Outsourcing Spurred by Strategic Competition

Yutian Chen, Pradeep Dubey

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1 Introduction

According to a survey of procurement professionals in Europe and the US, the value of contracts outsourced to low-cost countries is going to almost double over the next three years. The very heart of this paper is to shed light on a strategic reason underlying offshore outsourcing which has not been noticed before. Under economies of scale, outsourcing to a provider who is also a competitor for the final product is inferior to outsourcing to a provider outside of the final product market, generally like firms in these low-cost countries. This can be true even when these providers have higher cost compared with other potential providers.

2 A Model with Two Incumbents

Tow firms, F_1, F_2, are competing in quantities in the final product good B. The unique intermediate good needed to produce B is good A. Only F_1 can produce A inside. F_0 is a provider for A which is outside of the market for B. F_2 can either outsource to F_1 or outsource to F_0 for A.

F_1 and F_0 both have economies of scale in providing A, with cost function $C_i(q), i = 0, 1$ satisfying $C_i' > 0, C_i'' < 0$. Furthermore, one unit of A can produce one unit of B. F_1, F_2 have the same linear marginal cost in producing B from A, which is normalized to zero.

The game consists of three stages:

Stage one is the price competition stage. F_0 and F_1 announce their prices, $\{d_0, d_1\}$, for providing A simultaneously.

In stage two F_2 decides its quantity to outsource, and to which provider, F_0 or F_1 or both, to outsource. Binding contracts are signed in this stage between the provider and the outsourcer.

In stage three F_1 after observing F_2's strategy in stage two, determines either to produce inside or to outsource to F_0, or to do both, with its corresponding quantities.

Assume that F_0, F_1 have the same cost function for good A, given as

$$C_i(q) = \begin{cases} bq - cq^2 & \text{for } q \le \dfrac{b}{2c} \\ \dfrac{b^2}{4c^2} & \text{for } q > \dfrac{b}{2c} \end{cases} \quad i = 0, 1$$

The inverse demand function is $P = a - Q$. Below are assumptions on cost and demand function parameters.

A1. $b < a < \frac{b}{2c}$.

A2. $c \in (0, \frac{1}{2})$.

We are examing subgame perfect equilibrium of this game.

3 Model Analysis

In the last stage it is possible that F_1 partly outsources and partly produces inside. Let q_1 denote F_1's total quantity for good A, and q_1^i denote F_1's quantity outsourced to $F_i, i = 0, 1$, with $q_1 = q_1^0 + q_1^1$, it is possible that $q_1^0 > 0, q_1^1 > 0$. Here F_1 outsourcing to F_1 means that F_1 is producing inside.

In the second stage, it is also possible for F_2 to outsource to both F_0 and F_1, i.e. $q_2^0 > 0, q_2^1 > 0$, with q_2^i the quantity F_2 outsources to $F_i, i = 0, 1$ and $q_2 = q_2^0 + q_2^1$. Given F_2's strategy in the second stage as $\{q_2, q_2^1\}$, F_1's profit in the last stage is

$$\pi_1(q_1) = (a - q_1 - q_2)q_1 + d_1 q_2^1 - d_0(q_1 - q_1^1) - b(q_1^1 + q_2^1) + c(q_1^1 + q_2^1)^2.$$

Note that $\frac{d^2\pi_1(q_1)}{dq_1^2} = -2$ so we can use the first order condition to get the optimal $q_1(q_2)$ as

$$q_1(q_2) = \begin{cases} \dfrac{a - q_2 - d_0}{2} & \text{if } \dfrac{a - q_2 - d_0}{2} > 0 \\ 0 & \text{o.w.} \end{cases}$$

When $q_1(d_0)$ is positive, by substituting it into $\pi_1(q_1)$, it is true that $\frac{d^2\pi_1(q_2,q_1^1)}{dq_1^{1\,2}} = 2c > 0$. That means when F_1 is maximizing its profit with q_1 positive, the optimal q_1^1 is either $q_1^1 = 0$ or $q_1^1 = q_1$. F_1 will either fully produce inside or fully outsource to F_0. The possibility of $q_1^0 > 0, q_1^1 > 0$ is ruled out.

Similarly, given that F_1 is producing inside, the possibility of $q_2^0 > 0, q_2^1 > 0$ can be ruled out under A2. Given that F_1 is outsourcing to F_0, F_2 for sure outsources to F_0 if $d_0 < d_1$ and for sure outsources to F_1 if $d_1 < d_0$. Given that F_1 is outsourcing to F_0 and $d_1 = d_0$, it is true that F_2 outsources to F_0. To see this, suppose not. Suppose F_2 outsources some quantity x to F_1 with $0 < x \leq q_2$ and $(q_2 - x)$ to F_0. Then F_1 outsources $(q_1 + x)$ to F_0. For F_0 to be willing to provide, we have

$$\pi_0 = d_0(q_1 + x) - b(q_1 + x) + c(q_1 + x)^2 > 0 \Rightarrow d_0 > b - c(q_1 + x).$$

Thus

$$\begin{aligned}\pi_1 &= (a - q_1 - q_2)q_1 + d_1 x - d_0(q_1 + x) \\ &< (a - q_1 - q_2)q_1 + d_1 x - b(q_1 + x) + c(q_1 + x)^2.\end{aligned}$$

This means that F_1 is strictly better off to produce inside whatever it outsources to F_0. A contradiction to F_1 outsourcing to F_0.

Therefore, under A2, for any strategy followed by F_1, F_2 will either fully outsource to F_0 or fully outsource to F_1 when $q_2 > 0$. For the following analysis, we only need to focus on strategies of F_1 and F_2 in which they are either fully outsourcing to F_0 or to F_1.

