Communication through Noisy Channels

by

Penelope Hernandez, Amparo Urbano and Jose Vila.

Departamento de Fundamentos del Analisis Economico, Universidad de Alicante (Spain) and Departamento de Analisis Economico. Universidad de Valencia.(Spain).

E-mail: Penelope.Hernandez@merlin.fae.ua.es, Amparo.Urbano@uv.es and Jose.E.Vila@uv.es

ABSTRACT

We study two-player coordination games in which one player is better informed than the other and where the costs of miscoordination are different in distinct states of nature. Specifically, let $\Omega = \{\omega_1, \cdots, \omega_k\}$ be the states of nature, and for each $l = 1, 2, ..., k$, let $\Gamma^l = (A_1, A_2, u_1^l, u_2^l)$ be the strategic form game associated with ω_l, where A_i and u_i^l are the set of actions and payoff function of Γ^l, respectively, of player i. In this model, nature chooses one state ω_l (and hence the game Γ^l), using a commonly known probability distribution and informs only to player 1 of its choice. Then, the informed player transmits his private information to the uninformed one, trough a noisy channel, and actions are chosen and payoffs realized.

We assume that there is a unique optimal play, $(\widehat{a}_1^l, \widehat{a}_2^l) \in A_1 \times A_2$ in each Γ^l, and that players communicating through a discrete memoryless noisy channel, can use it repeatedly n times. Thus, we assume that signals or messages can be distorted in the communication process. A discrete channel is a system consisting of an input alphabet X and output alphabet Y, and a probability transition matrix $p(y|x)$, that expresses the probability of observing the output symbol y, given that the symbol x was sent. The channel is memoryless if the probability distribution of the output depends only on the input at that time and is conditionally independent of previous channel inputs or outputs.

Our aim is to analyze and characterize the set of achievable equilibrium outcomes for a given channel and a given and *finite* number of "uses" (repetitions) of the channel. We bound the efficiency loss of noisy communication in terms of the properties of the noisy channels such as the transition probability $p(y|x)$ and the channel capacity.

We define a *noisy communication protocol* which consists of a codification of the states of nature into a channel input sequences, $X^n(\omega_l)$, a noisy channel, $(X, p(y|x), Y)$, and a decodification rule mapping channel outputs into the uninformed player's actions. The noisy channel transforms the input sequence into an output sequence, that is random but has a distribution that depends on the input sequence. From the received output sequence, y^n, the uninformed player attempts to recover the transmitted message. Each of the possible input sequences induces a probability distribution on the output sequences. Since two different input sequences may give rise to the same output sequence, the inputs are confusable. The informed player wants to choose a "non-confusable" subset of input sequences so that with high probability, there is only one highly likely input that could have caused that particular output. The number of possible decodification functions is exponential with respect to n, since it is given by the number of partitions of the k subsets associated to each element of Ω, over the set of output sequences of length n. To cope with this problem, we borrow elements from Information Theory. More precisely we use the Typical Set methodology, which enables us to partition the set of sequences into two sets, the "typical set", where the sample entropy is close to the true entropy and the non-typical set, which contains the other sequences. The codification mapping consists of a random choice of a sequence of signals, for each state of nature, in the typical set. Then, we will decode a channel output y^n as state ω_l if the input sequence $X^n(\omega_l)$ is "jointly typical" with the received output sequence y^n, or in other words, if both sequences are highly probabilistically related.

Although the jointly typical decoding is suboptimal for a *finite* number of repetitions, it is a less complex way (polynomial with respect to n) to design noisy communication protocols than that of checking all feasible codifications to find the optimal one. However, being suboptimal, it still allows to transmit enough information to achieve coordination outcomes. Moreover, the Typical Set approach generates a deterministic decodification rule and thus, given a channel and a number of repetitions, we can design a communication protocol and characterize the equilibrium outcomes.