Towards a Characterization of Rational Expectations

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Abstract

R. J. Aumann and J. H. Drèze (2005) define a *rational expectation* of a game G as an expected payoff of some type of Player 1 in some belief system for G in which common knowledge of rationality and common priors obtain. Our goal is to characterize the set of rational expectations in terms of the game's payoff matrix. We provide such a characterization for a specific class of strategic games, which we call semi-elementary.

1 Definitions and Notations

Let G be a strategic n-person game, S_i the strategy set of player i, and U_i the payoff functions from $S_1 \times \ldots \times S_n$ to $\mathbf{R}$. A *belief system* B for G consists of:

(1) For each player i, a finite set T_i, whose members t_i are called *types* of i.

(2) For each *type* t_i of each player i,

(a) a strategy of i in G, denoted $s_i(t_i)$, and

(b) a probability distribution on $(n-1)$-tuples of types of the other players, called t_i's *theory*.

A *common prior* (CP) is a probability distribution π on $T_1 \times \ldots \times T_n$ that assigns positive probability to each type of each player, and such that the theory of each type of each player is the conditional of π given that the player is of that type. A type of a player is *rational* if the strategy it prescribes maximizes his expected payoff given his theory. Rationality is *commonly known* (CKR) if this is so for all types of all players.

We analyze G from the viewpoint of Player 1. A *rational expectation* in G is the expected payoff of some type of the Player 1 in some belief system for G in which CKR and CP obtain. We wish to characterize the set of rational expectations.

The *doubled* game $2G$ is the n-person game in which Player 1's strategy set is $S_1 \times \{1, 2\}$. That is, there are two copies of each of Player 1 strategies, the payoff functions are identical to the original game functions and do not depend on which copy is used.

Notations

Let G be a strategic game and μ a correlated equilibrium of G.

(1) For every $s_1 \in S_1$ s.t. $\mu(s_1) := \sum_{s_{-1} \in S_{-1}} \mu(s_1, s_{-1}) > 0$ let $(\mu \mid s_1)$ be the conditional probability distribution vector over S_{-1} given μ.

That is, $(\mu \mid s_1) := \sum_{s_{-1} \in S_{-1}} [\mu(s_1, s_{-1})/\mu(s_1)] e_{s_{-1}}$ (where $e_{s_{-1}}$ is the appropriate unit vector in $\mathbf{R}^{|S_{-1}|}$).

(2) Let v be a probability distribution vector over S_{-1}. For every $s_1 \in S_1$ we define $H_{s_1}(v)$ to be the payoff on s_1 given v.

That is, $H_{s_1}(v) := \sum_{s_{-1} \in S_{-1}} v_{s_{-1}} U_1(s_1, s_{-1})$.

(3) For every strategy $s_1 \in S_1$ we define a set $C(s_1) \subseteq \mathbf{R}$ as follows:

$\alpha \in C(s_1)$ if α is a conditional correlated equilibrium payoff for the strategy s_1, i.e., $\alpha \in C(s_1)$ iff there exists a correlated equilibrium μ of G s.t. $H_{s_1}(\mu \mid s_1) = \alpha$.

(4) We denote the set of conditional payoffs of Player 1 by $C(G)$. We note that $C(G) = \bigcup_{s_1 \in S_1} C(s_1)$.

Definitions

Given a strategic game G, we say that it is:

(1) *Elementary*, if it has a correlated equilibrium that assigns positive probability to each strategy of each player, and all the inequalities associated with this equilibrium are strict.

(2) *Full*, if it has a correlated equilibrium that assigns positive probability to each profile of strategies.

(3) *Semi-elementary*, if it is a *full* game and it has a correlated equilibrium s.t. all the inequalities related only to Player 1 are strict. That is, there exists a correlated equilibrium π s.t. $\pi(s) > 0$ for every $s \in \prod_{i \in N} S_i$, and

$$H_{s_1}(\pi \mid s_1) > H_{s_1^{'}}(\pi \mid s_1) for every s_1, s_1^{'} \in S_1, s_1 \neq s_1^{'}.$$

Before we quote our main results, we note that every *elementary* game is also a *semi-elementary* game.

