Timing games with informational externalities

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We analyze the following game of timing with incomplete information. At each stage $n \in \mathbf{N}$, each (of finitely many) player i has to choose whether to stay in or to drop out from the game. If she stays in, she receives a random payoff X_n^i. Once out, she receives 0 forever. The various payoffs X_n^i are conditionally independent given a state of nature Θ, selected at stage 0 according to some prior distribution.

A key feature of our game is the information structure. Payoffs are *private* information, while exit decisions are *publicly* observed. The interaction of the players is therefore of an informational nature: player i cares about player j's decisions, since these may reveal something about player j's private information on Θ.

This game relates to various strands of literature. From the viewpoint of *social learning* with endogenous timing (e.g., Chamley Gale (Econ, 1994)), the distinguishing feature of our game is that players keep receiving private signals as time proceeds. When compared with the *strategic experimentation* literature (e.g.,Bolton Harris (Econ, 1999)), our game is characterized by the fact that payoffs are *not* publicly observed, so that posterior beliefs are not common knowledge. On the other hand, our game can be viewed as a strategic version of *real options* problems (e.g., Dcamps, Mariotti,Villeneuve (2001)). Finally, it bears some relation to the *multi-armed bandit* problems studied in statistics (e.g. Ferguson (2004)).

Under minimal assumptions, we prove that *all* equilibria are in pure strategies and incorporate the public information (players' past decisions) in a particularly simple way. In addition, we provide a number of qualitative properties of these equilibria, and we fully describe the limit equilibrium, as the number of players increases to infinity. The intricacy of the posterior beliefs of different orders, makes it impossible to perform explicit computations.