Bargaining with History Dependent Preferences

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Abstract

We study perfect information bilateral bargaining game with an infinite alternating-offers procedure, in which we add an assumption of history dependent preference. A player will devalue a share which gives her strictly lower discounted utility than what she was offered in earlier stages of the bargaining. Under the strong version of the assumption, we characterize the essentially unique subgame perfect equilibrium path, which involves considerable delay and efficiency loss. We give different interpretations of the assumption. The assumption can also be weakened under the interpretation of loss aversion. We provide a sufficient condition under which the feature of the equilibrium from strong assumption remains.

JEL-codes: C72, C78.

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1 Introduction

1.1 Motivation and Approach

Rubinstein (1982) analyzed a bargaining model with an infinite alternating-offers procedure, and established uniqueness of subgame perfect equilibrium (SPE) when both parties' time preferences are represented by exponential discounting. He also showed that agreement is immediately reached. This contradicts

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our daily observation that bargaining almost always takes some time, and rarely appears to be efficient. Delays and even impasses are common in real bargaining. Much theoretical work has been done to account for this, and most of it relies on incomplete information as the driving force behind delays, i.e., the players have incomplete information about the "fundamentals" of the bargaining, such as the other party's patience, outside option, etc.. Under incomplete information settings, there are always multiple equilibria, and refinement criteria for the equilibria becomes a major issue.

Abreu and Gul (2000) introduced a small fraction of behavioral type into their bargaining model and studied two-sided reputation formation in bargaining. The behavioral bias they used was that the players might commit to a fixed share and only accept that amount or higher[1]. There is a unique sequential equilibrium in their model, and it entails delay, consequently efficiency loss. The type of each player is not revealed before the bargaining, and the efficiency loss in their model is effectively information-induced. The behavioral bias can be interpreted as being caused by an aberration of preference.

In this paper, we assume that players have history dependent preferences, that is, players' payoffs not only depend on the outcome of the bargaining, but also depend on the specific bargaining process leading to the outcome. What they get from a final agreement, together with what they have been offered and rejected determine their current payoff, which is then discounted due to impatience.

Experiments in game theory have been providing results inconsistent with game theoretical predictions. One may argue that subjects do not behave according to these predictions in many situations, and an alternative view is that the preferences of the subjects are not completely described by the outcome of the game. If two different plays bring same outcome for a subject, she may strictly prefer one to another. Weibull (2004) introduced a notion of "game protocol", and analyzed context-dependent preference and interpersonal preference dependence. Our preference assumption is in favor of the latter view mentioned above. It can be viewed as a special form of context-dependent preference. We are particularly interested in its application in bilateral bargaining game.

In our model, players still have intrinsic preferences over the outcome. As usual, this preference admits a separable representation, i.e., a concave utility function measuring current value of a share and a time-discounting term. This representation allows us to introduce a strong version of history dependent pref-

[1]The behavioral type with obstinate demand was first introduced into a bargaining model with one-sided reputation formation in Myerson (1991).

erence. We shall assume each player prefers impasse to any outcome, which are worse, in terms of discounted utility, than any offer she has rejected; and her preference over improving outcome is always measured by the intrinsic utility function. This preference is assumed to be common knowledge between the players. With such preferences, the players will strategically hold back in making offers, because once an offer is rejected the proposer has to keep improving it in order to reach agreement. Yet players are impatient, so they have a countervailing incentive to reach an early agreement. These two counteracting forces induce some interesting results. We still have essentially unique SPE, but it involves considerable delay. The equilibrium has a flavor of reciprocity, that is, the players will start from two extreme positions, each player will make some small concession at the beginning, after the opponent makes reciprocal responses, the step of the concession will increase, and they finally reach an agreement somewhere in the middle. Equilibrium play thus exhibits realistic features.

A weaker version of history dependent preference will also be studied. Before we continue, we would like to provide two different interpretations of the history dependent preferences.

1.2 A Delegated Bargaining Interpretation

A delegated bargaining is that principals delegate agents to bargain over some issues mainly concerning the principals. The principals care about the outcome of the bargaining in the way of the players in a basic bargaining problem, whereas the agents care about the outcome in the way specified in the contracts between principals and agents. Many real life bargainings take this form.

The simplest contract form one may think of will be that the principal and the agent share the outcome of the bargaining by a pre-specified percentage. There are two problems. One is that the principal and the agent may have different current utility functions and patience (discount factors), but the outcome will only depend on the agent's characteristics. If the principal has no information about the agent's characteristics, she loses control over the bargaining result by using such a contract. Another problem is that it is technically impossible to share the outcome in many real bargaining situations, for example, political bargaining.

Another possible contract will be that the principal asks for a fixed amount and the agent gets a fixed income by getting the committed amount successfully or zero if mission unfulfilled. It is well known that in one-sided delegated bargaining, the delegated side has a commitment advantage with such a contract, which actually permits the delegated side to get the whole pie when there is a prohibitive renegotiation (between the principal and the agent) cost. Even when the renegotiation cost is not prohibitively high, the delegated side still has an advantage as shown in Bester and Sakovics (2001). Then there is no reason for the other side not to seek to be delegated. When it is a two-sided delegated bargaining, if the principals choose an amount to commit to simultaneously, it simply becomes a Nash demand game.

The concept of *Justifiability* introduced by Spiegler (2002) provides us another possibility. After the bargaining, the agent has to explain to the principal, or, justify her performance during the bargaining process to an exogenous criterion. Being justifiable means the agent's performance satisfies the employed criterion. In Spiegler's model, the agent has to justify the optimality of her strategy choice based on a plausible conjecture of the opponent's strategy, Spiegler applied a simplicity criterion on plausibility, which works very well in some games with paradoxical SPEs, such as chain-store game and centipede game. We apply the idea in a different way. The justification is combined with a contract between the principal and the agent. The contract is monotone, i.e., the better is the discounted utility of the principal from the outcome of the bargaining, the better is the payment to the agent. The monotone contract prevents the agent from simply taking the first offer. About the justification, we think it is natural to have a criterion involving following rule: a performance is considered to be unsuccessful if the agent has forgone an offer and ends up with a worse offer in terms of discounted utility. Intuitively, the principal's logic is: the agent did not do a satisfactory job because she had chance(s) to get a better result, but did not take it. One notable point is that the outcome of the bargaining now depends only on the principal's characteristics, the utility function and the patience. Another point is that the justification requirement gives the agent a commitment advantage, but it is not so extreme as committing to a fixed share. When it is one-sided delegated bargaining under the current situation, the delegated side will get almost the whole pie due to the commitment advantage[2]. However in the two-sided delegated bargaining, now it does not simply result in an impasse. If we further assume the payment to the agent in case of unsuccessful performance is some negative value, the preference assumption we make fits completely

[2]If the non-delegated side moves first, the equilibrium share is $(1-\delta, \delta)$; if the delegated side moves first, the equilibrium share is $(1-\delta(1-\delta), \delta(1-\delta))$. In either case, there is no delay in the unique SPE.

into this delegated bargaining situation. Basically, the agents have endogenous lexicographic preferences under this setting, they prefer being successful (including impasse) to being unsuccessful, conditional on being successful, they prefer the agreement providing higher discounted utility than the one with lower discounted utility. We think this mirrors the behavior of politicians in real political bargaining situations.