Proof: Let G be an *elementary* game. By definition, G has a correlated equilibrium μ that assigns positive probability to each strategy of each player, and in which the associated inequalities are strict. Let S be the set of strategy profiles in G and θ a correlated strategy that assigns equal probabilities to all strategy profile. Then for sufficiently small $\varepsilon > 0$, $\lambda := (1-\varepsilon)\mu + \varepsilon\theta$ assigns positive probabilities to each strategy profile, and the associated inequalities are still strict. So G is a *semi-elementary* game.

Therefore the *semi-elementary* games is a larger family of games and all of our results, related to *semi-elementary* games, valid also for an *elementary* games.

The Main Results

Theorem 1: For every game G the set of rational expectations is closed.

Theorem 2: For a *semi-elementary* game G the set of rational expectations is the closed interval

$$[\max_{v\in\Delta(S_1)} \min_{s_{-1}\in S_{-1}} U_1(s_1, v), \max_{s\in S_1\times\ldots\times S_n} U_1(s)].$$

We will also show that Theorem 2 fails for $full$ games.

2 Proofs

Fundamental Proposition

Let G be a strategic game. We denote by $\pi(G)$ the convex polytope of the correlated equilibria set. Now let $\lambda \in \pi(G)$ be a correlated equilibrium and $s_1 \in S_1$ s.t. λ chooses s_1 with positive probability that is $\lambda(s_1) > 0$. Then there exist vertices of $\pi(G)$-$\mu^{'}, \mu^{''}$ s.t. $\mu^{'}$ and $\mu^{''}$ choose s_1 with positive probability and

$$H_{s_1}(\mu^{'} \mid s_1) \leq H_{s_1}(\lambda \mid s_1) \leq H_{s_1}(\mu^{''} \mid s_1).$$

In other words, for every correlated equilibrium $\lambda \in \pi(G)$ and a strategy s_1, chosen by λ in a positive probability, we can find vertices $\mu^{'}, \mu^{''} \in \pi(G)$ s.t. the conditional payoff of Player 1, given that the mediator tries to implement the strategy s_1, is grater if he uses $\mu^{''}$ and smaller if he uses $\mu^{'}$.

We will prove first the following lemma.

Lemma 2.1: Let $v_j \in \mathbf{R}^n_{++}$ ($\mathbf{R}^n_{++} = \mathbf{R}^n_+ \setminus \{0\}$), $j = 1, ..., m$, and let $\{a_j\}_{j=1,...,m}$ be positive constants and $u \in \mathbf{R}^n$. Denote $y = \sum\limits_{j=1}^{m} a_j v_j$, we define a function $E_u(\cdot) : \mathbf{R}^n_{++} \to \mathbf{R}$ by

$$E_u(v) = \frac{< v, u >}{|| \, v \, ||_1}$$

where $|| \cdot ||_1$ is simply the sum over the absolute value of the coordinates (note that $E_u(\cdot)$ is well defined on $\mathbf{R}^n_{++}$).

Then

$$\min_{1 \le j \le m} E_u(v_j) \le E_u(y) \le \max_{1 \le j \le m} E_u(v_j).$$

Proof: We use induction on m to show that $E_u(y)$ is a convex combination of $E_u(v_j)$. For $m = 1$ it is trivial. For $m = 2$, $y = a_1 v_1 + a_2 v_2$

$$E_u(y) = \frac{a_1 < v_1, u > + a_2 < v_2, u >}{a_1 \, || \, v_1 \, ||_1 + a_2 \, || \, v_2 \, ||_1}.$$

Multiply both numerator and denominator by $\frac{1}{||v_1||_1 ||v_2||_1}$ to get

$$E_u(y) = \frac{\frac{a_1 < v_1, u> + a_2 < v_2, u>}{||v_1||_1 ||v_2||_1}}{\frac{a_1}{||v_2||_1} + \frac{a_2}{||v_1||_1}}.$$

Now let $A_1 = \frac{a_1}{||v_2||_1}$ and $A_2 = \frac{a_2}{||v_1||_1}$, $A_j > 0$, $j = 1, 2$, so we get $E_u(y)$ as a convex combination of $E_u(v_1)$ and $E_u(v_2)$. That is,

$$E_u(y) = \frac{A_1 E_u(v_1) + A_2 E_u(v_2)}{A_1 + A_2}$$

and we're done.