3.1 F_1's Strategy in Stage Three

Depending on F_2's choice in the second stage, F_1 faces four possible cases in the last stage. In each case F_1 is maximizing its profit by choosing its quantity q_1.

Case I. F_2 outsources to F_1, then F_1 produces inside.

In this case F_1's profit is

$$\pi_1^I(q_1) = (a - q_1 - q_2)q_1 + d_1 q_2 - b(q_1 + q_2) + c(q_1 + q_2)^2.$$

Because $\frac{d^2\pi^I}{dq_1^2} = -2(1 - c) < 0$, there exists an unique optimal value of q_2 which maximizes $\pi_1^I(q_1)$, given by $q_1^I(q_2)$:

$$q_1^I(q_2) = \begin{cases} \dfrac{a-b-q_2+2cq_2}{2(1-c)} & \text{if } q_2 < \dfrac{a-b}{1-2c} \\ 0 & \text{o.w.} \end{cases}$$

Note that when $q_1^I(q_2) > 0$, $-1 < \frac{dq_1^I(q_2)}{dq_2} = -\frac{1-2c}{2(1-c)} < 0$.

Case II. F_2 outsources to F_0, then F_1 produces inside.

F_1's profit is

$$\pi_1^{II}(q_1) = (a - q_1 - q_2)q_1 - bq_1 + cq_1^2.$$

Because $\frac{d^2\pi^{II}}{dq_1^2} = -2(1-c) < 0$, there exists an unique optimal $q_1^{II}(q_2)$:

$$q_1^{II}(q_2) = \begin{cases} \dfrac{a-b-q_2}{2(1-c)} & \text{if } q_2 < a-b \\ 0 & \text{o.w.} \end{cases}$$

Note that when $q_1^{II}(q_2) > 0$, $-1 < \frac{dq_1^{II}(q_2)}{dq_2} = -\frac{1}{2(1-c)} < 0$.

Case III. F_2 outsources to F_0, then F_1 outsources to F_0 too.

F_1's profit function is

$$\pi_1^{III} = (a - q_1 - q_2 - d_0)q_1,$$

which is maximized at

$$q_1^{III}(q_2) = \begin{cases} \dfrac{a-d_0-q_2}{2} & \text{if } q_2 < a-d_0 \\ 0 & \text{o.w.} \end{cases}$$

Case IV. F_2 outsources to F_1, then F_1 outsources to F_0.

This case is impossible in equilibrium. Suppose case IV is in equilirbium, then for F_0 to be willing to provide,

$$\pi_0^{IV} = d_0(q_1+q_2) - b(q_1+q_2) + c(q_1+q_2)^2 > 0 \Rightarrow d_0 > b - c(q_1+q_2)$$

must be true. Thus

$$\begin{aligned} \pi_1^{IV} &= (a - q_1 - q_2)q_1 + d_1q_2 - d_0(q_1+q_2) \\ &< (a - q_1 - q_2)q_1 + d_1q_2 - [b - c(q_1+q_2)](q_1+q_2) \\ &= (a - q_1 - q_2)q_1 + d_1q_2 - b(q_1+q_2) + c(q_1+q_2)^2. \end{aligned}$$

But the last expression is F_1's profit when it produces inside. Therefore in any equilibrium when F_2 outsources to F_1, it must be that F_1 is producing inside.

3.2 F_2's Strategy in Stage Two

In this stage F_2 makes two decisions: To which one to outsource and how much to outsource. In a SPE it correctly expects F_1's reaction in the last stage, and accordingly chooses its optimal quantity q_2 to maximize its profit.

Case I. F_2 is outsourcing to F_1, then F_1 produces inside.

With $q_1^I(q_2)$ solved above, F_2's profit is

$$\pi_2^I(q_2) = (a - q_1^I(q_2) - q_2 - d_1)q_2$$
$$= \begin{cases} \dfrac{(a+b-2ac-q_2-2d_1+2cd_1)q_2}{2(1-c)} & \text{if } q_2 < \dfrac{a-b}{1-2c} \\ (a-q_2-d_1)q_2 & \text{o.w.} \end{cases}$$

Note $\frac{d^2\pi_2^I}{dq_2^2} = -\frac{1}{1-c} < 0$ when $q_2 < \frac{a-b}{1-2c}$. The optimal q_2 is solved as

$$q_2^I(d_1) = \begin{cases} 0 & \text{if } d_1 \geq \bar{d}_1 \\ \dfrac{a+b-2ac-2d_1+2cd_1}{2} & \text{if } d_{1l} < d_1 < \bar{d}_1 \\ \dfrac{a-b}{1-2c} & \text{if } d_{1r} \leq d_1 \leq d_{1l} \\ \dfrac{a-d_1}{2} & \text{o.w.} \end{cases}$$

Here $\bar{d}_1 = \frac{a+b-2ac}{2(1-c)}$, $d_{1l} = \frac{4ac^2+3b-2bc-a-4ac}{2(1-2c)(1-c)}$, $d_{1r} = \frac{2b-a-2ac}{1-2c}$. Substituting $q_2^I(d_1)$ into $q_1^I(q_2)$, the optimal q_1 produced in the last stage is given by $q_1^I(d_1)$:

$$q_1^I(d_1) = \begin{cases} \dfrac{a-b}{2(1-c)} & \text{if } d_1 \geq \bar{d}_1 \\ \dfrac{a-3b+4ac+2bc-4ac^2-6cd_1+2d_1+4c^2d_1}{4(1-c)} & \text{if } d_{1l} < d_1 < \bar{d}_1 \\ (d_1-b)q_2^I(d_1) + c[q_2^I(d_1)]^2 & \text{o.w.} \end{cases}$$

