

Lies and Slander: The Implementation of Truth-telling in Repeated Matching Games with Private Monitoring

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Abstract

This paper studies the implementation of truth-telling in a repeated random matching prisoners' dilemma where outcomes cannot be observed or verified by the public. It is well established that truthful information sharing can be a powerful device to overcome moral hazard problems. An equilibrium strategy can ask players to condition their behavior on this shared information, which creates strong incentives for cooperation (public information strategy). The paper, first, shows that there is no direct mechanism which implements truth-telling in Nash equilibrium, subgame perfect equilibrium or any other equilibrium solution concept if an equilibrium outcome in the repeated game is supported by a public information strategy. Second, there is a mechanism which implements truth-telling in subgame perfect equilibrium if an equilibrium strategy in the repeated game asks players to condition their behavior on both, public and private information. However, if an outcome can be supported by this strategy, it can also be supported by a strategy without information sharing.

Keywords: Truth-telling in repeated games, slander, implementation with complete information, perfect private monitoring.

1 Introduction

This paper builds directly on the papers by Greif (1994) and Kandori (1992). These papers study the role of information sharing in repeated matching games and establish that truthful information sharing may be a powerful device to overcome moral hazard problems. For example, Greif (1994) shows that the exchange of letters among Maghribi traders revealing information about the trustworthiness of overseas agents in merchant-agent relationships was an integral part of the incentive structure to keep agents honest. With an "information sharing institution" in place, one can achieve a desired outcome in the repeated game by playing strategies where players are asked to condition their behavior on this shared information (public information strategy). Then, the repeated matching game essentially collapses into a two-player game where each player plays a game with this "information sharing institution".¹ Equilibria based on a public information strategy have attractive properties because they do not depend on matching-rules (which in reality may be quite complicated), the number of players participating at the information-sharing institution, personal histories between specific players, etc. Obviously, key is that this "information sharing institution" produces truthful information which in the mentioned papers is assured by assumption. The present paper relaxes this assumption and studies the implementation of truth-telling in such games.

The implementation of truth-telling is not straightforward because of the presence of incentives to lie and slander: For example, Greif's "collectivistic strategy" is not negotiation proof which aligns matched players incentive to lie about a defection, since given that a defection occurred *both* players prefer to announce otherwise (see Section 2 for this case). On the other hand, in games where an equilibrium strategy is negotiation proof, players "like" to punish which gives players an incentive to slander (see Section 4 for this case). Thus, in either case to produce truthful information-sharing means to design a mechanism which helps players to overcome these incentive problems.

The analysis is based on an infinitely repeated prisoners' dilemma game where a finite number of players are randomly matched into pairs in each period. The sharing of information is modelled as a game or a direct mechanism g between the two matched players and a player's subsequent match. This setup captures the situation of search for references, where a player's current match inquiries about this player's play with his or her past match. A mechanism is direct because the strategy space in g for the two matched players simply consists in announcing the state of two players such as "guilty" or "innocent" (or any other state that needs to be reported to support an outcome in the repeated PD game). Announcements can be made simultaneously (Nash implementation) or sequentially (subgame perfect implementation). It is shown that a direct mechanism is entirely specified by the characteristics of the repeated game such as matching-rule and payoff structure, the equilibrium strategy used to support an outcome in the repeated game, and a rule (I call it an interpretation-rule)

¹For example, in Milgrom et al. (1990) the "Law Merchant" serves as an "information sharing institution".

which determines the state of each player for the continuation of the game as a function of the states announced by the players.

The main result of this paper establishes that there is *no* direct mechanism which implements truth-telling in Nash equilibrium, subgame perfect equilibrium or any other solution concept if an outcome in the repeated game is supported by a public information strategy. By "implementation" I refer to the standard definition of implementation where a desired outcome emerges as the *unique* equilibrium outcome of a mechanism g (Repullo 1986, Moore 1992). Important to note is that this result does not depend on the choice of any particulars, such as the stage game, matching-rules, the number of players (finite or infinite). Thus, this result is general. However, in some cases there is a mechanism g which implements the truth in a weaker form: For any possible state, there is a truthful equilibrium which coexists with other untruthful equilibria (which — however — often Pareto dominate the truthful one). Second, the paper shows that if the equilibrium strategy in the PD game asks players to condition their behavior on both, public *and* private information, there is a direct mechanism which implements the truth as a *unique* subgame perfect equilibrium. However, this result comes with a downside: If a strategy using both public and private information is an equilibrium in the repeated game then there is always an equilibrium strategy which supports the same outcome *without* information sharing. That is if one would like to be confident that information is shared truthfully by having a strong implementation requirement (e.g. truth as a unique equilibrium of the mechanism), then there is an alternative strategy which self-enforces cooperation without information sharing, which means that the benefits resulting from information-sharing as pointed out by the papers mentioned above disappear.

Truthful information sharing as an enforcement device is important in current applications as well: For example, it is reported to be important in small-firm clusters in developing countries helping to sustain inter-firm cooperation which makes these firms more successful on export markets (Woodruff 1998, Annen 2001, Annen 2003). More prominently, information sharing is also used on auction Web sites such as eBay, Yahoo, and Amazon (Ockenfels 2003, Resnick and Zeckhauser 2002). Buyers on these Web sites typically put dollars at risk since goods are shipped only after their payment has been received. These Web sites provide a forum where buyers can report their personal experiences about their sellers and vice versa. These reports presumably influence the buying decision of future buyers. This last example makes clear that the dramatic improvements in information technology extend the *technological* feasibility of this mechanism considerably. However, not much is known about the *economic* feasibility of this mechanism.

There are only a few papers that analyze incentive problems related to information sharing in repeated games with perfect private monitoring (see Ben-Porath and Kahneman 1996, Ben-Porath and Kahneman 2003, and Banerjee 2000). These papers achieve truth-telling by designing the following mechanism: First, players must announce their observations simultaneously. Second, if the players' announcements contradict, *all* announcing players get punished.

This mechanism in some cases implements truth-telling in the weaker form described above where truth is one equilibrium outcome in g among other untruthful ones. Truth-telling depends crucially on the assumption that players make their announcements simultaneously (Nash implementation). This mechanism lacks cutting power, and one may have doubts that players will play the truthful equilibrium because of the presence of a Pareto dominant untruthful equilibrium.

The remainder of the paper is organized as follows: Section 2 introduces a repeated random matching one-sided prisoners' dilemma game (OSPD) and defines an equilibrium based on a *private information* and *public information* strategy assuming information sharing is truthful. Section 3 defines direct mechanisms which implement truth-telling in the repeated OSPD game. Section 4 defines direct mechanisms which implement truth-telling in a two-sided prisoner's dilemma. Final remarks conclude the paper.

2 Moral Hazard in Economic Transactions

Consider the situation where in each period $t = 1, 2, \dots$ a seller $i \in N_S = \{1, \dots, n\}$ and a buyer $j \in N_B = \{1, \dots, n\}$ are matched according to uniform random matching. In each period t , the probability of a seller to be matched with a specific buyer is $1/n$. Let $\mu(i, t)$ denote seller i 's match at time t . In each period, pairs of players play a one-sided Prisoner's dilemma (OSPD) where both players i and $j = \mu(i, t)$ choose an action a_i and $a_j \in \{C, D\}$ sequentially as depicted in figure 1. Throughout the paper I will use i to denote the seller and $j = \mu(i, t)$ to denote the buyer. Payoffs are assumed to satisfy $a > c > 0 > b$.