For $m > 2$, $y = \sum\limits_{j=1}^{m} a_j v_j$. Let $y' = \sum\limits_{j=1}^{m-1} a_j v_j$. By the induction assumption there exists $\overline{a} \in \Delta^{m-2}$ s.t. $E_u(y') = \sum_{j=1}^{m-1} a_j E_u(v_j)$. But $y = 1 \cdot y' + a_m v_m$, so from the $m = 2$ case we get $0 \le \alpha \le 1$ s.t.

$$E_u(y) = \alpha E_u(y') + (1 - \alpha) E_u(v_m)$$

so

$$E_u(y) = \alpha \sum_{j=1}^{m-1} a_j E_u(v_j) + (1 - \alpha) E_u(v_m)$$

and we get y as a convex combination of $E_u(v_j)$.

This ends the proof of lemma 2.1.

Proof of the fundamental proposition: From the fact that $\pi(G)$ is a convex polytope we can write every $\lambda \in \pi(G)$ as a convex combination of the vertices of $\pi(G)$. Therefore $\lambda = \sum_{j=1}^{k} a_j \mu_j$, where $a = (a_1, ..., a_k) \in \Delta^{k-1}$ and $\{\mu_j\}_{1 \leq j \leq k}$ is the vertices set of $\pi(G)$.

Let λ^{s_1} be the reduced vector attained from λ by eliminating all the values that do not involve the strategy s_1 that is $\lambda^{s_1} = \sum_{s_{-1} \in S_1} \lambda(s_1, s_{-1}) e_{s_{-1}}$. Because λ is a distribution vector and $\lambda(s_1) > 0$ we deduce that $\lambda^{s_1} \in \mathbf{R}_{++}^{|S_{-1}|}$.

W.l.o.g. we can assume that $\mu_j^{s_1} \neq \overline{0}$ and $a_j > 0$ for every $1 \leq j \leq m$ where $m \leq k$. So we get μ^{s_1} as a positive combination of $\{\mu_j^{s_1}\}_{1 \leq j \leq m}$; that is, $\lambda^{s_1} = \sum_{j=1}^{k} a_j \mu_j^{s_1}$. We denote by $u \in S_{-1}$ the appropriate payoff vector to s_1, so

$$H_{s_1}(\lambda \mid s_1) = E_u(\lambda^{s_1})$$

Using lemma 2.1 we complete the proof of the proposition.

Corollaries from the fundamental Proposition

Corollary 2.1: The conditional correlated equilibrium payoffs set, $C(G)$, is closed.

We will prove corollary 2.1 based on the following lemma.

Lemma 2.2: For every $s_1 \in S_1$, $C(s_1)$ is a convex set.

Proof: Let $s_1 \in S_1$ if $C(s_1) = \emptyset$ then we are done. Suppose now $\alpha, \beta \in C^1(s_1)$, $\alpha \leq \beta$. We have to prove that $[\alpha, \beta] \subseteq C^1(s_1)$. Let μ_α, μ_β be a correlated equilibrium s.t. $H_{s_1}(\mu_{\alpha,\beta} \mid s_1) = \alpha, \beta$ respectively. From the convexity of the correlated equilibrium set we deduce that for every $0 \leq t \leq 1$, $\mu(t) = (1-t)\mu_\alpha + t\mu_\beta$ is a correlated equilibrium.

Define $f : [0,1] \to \mathbf{R}$ as follows: $f(t) = H_{s_1}(\pi(t) \mid s_1)$. f is a continuous function as a composition of continuous functions, and for every $0 \leq t \leq 1$,

$$f(t) \in C(s_1).$$

But $f(0) = \alpha$ and $f(1) = \beta$, so from the Intermediate Value Theorem for continuous functions we get

$$[\alpha, \beta] \subseteq Range(f) \subseteq C(s_1).$$

Proof of Corollary 2.1: Let $V(s_1)$ be the subset of vertices in $\pi(G)$ that ascribe positive probabilities to the strategy s_1. If $V(s_1)$ is a nonempty subset then we deduce from lemma 2.2 and the fundamental proposition lemma that

$$C(s_1) = [\min_{\mu^{'} \in V(s_1)} H_{s_1}(\mu^{'} \mid s_1), \max_{\mu^{'} \in V(s_1)} H_{s_1}(\mu^{'} \mid s_1)].$$

But $C(G) = \bigcup_{s_1 \in S_1} C(s_1)$ so it is closed as a finite union of closed sets.