This paper does not aim to discuss contract design. We simply assume that in delegated bargaining situation, there may exist such kind of arrangement explicitly or implicitly. The preference of the impasse to acceptance of a worse offer may come from some reputation concern instead of being specified in the contract. A principal can fire her agent after the unsuccessful performance without specifying it ex ante in their contract, and the agent also expects this during the bargaining.

1.3 A Behavioral Economics Interpretation

It has been well accepted that decisions are always made according to status quo or reference point. Theoretical and experimental research pioneered by Kahneman and Tversky have provided solid foundation for behavioral assumptions which deviate from the traditional rationality assumption. Our assumption also falls into this family.

Specifically, we would like to interpret the assumed preference as preference with loss aversion as defined by Tversky and Kahneman (1991). Each player takes the ever best offer in terms of discounted utility as her reference point. A share providing higher discounted utility will be valued as before, but a share with lower discounted utility, namely a loss, will be devalued. Our strong assumption corresponds to an extreme case of devaluation of the shares in the region of loss, any such share brings a negative payoff to the player. The magnitude of this negative payoff does not matter, the player simply prefer impasse to any loss. In other words, the players here are assumed to be extremely loss averse.

One notable feature here is that the reference point evolves endogenously. An improving offer from one side change the reference point of the other side in the future play, and without the improving offer, the reference point changes over time due to the discounting. There is a similar formation of reference level in Barberis, et al. (2001). In their model, investor gets utility from fluctuations of risky asset, and they take the reference level to be the current value of the asset scaled up by the risk-free interest rate.

Under this interpretation, it is quite natural to weaken the strong assumption. We can replace the discontinuity at the reference point by a kink. In the case of linear utility, what we do is simply to give the utility function a larger slope rate in the loss region. Our strong assumption corresponds to the case with infinite slope rate in the loss region. We also want to see what the equilibrium looks like when the loss aversion effect is moderate. In section 4, we will show that when the kink is sharp enough, or, the loss aversion effect is large enough, we can get similar unique SPE path with considerable delay. Thus, the discontinuity in the strong assumption is not necessary for us to obtain delay.

1.4 Related Literature

Some works have been done to explain the strategic delay in bargaining in complete information models. Ma and Manove (1993) studied a bargaining game with deadline and imperfect control over the timing of offers. They obtained a symmetric Markov-Perfect Equilibrium involving delay and positive probability of impasse. Perry and Reny (1993) and Sakovics (1993) considered continuous time case, simultaneous offer is the necessary condition to get delay in their models. More closely related, Fershtman and Seidman (1993) combined "endogenous commitment" assumption and "deadline effect" in their model. When players are sufficiently patient, there is unique and inefficient SPE, in which the agreement is delayed until the last period, and no concessions are made before that. A fair lottery determines one player who gets the whole pie in the last period. The endogenous commitment assumption, a player cannot accept any offer lower than what she has rejected, is similar to our assumption of history dependent preference. The key difference is that we scale up the commitment levels by the discount factors. More importantly, the "endogenous commitment" assumption in their model itself cannot induce delay without "deadline effect".

In our model, rejection of an offer from the opponent can also be view as a commitment tactic. There are other theoretical models concerning the strategic commitment, for example, Crawford (1982) and Muthoo (1996). The commitment in our model is different with theirs. They both assumed that once a player makes a demand for a specific share of the pie, she will have to pay a retreat cost if she settles with a share lower than her original demand. Crawford (1982) explained impasses in real bargaining based on this possibility of commitment, and Muthoo (1996) studied how the retreat cost functions affect the equilibrium share. In our model, the commitment comes from the rejection of offers. An offer to the opponent also means a demand from the proposer, thus, the key difference here is where the commitment effect comes from. If we consider the behavior of committing as a kind of reputation concern, the question will be which one hurts the reputation more, retreat or regret? From our point of view, there is no Yes or No answer for this question, they are just different perspectives to understand real life bargaining. Different commitment tactics work in different situations.

Another closely related paper is Admati and Perry (1991). Our model and some of the main results under the strong assumption share features with their contribution game. Both models have essentially unique SPE path and considerable delay in the equilibrium. The way we characterize the equilibrium is also similar to theirs. One important difference between the two models is about the inefficiency. In their model, the inefficiency is in the sense that the socially desirable project may not be completed. The delay is not the major concern of their model. Actually, given the convex investment cost functions, the delay is to some extent socially desirable. In our model, we refers to the inefficiency only in the sense of strategic delay as most of the bargaining literature do. Moreover, we also consider a more general setting, in which the players may have different concave utility functions and different discount factors.

More recently, Compte and Jehiel (2003) studied bargaining and contribution games under the assumption of history dependent outside options. The ideas of their paper and the current paper are similar, but the ways of modelling and the results are quite different.

The rest of the paper is organized as follows. In section 2, we lay out the model. We obtain the main results under the strong version of history dependent preference in section 3. Results under the loss aversion interpretation and the weaker version of history dependent preference is presented in section 4. Section 5 includes an informal discussion, in which we explore, without the assumption of history dependent preferences, the possibility of providing epistemic foundation for the equilibrium play we have obtained. Section 6 concludes.

2 The Model

We analyze a perfect information bargaining game. Two players bargain over a pie with size 1, the bargaining takes the infinite alternating-offers procedure introduced by Rubinstein (1982). The bargaining ends if one player accepts an offer from the other player. If no one ever accepts an offer, it is an impasse. For simplicity, we define an offer as the nominal share the proposer agrees to give to her opponent, it means the proposer gets whatever is left. A strategy is defined as following. Whenever it is her turn to make an offer, the player chooses an offer according to the specific history leading to the current node; whenever it is her turn to decide to accept an offer or not, the player chooses an acceptance rule according to the history. We only need to consider well-defined and "reasonable" rules, i.e., an acceptance rule is specified only as a lower bound of nominal offers to accept, any offer higher or equal to the lower bound will be accepted, otherwise rejected. Every history leads to a terminal node or a decision node, and a decision node in turn starts a subgame. A strategy specifies actions in any such subgame. We call the actions involved in a subgame as subgame strategy.

Initially, players have intrinsic utility functions $u(\cdot)$ and $\upsilon(\cdot)$ defined on $[0,1]$, $u(\cdot)$ and $\upsilon(\cdot)$ are strictly increasing and concave, and we also normalize $u(0) = \upsilon(0) = 0$, $u(1) = \upsilon(1) = 1$. The utility function specifies the current value of a share for the player. It is how the players value the share of the pie before they start to bargain. We can also define the initial set of feasible utility pairs as $U = \left\{(\alpha,\beta) \in [0,1]^2 : u^{-1}(\alpha) + \upsilon^{-1}(\beta) \leq 1\right\}$.

An agreement can be reached at $t = 0, 1, 2, ...$ The players are impatient, and the nominal utility from the share of the pie will be discounted over periods by $0 < \delta_1, \delta_2 < 1$, i.e., the discounted utility for player 1 (resp. player 2) from a share x (resp. y) attained in t^{th} period is $\delta_1^t u(x)$ (resp. $\delta_2^t \upsilon(y)$). As usual, the (discounted) utility from an impasse is simply zero. The key difference we have in the current model is that the payoff to the players will be history dependent. Basically, we do not always take the discounted utility from the share of the pie as the payoff of a player.

