Ordinal cheap talk*

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Abstract

Can comparative statements be credible even when absolute statements are not? For instance, can a professor credibly rank different students for a prospective employer even if she has an incentive to exaggerate the merits of each student? Or can an analyst credibly rank different stocks even if the client would be dubious about a recommendation to buy any one of them? We examine such problems in a multidimensional sender-receiver game where the sender has private information about multiple variables. We show that *ordinal cheap talk*, in which the variables are completely ordered by value or grouped into categories by value, can be credible even when interests are too opposed to support communication along any single dimension. Ordinal cheap talk is credible because it reveals both favorable and unfavorable information at the same time, thereby precluding any possibility of exaggeration. The communication gains from ordinal cheap talk can be substantial with only a couple of dimensions, and the payoffs from a complete ordering are asymptotically equivalent to full revelation as the number of variables becomes large. However, in some circumstances the sender can do better through a partial ordering that categorizes variables. Compared to other forms of cheap talk, ordinal cheap talk is exceedingly simple in that the sender only makes straightforward, comparative statements.

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1 Introduction

When can costless, unverifiable "cheap talk" be credible? When interests coincide there is no incentive to lie – collision-averse drivers gain nothing from deceptive use of their turn signals. But when interests are in conflict it is unclear how mere cheap talk can be credible. For instance, a salesperson might want a buyer to believe her product is of superior quality, or a professor might want to convince an employer that her student is highly qualified, regardless of the actual quality of the product or student. In signaling games the costliness of the signal reduces the incentive to exaggerate, but in cheap talk games the very ease of communication can make communication useless. When the sender wants the receiver to take a particular action regardless of their information, such as buy a good or hire a student, any claims become non-credible and cheap talk breaks down.

Crawford and Sobel (1982) investigate the intermediate case where sender and receiver interests are not so divergent. In a model that has been applied to a wide range of situations from announcing monetary policy (Stein, 1989) to making stock recommendations (Morgan and Stocken, 2003), they assume that a biased expert has private information about the realization of a random variable that affects what action a decision-maker should take. Remarkably, they find that the expert can often reveal which partition of the space the variable lies in even if the exact value cannot be credibly revealed. For instance, if a professor has an incentive to promote a student, the reader of a favorable recommendation letter may doubt the intensity of the letter's praise, but still be confident that a favorable letter indicates the student's ability is above some level. But even such "partition cheap talk" breaks down if the professor is willing to recommend a very bad student.

The Crawford and Sobel model and related cheap talk models generally assume that the sender has information about only one unknown variable. But in practice the sender may have information about many variables. For instance, a stock analyst has information about different stocks, a salesperson has information about different products, or a professor is in the position to evaluate multiple students. This paper examines the cheap talk possibilities that arise in such situations. In particular, we explore the potential for "ordinal cheap talk" regarding the *relative* magnitudes of multiple unknown variables, e.g. a professor might indicate that one student is better than another. Ordinal cheap talk has the potential to be credible because it simultaneously sends both favorable and unfavorable information about different issues. The ability to exaggerate is thereby eliminated.

In a multi-dimensional model that encompasses the Crawford and Sobel expert game we find that simple complementarity conditions are sufficient for ordinal cheap talk to be an equilibrium. The conditions ensure that the sender wants the receiver to take a higher action when the unknown variable is higher, and that the receiver wants to take a higher action when the unknown variable is expected to be higher. As a result the sender has no incentive to deviate from the true ranking. Even if the sender always wants the receiver to take a particular action in each dimension, ordinal cheap talk is credible as long as the complementarity conditions are satisfied. These conditions appear to fit many real world situations and are satisfied under the basic assumptions of the expert game. Whereas Crawford and Sobel show that cheap talk is only an equilibrium in the expert game if the expert's and decision-maker's interests are sufficiently aligned, in the multi-dimensional version of the game we find that ordinal cheap talk is always an equilibrium.