Proof of theorem 1: Aumann and Drèze proved an existence of a one-to-one correspondence between the *rational expectations* in the game G and the conditional correlated equilibrium payoffs in the doubled game $2G$. That is α is a rational expectation in G if and only if there exists a correlated equilibrium of the game $2G$ s.t. α is a conditional payoff in some strategy of Player one.

Therefore α is a rational expectation of Player 1 iff $\alpha \in C(2G)$. In particular, we get the *rational expectations* set as a closed set. So we get Theorem 1 as a corollary of the fundamental proposition.

Corollary 2.2: Using algorithms of linear programming we can characterize the *rational expectations* set.

Proof: We have just seen how $C(2G)$ is determined by the vertices of $\pi(2G)$. Now using the simplex algorithm (for example) we can run through the vertices and compute the relevant values.

Theorem 2

We will divide the proof of Theorem 2 into two parts. On *part a* we will prove the convexity of the *rational expectations* set for a *semi-elementary* games. On *part b* we will show that the *rational expectations* set is the interval

$$[\max_{v \in \Delta(S_1)} \min_{s_{-1} \in S_{-1}} U_1(s_1, v), \max_{s \in S_1 \times \ldots \times S_n} U_1(s)].$$

Definition: For a strategic game G, we will say that the strategy $s_1 \in S_1$ is a *best reply* for $v \in \Delta(S_{-1})$ if for every $s_1^{'} \in S_1 : H_{s_1}(v) \geq H_{s_1^{'}}(v)$.

Let G be a *semi-elementary* game. We will show that the *rational expectations* set is a convex (closed) set.

We will first prove the following proposition:

Proposition 2.1: For every *semi-elementary* game G and a *best-reply* distribution vector $v \in \Delta(S_{-1})$ for some strategy s_1. there exists a correlated equilibrium μ of $2G$, s.t. $(\mu \mid s_1^*) = v$ (where s_1^* , s_1^{**} are the two copies of the strategy s_1 in $2G$).

Proof: Let π be the correlated equilibrium attained from G being *semi-elementary*. We will define a correlated equilibrium μ on $2G$ s.t. $(\mu \mid s_1^*) = v$.

Let δ be s.t. $0 < \delta < \min_{s_{-1}\in S_{-1}}\{\pi(s_1, s_{-1}) \mid \pi(s_1, s_{-1}) > 0\}$. First we show that there exists a small enough $0 < \epsilon \leq \delta$ s.t. for every $s_1^{'} \neq s_1$:

$$(*) \sum_{s_{-1}\in S_{-1}} (\pi(s_1, s_{-1}) - \epsilon v_{s_{-1}}) \cdot U_1(s_1, s_{-1}) \geq \sum_{s_{-1}\in S_{-1}} (\pi(s_1^{'}, s_{-1}) - \epsilon v_{s_{-1}}) \cdot U_1(s_1^{'}, s_{-1})$$

Now both sides of $(*)$ are continuous functions of ϵ. For $\epsilon = 0$ the inequality in $(*)$ is strict and both sides of $(*)$ are monotonic in ϵ. As a result for every $s_1^{'} \neq s_1$ we can choose $0 < \epsilon\,(s_1^{'})$ s.t. the inequality in $(*)$ holds for every $0 \leq \varepsilon \leq \epsilon(s_1^{'})$. If we define

$$\epsilon = \min\{\epsilon(s_1^{'}) \mid s_1^{'} \in S_1, s_1^{'} \neq s_1\}$$

we will get the desired ϵ.

We define μ as follows:

For every $s_1^{'} \neq s_1$ and for every $s_{-1} \in S_{-1}$

$$\begin{aligned} \mu(s_1^{'*}, s_{-1}) &= \pi(s_1^{'}, s_{-1}) \\ \mu(s_1^{'**}, s_{-1}) &= 0 \end{aligned}$$

and for s_1

$$\begin{aligned} \mu(s_1^{**}, s_{-1}) &= \pi(s_1, s_{-1}) - \epsilon v_{s_{-1}} \\ \mu(s_1^{*}, s_{-1}) &= \epsilon v_{s_{-1}}. \end{aligned}$$

Lemma 2.3: The above μ is a correlated equilibrium of $2G$.

Proof: For any player other than player 1 all the required inequalities hold because π is a correlated equilibrium. Now for $s_1^{'} \neq s_1$ we have the same argument for the relevant $s_1^{'*}$. So what we have left to show is that the inequalities hold for the two copies of s_1.