We assume that if a player ends up with a share which gives her the highest discounted utility from whatever she has been offered and rejected, her payoff is just her discounted utility from that share. But if the player ends up with a share strictly worse in terms of discounted utility than any offer she rejected along the history path, say, an unsuccessful bargaining performance, her payoff will be negative. The payoff from impasse is zero. Therefore, impasse is strictly preferred to being unsuccessful. We can also put it this way, the players have lexicographic preferences over the results of the bargaining, in which a 0/1 indicator of being successful or not is the first argument, the discounted utility is the second argument, and the first argument has the higher priority. We think it makes sense in many political bargaining situations. Under such setting, it is obvious that in a possible subgame perfect equilibrium, each player will never accept an offer which gives her lower discounted utility than what she was offered before. This is what we call the Strong Assumption of History Dependent Preference. Before we formally state the assumption, we need one more notation, the state variables for the bargaining game.

Definition *State variable x_t (resp. y_t) at t^{th} period of the bargaining game is the smallest share player 1 (resp. player 2) needs to keep her discounted utility not lower than what she could get from any previous offer.*

We denote by c_s^i the offer made by player i in s^{th} period, then
$x_t = \max_{(s<t)}\{u^{-1}[u(c_s^2)/\delta_1^{t-s}]\}$, for any odd number $s < t$;
$y_t = \max_{(s<t)}\{v^{-1}\left[v(c_s^1)/\delta_2^{t-s}\right]\}$, for any even number $s < t$;
and $x_0 = y_0 = 0$.

In our model, all relevant information in a specific history is included in the state variables. We can denote a subgame starting with a decision node for one of the players to make an offer as $(x_t, y_t)_i$, the subscript i refers to the player who makes the first offer in the subgame.

Now we can give the Strong Assumption of History Dependent Preference formally as following.

Strong Assumption of History Dependent Preference

At t^{th} period of the bargaining game, the (current) utility functions from the share of the pie are:

Player 1: $\widetilde{u}^t(x) = \begin{cases} u(x) & if\ x \geq x_t, \\ -\varepsilon < 0 & if\ x < x_t; \end{cases}$

Player 2: $\widetilde{v}^t(\beta) = \begin{cases} v(y) & if\ y \geq y_t, \\ -\varepsilon < 0 & if\ y < y_t. \end{cases}$

What we are doing here is to truncate the utility functions, $u(\cdot)$ and $v(\cdot)$, according to the state variables every period. Another way to put it is to add a dynamic restriction on the feasible set of actions. It can be either one of the following two alternatives.

1. A player is not allowed to accept any offer which is worse (for herself) than what she has rejected along the history path in terms of discounted utility.

2. A player is not allowed to make any offer which is worse (for her opponent) than what she has offered and been rejected along the history path in terms of discounted utility.

It is obvious that this different way to put the assumption has no effect on the SPE path. We would like to take the assumption on the preference instead of on the feasible action set because it is easier to be extended as we will do in section 4.

3 Equilibrium under the Strong Assumption

3.1 A Simple Case

In this section, we discuss the benchmark case, i.e., two players have linear initial utility function, $u(x) = x$ and $\upsilon(y) = y$, and they also have common discount factor, $\delta \in (0,1)$. In this case, we simply have $x_t = \max\left[c_s^2/\delta^{t-s}\right]$, for any odd number $s < t$; $y_t = \max\left[c_s^1/\delta^{t-s}\right]$, for any even number $s < t$; and $x_0 = y_0 = 0$. c_s^i is the offer made by player i in s^{th} period.

Our purpose now is to characterize the SPE(s). The following straightforward lemmas lead to the main result.

Lemma 1 *Impasse is not a SPE outcome.*

Both players get zero payoff from impasse, while player 1 can simply offer anything larger than δ at the beginning, it will not be rejected and bring both players positive payoffs.

Lemma 2 *In the subgame $(x_t, y_t)_i$ with $x_t + y_t \leq 1$, the highest offer i will make in a SPE is* $\max[\delta - x_t, y_t]$ *for* $i = 1$, $\max[\delta - y_t, x_t]$ *for* $i = 2$.

Proof. *We consider $i = 1$, it is same for $i = 2$.*

It is obvious that in any subgame $(x_t, y_t)_i$ with $x_t + y_t > 1$, impasse is the only SPE outcome. Thus, when $x_t + y_t > \delta$, it is infeasible to get an agreement in next period, and player 2 will accept any offer higher or equal to y_t. Thus,it is optimal for player 1 to make offer y_t.

When $x_t + y_t \leq \delta$, any feasible agreement in next period has to give player 1 a share no lower than x_t/δ, by which player 2 gets the highest possible share $1 - x_t/\delta$. In terms of discounted utility, such a share is equivalent to a current share $\delta - x_t$. In other words, player 2 would not reject any offer higher than $\delta - x_t$ in a SPE, thus, it is also the upper bound of the offer player 1 would make.

Finally, $x_t + y_t \leq \delta$ is just $y_t \leq \delta - x_t$. ■

Lemma 3 *Any strategy involving acceptance of a non-highest offer is not a SPE strategy.*

Proof. This is also straightforward. We only need to consider a subgame $(x_t, y_t)_1$ with $x_t + y_t < \delta$, if player 1 offers c_t^1, with $y_t \leq c_t^1 < \delta - x_t$, player 2 will reject because she can at least counteroffer with $\max\left[\delta - c_t^1/\delta, x_t/\delta\right]$, the highest offer at $t+1^{th}$ period, and get a payoff higher than c_t^1.

To see this, if $\delta - c_t^1/\delta \geq x_t/\delta$, player 2's payoff in terms of utility in t^{th} period is

$$\delta(1 - (\delta - c_t^1/\delta)) = \delta - \delta^2 + c_t^1 > c_t^1;$$

if $\delta - c_t^1/\delta < x_t/\delta$, it is $\delta(1 - x_t^1/\delta) = \delta - x_t > c_t^1$. Thus, player 2 will reject any offer lower than $\delta - x_t$ in a SPE. ■

We notice that the lower bound of the offer not being rejected coincides with the upper bound of the offer being rejected. We name this offer as 'clinching offer'[3] with respect to the corresponding subgame. Obviously, a SPE of the bargaining game will end with a clinching offer being made and accepted. We are now ready for the main result of this section.

Proposition 1

(i) There exists an essentially unique subgame perfect equilibrium (SPE) path. When $\delta > \frac{\sqrt{5}-1}{2}$ (the golden number), there is delay in SPE.

(ii) The SPE delay, measured as the number n of time periods until agreement is reached, is a non-decreasing function of the common discount factor δ. As δ goes to 1, n converges to infinity.

(iii) The SPE share to player 1 (who makes the first offer) is $x = 1 - \delta^{n+1}$, while player 2 receives the share $y = \delta^{n+1}$. The associated payoffs are thus $\delta^n - \delta^{2n+1}$ and δ^{2n+1}, respectively.

