating from the optimal symmetric independent contract to such a profile of asymmetric independent contracts is not optimal for the firm and hence completes the proof of the above proposition.

□

H Appendix: Proof of Proposition 6

Proof:

The wage structure implied by ω^{AS*} implies that Worker i 's incentive constraint is

$$[w_i^H - w_i^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta p^{HH} \hat{\rho}.$$

Similarly Worker j 's incentive constraint is

$$[w_j^H - w_j^L] \geq \frac{\bar{c}}{\Delta p^H} - \beta (1 - p^{HH}) \hat{\rho}.$$

From the minimisation of the firm's expected wage cost we have $w_j^L = 0$, and taking into account Worker j 's incentive constraint we have:

$$w_j^H = \frac{\bar{c}}{\Delta p^H} - \beta (1 - p^{HH}) \hat{\rho}.$$

Then Worker i 's incentive constraint implies that

$$w_i^H \geq \frac{\bar{c}}{\Delta p^H} - \beta p^{HH} \hat{\rho} + w_i^L.$$

The condition $w_i^H > w_j^H$ implies that

$$w_i^H > \frac{\bar{c}}{\Delta p^H} - \beta (1 - p^{HH}) \hat{\rho}.$$

Hence Worker i 's incentive constraint is (not) binding iff

$$\beta (1 - 2p^{HH}) \hat{\rho} + w_i^L > (<) 0,$$

or

$$w_i^L > (<) -\beta (1 - 2p^{HH}) \hat{\rho}. \tag{20}$$

Firstly, let $p^{HH} \leq 1/2$, then the limited liability constraint implies that condition (20) is irrelevant because the *RHS* is negative. Hence, to minimise its expected wage cost the firm sets $w_i^L = \epsilon > 0$ but equal to the smallest monetary unit. From the cost minimisation and Worker i 's incentive constraint it follows that

$$w_i^H = \frac{\bar{c}}{\Delta p^H} - \beta p^{HH} \hat{\rho} + \epsilon.$$

This implies the profile of asymmetric wage contracts minimising the firm's expected wage cost is

$$\omega^{AS*} = ((w_i^H, w_i^L), (w_j^H, w_j^L)) = \left(\left(\frac{\bar{c}}{\Delta p^H} - \beta p^{HH} \hat{\rho} + \epsilon, \epsilon \right), \left(\frac{\bar{c}}{\Delta p^H} - \beta (1 - p^{HH}) \hat{\rho}, 0 \right) \right)$$

For $p^{HH} \leq 1/2$, the firm's minimum expected wage cost is

$$EWC(\omega^{AS*}) = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \frac{\hat{\rho}}{2} \right\} + \epsilon.$$

Secondly, let $p^{HH} > 1/2$, then *RHS* of condition (20) is strictly positive. Now, it is crucial whether the smallest monetary unit fulfills (20) or not. Suppose not, i.e.

$$\epsilon < -\beta (1 - 2p^{HH}) \hat{\rho}.$$

Then, it follows from the minimisation of the expected wage cost that $w_i^L = \epsilon$ and w_i^H is set such that $w_i^H > w_j^H$. Note, Worker i 's incentive constraint is not binding in this case. We have

$$w_i^H = w_j^H + \epsilon = \frac{\bar{c}}{\Delta p^H} - \beta (1 - p^{HH}) \hat{\rho} + \epsilon.$$

The firm's expected wage cost EWC is then:

$$EWC = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{1 - p^{HH}\} \hat{\rho} \right\} + \epsilon$$

Suppose next that the smallest monetary unit is such that

$$\epsilon \geq -\beta (1 - 2p^{HH}) \hat{\rho}.$$

Then, it follows from the minimisation of the expected wage cost that $w_i^L = \epsilon$ and w_i^H is set such that Worker i 's incentive constraint is binding in this case. We have

$$w_i^H = \frac{\bar{c}}{\Delta p^H} - \beta p^{HH} \hat{\rho} + \epsilon.$$

The firm's expected wage cost EWC is then:

$$EWC(\omega^{AS*}) = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \frac{\hat{\rho}}{2} \right\} + \epsilon.$$

Note, with $p^{HH} > 1/2$ we have that

$$EWC(\omega^{AS*}) > EWC = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{1 - p^{HH}\} \hat{\rho} \right\} + \epsilon.$$

□

I Appendix: Proof of Theorem 2

Proof: From Proposition (6) we know the EWC function with Asymmetric contracts: If $p^{HH} > \frac{1}{2}$, then EWC is described by the condition

$$EWC \geq 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \{1 - p^{HH}\} \hat{\rho} \right\} + \epsilon > EWC^S.$$

Hence with $p^{HH} > 1/2$ it is not profitable for the firm to deviate from its optimal symmetric independent contract to this profile of asymmetric independent wage contracts.

Secondly, let $p^{HH} \leq 1/2$, then

$$EWC(\omega^{AS*}) = 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \frac{\hat{\rho}}{2} \right\} + \epsilon.$$

$EWC(\omega^{AS*}) < EWC^S$ iff

$$2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta \frac{\hat{\rho}}{2} \right\} + \epsilon < 2p^{HH} \left\{ \frac{\bar{c}}{\Delta p^H} - \beta [(1 - p^{HH})(\hat{\rho} - \tilde{\rho}) + p^{HH}\tilde{\rho}] \right\}$$

or simplified

$$\epsilon < \beta p^{HH} [1 - 2p^{HH}] [2\tilde{\rho} - \hat{\rho}].$$

If $p^{HH} < 1/2$, then the *RHS* of this inequality is strictly positive iff

$$2\tilde{\rho} > \hat{\rho},$$

completing the proof of the above theorem.

□