The communication gains from ordinal cheap talk can be substantial. As the number of issues grow, the sender's ranking becomes a very accurate signal of each variable's value. For instance, the smallest of two variables is likely to be below average, but the smallest of 100 variables is not only likely to be below average, but its distribution is heavily concentrated near the bottom of the original distribution. In general, for any $p \in (0, 1)$, as the number of issues N becomes large the pN-th issue is very likely to be very close to the p-th percentile of the original distribution. As a result, revealing the sender's comparative information through a complete ordering is asymptotically equivalent to revealing *all* of the sender's private information.

Ordinal cheap talk as we define it includes both a complete ordering in which all the variables are ranked, and a partial ordering in which the variables are categorized by relative size. For instance, a partial ordering could involve dividing students into two groups in which all members of one group are at least as good as those in another group, but no finer information is revealed. The same complementarity conditions determine whether a complete ordering or partial ordering is an equilibrium, but a partial ordering clearly transmits less information if there are more than two issues.

While the receiver is never worse off from having more information, partial orderings can help maintain some ambiguity for the sender and serve as an attractive compromise between no revelation and a complete ordering. For instance, if there are three students and none are likely to be hired without additional information from a professor, then a complete ranking can push the best student over the top. But if the middle student will not quite make it based on the complete ordering an alternative is to put the top two students in a group and not differentiate between them. The average quality of the top two students might then be sufficiently high that both are hired. As the number of students increases, such groupings can be used more and more effectively to maximize the sender's payoffs.[1]

With the exception of allowing for multiple dimensions and allowing for divergent interests between the sender and receiver, we follow the standard assumptions of the Crawford-Sobel framework. In particular, there is only one sender and one receiver who engage in a one-shot game, only the receiver takes an action, and the receiver does not have any private information. Clearly these assumptions do not apply in many situations and they have all been relaxed within the literature.[2] When there are multiple senders, competition between the senders can lead to more information revelation (Krishna and Morgan, 2001). Battaglini (2002) finds the strong result that in a multi-dimensional setting a receiver can structure such competition to induce full information revelation when the multiple senders have different preferences. Surprisingly, the result still holds when sender and receiver interests are insufficiently aligned to support even partition cheap talk in a single dimension. However, the communication still depends on some commonality of interests in that the senders do not always want the receiver to take a particular action, e.g. the maximal or minimal action, regardless of the state.[3]

The assumption that only the receiver can take actions does not cover coordination games, such as drivers encountering each other at an intersection, where cheap talk can sometimes resolve strategic uncertainty over which action each side intends to take (Farrell, 1987; Farrell and Rabin, 1996). Baliga and Morris (2002) consider coordination games in which, in addition to the strategic uncertainty, there is one-sided uncertainty about payoffs, thereby incorporating elements of the expert game and the coordination game. For instance, two firms must each decide whether to invest in complementary research projects and one company has private information about the profitability of their own investment. They find the strong negative

[1] It is a common strategy for highly ranked schools to obscure the relative quality of their graduates, either by grade inflation, e.g. Ivy League undergraduate programs, or by withholding grades from employers, e.g. some elite M.B.A. programs. For a related analysis see Ostrovsky and Schwarz (2003).

[2] Additional assumptions we follow that have been relaxed elsewhere in the literature are that there is only one stage of cheap talk (Aumann and Hart, 2002; Krishna and Morgan, 2002), that the sender and receiver are fully rational (Crawford, 2003; Ottaviani and Squintani, 2002), and that there are no third-party receivers whom the players wish to impress (Groseclose and McCarty, 2001).

[3] Battaglini defines sender and receiver preferences on an outcome space where the outcome is the sum of the state and the receiver's action. The receiver action that each player prefers varies with the state and in the same direction across players. We define preferences on action-state pairs and preferred actions may or may not vary with the state.

result that in a binary action game if the informed side always wants the uninformed side to take a particular action, e.g. wants the other firm to invest, then no cheap talk of any kind is possible. Note that this result applies to games in a single dimension. If the two firms are considering several different projects, it is straightforward to show that ordinal cheap talk can be used to credibly rank the different projects and thereby increase investment efficiency. We will not formally analyze such a model in this paper but rather stick to the Crawford-Sobel framework in which only the sender takes an action.