(iv) As δ goes to 1, δ^n (where n is a function of δ) converges to 1/2 from above, i.e., the efficiency loss converges to 1/2.

Proof. (i) From the lemmas, we know any SPE ends the bargaining with acceptance of the clinching offer with respect to the state variables, thus, we need to see when it is optimal for a player to make the clinching offer.

When $x_t + y_t > \delta$, it is obvious that players would make the clinching offer defined as above.

When $x_t + y_t \leq \delta$, by making the clinching offer, the proposer, say, player 1 gets a current share $1 - (\delta - x_t)$; if player 1 does not make the clinching offer, the best outcome for her is to get a share $\delta - y_t/\delta$, the clinching offer, in next period.

[3] Credit goes to Jeff Ely for the use of this intuitive term.

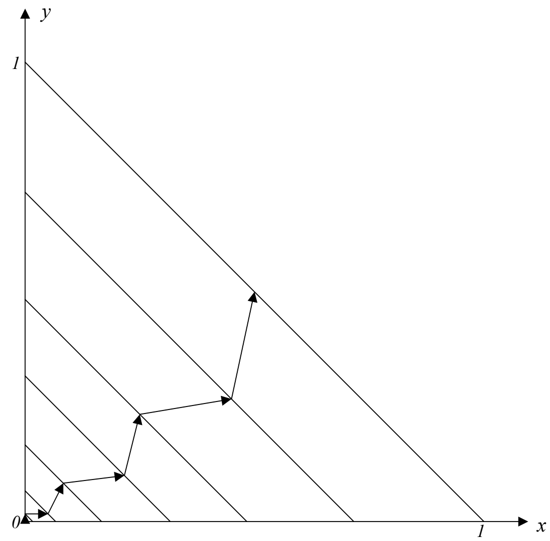

Figure 1: Simple Case

Then it is obvious that when $x_t + y_t \geq \delta^2 + \delta - 1$, for the current proposer[4], it is better, in terms of discounted utility, to make the clinching offer (being accepted immediately) than to wait for the clinching offer in next period. Therefore when the state variables get into this grid (See Figure 1), say, grid 1, the bargaining will end with an immediate agreement in any SPE. We define grid 1 as

$$G_1 = \left\{ (\alpha, \beta) \in [0,1]^2 : \delta^2 + \delta - 1 \leq \alpha + \beta \leq 1 \right\}.$$

Formally speaking, any subgame $(x_t, y_t)_i$ with $(x_t, y_t) \in G_1$ has unique SPE, in which the clinching offer is made by i, and accepted by $-i$. Moreover, the equilibrium share will be $(1 - \delta + x_t, \delta - x_t)$ when $x_t + y_t < \delta$ and it is player 1's turn ($i = 1$) to make an offer, and vice versa.

When (x_t, y_t) is out of G_1, the current proposer, say, player 1, will compare the following two choices: one is to make the smallest offer such that (x_{t+1}, y_{t+1}) will be in G_1; the other is to make the basic offer, y_t, or a lousy offer, lower than y_t, and in the next period player 2 makes an offer such that (x_{t+2}, y_{t+2}) will be in G_1. We know once a proposer realizes she is in G_1, she will make the highest offer in any SPE. It is easy to see when $\delta(\delta^2 + \delta - 1) \leq x_t + y_t < \delta^2 + \delta - 1$, there is no difference between the two alternatives; and when $\delta^4 + \delta^3 - \delta \leq x_t + y_t < \delta(\delta^2 + \delta - 1)$, the

[4] An important point here is that the condition is same for both players, which differs with the general case we will discuss later.

former choice gives a higher discounted utility. Thus we can again define grid 2 as

$$G_2 = \left\{ (\alpha, \beta) \in [0,1]^2 : \delta^4 + \delta^3 - \delta \le \alpha + \beta < \delta^2 + \delta - 1 \right\}$$

For $(x_t, y_t)_1 \in G_2$, the SPE path will start with player 1's making offer $\max\left[\delta(\delta^2 + \delta - 1 - x_t/\delta), y_t\right]$ and for player 2, it is still to reject any non-clinching offer. After the rejection from player 2, we will have $(x_{t+1}, y_{t+1})_2 \in G_1$. If $x_t + y_t < \delta(\delta^2 + \delta - 1)$, the equilibrium share will be $\left(1 - \delta^2 + x_t/\delta, \delta^2 - x_t/\delta\right)$. The analysis will be similar for $(x_t, y_t)_2 \in G_2$.

By doing this recursively, we can find a series of such grids with critical values of form $A_n = \delta^{2n} + \delta^{2n-1} - \delta^{n-1}$, i.e.,

$$G_n = \left\{ (\alpha, \beta) \in [0,1]^2 : A_n \le \alpha + \beta < A_{n-1} \right\}.$$

When $(x_t, y_t)_1 \in G_n$, the SPE path will start with player 1's making offer

$$\max\left[\delta(\delta^{2(n-1)} + \delta^{2(n-1)-1} - \delta^{n-2} - x_t/\delta), y_t\right],$$

and rejection of any non-clinching offer by player 2, then we have $(x_{t+1}, y_{t+1})_2 \in G_{n-1}$, and so on. When

$$x_t + y_t < \delta(\delta^{2(n-1)} + \delta^{2(n-1)-1} - \delta^{n-2}),$$

the equilibrium share will be $\left(1 - \delta^n + x_t/\delta^{n-1}, \delta^n - x_t/\delta^{n-1}\right)$. The analysis will be similar for $(x_t, y_t)_2 \in G_n$.

Now we can characterize the equilibrium path as following:

(a) If for some n, we have

$$\delta^{2n} + \delta^{2n-1} - \delta^{n-1} > 0 > \delta^{2(n+1)} + \delta^{2(n+1)-1} - \delta^n,$$

player 1 makes her first offer $c_0^1 = \delta(\delta^{2n} + \delta^{2n-1} - \delta^{n-1})$, which makes $x_1 = 0$ and $y_1 = \delta^{2n} + \delta^{2n-1} - \delta^{n-1}$, then it follows the SPE path we have defined. In this case, this is the unique SPE path[5];

[5]We call it the unique SPE path instead of SPE because off equilibrium path, when a player only need to make basic offer (specified by state variables) to get into next grid in next period, she can also make any lousy offer, say, the offers lower than the state variable. It has no effect on the outcome.

(b) If we have $\delta^{2(n+1)} + \delta^{2(n+1)-1} - \delta^n = 0$, in addition to the SPE path we specified in (a), we have another SPE path, in which player 1's first offer is $c_0^1 = 0$, followed by the previous SPE path starting with player 2. Indifference of player 1 between the two outcomes may induce one more period of delay.

When $\delta^2 + \delta - 1 > 0$, i.e., $\delta > \frac{\sqrt{5}-1}{2}$, there will be at least one period of delay. In the case with delay, the equilibrium path is basically that the players take turn to jump from the lower boundary of one grid to another.

(ii) When

$$\delta^{2n} + \delta^{2n-1} - \delta^{n-1} > 0 \geq \delta^{2(n+1)} + \delta^{2(n+1)-1} - \delta^n,$$

n is the number of periods delayed in the essentially unique SPE path (the more efficient one in case of two). In other words, in the SPE, it takes $n+1$ periods of offering and counteroffering to reach an agreement.