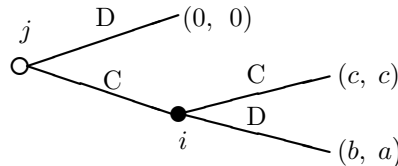


Figure 1: One-sided Prisoner's Dilemma

The one-shot OSPD game has a unique Nash equilibrium which is (D,D). Internet transaction on Web sites like eBay, Yahoo, or Amazon have the characteristic of a game with a one-sided incentive problem. First, Resnick and Zeckhauser (2002, p. 139) show in their empirical analysis of data on buyer-seller transactions on eBay that most sellers are "professional" sellers, i.e. players who are deliberately acquiring items from other sources in order to sell them on eBay. If a low feedback score of a seller indicates his status as an amateur seller, then only about 18% of of all seller are amateur sellers. Thus, there seems to be a rather clear separation of roles between sellers and buyers as it is part of the definition of a game with a one-sided incentive problem: Sellers and buyers match out of two difference sets of players. Second, internet transactions have the characteristic of a game with a one-sided incentive problem because buyers

typically pay for a good before they have the chance to inspect or receive it. This leaves sellers with the temptation of not fulfilling their part of the transaction by either not shipping items, shipping them late, or shipping items in a different condition than promised. Only sellers have an incentive problem. The buyer has no incentive problem. If she chooses D, then no transaction takes place and both players earn their reservation payoff which is assumed to be zero.

The following assumption holds throughout the paper:

Assumption 1 (Perfect Private Monitoring). *In any match, the outcome in the OSPD stage game is perfectly observable by player i and $\mu(i, t)$. This outcome is not observable for any other player, and outcomes are not verifiable ex post.*

The objective is to support the outcome (C,C) as an equilibrium outcome by infinitely repeating the OSPD game. It is assumed that players discount their payoffs with the discount factor δ , which is assumed to be the same for all players. It is easy to show that (C,C) can be supported as an equilibrium outcome based on personal retaliation even when players have a low probability of being rematched if players are patient enough. However, the minimal discount factor needed to support (C,C) as an equilibrium outcome may decrease substantially if there is a mechanism or an institution in place which allows players to share information. Then, an equilibrium strategy may ask players to choose their actions conditional of this shared information. For example, the strategy may ask you to play D when ever your partner is said to have cheated at least once in his or her past. Kandori (1992) shows that in a setting with a one-sided incentive problem, (C,C) can be sustained by an equilibrium where only defectors are punished if $\delta \in [\delta^*, 1)$, where δ^* is independent of the matching-rule and the population size n if there is an institution in place which processes information honestly so that players in any given match are assured to have information about each other's history. Note that this information processing institution does not need to provide information about the history of *all* players in the game but only about the history of a player's current partner. Information sharing is local. As in Kandori (1992), I assume throughout the paper that information sharing in the repeated OSPD matching game is local.

Assumption 2 (Local Information Sharing). *In each period t , before a pair of players i and $\mu(i, t)$ plays the OSPD stage game, information sharing between the players i , $\mu(i, t - 1)$, and $\mu(i, t)$ takes place.*

Assumption 2 implies that each player potentially knows the history of only a fraction of all players in the game. In this situation, a player has no information about the state of most players. More specifically, here seller i , i 's previous match and i 's current match share information with each other. This setting captures the situation of giving references. Before playing the OSPD game with i , one inquires with the person who had a last experience with i . An implication of assumption 2 is that in order to determine players' best responses one will need to specify their beliefs about the state of all the players for which they do not have information regarding their state. However, in the OSPD game

studied here, a buyer's best response is unaffected by different beliefs regarding the number of sellers who are on the punishment path.² Since player j 's choice in a given match does not affect his or her continuation payoff, j always plays one-shot best responses given seller i 's action.

As in any repeated game, there are many equilibrium strategies that support (C,C) as an outcome if players are patient enough. The analysis considers a simple strategy which is to play D once with a player who unilaterally deviated from playing C (one period Nash reversion, NR).³ This strategy is *forgiving* since the punishment after a unilateral defection last for only one period.

Definition 1 (NR-Strategy). *At time t , player i is in the state $s^i(t) \in \{\theta, \phi\}$, where $s^i(1) = \theta$, $\forall i \in N_S$ and N_B . The state of a match between i and $j = \mu(i, t)$ is denoted by $s = (s^i, s^j)$. The NR-strategy is defined by the triple $(\sigma(s), \rho(a_i), t^*)$. The action-rule, $\sigma(s)$, is specified as follows:*

$$a_i(t) = \sigma(s) = \begin{cases} C & \text{if } s(t^*) = (\theta, \theta), \\ D & \text{otherwise.} \end{cases}$$

The transition-function, $\rho(a_i)$, is specified as follows:

$$s^i(t+1) = \rho(a_i) = \begin{cases} \theta & \text{if } a_i(t) = \sigma(s), \\ \phi & \text{otherwise.} \end{cases}$$

The state $s^i = \theta$ indicates that player i is "innocent" while the state $s^i = \phi$ indicates that player i is "guilty".⁴ The fact that NR is forgiving by punishing a unilateral defection by a one-period Nash reversion only, does not affect the result of the paper. It simplifies notation because players can have only one of two possible states, namely "guilty" or "innocent". The action rule σ prescribes an action for each player as a function of the state s of a match, where $s \in S = \{(\theta, \theta), (\phi, \theta), (\theta, \phi), (\phi, \phi)\}$. The knowledge of the state s before playing the OSPD game can come from two sources:

- First, it can be a result of *personal experience*. That is, did a player's partner deviate from σ the last time they were matched. In this case t^* refers to the time period when player i and $\mu(i, t)$ were matched the last time, denoted by t^P . If players meet the first time, $t^P = 0$. I call a strategy which asks players to condition their behavior on their private information a *private information strategy* denoted by NR-P = $(\sigma, \rho, t^* = t^P + 1)$.

²Note that in the two-sided case, a player's best-response when matched with an innocent partner may be to choose D, if this player believes that he or she will be matched mostly with guilty players in subsequent periods.

³Note that this strategy does not constitute an *optimal penal code* (Abreu 1988). An optimal penal code is the punishment path with the lowest possible payoff for the punished player included in the set of all subgame perfect punishment paths. To play D in all subsequent periods after a unilateral defection constitutes an optimal penal code.

⁴The definition of NR provides for the possibility that a buyer may be "guilty", even though he or she has no incentive problem. To allow the possibility of guilty buyers may be important for providing incentives for truth-telling. For example, buyers may have an incentive to slander a seller.

- Second, the knowledge of whether a player is guilty or innocent can be based on *experience made by others* which is conveyed in a public label. The public label is the result of an information sharing process as described in assumption 2. In this case $t^* = t$ since by assumption 2 the history of a player is tracked without gap. I call a strategy which asks players to condition their behavior on public information revealed in a public label a *public information strategy* denoted by NR-L= $(\sigma, \rho, t^* = t)$.

In the first case, players are deterred from deviating from NR based on personal retaliation (in case of a rematch, players get punished if guilty), while in the latter case players are deterred from deviating from NR because of a punishment of the community of buyers (all subsequent matches will play D if guilty). In the terminology of Kandori (1992), the public information strategy NR-L is called a *straightforward* strategy when public labels contain all necessary information to sustain the outcome (C,C) as an equilibrium outcome.

A public information strategy has some attractive properties: First, once the OSPD game is played and states are reported, the game is entirely public. It implies that an equilibrium based on a public information strategy does not depend on the matching-rule. No personal retaliation is needed to support cooperation. Incentives to build reputations do not come from personal ties to specific players, but simply from an "information sharing institution". Within that institution, personal links do not matter. For example, even in case that players will never be rematched again (as for example with the the matching rule known as Townsend matching⁵), this strategy will be able to support the outcome (C,C). The player's incentive to build a reputation for being cooperative comes from the link to the information-sharing institution and not from personal links within that institution. In this sense, it is a more anonymous form of trade than personalized trade where incentives to be cooperative emerge from the future with a specific partner. Second, a public information strategy has also the property that it is transparent. All necessary information is contained in the public label. It does not allow for ambiguities such as the play of games involving "public lies and private truth". If new players join the game, then they can rely on the public label and need not to have a private personal history with a player in order to reach the cooperative outcome.