Cheap talk with both sender and receiver actions can also be important when actions are sequential as in bargaining between an executive and legislature (Matthews, 1989). Chakraborty and Harbaugh (2003) consider a bargaining game in which an offeree first indicates the relative intensity of her preferences regarding two issues, the offerer then proposes concessions on the two issues, and finally the offeree accepts or rejects the offers. Credible communication is possible, but only if the two issues are bundled together in a single offer that must be accepted or rejected in its entirety.[4,5]

The assumption of only one, uninformed receiver is clearly inappropriate in many selling environments.[6] Consider a multi-object auction in which an informed seller makes cheap talk statements about her goods to multiple buyers with private information. If credible, such revelation of seller information increases expected revenues by narrowing information differences among buyers (Milgrom and Weber, 1982), but the seller has an incentive to only reveal good information or even lie about bad information. Chakraborty, Gupta, and Harbaugh (2002a and 2002b) show that ordinal cheap talk can be credible in such circumstances.

Finally, the main results of this paper are based on the assumption of a one-shot game since such a game is least conducive to cheap talk. In a repeated game if the sender is sufficiently patient, there is a sufficiently high probability of continuation, and information is fully revealed between periods, reputation can support full revelation (Sobel 1985; Stocken, 2000).[7] As an

[4]Applied to bargaining between an executive and legislature, this result implies that the executive will more accurately communicate her preferences when spending bills are required by law to be bundled than when a line-item veto is possible.

[5]Note that since the game is still a cheap talk game, the offerer does not pre-commit to a set of offers based on offeree statements but is free to make any offer. In mechanism design problems, where such commitment is possible, Jackson and Sonnenschein (2003) show in general that linking together multiple problems strictly increases efficiency.

[6]It is also inappropriate in voting games where each voter may be both a sender and receiver who conveys her private information to other voters and also listens to other voters (Austen-Smith, 1990).

[7]The career concerns literature starting with Holmstrom (1982/1999) shows that in some cases a concern for

extension of the main results of this paper we investigate whether a reputation for honest ordinal cheap talk is easier to sustain than a reputation for fully revealing information. For a large number of issues the long-term value of a reputation for honest ordinal statements is almost the same as the value of a reputation for fully revealing information. But the short-term gain from lying is always less for ordinal cheap talk. In a game where ordinal cheap talk is not credible without reputational considerations, we find that for a sufficiently large number of issues a lower discount rate supports ordinal cheap talk.

In general, ordinal cheap talk has several attractive features for understanding communication. First, it can be credible when other forms of cheap talk are not. Second, it is exceedingly simple in that the sender only makes straightforward, comparative statements. Third, it can reveal a large amount of information even for a limited number of dimensions and, as the number of dimensions increases, it is asymptotically equivalent to full information revelation. Finally, it appears to offer insight into the real-world prevalence of comparative statements (Rubinstein, 1996).

2 The model

We consider a multi-dimensional game in which player S (the sender) possesses private information about $N \geq 2$ different issues. The sender's private information about issue $k = 1, ..., N$ is represented by a random variable $\theta_k \in \Theta$ where the θ_k are independently and identically distributed. We assume that Θ is a compact convex subset of $\mathbb{R}$. Let $\theta = (\theta_1, ..., \theta_N)$ and let F denote the distribution of θ_k. For simplicity, we assume that F has a positive density f so that F is invertible.

At the beginning of the game the sender sends a message m from a finite set M that is heard by player R (the receiver).[8] Subsequently, for each issue k player R chooses an action a_k from a set A that is independent of k. We assume that A is a compact convex subset of $\mathbb{R}$. Let $a = (a_1, ..., a_N)$ denote the action profile chosen by the receiver.[9]

The payoff from issue k to player $i \in \{S, R\}$ is given by a function $u^i : \Theta \times A \rightarrow \mathbb{R}$ that is independent of k. We assume that u^i is continuous in each argument, for all $i \in \{S, R\}$.

reputation can paradoxically distort behavior. Morris (2001) shows that it can distort cheap talk messages.