By substituting $q_1^I(d_1), q_2^I(d_1)$ into the profit functions, we have the maximized profits for F_1, F_2 as $\pi_1^I(d_1)$ and $\pi_2^I(d_1)$ respectively:

$$\pi_1^I(d_1) = \begin{cases} \pi_1^M = \dfrac{(a-b)^2}{4(1-c)} & \text{if } d_1 \geq \bar{d}_1 \\ \pi_1^C(d_1) & \text{if } d_{1l} < d_1 < \bar{d}_1 \\ 0 & \text{o.w.} \end{cases}$$

$$\pi_2^I(d_1) = \begin{cases} 0 & \text{if } d_1 \geq \bar{d}_1 \\ \pi_2^C(d_1) = \dfrac{(a+b-2ac-2d_1+2cd_1)^2}{8(1-c)} & \text{if } d_{1l} < d_1 < \bar{d}_1 \\ \dfrac{(a-b)(b-2ac-d_1+2cd_1)}{(1-2c)^2} & \text{if } d_{1r} \leq d_1 \leq d_{1l} \\ \pi_2^M(d_1) = \dfrac{(a-d_1)^2}{4} & \text{if } d_1 < d_{1r} \end{cases}$$

Here $\pi_1^C(d_1), \pi_2^C(d_1)$ is F_1, F_2's profits when both are producing positive quantities. $\pi_1^C(d_1)$ is a long expression so is omitted here. Since $\frac{d^2\pi_1^C(d_1)}{dd_1^2} = \frac{3}{2}(c-1)$, $\frac{d^2\pi_2^C(d_1)}{dd_1^2} = 1-c$, $\pi_1^C(d_1)$ is increasing and strictly concave in d_1; $\pi_2^C(d_1)$ is decreasing and strictly convex in d_1.

Case II. F_2 is outsourcing to F_0, then F_1 produces inside.

F_2's profit function is

$$\begin{aligned} \pi_2^{II}(q_2) &= (a - q_1^{II}(q_2) - q_2 - d_0)q_2 \\ &= \begin{cases} \dfrac{(a+b-2ac-q_2+2cq_2-2d_0+2cd_0)q_2}{2(1-c)} & \text{if } q_2 < a-b \\ (a-q_2-d_0)q_2 & \text{o.w.} \end{cases} \end{aligned}$$

Note when $q_2 < a-b$, $\frac{d^2\pi_2^{II}}{dq_2^2} = -\frac{1-2c}{1-c} < 0$, the profit function is strictly concave in q_2. The optimal quantity $q_2^{II}(d_0)$ is:

$$q_2^{II}(d_0) = \begin{cases} 0 & \text{if } d_0 \geq \bar{d}_0 \\ \dfrac{a+b-2ac-2d_0+2cd_0}{2(1-c)} & \text{if } d_{0l} < d_0 < \bar{d}_0 \\ a-b & \text{if } d_{0r} \leq d_0 \leq d_{0l} \\ \dfrac{a-d_0}{2} & \text{o.w.} \end{cases}$$

Here $\bar{d}_0 = \frac{a+b-2ac}{2(1-c)}$, $d_{0l} = \frac{2ac-a+3b-4bc}{2(1-c)}$, $d_{0r} = 2b-a$. Substituting $q_2^{II}(d_0)$ into $q_1^{II}(q_2)$, the optimal q_1 produced in the last stage is given by $q_1^{II}(d_0)$:

$$q_1^{II}(d_0) = \begin{cases} \dfrac{a-b}{2(1-c)} & \text{if } d_0 \geq \bar{d}_0 \\ \dfrac{a-3b-2ac+4bc+2d_0-2cd_0}{4(1-c)(1-2c)} & \text{if } d_{0l} < d_0 < \bar{d}_0 \\ 0 & \text{o.w.} \end{cases}$$

By substituting $q_1^{II}(d_0), q_2^{II}(d_0)$ into the profit functions, we have the maximized profits for F_1, F_2 as $\pi_1^{II}(d_0)$ and $\pi_2^{II}(d_0)$ respectively:

$$\pi_1^{II}(d_0) = \begin{cases} \pi_1^M = \dfrac{(a-b)^2}{4(1-c)} \text{ if } d_0 \geq \bar{d}_0 \\ \pi_1^C(d_0) = \dfrac{(a+4bc-2ac-3b+2d_0-2cd_0)^2}{16(1-c)(1-2c)^2} \text{ if } d_{0l} < d_0 < \bar{d}_0 \\ 0 \qquad \text{o.w.} \end{cases}$$

$$\pi_2^{II}(d_0) = \begin{cases} 0 \qquad \text{if } d_0 \geq \bar{d}_0 \\ \pi_2^C(d_0) = \dfrac{(a+b-2ac-2d_0+2cd_0)^2}{8(1-2c)(1-c)} \text{ if } d_{0l} < d_0 < \bar{d}_0 \\ (a-b)(b-d_0) \qquad \text{if } d_{0r} \leq d_0 \leq d_{0l} \\ \pi_2^M(d_0) = \dfrac{(a-d_0)^2}{4} \qquad \text{if } d_0 < d_{0r} \end{cases}$$

Here $\pi_1^C(d_0), \pi_2^C(d_0)$ is F_1, F_2's profits when both are producing positive quantities. Since $\frac{d^2\pi_1^C(d_0)}{dd_0^2} = \frac{1-c}{2(1-2c)^2}$, $\frac{d^2\pi_2^C(d_0)}{dd_0^2} = \frac{1-c}{1-2c}$, $\pi_1^C(d_0)$ is increasing and strictly convex in d_0; $\pi_2^C(d_0)$ is decreasing and strictly convex in d_0.