Proposition 1. *Under assumptions 1 and 2, and if $a < 2c$, NR-P is a subgame perfect equilibrium in the repeated OSPD random matching game iff $\delta \in [\delta^P, 1)$ where $\delta^P = \frac{n(a-c)}{a(n-1)-c(n-2)}$. When information sharing is assumed to be honest, NR-L is a sequential equilibrium⁶ in the repeated OSPD random matching game iff $\delta \in [\delta^L, 1)$ where $\delta^L = \frac{a-c}{c}$. For $n > 1$, $\delta^P - \delta^L > 0$, and this difference increases in n .*

⁵For example, Milgrom et al. (1990) use Townsend matching in their application of the Law Merchant in medieval Europe.

⁶Note due to the one-sidedness of the incentive structure, to play C with an innocent player is always optimal no matter whether buyer j believes he or she will be matched with a few or many guilty sellers in subsequent periods.

Proof. See appendix. □

Note that the payoff restriction $a < 2c$ is needed because otherwise to play D all the time is clearly better than playing NR. The reason is that due to the strong forgiveness of NR, by always playing D player i is able to get a every second period with information sharing (or every second rematch without information sharing). Note also that the benefits from information-sharing increases in the number of players, since the power of private retaliation decreases as the number of players increases.

Proposition 1 implies that the existence or non-existence of truth-full information sharing may decide of whether cooperation among a group of players is sustainable or not.⁷ Without information sharing, incentives may be too weak to support (C,C) among the players. The key question, however, is whether there is an information sharing institution or mechanism that is able to produce truth-telling as an equilibrium outcome. Obviously information sharing without truth-telling gives no incentives to play C. The players' incentive to invest in their reputation depends crucially on the ability of an information sharing process to produce information which is truthful.

3 Implementing Truth-telling

The objective is to find a revelation mechanism or a *direct mechanism* g which gives players an incentive for truth-telling. Information-sharing constitutes an implementation problem with complete information.⁸ I focus on direct mechanism by limiting the attention to games in which the strategy set in g consists of the set of states S in the OSPD game. Thus, the mechanism is a function $g : S \times S \rightarrow S$.

Consider the OSPD game introduced before in which we seek to support the outcome (C,C) by playing NR-L= $(\sigma, \rho, t^* = t)$. Here, buyers and sellers in any given match need to know the state $s^i \in \{\theta, \phi\}$ of their partner i . In each match there are four possible outcomes characterizing the state of each player, given by $s \in S = \{(\theta, \theta), (\phi, \theta), (\theta, \phi), (\phi, \phi)\}$. The objective is to implement a function $f(s)$, which satisfies

$$f(s) = s.$$

That is, f specifies the public label s as a function of the true state s . For example, if the true state is $s = (\theta, \theta)$, then f specifies the public label $s = (\theta, \theta)$.

Since I focus on direct mechanisms, each player i in g chooses a message $s_i = (s_i^i, s_i^j)$, where I use subscripts to denote the player who sends the message, and superscripts to denote the player whose state is reported in the message. I shall introduce the convention that the symbol to the left of the comma refers to the state of the seller i and the symbol to the right of the comma refers to the state of the buyer j . In a match between a seller i and a buyer $j = \mu(i, t)$, for

⁷This point has been made by Okuno-Fujiwara and Postlewaite (1989) and Kandori (1992).

⁸See Moore (1992) for a very helpful overview of the literature on implementation with complete information.

example, the message $s_j = (\phi, \theta)$ means that buyer j announces that seller i is guilty and he or she is innocent. And the statement $s_i^j = \theta$ indicates that player i announces that player j is innocent. Furthermore, I introduce the convention that states without a subscript denote true states or public labels used for the continuation of the OSPD game. $s^i = \theta$, for example, indicates that the true state for player i is "innocent", or the public label relevant for the continuation of the OSPD game indicates that player i is innocent.

The direct mechanism g can be a *strategic form game* or an *extensive form game*, which will be partly dictated by the application to which the implementation problem applies. More specifically, as any game, the direct mechanism g is defined by the following components:

- First, the set of players in g is given by $N^g = \{i, \mu(i, t - 1), \mu(i, t)\}$ as defined in Assumption 2.
- Second, if g is a strategic form game, players i and $j = \mu(i, t - 1)$ send simultaneously a message s_i and $s_j \in S$ respectively. If g is a extensive form game, a set of sequences or histories H needs to be specified, where the empty sequence \emptyset is a member of H , and each component of a history h is a message s_i or s_j sent by a player as specified in a player function $\iota(h)$. For example, if $\iota(\emptyset) = i$, player i moves first. Note that player $\mu(i, t)$ has a passive role in this game: He simply listens to the messages and converts those messages into the state for the continuation of the OSPD game according to the interpretation-rule, $\tau(s_i, s_j)$, which will be defined below.
- Finally, a weak preference relation $R_i(s)$ and $R_j(s)$ depending on the true state s on the message space $S \times S$ in case of a strategic form game or a preference relation $R_i(s)$ and $R_j(s)$ on set of final histories in case of an extensive form game needs to be specified. A strict preference relation and indifference is denoted by $P_i(s)$ and $I_i(s)$ respectively.

Note that the preference ordering $R_i(s)$ is defined by the *continuation payoffs* in the OSPD game. That is $R_i(s)$ depends on properties of the game such as payoffs and the matching rule, and the equilibrium strategy defined to support a given outcome in the OSPD game. Since both players send messages about the state of the match s , the preference ordering $R(s)$ will also depend on how messages of the two players are translated into states for the continuation of the OSPD game. Thus, we need a rule — say an interpretation rule τ — that specifies the state for the continuation of the OSPD game as a function of the messages: $\tau : S \times S \rightarrow S$.⁹ The rule τ is a crucial part of an implementation mechanism. Then, together with the specification of the OSPD game (i.e. payoffs, matching-rule) and the equilibrium strategy NR-L= $(\sigma, \rho, t^* = t)$, the preference ordering in g is completely defined: The transition functions $\rho(s^i)$ and $\rho(s^j)$ specify player i 's and j 's message depending on their state before they

⁹Note that if we should consider the message of only one player, then that will be specified in τ .

played the OSPD game and the action chosen in the OSPD game. Then, the interpretation-rule $\tau(s)$ specifies the states of player i and j for the continuation of the game. And finally, the action-rule $\sigma(s)$ specifies continuation payoffs.

Once the mechanism g with complete information is defined, we can find the equilibria in g . If g is a normal form game, then I will be interested in Nash equilibria. If g is an extensive form game, then I will be interested in subgame perfect equilibria. Denote the set of equilibria outcomes (given the choice of an equilibrium solution concept such as Nash equilibrium or subgame perfect equilibrium) in g given the true state $s \in S$ by $E_g(s)$. Before I state a first result, the following two definitions are important (see Repullo (1986)):

Definition 2 ("To implement f "). *The direct mechanism g "implements f " in Nash equilibrium or subgame perfect equilibrium if for every true state $s \in S$*

$$E_g(s) \neq \emptyset, \text{ and}$$

$$E_g(s) = \{f(s)\}.$$

Definition 3 ("To weakly implement f "). *The direct mechanism g "weakly implements f " in Nash equilibrium or subgame perfect equilibrium if for every true state $s \in S$*

$$E_g(s) \neq \emptyset, \text{ and}$$

$$f(s) \in E_g(s).$$

Note that to "implement f " (Definition 2) is the standard definition of "implementation" and is a stronger concept than to "weakly implement f " (Definition 3). Since $f(s)$ is single valued, to "implement f " amounts to the requirement that the equilibrium in g for any given true state s to be unique. In contrast, to "weakly implement f " does not rule out the existence of other equilibria in g which are untruthful. Note that results based on this latter concept may be rather weak because in this case there is no guarantee that players will choose to play the truthful equilibrium if there are untruthful equilibria present (Repullo 1986). As shown below, in the application discussed here, in most states s there is an untruthful equilibrium which strictly dominates the truthful one. One, therefore, would expect players to play the untruthful equilibrium. Obviously a mechanism which "implements f " is much more robust, and one may hope to get truth-telling based on this stronger concept. However, this negative result follows:

Proposition 2. *i) If the outcome (C,C) in the OSPD game is supported by a public information strategy then no direct mechanism g "implements f " in Nash equilibrium or subgame perfect equilibrium.*

ii) If the outcome (C,C) in the OSPD game is supported by NR-L, no direct mechanism "weakly implements f " in subgame perfect equilibrium.