Claim n is a nondecreasing function of δ.

Suppose not, $\exists\ 1 > \delta_1 > \delta_2 > 0$, and $n_1 < n_2$, that is,

$$\delta_1^{2n_2} + \delta_1^{2n_2-1} - \delta_1^{n_2-1} \leq 0 \;\; \textit{and} \;\; \delta_2^{2n_2} + \delta_2^{2n_2-1} - \delta_2^{n_2-1} > 0,$$

then

$$\delta_1^{n_2} \leq \frac{1}{1+\delta_1} \;\; \textit{and} \;\; \delta_2^{n_2} > \frac{1}{1+\delta_2}.$$

Since $\delta_1 > \delta_2$, we have a contradiction.

It is straightforward to show that number of periods delayed goes to infinity as δ goes to 1.

(iii)&(iv)

It is easy to see the equilibrium shares will be $1-\delta^{n+1}$ and δ^{n+1} for player 1 and 2 respectively, and the payoffs are $\delta^n - \delta^{2n+1}$ and δ^{2n+1}. Moreover, $\delta^n > \frac{1}{1+\delta} > \frac{1}{2}$, as δ goes to one, δ^n goes to $\frac{1}{2}$ from above. The delay, or, the efficiency loss in the current model is substantial. More specifically, the efficiency loss converges to $\frac{1}{2}$[6]. ■

In Rubinstein's model, the equilibrium share for this simple case is $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$; and in our model, the equilibrium share lies between $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ and $\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right)$. $\frac{\delta}{1+\delta}$ is the lower bound of player 1's equilibrium share, and $\frac{1}{1+\delta}$ is the upper bound. Thus, there is no first mover advantage in our model. There is not necessarily first mover disadvantage either. When the $(0, 0)$ is close to or on the lower boundary

[6]The efficiency loss is not an increasing function of δ due to the reason that the number of periods delayed, n, is not strictly increasing with δ.

of G_n, there is first mover disadvantage; when it is close to the upper boundary, the first mover disadvantage gets smaller and even becomes advantage. As δ goes to 1, both $\frac{\delta}{1+\delta}$ and $\frac{1}{1+\delta}$ go to $\frac{1}{2}$, which indicates that in the limit the equilibrium share goes to Nash solution.

It is not hard to see that the steps by which the players jump (improve their offers) are increasing. This is quite intuitive, the higher is the current offer, the higher is the waiting cost from the proposer's point of view since she has to keep her opponent at least the same discounted utility to get an agreement. Thus, players have incentive to improve their offers by increasing steps. We think it is also quite a realistic prediction. In political bargaining, we always observe two parties start from two extreme positions, and at the beginning they are quite insistent on their positions, as times goes by, they start to make small concessions, the size of concessions tends to increase over time until they meet somewhere in the middle.

3.2 General Case

We have studied the case with homogeneous and risk neutral players. We wonder if the main result is robust to the introduction of asymmetry and risk aversion. Thus, we consider a general case next, in which two players have different concave utility functions and different discount factors. We will follow the same logic as in the benchmark model. Given any subgame $(x_t, y_t)_i$, we first specify the 'clinching offer' in the following lemma. Same as before, 'clinching offer' is the lower bound of the offers, which the opponent cannot reject in a SPE; and it is also the upper bound of the offers, which the opponent always rejects in a SPE. A SPE path should end the bargaining with a 'clinching offer' being made and accepted.

Lemma 4 *For any subgame* $(x_t, y_t)_i$, *with* $u^{-1}(u(x_t)/\delta_1) + \upsilon^{-1}(\upsilon(y_t)/\delta_2) \leq 1$[7], *the clinching offer is:*

$$\begin{aligned} \upsilon^{-1}(\delta_2 \upsilon(1 - u^{-1}(u(x_t)/\delta_1))),\ if\ i &= 1; \\ u^{-1}(\delta_1 u(1 - \upsilon^{-1}(\upsilon(y_t)/\delta_2))),\ if\ i &= 2. \end{aligned}$$

For tractability, we focus on the family of concave power functions, i.e., $u(x) = x^{\lambda_1}$ and $\upsilon(y) = y^{\lambda_2}$, $\lambda_1, \lambda_2 \in (0, 1]$. The set of feasible utility pairs now is $U = \left\{(\alpha, \beta) \in [0,1]^2 : \alpha^{\frac{1}{\lambda_1}} + \beta^{\frac{1}{\lambda_2}} \leq 1\right\}$. The discount factors remain as δ_1, $\delta_2 \in (0, 1)$.

[7]This condition means that without making any further concession in the current period, an agreement is still feasible in next period.

Now we pin down the conditions on the state variables under which the players will make the clinching offer. For player 1, making the clinching offer means payoff is $u(1-\upsilon^{-1}(\delta_2\upsilon(1-u^{-1}(u(x_t)/\delta_1))))$, waiting for the clinching offer in next period means payoff is $\delta_1^2 u(1-\upsilon^{-1}(\upsilon(y_t)/\delta_2^2))$.Comparing the payoffs, we can define the grid 1 for player 1, i.e., G_1^1.

$$G_1^1 \equiv \left\{(\alpha,\beta)\in[0,1]^2 : A_1^1 \le \delta_2^{\frac{3}{\lambda_2}}\alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3}{\lambda_1}}\beta^{\frac{1}{\lambda_2}} \le 1\right\},$$

where

$$A_1^1 = \delta_1^{\frac{1}{\lambda_1}}\delta_2^{\frac{3}{\lambda_2}} + \delta_2^{\frac{2}{\lambda_2}}\delta_1^{\frac{3}{\lambda_1}} - \delta_1^{\frac{1}{\lambda_1}}\delta_2^{\frac{2}{\lambda_2}}.$$

For player 2, making the clinching offer means payoff is $\upsilon(1-u^{-1}(\delta_1 u(1-\upsilon^{-1}(\upsilon(y_t)/\delta_2))))$, waiting for the clinching offer in next period means payoff is $\delta_2^2\upsilon(1-u^{-1}(u(x_t)/\delta_1^2))$. Comparing the payoffs, we can define the grid 1 for player 2, i.e., G_1^2.

$$G_1^2 \equiv \left\{(\alpha,\beta)\in[0,1]^2 : A_1^2 \le \delta_2^{\frac{3}{\lambda_2}}\alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3}{\lambda_1}}\beta^{\frac{1}{\lambda_2}} \le 1\right\},$$

where

$$A_1^2 = \delta_2^{\frac{1}{\lambda_1}}\delta_1^{\frac{3}{\lambda_2}} + \delta_1^{\frac{2}{\lambda_2}}\delta_2^{\frac{3}{\lambda_1}} - \delta_2^{\frac{1}{\lambda_1}}\delta_1^{\frac{2}{\lambda_2}}.$$