Case III. F_2 is outsourcing to F_0, then F_1 outsources to F_0 too.

F_2's profit is given by

$$\pi_2^{III} = \begin{cases} (a - q_1^{III}(q_2) - q_2 - d_0)q_2 \text{ if } q_2 < a - d_0 \\ 0 \qquad \text{o.w.} \end{cases}$$

When $d_0 < a$, it is maximized at $q_2^{III}(d_0) = \frac{a-d_0}{2}$, otherwise $q_2^{III}(d_0) = 0$. F_1's production in the last stage is $q_1^{III}(d_0) = \frac{a-d_0}{4}$ for $d_0 < a$, and zero otherwise. The corresponding profits for F_1 and F_2 are

$$\pi_1^{III}(d_0) = \begin{cases} \dfrac{(a-d_0)^2}{16} \text{ if } d_0 < a \\ 0 \qquad \text{o.w.} \end{cases}$$

$$\pi_2^{III}(d_0) = \begin{cases} \dfrac{(a-d_0)^2}{8} \text{ if } d_0 < a \\ 0 \qquad \text{o.w.} \end{cases}$$

In case I the highest value of d_1 for F_2 to produce is $\bar{d}_1 = \frac{a+b-2ac}{2(1-c)}$, the lowest d_1 for F_1 to produce positive quantity of B is $d_{1l} = \frac{4ac^2+3b-2bc-a-4ac}{2(1-2c)(1-c)}$.

In case II the highest value of d_0 for F_2 to produce is $\bar{d}_0 = \frac{a+b-2ac}{2(1-c)}$, the lowest d_0 for F_1 to produce positive quantity of B is $d_{0l} = \frac{2ac-a+3b-4bc}{2(1-c)}$. Profits of F_1, F_2 in cases I, II and III are illustrated by Figure 1.

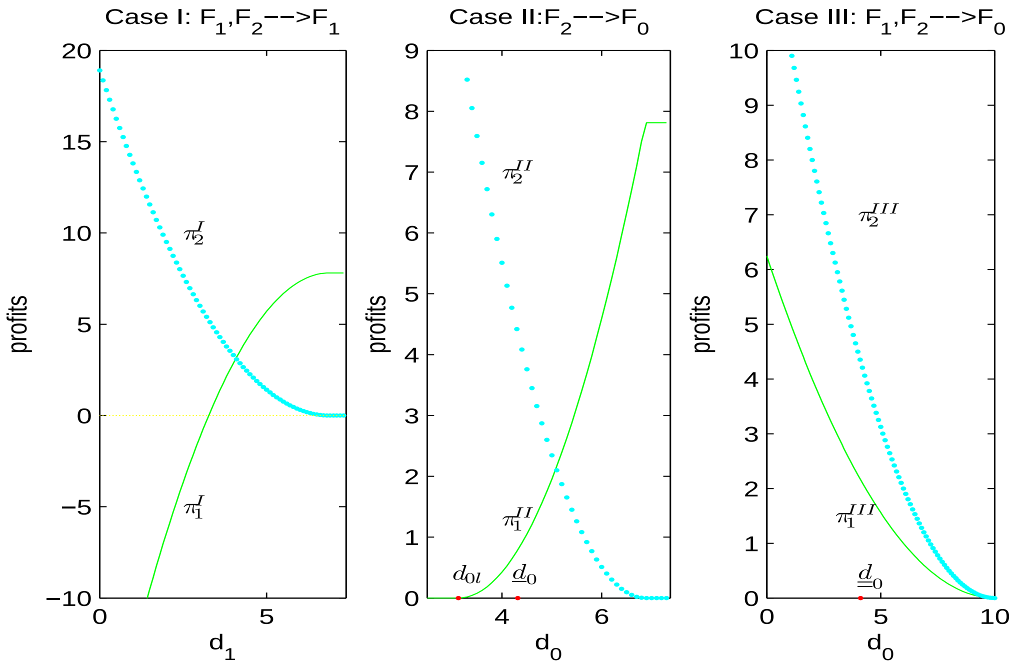

Figure 1: Profits of F_1, F_2 in case I, II and III. Parameters are set as a=10, b=5, c=0.2. d_{1l} is negative here.

Denote profits of F_1 and F_2 in case I, II, and III as π_i^j correspondingly, with $i = 1, 2$, $j = I, II, III$. Figure 2 illustrates F_1's profits in case II and III. There exists an unique d_0 at which F_1 is indifferent between case II and case III, solved from $\pi_1^{II}(d_0) = \pi_1^{III}(d_0)$ as

$$\hat{d}_0 = \frac{a(1-3c+2c^2) - \sqrt{1-c}(a-3b-2ac+4bc)}{(1-c)(1+2\sqrt{1-c}-2c)}.$$

Note that $d_{0l} < \hat{d}_0 < \bar{d}_0$ under A2. See Figure 3.