From the definition of ϵ and the fact that the inequalities in $(*)$ hold, for every $s_1^{'} \neq s_1$ we have

$$H_{s_1}(\mu \mid s_1^{**}) \geq H_{s_1^{'}}(\mu \mid s_1^{**}).$$

From the fact that v is a *best reply* to s_1 we deduce directly from the definition that

$$H_{s_1^*}(\mu \mid s_1^*) = H_{s_1^*}(v) \geq H_{s_1^{'}}(v) = H_{s_1^{'}}(\mu \mid s_1^*).$$

We get μ as a correlated equilibrium of $2G$ and we proved Lemma 2.3. But $(\mu \mid s_1^*) = v$ so we have also proved proposition 2.1.

Part a of Theorem 2:

proof: Let G be a *semi-elementary* game and let α, β be *rational expectations* of G, $\alpha \leq \beta$. We would like to show that the interval $[\alpha, \beta]$ is included in the *rational expectations* set of G.

From the fact that α, β are *rational expectations* we got μ_α and μ_β correlated equilibria of $2G$ and $s_1, s_1^{'} \in S_1$ s.t.

$$\begin{array}{lcl} H_{s_1}(\mu_\alpha & \mid & s_1) = \alpha and \\ H_{s_1^{'}}(\mu_\beta & \mid & s_1^{'}) = \beta. \end{array}$$

We define functions $v(t) : [0,1] \rightarrow \mathbf{\Delta}^{|S_{-1}|-1}$ and $f(t) : [0,1] \rightarrow \mathbf{R}$ as follows:

$$\begin{array}{rcl} v(t) & = & t(\mu_\beta \mid s_1^{'}) + (1-t)(\mu_\alpha \mid s_1) \\ f(t) & = & \max_{s_1 \in S_1} H_{s_1}(v(t)). \end{array}$$

$H_{s_1}(v(t))$ is a continuous function for every $s_1 \in S_1$. Therefore $f(t)$ is a continuous function as a maximum over a finite set of continuous functions. Now for every $0 \leq t \leq 1$ $v(t)$ is a *best-reply* distribution vector for some $s_1 \in S_1$.

From proposition 2.1 we've got a correlated equilibrium $\lambda(t)$ of $2G$ s.t.

$$(\lambda(t) \mid s_1^*) = v(t).$$

So $f(t)$ is a *rational expectation* for every $0 \leq t \leq 1$, that is $f(t) \in C(2G)$. But $f(0) = \alpha$ and $f(1) = \beta$ so we can deduce from the continuity of $f(t)$ that $[\alpha, \beta] \subseteq C(2G)$.

Part b of Theorem 2

Let $\widetilde{G}$ be the two-person zero-sum game derived from G where the strategy set of the row player is S_1 and the strategy set of the column player is S_{-1}. The payoff function is $g(s_1, s_{-1}) = U_1(s_1, s_{-1})$. Let

$$a = \max\min \widetilde{G}, b = \max\{U_1(s) : s \in S_1 \times \ldots \times S_N\}$$

Proof of part b: Aumann and Dreze showed that for every game G, $C(2G)$ is bounded from below by a and that in an *elementary* game $b \in C(2G)$. Using proposition 2.1 it will be easy to generalize this also to *semi-elementary* games.

Lemma 2.4. For every *semi-elementary* game G, $b \in R(G)$.

Proof: Let G be a *semi-elementary* game and let $s_1 \in S_1$, $s_{-1} \in S_{-1}$ s.t. $b = U_1(s_1, s_{-1})$. Now let $v \in \Delta(S_{-1})$ be defined by

$$v_{s_{-1}} = 1 and v_{s'_{-1}} = 0, for s'_{-1} \neq s_{-1}.$$

By the definition of v we get

$$b = H_{s_1}(v) \geq H_{s'_1}(v) for every s'_1 \in S_1.$$

Therefore v is *best reply* to s_1. So by proposition 2.1 there exists a correlated equilibrium π of $2G$ s.t. $(\pi \mid s_1^*) = v$ and we get b as a rational expectation of Player 1, $b \in C(2G)$.

Lemma 2.5. For a *semi-elementary* game G, $a \in C(2G)$.