Here comes the main difference between the current case and the benchmark case, G_1^1 does not coincide with G_1^2. Nevertheless, there is still an inclusion relation between G_1^1 and G_1^2, which depends on $A_1^1 \gtrless A_1^2$. Without loss of generality, we assume $A_1^1 > A_1^2$, then $G_1^1 \subset G_1^2$. For any subgame $(x_t, y_t)_i$, with $(u(x_t), \upsilon(y_t)) \in G_1^1$, the SPE path will start with that player i makes the clinching offer, which will be accepted by player $-i$. For $(x_t, y_t)_i$, with $(u(x_t), \upsilon(y_t)) \in G_1^2 \backslash G_1^1$, if $i = 2$, it is still that player 2 makes the clinching offer, and player 1 accepts; if $i = 1$, player 1 will choose to wait, i.e., making an offer equal to y_t or less, and player 2 will reject and make the clinching offer in the following period. Similar to the simple case, we can define two families of grids, G_n^1 and G_n^2, recursively as following:

$$G_n^1 \equiv \left\{(\alpha,\beta)\in[0,1]^2 : A_n^1 \le \delta_2^{\frac{3n}{\lambda_2}}\alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3n}{\lambda_1}}\beta^{\frac{1}{\lambda_2}} < \min[A_{n-1}^1, A_{n-1}^2]\right\},$$

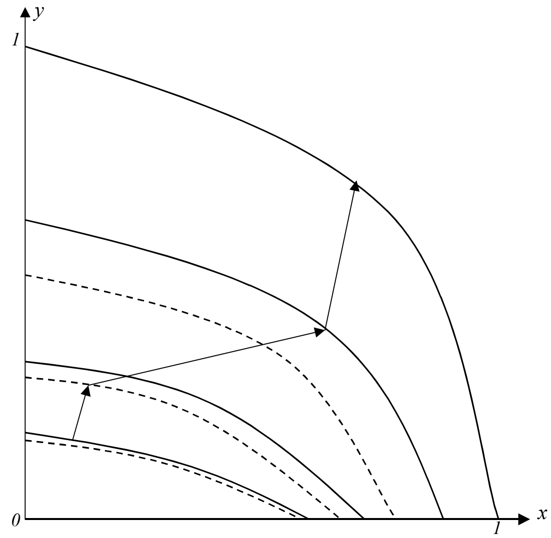

Figure 2: General Case

and

$$G_n^2 \equiv \left\{ (\alpha, \beta) \in [0,1]^2 : A_n^2 \leq \delta_2^{\frac{3n}{\lambda_2}} \alpha^{\frac{1}{\lambda_1}} + \delta_1^{\frac{3n}{\lambda_1}} \beta^{\frac{1}{\lambda_2}} < \min[A_{n-1}^1, A_{n-1}^2] \right\},$$

where

$$A_n^1 = \delta_1^{\frac{2n-1}{\lambda_1}} \delta_2^{\frac{3n}{\lambda_2}} + \delta_2^{\frac{2n}{\lambda_2}} \delta_1^{\frac{3n}{\lambda_1}} - \delta_1^{\frac{2n-1}{\lambda_1}} \delta_2^{\frac{2n}{\lambda_2}},$$
$$A_n^2 = \delta_2^{\frac{2n-1}{\lambda_1}} \delta_1^{\frac{3n}{\lambda_2}} + \delta_1^{\frac{2n}{\lambda_2}} \delta_2^{\frac{3n}{\lambda_1}} - \delta_2^{\frac{2n-1}{\lambda_1}} \delta_1^{\frac{2n}{\lambda_2}}.$$

The inequality between A_n^1 and A_n^2, thus, the inclusion relation between G_n^1 and G_n^2, will not change with n. This is given in the following Lemma.

Lemma 5 *$A_n^1 \geqslant A_n^2$ for any n if and only if $\delta_1^{\frac{1}{\lambda_1}} \geqslant \delta_2^{\frac{1}{\lambda_2}}$.*

Proof. See appendix.

Given the inclusion relation of G_n^1 and G_n^2, we can specify the SPE path for any subgame $(x_t, y_t)_i$. Here we only consider the case with $G_n^1 \subset G_n^2$. If $(u(x_t), \upsilon(y_t)) \in G_n^1$, player i will make the offer such that $(u(x_{t+1}), \upsilon(y_{t+1}))$ is on the lower boundary of G_{n-1}^2,player $-i$ will reject the offer unless $n = 1$. If $(u(x_t), \upsilon(y_t)) \in G_n^2 \backslash G_n^1$, and $i = 2$, it is same to the above; if $i = 1$, player 1 will now choose to wait, i.e., making an offer no larger than y_t, and it will become the same as above from next period. Finally we can find some n such that $(0,0) \in G^i_{n+1}$, and define the essentially unique SPE path for the whole game as before. When $(0,0)$ is on the lower boundary of G^1_n and $G^1_n \subset G^2_n$, we have two SPE paths with player 1's choice of waiting or not at the beginning of the game.

Another important feature (See Figure 2) is that for each n, the lower boundaries of G^1_n and G^2_n are parallel, but they are not parallel to the boundary of the set of feasible utility pairs. The leaning of the lower boundaries will keep to the same direction as n increases, and the direction of leaning depends on $\delta_1^{\frac{1}{\lambda_1}} \lesseqgtr \delta_2^{\frac{1}{\lambda_2}}$. From Figure 2, we can see that the direction of the leaning is in favor of the player with the larger $\delta_i^{\frac{1}{\lambda_i}}$ in terms of the final share in the SPE. The inclusion relation between G^1_n and G^2_n also has this effect. It is therefore natural to take $\delta_i^{\frac{1}{\lambda_i}}$ as the measure of 'bargaining power'. It is increasing with respect to both δ_i and λ_i.

The results for the general case are summarized in the following Proposition.

Proposition 2

(i) There is an essentially unique SPE path, the SPE strategies are specified as above given the location of (x_0, y_0) *or* $(0,0)$*;*

(ii) Either δ_1 *or* δ_2 *goes to* 1, *the number of periods delayed goes to infinity;*

(iii) If $G^1_{n+1} \subseteq G^2_{n+1}$, *when* $(0,0) \in G^1_{n+1}$, *the equilibrium share is* $(1 - \delta_2^{\frac{n+1}{\lambda_2}}, \delta_2^{\frac{n+1}{\lambda_2}})$ with n periods of delay; *when* $(0,0) \in G^2_{n+1} \backslash G^1_{n+1}$, *the equilibrium is* $(\delta_1^{\frac{n+1}{\lambda_1}}, 1 - \delta_1^{\frac{n+1}{\lambda_1}})$ with $n+1$ periods of delay. *If* $G^2_{n+1} \subseteq G^1_{n+1}$ *and* $(0,0) \in G^1_{n+1}$, *the equilibrium share is* $(1 - \delta_2^{\frac{n+1}{\lambda_2}}, \delta_2^{\frac{n+1}{\lambda_2}})$ with n periods of delay.

4 Results under Weaker Assumption

In this section, we take the interpretation of loss aversion to explore the possibility of obtaining similar results under weaker assumption. We are interested in the effects on the SPE path of weakening the assumption from the extremely loss aversion to some moderate level of loss aversion. We work with the simple case, i.e., linear utility and common discount factor, for tractability.