Lemma 1. *Given that F_2 outsources to F_0, in stage three F_1 produces inside if $d_0 > \hat{d}_0$ and outsources to F_0 if $d_0 < \hat{d}_0$. If $d_0 = \hat{d}_0$, F_1 is indifferent.*

Suppose $d_0 > \hat{d}_0$. That means if F_2 outsources to F_0 in the second stage, case II will be the outcome, and F_2 knows this. F_2 is comparing its profits

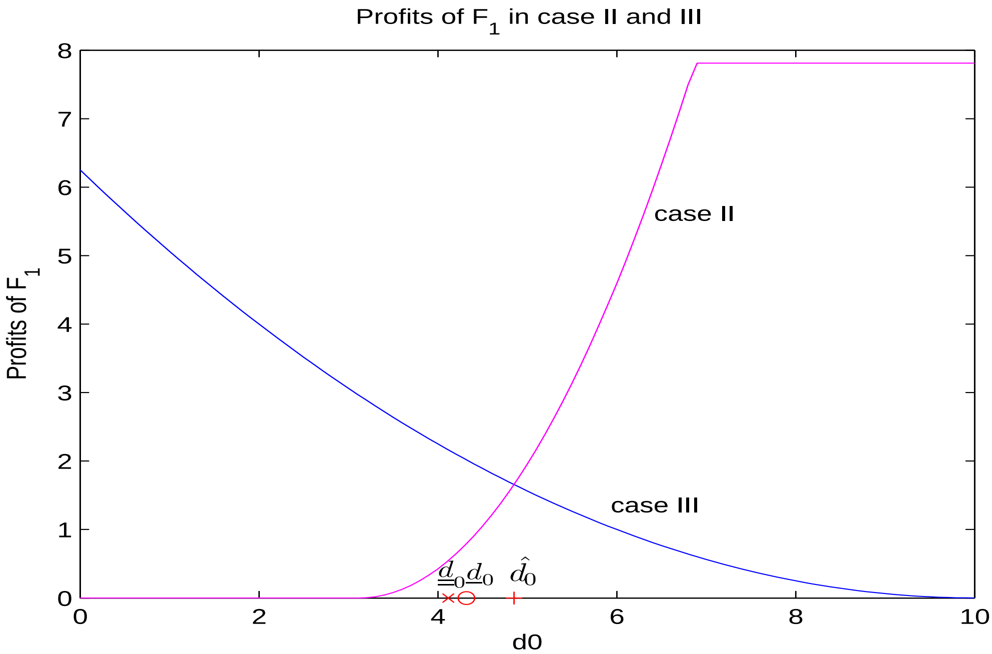

Figure 2: Parameters are set as a=10, b=5, c=0.2.

in case I and II when deciding to which one to outsource. The condition for F_2 to be willing to outsource to F_1 is given by

$$\pi_2^I(d_1) \geq \pi_2^{II}(d_0).$$

When equality holds, F_2 is indifferent between outsourcing to F_0 or to F_1. Because $\pi_2^I(d_1) < \pi_2^{II}(d_0)$ everywhere whenever $\hat{d}_0 < d_1 = d_0 < \frac{a+b-2ac}{2(1-c)}$, there is no intersection of $\pi_2^I(d_1)$ and $\pi_2^{II}(d_0)$. Given any $\hat{d}_0 < d_0 < \bar{d}_0$, there exists a unique d_1 which solves $\pi_2^I(d_1) = \pi_2^{II}(d_0)$, and it is a function of d_0 and is denoted as $\alpha(d_0)$.

$$\alpha(d_0) = \frac{a+b-2ac}{2(1-c)} - \frac{a+b-2ac-2d_0+2cd_0}{2\sqrt{1-2c}(1-c)}.$$

It is true that $\alpha(d_0) < d_0$ and it is increasing in d_0 whenever $d_0 < \bar{d}_0$. We have Lemma 2 below.

Lemma 2. *Suppose $d_0 > \hat{d}_0$. If $d_1 > \alpha(d_0)$, F_2 outsources to F_0 for sure; if $d_1 < \alpha(d_0)$, F_2 outsources to F_1 for sure.*

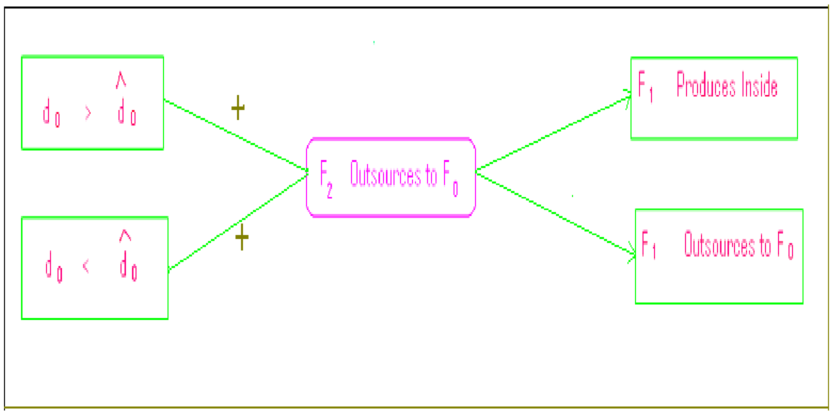

Figure 3: Lemma 1.

Secondly suppose $d_0 < \hat{d}_0$. Thus if F_2 outsources to F_0, in the last stage F_1 outsources to F_0 too, and F_2 knows this. F_2 compares its profits in case I and III to decide to which one to outsource. The condition for F_2 to be willing to outsource to F_1 is

$$\pi_2^I(d_1) > \pi_2^{III}(d_0).$$

When equality holds, F_2 is indifferent between these two cases. The left hand side is strictly decreasing in d_1 and the right hand side is strictly decreasing in d_0. For any given $d_0 < \hat{d}_0$, there exists a unique d_1 solving the equality, which is a function of d_0, denoted as $\beta(d_0)$:

$$\beta(d_0) = \frac{a + b - 2ac - (a - d_0)\sqrt{1-c}}{2(1-c)}.$$

$\beta(d_0)$ is increasing in d_0. When $d_0 = \hat{d}_0$, $\alpha(\hat{d}_0) > \beta(\hat{d}_0)$. F_2 knows that F_1 may produce inside or outsource to F_0 with arbitrary probability, thus it must be that only when $d_1 < \beta(\hat{d}_0)$, F_2 will outsource to F_1 for sure.