Proof. For any direct mechanism g used to "implement f " or to "weakly implement f " in order to support the outcome (C,C) by playing a *public information strategy*, the following holds:

$$R_i(s) = R_i(s'), \text{ and} \tag{1}$$

$$R_j(s) = R_j(s'), \text{ where } s \neq s', \forall s, s' \in S.$$

That is, independent of what happened in a match, the players' preference ordering is identical for any possible true state $s \in S$. The players' message space is identical for any state s . And since the equilibrium strategy asks all players to condition their behavior on public labels only, the continuation payoffs of the OSPD game will depend on the messages s_i and s_j and the interpretation rule τ . This implies that if s' is an equilibrium outcome in the true state s' , i.e. $s' \in E_g(s')$, then s' must also be an equilibrium outcome in the true state s , because the preference ordering has not changed by going from state s' to state s . Thus, g cannot be used to "implement f " (Definition 2) since $\{s, s'\} \subseteq E_g(s)$. This proves part *i*) of Proposition 2.

Assume contrary to Proposition 2 part *ii*) that g "weakly implements f " in subgame perfect equilibrium. Then because of (1), the following has to be true:

$$\{(\theta, \theta), (\phi, \theta), (\theta, \phi), (\phi, \phi)\} \subseteq E_g(s), \forall s \in S.$$

But this is only possible if $(\theta, \theta)I_k(s)(\phi, \theta)I_k(s)(\theta, \phi)I_k(s)(\phi, \phi)$ where $k = \iota(\theta)$ in g . But we know that $(\theta, \theta)P_i(s)(\phi, \theta)$ for all $s \in S$. Player i prefers being innocent than being guilty for the continuation of the game. It is also the case that $(\theta, \theta)P_j(s)(\phi, \theta)$ for all $s \in S$. Because of the probability of a rematch, buyer j prefers the situation in which seller i is innocent to the situation in which he or she is guilty. So the equilibrium outcome (ϕ, θ) fails to be an equilibrium outcome in an extensive form game whether seller i or buyer j moves first. That is $f((\phi, \theta)) \notin E_g(s) \forall s \in S$. Thus, g fails to "weakly implement f " in subgame perfect equilibrium. This proves part *ii*) of Proposition 2. \square

Note that Part *i*) of Proposition 2 is not driven by any of the specifications of the game I am analyzing here. That the preference ordering in g is identical in each true state $s \in S$ holds for any game in which an outcome is supported by a public information strategy. Thus, this result does not depend on characteristics such as the matching-rule, the number of players, and the characteristics of the stage game.

In contrast, Part *ii*) of Proposition 2 is driven by the particular game chosen: The positive probability of a rematch (due to the choice of the matching rule and the fact that the population size is finite) reduces continuation payoffs for *all* players in case of a punishment because NR-L is not negotiation proof. Given that a defection occurred, both players would be better off announcing that a defection did not occur. In contrast, when the probability of a rematch is zero, there is a mechanism which "weakly implements f " in subgame perfect equilibrium. In this case, $(\theta, \theta)I_j(s)(\phi, \theta)$, so that truth-telling can be established as a subgame perfect equilibrium if $\iota(\theta) = j$.

3.1 "Weakly Implementing f " in Nash Equilibrium

Proposition 2 suggests that we can find a direct mechanism g which "weakly implements f " in Nash equilibrium. This mechanism will have the property

that $\{(\theta, \theta), (\phi, \theta), (\theta, \phi), (\phi, \phi)\} \subseteq E_g(s)$ for all $s \in S$. One way to achieve that is to set up g such that players have to coordinate their messages, and if they fail to coordinate for both players something bad will happen. They may be both sent on the punishment path. This mechanism has for example been used by Ben-Porath and Kahneman (1996, 2003) in their n -persons repeated game with communication and perfect private monitoring (however, without random matching). Since I analyze a game with a one-sided incentive problem, we can design a mechanism based on the rule that only the player with an incentive problem is punished in case that messages are not coherent. The mechanism g is setup as follows: First, seller i and buyer $j = \mu(i, t)$ send simultaneously a message $s_i, s_j \in S = \{(\theta, \theta), (\phi, \theta), (\theta, \phi), (\phi, \phi)\}$. Second, a rule is imposed that if messages are not consistent, i.e. $s_i \neq s_j$, then the player with an incentive problem will be sent on the punishment path. That is, τ is defined as follows:

$$\tau(\cdot) = \begin{cases} (s_i^i, \theta) & \text{if } s_i^i = s_j^i, \\ (\phi, \theta) & \text{otherwise.} \end{cases}$$

The interpretation-rule τ checks for possible motivations of players to send messages. In particular, it recognizes that player j has no incentive problem, so clearly he or she could not have cheated even when players announce so. That is, players simply announce the state of seller i .

The game g is depicted in Figure 2 for the two possible true states $s^i = \theta$ and $s^i = \phi$.

	True State: $s^i = \theta$	True State: $s^i = \phi$																				
	<table style="margin: auto;"> <tr> <td></td> <td colspan="2">j</td> </tr> <tr> <td></td> <td>θ</td> <td>ϕ</td> </tr> <tr> <td>i</td> <td>θ</td> <td>ϕ</td> </tr> <tr> <td></td> <td><table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>c, c</td><td>p, c'</td></tr> <tr><td>p, c'</td><td>p, c'</td></tr> </table></td> <td><table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>c, c</td><td>p, c'</td></tr> <tr><td>p, c'</td><td>p, c'</td></tr> </table></td> </tr> </table>			j			θ	ϕ	i	θ	ϕ		<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>c, c</td><td>p, c'</td></tr> <tr><td>p, c'</td><td>p, c'</td></tr> </table>	c, c	p, c'	p, c'	p, c'	<table border="1" style="display: inline-table; border-collapse: collapse; text-align: center;"> <tr><td>c, c</td><td>p, c'</td></tr> <tr><td>p, c'</td><td>p, c'</td></tr> </table>	c, c	p, c'	p, c'	p, c'
	j																					
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c, c	p, c'																					
p, c'	p, c'																					
c, c	p, c'																					
p, c'	p, c'																					

Figure 2: Weak Nash Implementation

The fields indicate the continuation payoff generated in the OSPD when using τ and σ for the continuation of the game for player i and j respectively. If player i gets punished his continuation payoff is $p = \delta c / (1 - \delta) < c = c / (1 - \delta)$. Note that $c' = c(n - 1) / n + \delta c / (1 - \delta) < c$. The continuation payoff decreases because of the probability of being re-matched with the current partner in which case NR asks the players to carry out the punishment.