Given the state variables (x_t, y_t), the (current) utility functions from the share of the pie are:

$$\textit{Player 1: } \widetilde{u}^t(x) = \begin{cases} x & if \;\; x \geq x_t, \\ x_t + \beta(x - x_t) & if \;\; x < x_t; \end{cases}$$

Player 2: $\widetilde{v}^t(y) = \begin{cases} y & if \;\; y \geq y_t, \\ y_t + \;\beta(y - y_t) & if \;\; y < y_t. \end{cases}$

When $\beta = 1$, it is the standard setting in Rubinstein(1982); when β goes to infinity, it goes to the case with strong assumption as we discussed in section 3. We are now interested in the case with $1 < \beta < \infty$. Basically, what we are doing here is to replace the discontinuity of the utility function under the strong assumption by a kink. Reducing the sharpness of the kink is equivalent to weakening the assumption, or reducing the extent of the loss aversion. Our next Proposition says when β is large enough, we can have essentially unique SPE path with considerable delay, which is similar to the case under the strong assumption.

Proposition 3

When $\beta \geq \frac{1}{1-\delta}$,there is an essentially unique SPE path with considerable delay, and the number of periods delayed is same to the case under the strong assumption for the same δ.

Proof. For $\beta \geq \frac{1}{1-\delta}$, we have $\frac{\beta-1}{\beta\delta} \geq 1$.

Given the current utility functions with loss aversion, impasse is the only SPE in a subgame $(x_t, y_t)_i$ with $\frac{\beta-1}{\beta}(x_t + y_t) > 1$. For subgame $(x_t, y_t)_i$ with $\frac{\beta-1}{\beta}(x_t + y_t) = 1$, we have either impasse or the immediate agreement $(\frac{\beta-1}{\beta}x_t, \frac{\beta-1}{\beta}y_t)$, both have payoff $(0, 0)$.

Consider a possible clinching offer $(A(x_t) - x_t)$ for player 1 in $(x_t, y_t)_1$, where $\delta \leq A(x_t) < 1$.

If $\frac{\beta\delta}{\beta-1} < A(x_t) < 1$, $A(x_t) - x_t$ will be accepted since $\frac{\beta-1}{\beta}(x_{t+1} + y_{t+1}) = \frac{\beta-1}{\beta}\frac{A(x_t)}{\delta} > 1$. (Impasse if rejected)

If $\delta \leq A(x_t) \leq \frac{\beta\delta}{\beta-1}$:

if rejected, player 2 has to offer $\frac{\beta-1}{\beta}\frac{x_t}{\delta}$(zero payoff) in next period, which will not be rejected by player 1 since there will be impasse in following period. Player 2 then has $y^* = 1 - \frac{\beta-1}{\beta}\frac{x_t}{\delta}$. Choose minimum $A(x_t)$ such that $y^* \leq \frac{[A(x_t)-x_t]}{\delta}$, we get $A(x_t) \geq \delta + x_t/\beta$.

Now we can specify the clinching offer as following: $A(x_t) - x_t$,

where

$$A(x_t) = \begin{cases} \delta + x_t/\beta & if \;\; x_t \leq \frac{\beta\delta}{\beta-1} \\ x_t & if \;\; x_t \geq \frac{\beta\delta}{\beta-1}. \end{cases}$$

In any $(x_t, y_t)_1$, once the offer $A(x_t) - x_t$ is made by player 1, player 2 will accept immediately. On the other hand, any offer lower will be rejected because a similar counteroffer by player 2 in next period will not be rejected by player 1, and the payoff will be better for player 2. Similar clinching offer can be defined for player 2.

When will a player make the clinching offer?

Given $(x_t, y_t)_i$:

$$1 - [A(x_t) - x_t] \geq \delta[A(y_t/\delta) - y_t/\delta], for\ i = 1,$$
$$1 - [A(y_t) - y_t] \geq \delta[A(x_t/\delta) - x_t/\delta], for\ i = 2.$$

We get the following condition:

$$x_t + y_t \geq \frac{\beta}{\beta - 1}(\delta^2 + \delta - 1).$$

After this, it will be same to the case under strong assumption. A family of grids can be defined recursively as following:

$$G_n = \left\{(\alpha, \beta) \in [0, 1]^2 : A_n \leq \alpha + \beta < A_{n-1}\right\},$$

where

$$A_n = \frac{\beta}{\beta - 1}(\delta^{2n} + \delta^{2n-1} - \delta^{n-1}).$$

Therefore, we get essentially unique SPE path again, and the equilibrium share and the number of periods delayed are same as before. ■

It is natural to ask the following questions now: (1) Will there be delay in SPE path when β is small? (2) Will the SPE path converge to Rubinstein's result when β goes to 1? (3) Is there necessarily delay in SPE for any $\beta > 1$? We cannot give clear answers for these questions at this stage. Based on our preliminary work on these questions, our conjecture now is that at least for not very large β, say, β is close to 2, there is still delay in the bargaining game with parameter (β, δ), and δ is close enough to 1.

5 Discussion

One interesting observation is that the equilibrium play we obtain has a flavor of forward induction. When a player believes that her opponent is a Bayesian maximizer, she has a reason to believe that her opponent is looking forward to a better payoff after observing a rejection, therefore, it is also likely that she believes that she has to improve the offer in order to reach an agreement. However, it is not exactly the forward induction as stated in Van Damme (1989). In his informal definition, if a player chooses between an outside option and a subgame with a unique and viable equilibrium, which is strictly better than the outside option, the equilibrium with players' playing the subgame is the only self-enforcing one. The SPE in Rubinstein's model satisfies this forward induction requirement, and an important reason is that the bargaining game in his model is isomorphic at every decision node at which an offer is made. The reason that uniqueness is needed is that there maybe a coordination problem in the subgame. Our bargaining model is a perfect information game, this should not be a problem. The viability concerns the situation, in which there maybe further moves by both players in the subgame after the deviation from the outside option. In the bargaining game with infinite alternating-offers procedure, if we want to justify the deviation, i.e., the rejection of the SPE offer in Rubinstein's model, as a signal about future play, the viability requirement cannot be satisfied. Another problem is that there exists an outcome in the subgame, which is indifferent with the current offer for the player who has to decide to accept or not[8]. To incorporate the forward induction analysis into bargaining model is not new. Dekel (1990) discussed the power of forward induction and stability in a two-period simultaneous bargaining game. We are trying to understand how the idea of forward induction could work in an infinite horizon game such as the bargaining game with alternating-offers procedure.

Another closely related issue is about the common belief of rationality in extensive games. It is well known that we have counterintuitive SPEs in many cases, for example, the chain-store paradox in Selten (1978) and the centipede game in Rosenthal (1981). Kreps *et al.* (1982) obtain considerable cooperations in a finitely repeated prisoner's dilemma game by adding a small dose of "Tit-for-Tat" players. An exogenous lack of common belief of rationality results in a rational play which is inconsistent with the SPE. Reny (1992) argued that even there is common belief of rationality at the beginning of the game, it might still be possible to have a rational play inconsistent with the SPE. The lack of common belief of rationality may arise endogenously. In a perfect information game, the concept of SPE depends on the assumption of common knowledge of rationality, or, common belief of rationality at every information set. Reny (1992) showed that in the games with paradoxical SPEs, such as the "Take it or leave

[8]This is actually the key point of Rubinstein's result.

it" game, once a player deviates from the SPE path, it is impossible to retain common belief of rationality. Reny (1993) showed that in most two-player games with perfect information, it is impossible to have common belief of rationality everywhere. This challenged the salience of the concept of SPE and the theory of rationalizability in extensive form games developed by Bernheim (1984) and Pearce (1984) as well.