Lemma 3. *Suppose $d_0 \leq \hat{d}_0$. If $d_1 > \beta(d_0)$, F_2 outsources to F_0 for sure; if $d_1 < \beta(d_0)$, F_2 outsources to F_1 for sure.*

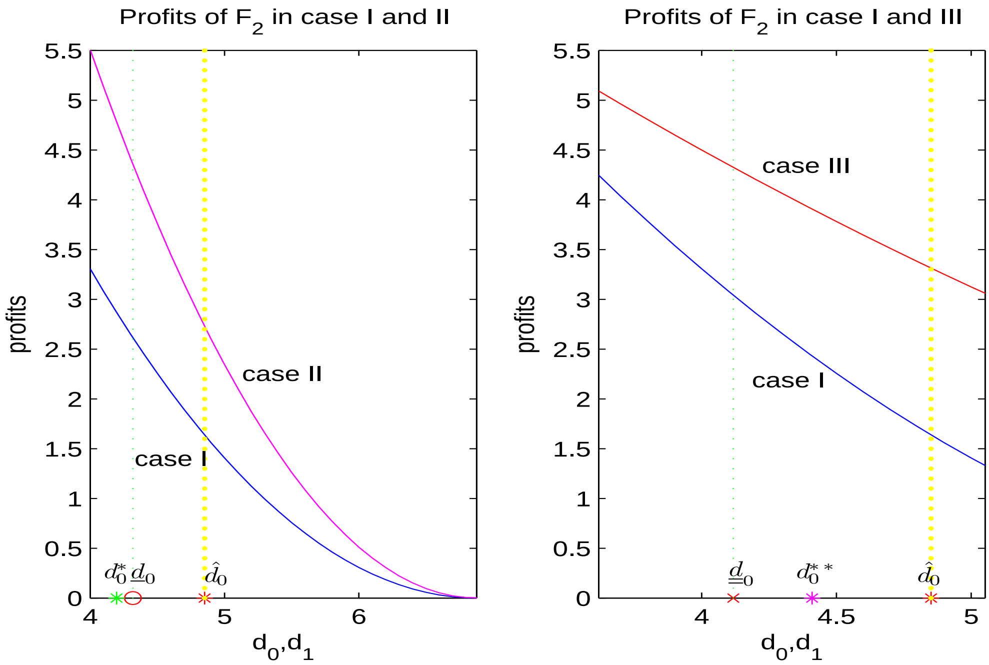

Figure 4: Parameters are set as a=10, b=5, c=0.2.

3.3 Strategies in Stage One

In stage one F_0, F_1 are expecting their future payoffs determined in stage two and three. If F_2 outsources to F_1 in the second stage, F_0 gets zero profit. Therefore F_0 is grimly competing F_1 in order to attract F_2 as long as it can achieve a positive profit through providing A.

Proposition 1. *If in any SPE F_2 is outsourcing to F_1, then in the first stage $\{d_0, d_1\}$ must take either one of the form:*

(1) $\{d_0, \alpha(d_0)\}$ if $d_0 > \hat{d}_0$;

(2) $\{d_0, \beta(d_0)\}$ if $d_0 \leq \hat{d}_0$.

Proof. We have proved that if F_2 is outsourcing to F_1, in any equilibrium it must be that F_1 is producing inside, i.e. F_1 and F_2 are in case I. Suppose $d_0 > \hat{d}_0$. If $d_1 > \alpha(d_0)$, by Lemma 2 we know that F_2 will outsource to F_0, a contradiction; if $d_1 < \alpha(d_0)$, since $\pi_1^I(d_1)$ is strictly increasing in d_1, and F_2

has no incentive to deviate to outsourcing to F_0 as long as $d_1 \leq \alpha(d_0)$, F_1 will deviate to $d_1 = \alpha(d_0)$, again a contradiction. Thus if there exists such a SPE, it must be $d_1 = \alpha(d_0)$ if $d_0 > \hat{d}_0$. Similarly, we can prove (2). ||

Consider the case when $d_0 < \hat{d}_0$. F_1's reservation profit when it price competes F_0 in the first stage is $\pi_1^{III}(d_0)$. F_1 compares $\pi_1^{III}(d_0)$ and $\pi_1^{I}(\beta(d_0))$ when deciding whether or not to compete F_0 by charging a d_1 attractive to F_2. The value of d_0 which makes F_1 indifferent is solved by

$$\pi_1^{III}(d_0) = \pi_1^{I}(\beta(d_0)) \Rightarrow d_0 = d_0^{**} = a - \frac{a-b}{\sqrt{1-c}}.$$

If $d_0 < d_0^{**}$, F_1 is strictly better off in case III, which means that F_1 becomes unwilling to compete F_0; if $d_0 > d_0^{**}$, F_1 is strictly better off in case I, thus F_1 has incentive to charge d_1 a little bit less than $\beta(d_0)$ to attract F_2. On the other side, F_0's profit is given by

$$\pi_0^{III}(d_0) = (d_0 - b)[q_1^{III}(d_0) + q_2^{III}(d_0)] + c[q_1^{III}(d_0) + q_2^{III}(d_0)]^2,$$

which is strictly concave in d_0 as long as $c < \frac{4}{3}$. The lowest d_0 which F_0 is willing to charge is solved from $\pi_0^{III}(d_0) = 0$ as

$$\underline{\underline{d}}_0 = \frac{4b - 3ac}{4 - 3c}.$$

Under A1, $\underline{\underline{d}}_0 > 0$. Note that $\underline{\underline{d}}_0 < d_0^{**} < \hat{d}_0$ under $A2$, which means that for $d_0 \leq d_0^{**}$, outsourcing to F_0 is a dominant strategy of F_1. By charging $d_0 \in [\underline{\underline{d}}_0, d_0^{**}]$, F_0 can achieve a positive profit, because now F_1 is unwilling to decrease d_1 to be less than $\beta(d_0)$. Thus F_0 for sure beats F_1 in the first stage, and in the following stages both F_1 and F_2 are outsourcing to F_0.