Proof: a is the value of the game $\widetilde{G}$ defined above. So by the min max theorem we have $\overline{y}^* = \{y^*_{s_{-1}}\}_{s_{-1} \in S_{-1}}$ an optimal strategy for the column player that assures him an expected payoff smaller than the value for every strategy of the row player. $\overline{x}^* = \{x^*_{s_1}\}_{s_1 \in S_1}$ an optimal strategy for the row player that assures him an expected payoff greater than the value for every strategy of the column player. So we have

$$(\#) \sum_{s_{-1} \in S_{-1}} \overline{y}^*_{s_{-1}} g(s_1, s_{-1}) = H_{s_1}(\overline{y}) \leq a, for every s_1 \in S_1.$$

On the other hand, for $s_1 \in S_1$ s.t. $x^*_{s_1} > 0$ we have of course equality in (#), so we get $\overline{y}^*$ as a *best-reply* vector for that s_1. According to proposition 2.1. we have a correlated equilibrium π of $2G$ s.t. $(\pi \mid s_1^*) = \overline{y}^*$. So we get a as a rational expectation, $a \in C(2G)$.

We proved that $a, b \in C(2G)$, and they are also the boundaries of $C(2G)$ from below and above respectively. From part a of Theorem 2 (convexity of $C(2G)$ for *semi-elementary* games) we deduce that $C(2G) = [a, b]$.

3 Failure of Theorem 2 without Semi-elementarity

We ask ourselves whether we can go one step further and abandon the demand for semi elementarity, i.e., whether the conclusion of Theorem 2 holds for $full$ games that are not *semi-elementary* games.

The answer to this question is, unfortunately, no.

Take for example the following game:

$$G = \begin{array}{|c|c|}\hline 1,-1 & -1,1 \\ \hline -1,1 & 1,-1 \\ \hline -4,0 & 2,0 \\ \hline \end{array}$$

We note first that the game $G' = \begin{array}{|c|c|}\hline 1,-1 & -1,1 \\ \hline -1,1 & 1,-1 \\ \hline \end{array}$ is a two-person zero-sum game, that has a unique correlated equilibrium, that is also a Nash equilibrium assigns equal probability of $\frac{1}{4}$ to every profile of strategies in G'. Secondly, every correlated equilibrium of the game G that assigns positive probability to one of the first two strategies has to satisfy the same constraint of the game G', i.e., the reduction of every correlated equilibria of the game G, that assigns positive probability to one of the first two strategies, to the game G' is a correlated equilibrium of G'.

Therefore we can deduce that the set of correlated equilibria of G is the following:

$\begin{array}{|c|c|}\hline \frac{(1-\alpha)}{4} & \frac{(1-\alpha)}{4} \\ \hline \frac{(1-\alpha)}{4} & \frac{(1-\alpha)}{4} \\ \hline \alpha\beta & \alpha(1-\beta) \\ \hline \end{array}$ for $0 \leq \alpha \leq 1, 0 \leq \beta \leq \frac{1}{4}$.

Proposition: The set of conditional correlated equilibrium payoffs for Player 1 in the game $2G$ is the same as in the game G.

Proof: It is clear that the set of payoffs for the two copies of the third strategy is the same in G and in $2G$. This follows from the fact that for every distribution vector $v \in \Delta(S_{-1})$ where the third strategy is a *best-reply* to it,there exist a correlated equilibrium π of G s.t. $(\pi \mid s_1^3) = v$.

Now let $\mu = \{\mu_{ij}\}_{1 \leq i \leq 6, 1 \leq j \leq 2}$ be a correlated equilibrium of $2G$ that ascribes a positive probability to one of the copies of the first two strategies. Let $a = \sum_{i,j=1}^{2} \mu_{ij}$, $a > 0$. we can define a correlated equilibrium λ of $2G'$ using μ as follows:

$$\lambda_{ij} = \frac{1}{a}\mu_{ij} for 1 \leq i \leq 4, 1 \leq j \leq 2$$

The fact that λ is a correlated equilibria follows from:

a. μ being a correlated equilibrium of $2G$.

b. Given that Player one is playing the third strategy the payoff of Player two is zero.

Now every two-person zero-sum game has a unique *rational expectation*, (Aumann and Drèze) which is the value. In this case namely zero thus

$$C(G) = C(2G) = \{0\} \cup [\frac{1}{2}, 2].$$

And convexity fails.

4 References

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