We think it is also reasonable to include the bargaining game into this family of extensive form games with counterintuitive SPEs. The unique SPE in Rubinstein's bargaining game also depends on the common belief of rationality at every decision node. This point can be made clear by looking at the concept of iterated conditional dominance introduced in Fudenberg and Tirole (1991). By iterative elimination of conditional dominated actions, the unique SPE can be obtained in Rubinstein's bargaining game. If we put the belief about rationality explicitly, the logic will be like this: player 1 believes player 2 is rational, thus she will not make an offer higher than δ; player 1 believes that player 2 believes player 1 is rational, thus she will not make an offer lower than $\delta(1-\delta)$; etc. This logic goes as an infinite sequence, the unique SPE will be the convergent point, which corresponds to the argument, *player 1 believes player 2 believes player 1...is rational*, with infinite length. Since the common belief of rationality gives the unique prediction of the play, the SPE path, the deviation from it will be the violation of the common belief of rationality, it will not hold any more. Once there is no common belief of rationality, it will be an important issue that how the players update their belief, belief about belief, etc., then given the belief updating rule, it may be rational to choose a play inconsistent with the SPE at the first place. In Rubinstein's model, there is no such updating, the common belief of rationality remains after any history, we take it as an extreme case of the belief updating. Our assumption is equivalent to another extreme case of belief updating, the players update their believes such that the history is always rationalizable under the new belief. Given some well-specified intermediate belief updating rule, our conjecture will be that a rational play will still involve delay. It will be of our interest for further study to formally model the idea included in this discussion. Our inclination now is that the relation between the Abreu and Gul (2000) and this possible line of research on bargaining will be parallel to that between Kreps *et al.* (1982) and the sequel papers by Reny (1992a, 1992b and 1993).

Yildiz (2003) also discussed the role of belief in bargaining game. The belief in his model is about the future position in the bargaining. The collective optimism, an assumption of heterogenous priors, may cause delay; but in general case, the collective optimism remains for sufficient long time, an immediate agreement result is obtained.

Our last point is about the discretization. It is well-known that Rubinstein's result is not robust to discretization. As shown by Van Damme et al. (1990) and Muthoo (1991), any Pareto efficient agreement can be supported as the outcome of a SPE if the time interval between two offers is sufficiently small. It also results from the fact that in Rubinstein's model, the bargaining game is isomorphic at any decision node where an offer should be made. Under the setting of our model, the discretization does not change our result so drastically. Actually, it does not change the uniqueness of the SPE path.

6 Conclusion

This paper is trying to provide an explanation for the delay in real life bargaining as an alternative of the usual incomplete information approach. Our point of view is that in many serious bargaining situation, information asymmetry may not be the main underlying force for the delay, endogenous preferences or strategic commitment effects should be taken into account. Our behavioral assumption will be taken as incredible threat in classic analysis, and we try to provide a rationale for such kind of behavioral type to make the incredible threat credible. We think both the delegated bargaining story and the loss aversion treatment have some explaining power.

Meanwhile, the flavor of forward induction and the issues about rationality, belief, and belief updating are intriguing topics to us. The pioneering works in these areas focused on finite extensive form games, while as the bargaining game has two dimensional infinity, infinite horizon and infinite actions at each decision node for an offer to be made. This imposes the great difficulty on analysis. We leave it for further investigation.

APPENDIX

Proof of Lemma 4

Let $\theta_1 = \delta_1^{\frac{1}{\lambda_1}}$, $\theta_2 = \delta_2^{\frac{1}{\lambda_2}}$.

$A_n^1 - A_n^2$

$= (\theta_1^{3n}\theta_2^{2n} - \theta_2^{3n}\theta_1^{2n}) + (\theta_1^{2n-1}\delta_2^{3n} - \theta_2^{2n-1}\theta_1^{3n}) - (\theta_1^{2n-1}\theta_2^{2n} - \theta_2^{2n-1}\theta_1^{2n})$

Divide both sides by $\theta_1^{2n-1}\theta_2^{2n-1} > 0$

$(A_n^1 - A_n^2)/\theta_1^{2n-1}\theta_2^{2n-1}$

$= (\theta_1^{n+1}\theta_2 - \theta_2^{n+1}\theta_1) + (\theta_1 - \theta_2) - (\theta_1^{n+1} - \theta_2^{n+1})$
$= \theta_1\theta_2(\theta_1 - \theta_2)\sum_{i=0}^{n-1}\theta_1^i\theta_2^{n-1-i} + (\theta_1 - \theta_2) - (\theta_1 - \theta_2)\sum_{i=0}^{n}\theta_1^i\theta_2^{n-i}$
$= (\theta_1 - \theta_2)(\theta_1\theta_2\sum_{i=0}^{n-1}\theta_1^i\theta_2^{n-1-i} + 1 - \sum_{i=0}^{n}\theta_1^i\theta_2^{n-i})$
$= (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}\theta_1^{i+1}\theta_2^{n-i} - \sum_{i=0}^{n}\theta_1^i\theta_2^{n-i})$
$= (1) = (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}(\theta_1^{i+1}\theta_2^{n-i} - \theta_1^i\theta_2^{n-i}) + 1 - \theta_1^n)$
$= (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}\theta_1^i\theta_2^{n-i}(\theta_1 - 1) + (1 - \theta_1)\sum_{i=0}^{n-1}\theta_1^{n-1-i})$
$= (\theta_1 - \theta_2)(1 - \theta_1)\sum_{i=0}^{n-1}(\theta_1^{n-1-i} - \theta_1^i\theta_2^{n-i})$
or $= (2) = (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}(\theta_2^{i+1}\theta_1^{n-i} - \theta_2^i\theta_1^{n-i}) + 1 - \theta_2^n)$
$= (\theta_1 - \theta_2)(\sum_{i=0}^{n-1}\theta_2^i\theta_1^{n-i}(\theta_2 - 1) + (1 - \theta_2)\sum_{i=0}^{n-1}\theta_2^{n-1-i})$
$= (\theta_1 - \theta_2)(1 - \theta_2)\sum_{i=0}^{n-1}(\theta_2^{n-1-i} - \theta_2^i\theta_1^{n-i})$
If $\theta_1 \geq \theta_2$, $(A_n^1 - A_n^2)/\theta_1^{2n-1}\theta_2^{2n-1} \geq 0$ by (1);
if $\theta_1 < \theta_2$, $(A_n^1 - A_n^2)/\theta_1^{2n-1}\theta_2^{2n-1} < 0$ by (2).
Thus, $A_n^1 \geqslant A_n^2$ for any n if and only if $\delta_1^{\frac{1}{\lambda_1}} \geqslant \delta_2^{\frac{1}{\lambda_2}}$.

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