Proposition 2. *There does not exist a SPE in which F_2 is outsourcing to F_1.*

Proof. Suppose in some SPE F_2 outsources to F_1. Firstly, suppose in the first stage $d_0 \geq d_0^{**}$. If $d_0 > \hat{d}_0$, by Proposition 1, in the SPE it must be $d_1 = \alpha(d_0)$. Since it is true that $\beta(d_0^{**}) < \alpha(\hat{d}_0)$ and $\alpha(d_0)$ is increasing in d_0, by deviating to $d_0 \in (\underline{\underline{d}}_0, d_0^{**})$, F_0 can attract both F_2 and F_1 and achieve a positive profit. Thus F_0 will deviate. $\{d_0, \alpha(d_0)\}$ with $d_0 > \hat{d}_0$ can not be a SPE. Similarly, if $d_0^{**} \leq d_0 \leq \hat{d}_0$, $\{d_0, \beta(d_0)\}$ can not be a SPE, because F_0 will also deviate to $d_0 \in (\underline{\underline{d}}_0, d_0^{**})$ to win a positive profit since $\beta(d_0)$ is increasing in d_0. Secondly, suppose $d_0 < d_0^{**}$. By Proposition 1, in any SPE in which F_2 outsources to F_1, it must be that $d_1 = \beta(d_0)$. However, F_1 has incentive to deviate to $d_1 > \beta(d_0)$, because it is better off in case III than in case I, i.e. it is better off outsourcing to F_0 together with F_2 than beating F_0 with a low enough d_1. ||

Next consider the case $d_0 > \hat{d}_0$. If F_0 wins F_2 and case II is the outcome, F_0's profit $\pi_0^{II}(d_0)$ is given by

$$\pi_0^{II}(d_0) = (d_0 - b)q_2^{II}(d_0) + c(q_2^{II}(d_0))^2.$$

For $d_{0l} < d_0 < \bar{d}_0$, we have $\frac{d^2\pi_0^{II}(d_0)}{dd_0^2} = -\frac{2(1-c)(c^2-3c+1)}{(1-2c)^2}$, F_0 is strictly concave in d_0 for $c < \frac{3-\sqrt{5}}{2}$ and convex otherwise. The lowest d_0 at which F_0 is willing to provide F_2, is solved by $\pi_0(d_0) = 0$ as $\underline{d}_0$. F_0's profits in case II and III are illustrated in Figure 5.

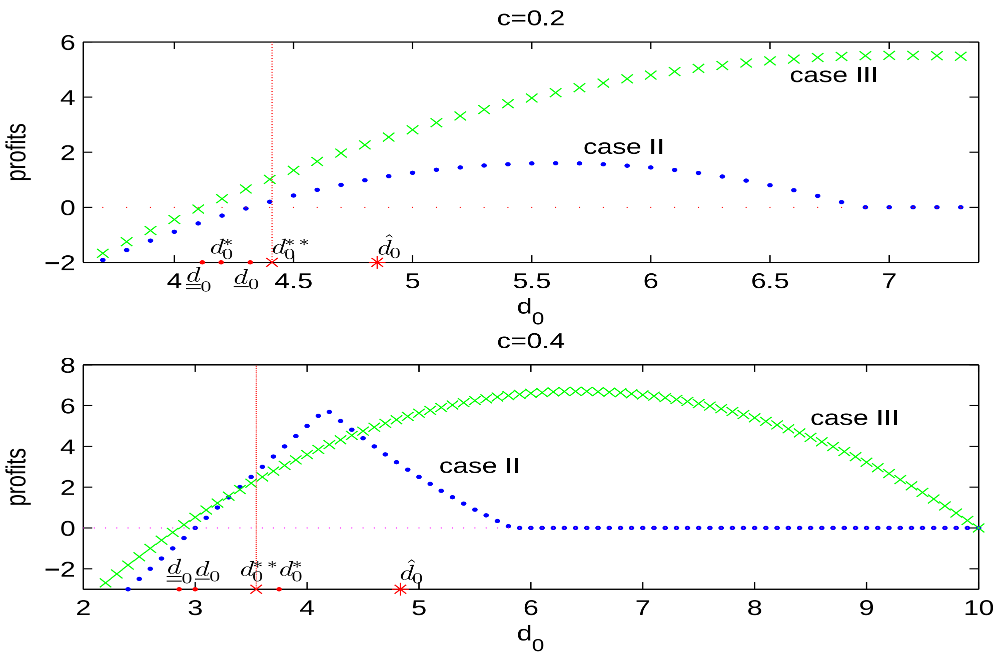

Figure 5: F_0's profit in case II and III with $c = 0.2$ or $c = 0.4$. Other parameters are set as $a = 10, b = 5$.

Given any $d_0 > \hat{d}_0$, F_1's reservation profit is $\pi^{II}(d_0)$. Upper bound of d_0

which can attract F_2 solves the following problem:

$$\pi_1^{II}(d_0) = \pi_1^{I}(\alpha(d_0)) \Rightarrow d_0 = d_0^* = \frac{6ac^2 - 3ac - 7cb + 4b}{2(3c^2 - 5c + 2)}.$$

However, it is true that $\hat{d}_0 > d_0^*$, thus for $d_0 > \hat{d}_0$, F_0 can not beat F_1 to attract F_2, and case II will be the outcome.