Figure 2 highlights the following points stated in the proof of Proposition 2: First, the payoff structure in g remains unaffected by the true state s . Second, the outcomes necessary to weakly implement $f(s)$ are an equilibrium outcome in g , namely (θ, θ) and (θ, ϕ) . Third, since $c > c'$, players prefer the equilibrium (θ, θ) , and, thus, would choose to play that equilibrium if they were allowed to coordinate their messages. Thus, if g would be set up as a sequential game, then (θ, θ) is the unique equilibrium independent of the true state — which means that it is not possible to "weakly implement f " in subgame perfect equilibrium

(Proposition 2). But since players are not able to coordinate their messages, truth-telling can be implemented by this mechanism: The equilibrium strategy will simply prescribe: "Send a message according to the rule ρ ". That means that in the true state $s^i = \theta$ only the outcomes in the row and column θ in the left-hand side matrix will be relevant because players check for unilateral deviations (check the benefit of telling a lie given the other player tells the truth). For this range of outcomes, (θ, θ) is the only Nash equilibrium. Similarly, if the true state is $s^i = \phi$ (i.e. seller cheated), then only the outcomes in row and column ϕ will be relevant, and in this range the only Nash equilibrium is (ϕ, ϕ) , which following τ produces the outcome (ϕ, θ) .

Is the message-interpretation rule τ reasonable? I would argue yes. First, note that this rule is different than simply stating believe the buyer j 's statement. The reason is that if $c > c'$ then the buyer would prefer to announce that the seller is innocent in order to avoid costly punishment in case of a rematch. Accordingly, τ ignores the buyer j 's statement $s_j^i = \theta$ if the seller turns himself or herself in and announces $s_i^i = \phi$. On the other hand, the rule ignores the seller i 's message $s_i^i = \theta$ if j announces $s_j^i = \phi$ (which in terms of continuation payoffs is costly to announce). However, the existence of truth-telling depends crucially on the choice of the solution concept of the implementation mechanism. That messages are sent simultaneously is essential. But because the mechanism has also an untruthful equilibrium in all true states, it has almost no cutting power.

3.2 Subgame Perfect Implementation

In many applications messages are sent sequentially. For example, on eBay players report their mutual experiences sequentially. One therefore, may want to impose on a mechanism g that truth-telling is a subgame perfect equilibrium in a sequential game. Since we know that the implementation problem depends on the strategy played in the OSPD game, one can work backwards and try to find a strategy based on with truth-telling can be implemented as a subgame perfect equilibrium. Consider the following strategy which asks players to take private and public information into account. It asks players to condition their behavior on their private information in cases where private information contradicts the information revealed by the public label.

Definition 4 (NR-PL Strategy). *At time t , player i is in the state $s^i(t) \in \{\theta, \phi\}$, where $s^i(1) = \theta$, $\forall i \in N_S$ and N_B . The state of a match between i and $j = \mu(i, t)$ is denoted by $s = (s^i, s^j)$. The NR-PL is defined by $(\sigma(s), \rho(a_i), t, t^P)$. The action-rule, $\sigma(s)$, is specified as follows:*

$$a_i(t) = \sigma(s) = \begin{cases} C & \text{if } s(t) = (\theta, \theta), \text{ and if } \{s^j(t^P + 1) = \phi | a_j(t^P) \neq \sigma(s)\} \\ & \text{or if } \{s^j(t^P) = \theta | a_j(t^P) = \sigma(s)\} \\ D & \text{otherwise,} \end{cases}$$

where t^P denotes the time period in which player i and j were previously matched.

The transition-function, $\rho(a_i)$, is specified as follows:

$$s^i(t+1) = \rho(a_i) = \begin{cases} \theta & \text{if } a_i(t) = \sigma(s), \\ \phi & \text{otherwise.} \end{cases}$$

The implication of this strategy is that when ever a guilty player failed to be labelled accordingly, a personal punishment is triggered which consists in playing D the next time these players are matched. That is, if a guilty player is not punished by the community of players, the action rule asks the cheated player to retaliate him or herself by playing D in the subsequent rematch.

Proposition 3. *i) If the outcome (C,C) in the OSPD game is supported by NR-PL, there is a direct mechanism g which "implements f " in subgame perfect equilibrium.*

ii) If NR-PL is a sequential equilibrium in the OSPD game then NR-P is a subgame perfect equilibrium in the OSPD game.

Proof. The inclusion of private retaliation changes the preference ordering in g in a substantial way: For at least one player, *preference reversal* takes place. One can use this preference reversal to "implement f " in subgame perfect equilibrium (Moore 1992). The following holds:

$$(\theta, \theta)P_j(\theta, \theta)(\phi, \theta), \text{ and} \tag{2}$$

$$(\phi, \theta)P_j(\phi, \theta)(\theta, \theta).$$

The interpretation-rule τ is defined as before. Then, continuation payoffs in the OSPD game define the following payoff matrix of g :

True State: $s^i = \theta$	True State: $s^i = \phi$																														
<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td colspan="2"></td> <td colspan="2" style="padding: 2px;">j</td> </tr> <tr> <td colspan="2"></td> <td style="padding: 2px;">θ</td> <td style="padding: 2px;">ϕ</td> </tr> <tr> <td rowspan="2" style="padding: 2px;">i</td> <td style="padding: 2px;">θ</td> <td style="padding: 2px;">c, c</td> <td style="padding: 2px;">p, c'</td> </tr> <tr> <td style="padding: 2px;">ϕ</td> <td style="padding: 2px;">p, c'</td> <td style="padding: 2px;">p, c'</td> </tr> </table>			j				θ	ϕ	i	θ	c, c	p, c'	ϕ	p, c'	p, c'	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td colspan="2"></td> <td colspan="2" style="padding: 2px;">j</td> </tr> <tr> <td colspan="2"></td> <td style="padding: 2px;">θ</td> <td style="padding: 2px;">ϕ</td> </tr> <tr> <td rowspan="2" style="padding: 2px;">i</td> <td style="padding: 2px;">θ</td> <td style="padding: 2px;">p'', p''</td> <td style="padding: 2px;">p, c'</td> </tr> <tr> <td style="padding: 2px;">ϕ</td> <td style="padding: 2px;">p, c'</td> <td style="padding: 2px;">p, c'</td> </tr> </table>			j				θ	ϕ	i	θ	p'', p''	p, c'	ϕ	p, c'	p, c'
		j																													
		θ	ϕ																												
i	θ	c, c	p, c'																												
	ϕ	p, c'	p, c'																												
		j																													
		θ	ϕ																												
i	θ	p'', p''	p, c'																												
	ϕ	p, c'	p, c'																												

Figure 3: Subgame Perfect Implementation under NR-PL

The fields indicate continuation payoffs. c , c' , and p are defined as before. p'' is the continuation payoff for both players if private retaliation takes place. This payoff equals $p'' = \frac{c}{1-\delta} - \frac{c}{n(1-\delta(\frac{n-1}{n}))}$. The latter term is the forgone cooperation payoff in case players meet again for the first time, for all periods t . It is apparent that the payoff matrix is different in the two states. For player j there is a preference reversal when switching from state $s^i = \theta$ to state $s^i = \phi$ because $p'' < c' < c$. In state $s^i = \theta$, g has two Nash equilibria, namely (θ, θ) and (ϕ, ϕ) . But in both states, it has a unique subgame perfect equilibrium outcome if g is setup as a sequential game where player i or player j moves first. In state $s^i = \theta$ the unique subgame perfect equilibrium is (θ, θ) with the outcome

$s^i = \theta$, and in state $s^i = \phi$ the unique subgame perfect equilibrium outcome is $s^i = \phi$. Here, $E_g((\theta, \theta)) = \{(\theta, \theta)\}$ and $E_g((\phi, \theta)) = \{(\phi, \theta)\}$. This implies that g "implements f " in subgame perfect equilibrium as defined in Definition 3. The untruthful Nash equilibrium outcome gets knocked out by setting up g sequentially.

