Markets with complementarities and Mixed-Bundling Pricing.

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ABSTRACT

We show the existence of subgame perfect Nash equilibrium prices in two market structures with some specific geometry-based complementarities. Namely, markets of compatible systems and spatial markets with interval-based complementarities.

Arribas and Urbano (2003) analyze a model with price competition among n multiproduct firms and one representative buyer, characterized by her willingness to pay -in monetary terms- for every subset of products. They show that a mixed bundling subgame perfect equilibrium outcome always exists and it is efficient in the sense of maximizing the social surplus. The set of subgame–perfect Nash equilibrium outcomes is equivalent to integer-valued solutions of the linear relaxation of a package assignment problem. However, the existence of equilibrium outcomes for an m-buyers' model under general value functions is hardly guaranteed (see also Bikhchandani and Ostroy, 2002, for the Walrasian Equilibrium set up). Moreover, recent contributions to discrete convexity analysis (see, for instance, Danilov, Koshevoy and Murota, 2001), point out, the very restrictive assumptions on the buyers' value functions to achieve equilibrium outcomes in such models. Specifically, those value functions which imply the gross-substitution property.

We take a different viewpoint and offer results in models with complementaries, which hinges upon market structures as well as on the buyers' value functions. A *market with compatible systems* is defined as follows. A set $B = \{1, 2, ..., m\}$ of consumers and a set $N = \{1, 2, ..., n\}$ of firms, where each firm produces two complementary components of a system, which are substitutes of the two products produced by its rival firms. Thus, let $N_i = \{l_i, r_i\}$ be firm i's set of products, with $l_i \in L = \{l_1, l_2, \ldots, l_n\}$ and $r_i \in R = \{r_1, r_2, \ldots, r_n\}$, and where each product in L is complement of any other product in R and viceversa. Define the set of products of the market as $\Omega = L \cup R$ and let $C = L \times R = \{\{l, r\} | l \in L, r \in R\}$ be the set of possible

consumptions or systems. Given a system $\{l_i, r_j\} \in C$, is a pure system if $i = j$ and it is a mixed system if $i \neq j$. Each consumer $b \in B$ is characterized by a value function over any system $S \in C$. Function $v_b(S)$, represents her total willingness to pay for consumption set S, with $v_b(\emptyset) = 0$. Each firm i sets prices for its two products however, it can offer both of them as a bundle for a special price.

In *spatial markets with interval-based complementarities*, there is a total ordering among products, i.e., $\Omega = \{1, 2, ..., h\}$ and consumers preferences are only positively defined for consecutive products, i.e., $v_b(S) > 0$ if and only if $S = \{i, i+1, ..., j-1, j\}$ for all $1 \leq i \leq j \leq h$. Here then, the set of consumptions is defined as $C = \{S \subseteq \Omega | S = \{i, i+1, ..., j-1, j\}, 1 \leq i \leq j \leq h\}$. As above a consumption set is pure if all products in the interval belongs to only one firm and it is mixed otherwise.

The two market structures can be modeled as an integer package assignment problem, where the set of restrictions defines a polyhedron belonging to the class of polymatroids. This guarantees an efficient integer partition of the goods. Moreover, if the buyers' value functions satisfy that $\forall S, T \in C$ where $S \cap T = \emptyset$, there exists $i \in \arg\max_b\{v_b(S)\}$ and $j \in \arg\max_b\{v_b(T)\}$ such that $i \neq j$, i.e., consumers' preferences are sufficiently compatible, then the existence of the dual of an associated linear programming problem providing the equilibrium (mixed-bundling) prices is also assured. The equilibrium assignment is efficient since it maximizes the social surplus and may consists of both pure and mixed consumption sets.

References:

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