Theorem 1. *Under A1 and A2, there is a unique SPE in which F_1, F_2 both outsource to F_0, and prices are $\{d_0 = d_0^{**}, d_1 = \beta(d_0^{**})\}$.*

Proof. Firstly we want to show that F_1, F_2 both outsourcing to F_0 under $\{d_0 = d_0^{**}, d_1 = \beta(d_0^{**})\}$ is a SPE. Given that $d_0 = d_0^{**}$, if F_1 deviates to $d_1 < \beta(d_0^{**})$, F_2 is going to outsource to F_1. However, F_1 is worse off providing F_2 when $d_0 \leq d_0^{**}$. F_1 will not deviate. On the other side, given $d_1 = \beta(d_0^{**})$, F_0 will not deviate to $d_0 > d_0^{**}$, because it will loss F_2 in the second stage and therefore be worse off. Furthermore, F_0 will not deviate to $d_0 < d_0^{**}$. The reason lies on the fact that $\pi_0^{III}(d_0)$ is strictly concave in d_0. The optimal d_0 solved from $\frac{d\pi_0^{III}(d_0)}{dd_0} = 0$ is $\frac{2a+2b-3ac}{4-3c}$. Since $d_0^{**} < \frac{2a+2b-3ac}{4-3c}$, $\pi_0^{III}(d_0)$ is strictly increasing in d_0 for $d_0 \leq d_0^{**}$. F_0 can guarantee that F_1, F_2 will outsource to F_0 by charging $d_0 \leq d_0^{**}$, thus F_0 will charge $d_0 = d_0^{**}$ to maximize its profit. F_0 will not deviate. Given $d_1 \geq \beta(d_0)$, F_2 has no incentive to deviate to outsourcing to F_1; Given that $d_0^{**} < \hat{d}_0$ and F_2 outsources to F_0, F_1 has no incentive to deviate to producing inside. Thus F_1, F_2 both outsourcing to F_0 with $\{d_0 = d_0^{**}, d_1 = \beta(d_0^{**})\}$ is a SPE.

Secondly we want to show that there is no SPE other than the SPE stated above. By Proposition 2, there does not exist any SPE in which F_2 outsources to F_1. We need to analyze the possibility that case II is a SPE. If it is, it must be true that $d_0 \geq \hat{d}_0$, otherwise given that F_2 is outsourcing to F_0, F_1 will deviate to outsourcing to F_0. It must also be true that $d_1 \geq \beta(\hat{d}_0)$ for $d_0 = \hat{d}_0$, or $d_1 \geq \alpha(d_0)$ for $d_0 > \hat{d}_0$, otherwise F_2 will deviate. However, since $d_0^* < \hat{d}_0$, F_1 is better off to beat F_0 by deviating to $d_1 < \beta(\hat{d}_0)(\alpha(d_0))$ for $d_0 = (>)\hat{d}_0$, to attract F_2. Thus case II can not be SPE, either. ||

4 When F_0 Has Some Cost Disadvantage

Suppose F_0 has some cost disadvantage compared with F_1 in producing A. Strategies in stage two and three will not be affected, the only change which matters is that F_0 has a higher $\underline{d}_0$. However, as long as $\underline{d}_0 < d_0^{**}$, Theorem 1 still holds. For example, suppose now cost of F_0 in providing A is

$$C_0(q) = (b+\epsilon)q - cq^2,$$

with ϵ a small positive value. Now the lowest d_0 which F_0 is willing to charge in case III is

$$\pi_0^{III}(\underline{d}_0(\epsilon)) = 0 \Rightarrow \underline{d}_0(\epsilon) = \frac{4(b+\epsilon) - 3ac}{4-3c},$$

and

$$\underline{d}_0(\epsilon) < d_0^{**} \Rightarrow \epsilon < \epsilon_1 = (a-b)(1 - \frac{4-3c}{4\sqrt{1-c}})$$

because $\frac{d(d_0^{**} - \underline{d}_0)}{d\epsilon} = -\frac{4}{4-3c} < 0$. Note that $\epsilon_1 > 0$ under A1, A2. Furthermore, ϵ_1 is increasing in values of a and c. I.e., when the market size or economies of scale for producing A is bigger, F_0 can have a bigger cost disadvantage while can still attract F_1 and F_2.

Similarly, if F_0's cost disadvantage is reflected in a smaller economies of scale, i.e. when $C_0(q) = bq - (c-\epsilon)q^2$, with ϵ a small positive value, we can reach the same conclusion that as long as $\epsilon < \epsilon_2 = \frac{4\sqrt{1-c}-(4-3c)}{3}$, which is a positive value under A2, Theorem 1 still holds. And ϵ_2 is increasing in c.

Theorem 2. *Under A1 and A2, Theorem 1 holds even when F_0 has small cost disadvantage compared with F_1 in providing A.*

5 When $n > 2$

When $n > 2$, with the two assumptions below,

A1. $b < a < \frac{b(1-2c-2c^2)}{2c(1-2c)}$;

A2. $c \in (0, \frac{2-\sqrt{2}}{2})$,

we have Theorem 2 and 3.

Theorem 3. *In the unique SPE $F_1, F_2, ..., F_n$ all outsource to F_0 and prices satisfy $\{d_0 = d_0^{**}, d_1 = \beta(d_0^{**})\}$.*

Theorem 4. *$F_1, F_2, ..., F_n$ all outsources to F_0 in any SPE even when F_0 has some cost disadvantage compared to F_1. Furthermore, the allowed cost disadvantage is increasing in n.*