To establish NR-PL as a sequential equilibrium in the OSPD game, it must be optimal for a player i to play NR-PL after any possible history of the game when player i 's information set is reached. One such possible history is that seller i gets privately punished by all but one buyer. After this history it is optimal for seller i to play C iff

$$c + \frac{\delta c}{n(1-\delta)} \geq a + \frac{\delta c}{n(1-\delta)} - \frac{\delta c}{n(1-\delta(\frac{n-1}{n}))}, \quad (3)$$

which is the same incentive compatibility constraint as the one found in order to support (C,C) by playing NR-P (see proof of Propostion 1 in the appendix). Solving for δ when (4) holds with equality, yields the critical discount factor $\delta^* = \frac{n(a-c)}{a(n-1)-c(n-2)} = \delta^P$. If it is optimal for player i to play C after this history, it is optimal for player i to play C after any other history. Thus, if NR-PL is a sequential equilibrium in the OSPD game then NR-P is a subgame perfect equilibrium in this game as claimed. □

One can be quite confident that this mechanism induces players to tell the truth, since they have strong incentives to do so. In addition, truth-telling does not depend on which player moves first in g . Crucial for the result is that there is a probability that players are rematched in the future. Private retaliation is essential to get truth-telling as the unique equilibrium outcome of this mechanism. However, to support the outcome (C,C) in the OSPD game by this strategy comes at a cost. The presence of private retaliation weakens the power of the punishment by the public. The consequence is that if (C,C) is supported by NR-PL as a sequential equilibrium then (C,C) can also be supported by NR-P where incentive for cooperation are entirely based on private retaliation without information-sharing. Thus, the benefit coming from information-sharing disappears.

That the minimal discount factor needed to support (C,C) is the same under NR-P and NR-PL comes from the fact that the equilibrium solution concept asks an equilibrium strategy to be optimal after any possible history of the game (including the ones off the equilibrium path). Why should we consider the history in which a player is privately punished by all but one buyer as relevant when finding an equilibrium. We could impose a restriction to rule out this history. Assume, for example, that a player can be punished by $x < n$ buyers at most.¹⁰ In this case, the minimal discount factor needed to support

¹⁰Note that such restrictions are common in repeated games which assume complete public information. For example, the restriction is introduced that at most one player is punished, or if new deviation occurs, players currently on the punishment path are released from their pun-

NR-PL will decrease (so that part ii) of Proposition 3 does not hold any longer) but it will be still higher than the minimal discount factor needed under NR-L. That is, in any case to get truth-telling by a mechanism in which one is confident that players will actually tell the truth comes at the cost of needing a higher minimal discount-factor to get cooperation in a community of players.

4 Games with a Two-sided Incentive Problem

In a two-sided prisoners dilemma, the strategy NR is not able to support the outcome (C,C) under local information sharing. In this situation to play C is not a best response for histories off the equilibrium path where many players are guilty. A player is better off playing D in a match with an innocent player if he or she believes to be matched with many guilty players in subsequent matches. This incentive is removed by introducing equilibrium strategies which ask guilty players to repent so that punishing players becomes less costly or even advantageous to innocent players (Kandori 1992). However, this aspect changes the nature of the implementation problem substantially. Here, an innocent player "likes" to punish, and a guilty player obviously prefers not to be punished. Players have a motive for slander.

Consider the situation where in each period $t = 1, 2, \dots, n$ players are matched into pairs according to uniform random matching. In each period, the probability of a player i to be matched with player $j \neq i$ equals $1/(n-1)$. As before, let $\mu(i, t)$ denote player i 's match at time t . In each period, pairs of players play a Prisoner's dilemma (PD) where both players i and $j = \mu(i, t)$ simultaneously choose an action a_i and $a_j \in \{C, D\}$ as depicted in figure 4.

		j	
		C	D
i	C	c, c	b, a
	D	a, b	$0, 0$

Figure 4: Prisoners' Dilemma Game

The payoffs are assumed to satisfy the conditions $a > c > 0 > b$ and $(a + b)/2 < c$. Assumptions 1 and 2 apply. Tit-for-Tat is a strategy which does not incur a cost to a player in case he or she is asked to punish another player. On the contrary: Players actually "like" to punish other players. After a unilateral defection, the punishment of a "guilty" player consists of playing him or her C, while the punishing player plays D and receives the stage game payoff $a > c$.

Definition 5 (TFT-Strategy). *At time t , player i is in the state $s^i(t) \in \{\theta, \phi\}$, where for all players i , $s^i(1) = \theta$. For any player i , the TFT-strategy is defined*

ishment etc. (see for example Kandori 1992, Abreu 1988). However, under *local* information sharing, it is not clear how such a restriction could be implemented.

by the triple $(\sigma(s^j), \rho(a_i), t^*)$. The action-rule, $\sigma(s^j)$, is specified as follows:

$$a_i(t) = \sigma(s^j) = \begin{cases} C & \text{if } s^j(t^*) = \theta, \\ D & \text{otherwise.} \end{cases}$$

The transition-function, $\rho(a_i)$, is specified as follows:

$$s^i(t+1) = \rho(a_i) = \begin{cases} \theta & \text{if } a_i(t) = \sigma(s^j), \\ \phi & \text{otherwise.} \end{cases}$$

Again, the state $s^i = \theta$ indicates that player i is "innocent" while the state $s^i = \phi$ indicates that player i is "guilty". But in contrast to before, a guilty player in a match with an innocent players has to play C while the latter is asked to play D. That is, a guilty player has to show that he or she is repenting by letting his or her partner to play D while playing C. As before, TFT can be played conditional on information revealed in a public label, in which case by assumption 2, $t^* = t$. Denote this strategy TFT-L. On the other hand, TFT can be played conditional on private information, in which case $t^* = t^P + 1$. Denote this strategy TFT-P.

Proposition 4. *Under assumptions 1 and 2, and if $a > c - b$, TFT-P is a subgame perfect equilibrium in the repeated PD random matching game iff $\delta \in [\delta^P, 1)$ where $\delta^P = \frac{(n-1)(a-c)}{c-b+(n-2)(a-c)}$. When information sharing is assumed to be honest, TFT-L is a sequential equilibrium in the repeated PD random matching game iff $\delta \in [\delta^L, 1)$ where $\delta^L = \frac{a-c}{c-b}$. For $n > 2$, $\delta^P - \delta^L > 0$, and this difference increases in n .*

Proof. See appendix. □

Proposition 4 implies again — as in Propostion 1 — that the existence or non-existence of truth-full information sharing may decide of whether cooperation in a group of players is sustainable or not. In contrast to before, the critical discount factor δ supporting the cooperative outcome is constrained in two ways: First, it has to be high enough so that players prefer the discounted payoff from cooperation compared to the payoff from defection (and subsequent punishment). The second constraint comes from the fact, that the discounted payoff of a punishment path cannot be lower than zero, because then a player prefers to defect all the time and get his reservation payoff zero instead of going along with the punishment. If $a > c - b$, then this latter constraint does not bind.¹¹

4.1 Truth-telling in Nash Equilibrium

If the assumption of truthful information sharing is relaxed, the question again emerges if there is a revelation mechanism g which gives players an incentive for

¹¹This condition is easier satisfied the larger b (which is assumed to be < 0). This means to get punished is not as dramatic. However, note that b cannot be too large, since the condition $(a+b)/2 < c$ needs to be satisfied. That is, payoffs satisfying $c-b < a < 2c-b$, where $b < 0$ are considered here.

truth-telling. If (C,C) is supported by TFT-L, truth-telling cannot be implemented in the stronger sense (Definition 3) nor in the weaker sense (Definition 2). The following negative result holds:

Proposition 5. *If the outcome (C,C) in the PD game is supported by TFT-L then no direct mechanism g "weakly implements f " in Nash equilibrium or subgame perfect equilibrium.*

Proof. Since (C,C) is supported by a *public information strategy* then again the preference ordering for each player is identical in each true state $s \in S$, — that is, (1) holds. TFT has the property that $(\phi, \phi)R_i(s)(\phi, \theta)$ and $(\phi, \phi)R_j(s)(\theta, \phi)$, $\forall s \in S$. The punishment is less drastic if two players are on the punishment path instead of one player. In a match between two guilty players TFT asks both players to play D yielding a payoff of zero which is larger than b . Assume that the true state is $s = (\phi, \theta)$. For a τ sending both players on the punishment path if $s_i \neq s_j$, the guilty player i is better off announcing an untruthful message given the innocent player j announces truthfully. That is, $s = (\phi, \theta) \notin E_g(s)$ (see Figure 5a). The fields in the figure indicate the continuation payoffs given τ , where $c' = (\frac{n-2}{n-1})c + (\frac{1}{n-1})a + \frac{\delta c}{1-\delta} > c = \frac{c}{1-\delta}$, and $p = b + \frac{\delta c}{1-\delta} < p' = (\frac{n-2}{n-1})b + \frac{\delta c}{1-\delta}$. Thus, $((\phi, \theta), (\phi, \theta))$ is not a Nash equilibrium.

a) True state: $s = (\phi, \theta)$		b)									
Interpretation rule τ				Interpretation rule τ'							
		j				j					
		(θ, θ)	(ϕ, θ)	(θ, ϕ)	(ϕ, ϕ)	(θ, θ)	(ϕ, θ)	(θ, ϕ)	(ϕ, ϕ)		
i	(θ, θ)	c, c	p', p'	p', p'	p', p'	i	(θ, θ)	c, c	p, c'	?	?
	(ϕ, θ)	p', p'	p, c'	p', p'	p', p'		(ϕ, θ)	?	p, c'	?	?
	(θ, ϕ)	p', p'	p', p'	c', p	p', p'		(θ, ϕ)	?	p, c'	c', p	?
	(ϕ, ϕ)	p', p'	p', p'	p', p'	p', p'		(ϕ, ϕ)	?	p, c'	?	p', p'

Figure 5: Nash Implementation under TFT-L

Consider a different interpretation-rule τ' which gives i an incentive to tell the truth given j tells the truth if the true state is $s = (\phi, \theta)$. Then, what ever i announces he or she should get at most p . But given the TFT strategy, i can only get p if j gets c' . But then, given that any interpretation rule will need to consider two coherent announcements as an indication for the true state, j will always announce $s_j^i = \phi$ so that $s = (\theta, \theta)$ ceases to be an equilibrium outcome if the true state is (θ, θ) (see Figure 5b for this situation). Thus, no mechanism "weakly implements f " as claimed. \square

TFT has the property that players "like" to punish. If a player has the power to determine the state of his or her partner for the continuation of the game, he or she has an incentive to slander. If one tries to remove this power by sending both players on the punishment path if messages contradict, then both players

prefer the situation in which both get punished compared to the situation in which each of them is the only one getting punished. A guilty player prefers to lie given his or her partner announces the truth.

Note that this negative result does not hold if the probability of being re-matched is zero. Then, $c=c'$, and $p=p'$, implying that the mechanism g is able to "weakly implement f " in Nash equilibrium. Another solution is to prescribe a stronger punishment than the one prescribed under TFT in case that messages are not coherent. Then, in order to avoid that strong punishment, players are better off coordinating their messages.

4.2 Truth-telling in Subgame Perfect Equilibrium

By considering an equilibrium strategy that asks players to condition their behavior on public *and* private information, we are able to implement truth-telling as a unique subgame-perfect equilibrium in g for each true state $s \in S$. However, this time the truth cannot be implemented by defining a strategy which is a combination of TFT-L and TFT-P because based on this strategy, players "like" do punish, and if they have the choice between triggering a public or a private punishment, they will choose the latter one. The only way to give incentives for truth-telling is making private punishment costly so that a player prefers to delegate the punishment onto the community of players. Consider the following strategy which is a combination between TFT-L and permanent Nash reversion as the private punishment path.

Definition 6 (TFT-NR-Strategy). *At time t , player i is in the state $s^i(t) \in \{\theta, \phi\}$, where for all players i , $s^i(1) = \theta$. For any player i , the TFT-NR-strategy is defined by $(\sigma(s), \rho(a_i), t, T^P)$. The action-rule, $\sigma(s)$, is specified as follows:*

$$a_i(t) = \sigma(s) = \begin{cases} C & \text{if } s^j(t) = \theta, \text{ and if } s_i(t') = s_j(t') = \rho(t'), \forall t' \in T^P, \\ D & \text{otherwise,} \end{cases}$$

where T^P is a set which consists of all time periods when i and j were previously matched. The transition-function, $\rho(a_i)$, is specified as follows:

$$s^i(t+1) = \rho(a_i) = \begin{cases} \theta & \text{if } a_i(t) = \sigma(s^j), \\ \phi & \text{otherwise.} \end{cases}$$

This strategy prescribes the players to retaliate by playing D when ever one or both player failed to sent a message according to ρ at least once. If this private punishment is strong enough, there is a mechanism g that gives players an incentive for truth-telling as the unique subgame perfect equilibrium.

Proposition 6. *i) If the outcome (C,C) in the PD game is supported by TFT-NR, then there is a mechanism g which "weakly implements f " in Nash equilibrium.*

ii) If $a > A$ or if $a < A$ and $n < n^$, where $A = (c-b) + \frac{c^2}{2c-b}$ and $n^* = 1 + \frac{c(2c-b-a)}{b(a+b)-c(2a+3b-3c)}$ then there is a direct mechanism g which "implements f " in subgame perfect equilibrium.*

iii) If TFT-NR is a sequential equilibrium in the PD game then TFT-P is a subgame perfect equilibrium in the PD game.

Proof. Consider an interpretation-rule τ which is defined as follows:

$$\tau(\cdot) = \begin{cases} s_i & \text{if } s_i = s_j, \\ (\phi, \phi) & \text{otherwise.} \end{cases}$$

The payoff matrix in g given TFT-NR and τ is as follows:

		True State: (θ, θ)				True State: (ϕ, ϕ)			
		j				j			
		(θ, θ)	(ϕ, θ)	(θ, ϕ)	(ϕ, ϕ)	(θ, θ)	(ϕ, θ)	(θ, ϕ)	(ϕ, ϕ)
i	(θ, θ)	c, c	p', p'	p', p'	p', p'	p'', p''	p', p'	p', p'	p', p'
	(ϕ, θ)	p', p'	p', p''	p', p'	p', p'	p', p'	p', p''	p', p'	p', p'
	(θ, ϕ)	p', p'	p', p'	p'', p'	p', p'	p', p'	p', p'	p'', p'	p', p'
	(ϕ, ϕ)	p', p'	p', p'	p', p'	p', p'	p', p'	p', p'	p', p'	p, p
		True State: (ϕ, θ)				True State: (θ, ϕ)			
		j				j			
		(θ, θ)	(ϕ, θ)	(θ, ϕ)	(ϕ, ϕ)	(θ, θ)	(ϕ, θ)	(θ, ϕ)	(ϕ, ϕ)
i	(θ, θ)	p'', p''	p', p'	p', p'	p', p'	p'', p''	p', p'	p', p'	p', p'
	(ϕ, θ)	p', p'	p, c'	p', p'	p', p'	p', p'	p', p''	p', p'	p', p'
	(θ, ϕ)	p', p'	p', p'	p'', p'	p', p'	p', p'	p', p'	c', p	p', p'
	(ϕ, ϕ)	p', p'	p', p'	p', p'	p', p'	p', p'	p', p'	p', p'	p', p'

