Mutivalued dynamics and games

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Abstract

We present several properties related to dynamical systems on the strategy spaces governed by a differential inclusion of the form

$$\dot{x}(t) \in F(x(t)) \qquad (*)$$

where F is an uper semi continuous correspondance with compact convex values.

1) The first result is the convergence of the (continuous) best reply dynamics for two person zero-sum continuous concave-convex games: the product set of optimal strategies is a global uniform attractor.
In particular this implies convergence of generalized fictitious play processes to this case.
This is a joint work with Josef Hofbauer.

2) The next advances extend some results of the theory of stochastic approximation (in the spirit of Benaïm and Hirsch) to the above framework. A perturbed solution y of (*) satisfies

$$\dot{y}(t) \in F^{\delta(t)}(y(t)) + U(t)$$

for suitable vanishing approximation $\delta(t)$ and noise $U(t)$. The limit set

$$L(y) = \bigcap\nolimits_{t \geq 0} \overline{\bigcup\nolimits_{s \geq t} y(s)}$$

of a bounded perturbed solution is an internally chain recurrent set for the multivalued flow associated to (*).
Applications are convergence of generalized approchability processes and convergence of fictitious play for concave continuous potential games.
This is an outcome of a joint research with Michel Benaïm and Josef Hofbauer.

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