Figure 6: Subgame Perfect Implementation under TFT-NR

Again, the fields indicate continuation payoffs for player i and j . c is the continuation payoff in the absence of any punishment, which equals $c = c/(1-\delta)$. c' is the continuation payoff for an innocent player participating in the public punishment: $c' = (\frac{n-2}{n-1})c + (\frac{1}{n-1})a + \delta c/(1-\delta)$. Players "like" to punish since $c' > c$. p' is the continuation payoff in case of public and private punishment, $p' = (\frac{n-2}{n-1})p$. p is the continuation payoff in case of a public punishment which equals $p = b + \delta c/(1-\delta)$. Thus, $p > p'$. p'' is the continuation payoff in case of a private punishment without public punishment, which equals $p'' = (\frac{n-2}{(n-1)(1-\delta)})c$. Thus, $p'' > p'$. $\{(\theta, \theta), (\phi, \theta), (\theta, \phi), (\phi, \phi)\}$ are all Nash equilibrium outcomes in g for all $s \in S$. Thus, this mechanism "weakly implements f " in Nash equilibrium. This proves part i) of Proposition 6. Again, in any state s there are a large amount of untruthful equilibria. One may be able to knock out the untruthful equilibria by playing g sequentially.

From Figure 6 one can see that since $c' > p''$ and if $p > p''$ then $E_g(s) = \{s\}$ for all true states $s \in S$, where $E_g(s)$ denotes the set of subgame perfect equilibrium outcomes in true state s . That is, if a guilty player prefers the public punishment to the private one, truth-telling can be implemented as a unique subgame perfect equilibrium. And again, the equilibrium does not depend on the choice of the player who announces first. $p - p'' = \frac{c(\delta - 1 + \pi) + b(1 - \delta)}{1 - \delta}$, where $\pi = 1/(n - 1)$. $p - p''$ increases in δ . Denote the critical δ for which $p - p'' = 0$ by $\delta^* = \frac{b - (1 - \pi)c}{b - c}$. Compare δ^* with the minimal discount factor needed to support TFT-NR, which equals δ^P (see part iii) in this proof). It is the case, that $\delta^* < \delta^P$ for any n if $a > A$. If $a < A$ then $\delta^* < \delta^P$ as long as $n < n^*$. This proves part ii) of proposition 6.

To establish TFT-NR as a sequential equilibrium in the PD game, it must be optimal for a player i to play TFT-NR after any possible history of the game when player i 's information set is reached. One such possible history is that seller i gets privately punished by all but one buyer. After this history it is optimal for seller i to play C iff

$$c + \frac{\delta\pi c}{(1 - \delta)} \geq a + \pi\delta\left(\frac{c}{1 - \delta} - \frac{b - c}{1 - \delta(1 - \pi)}\right), \quad (4)$$

which is the same incentive compatibility constraint as the one found in order to support (C,C) by playing TFT-P (see proof of Proposition 4 in the appendix). Solving for δ when (4) holds with equality, yields $\delta^P = \frac{(n-1)(a-c)}{c-b+(n-2)(a-c)}$. If it is optimal for player i to play C after this history, it is optimal for player i to play C after any other history. Thus, if TFT-NR is a sequential equilibrium in the repeated PD game then TFT-P is a subgame perfect equilibrium in this game as claimed. \square

Note that A is always larger than $c - b$, and always smaller than $2c - b$. If $a < A$ then the ability to implement truth-telling as a unique equilibrium depends on the number of players. $n^* > 2$ if $A > a > c - b$ and it increases with an increasing b , which makes the public punishment less drastic. If $a > A$, then truth-telling can be implemented as a unique subgame perfect equilibrium for any number of players. The intuition behind this is that if the defection payoff, a , is higher than the threshold value A , the minimal discount factor needed to support TFT-NR is high enough that a private punishment is always more drastic than the punishment by the public for any number of players. A increases in c — since a higher c makes the private punishment more drastic — and decreases in b — since a higher b makes the public punishment less drastic.

5 Conclusions

This paper makes the general point that there is *no* direct mechanism which implements truth-telling in a repeated matching games with perfect private monitoring if an outcome in such a game is supported by a public information

strategy. The crux is that the benefit from information-sharing as pointed out in previous literature comes precisely from the possibility to achieve a desired outcome by playing such a strategy. This result is general and does not depend on the matching-rule, the characteristics of the stage game, and the number of players.

Next, the paper shows that if an outcome in the repeated game is supported by a strategy where players condition their behavior on public and private information, there is a mechanism which implements truth-telling in subgame perfect equilibrium. However, this comes at a considerable cost: The minimal discount factor needed to support such a strategy as an equilibrium identical to the minimal discount factor needed to support the strategy *without* information-sharing as an equilibrium. Thus, benefits resulting from information-sharing are essentially removed. This negative result may be weakened by imposing some restriction on the play off-the equilibrium path, as for example, that a player can be privately punished by a limited number of players at once. However, it is hard to implement such a restriction because of the nature of these punishments. They are private and unknown by the public.

Appendix

Proof of Proposition 1. In order to check whether NR-P is a subgame perfect equilibrium, one has to assure that a unilateral one-shot deviation from NR-P is not beneficial to any player in the game. To follow NR-P and play $a_i = C$ in each match with player $j = \mu(i, t)$ yields an expected discounted payoff of $c + \frac{\delta c}{n(1-\delta)}$. To deviate from NR-P once and play $a_i = D$ yields an expected discounted payoff of $a + \frac{\delta c}{n(1-\delta)} - \frac{\delta c}{n(1-\delta(\frac{n-1}{n}))}$. The latter term is the forgone cooperation payoff in case players meet again for the first time in a period, for all periods t . Setting both payoffs equal and solving for δ yields the critical delta $\delta^P = \frac{n(a-c)}{a(n-1)-c(n-2)}$. With truthful information sharing, the game turns into a game in which i plays the OSPD game with one other player, namely the "information sharing institution". Set n equal to one, and one gets the critical value $\delta^L = \frac{a-c}{c}$. \square

Proof of Proposition 4. To play $a_i = C$ in a given match with $j = \mu(i, t)$ yields a payoff of $c + \frac{\delta \pi c}{(1-\delta)}$. To play $a_i = D$ yields a payoff of $a + \pi \delta (\frac{c}{1-\delta} - \frac{b-c}{1-\delta(1-\pi)})$, where $\pi = 1/(n-1)$. The last term in this expression accounts for the possibility that players rematch for the first time for all time periods, in which case player i gets b instead of c . Setting both payoffs equal and solving for delta yields the critical delta $\delta^P = \frac{(n-1)(a-c)}{c-b+(n-2)(a-c)}$. With information sharing, the game turns into a two player game between player i and the "information sharing institution". Set $n = 2$, and one gets the critical value $\delta^L = \frac{a-c}{c-b}$.

The punishment path of TFT-L is part of a sequential equilibrium only if it yields a discounted payoff of at least zero. This is the reservation payoff a player can get by playing D in each period. Setting $b + \delta c / (1 - \delta)$ equal to zero

and solving for δ yields $\delta^{L'} = b/(b - c)$. $\delta^{L'} < \delta^L$ if $a > c - b$ as assumed in Proposition 4. Thus, a guilty player will follow the punishment prescribed by TFT-L. Denote by $\delta^{P'}$ the critical delta making an individual indifferent between following the punishment or deviating under TFT-P. If $\delta^{L'} < \delta^L$ then it must be the case that $\delta^{P'} < \delta^P$. Thus, a guilty player will follow the punishment prescribed by TFT-P. \square

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