[8]The results are robust to allowing different receivers for each issue, as long as the sender's message is a public message.

[9]Note that the receiver can take actions independently on each issue. In some cases bundling the issues so that actions are interdependent can be more efficient as shown in Chakraborty and Harbaugh (2003).

For each θ_k let $a(\theta_k)$ be the unique maximand of $u^R(\theta_k, a_k)$ with respect to a_k. We denote by $U^i(\theta, a)$ the total payoff to player i from an action profile a in a state of the world θ, and assume that it is additive across issues so that $U^i(\theta, a) = \sum_k u^i(\theta_k, a_k)$. Notice that the payoffs of either player do not directly depend on the message m that is sent by player S. In other words, the sender's message is pure cheap talk.

For any set X let $\Delta(X)$ denote the set of probability distributions on X. A strategy for the sender is a function $\mu : \Theta^N \to \Delta(M)$ and a strategy for the receiver is a function $\sigma : M \to \Delta(A^N)$. Beliefs of the receiver over Θ^N (inferred from a message m) are given by a function $\phi : M \to \Delta(\Theta^N)$. Our equilibrium notion is that of weak perfect Bayesian equilibrium.[10]

Definition 1 *A tuple (μ, σ, ϕ) is an equilibrium if:*

1. *For all θ, $\mu(m|\theta) > 0 \Rightarrow m \in \arg\max_{m'} E[U^S(\theta, a)|\theta, m']$*
2. *For all m, $\sigma(a|m) > 0 \Rightarrow a \in \arg\max_{a'} E[U^R(\theta, a')|m]$*
3. *The beliefs ϕ are derived from μ and F via Bayes' Rule whenever possible.*

We are interested in the possibility of informative cheap talk equilibria where the sender's message consists of disclosing a partial or complete order on her private information $\theta_1, ..., \theta_N$ about the N issues. Such a message contains information about each issue that is not independent of the information it contains about other issues. As we will show below, this implies that even when there is a strong conflict of interest between the sender and receiver with regard to the optimal action that should be taken on each issue, informative communication will still be possible. We call such strategies ordinal cheap talk strategies.

Formally, let $\theta_{i:N}$ indicate the ith smallest realization of the N different θ_k. Let $C = (C_1, ..., C_{|C|})$ denote an ordering of $\theta_1, ..., \theta_N$ into $|C| \leq N$ elements or categories such that category $j = 1, ..., |C|$ contains $|C_j| \geq 1$ issues with $\sum_j |C_j| = N$. Thus, the category $C_1 = \{\theta_{1:N}, ..., \theta_{|C_1|:N}\}$ contains the lowest $|C_1|$ of the θ's, the category $C_2 = \{\theta_{|C_1|+1:N}, ..., \theta_{|C_1|+|C_2|:N}\}$ contains the next set of the $|C_2|$ lowest θ's, and so on. For $a \leq b$, $a, b \in \{1, ..., N\}$, let $c_j = \{a, ..., b\}$ denote the set of indices of the elements in the set $C_j = \{\theta_{a:N}, ..., \theta_{b:N}\}$.

The ordinal cheap talk strategy that corresponds to the ordering C is described as follows. For each realization of θ, the sender announces that the $|c_1|$ issues with the lowest values of θ_k

[10]Note that the equilibrium notion states that R's action has to be optimal given his inference about θ upon hearing m. This distinguishes the cheap talk model from a screening problem where R first commits to a menu of actions for each message and S chooses among them.

are in category one, the next $|c_2|$ issues are in category two and so on. If there are ties between some of the θ_k's, the sender uniformly randomizes when she sorts those issues into different categories. Consequently, the receiver knows that for issues in higher categories the sender's private information has a weakly higher value and cannot distinguish between issues within a category.

We denote an ordinal cheap talk strategy by the corresponding ordering C which is fixed and does not depend on the realization of θ. The finest possible ordering, $C = (\{\theta_{1:N}\}, ..., \{\theta_{N:N}\})$ or $c_j = \{j\}$ for all $j = 1, ..., N$, is when the sender completely orders the N issues. On the other hand, the coarsest possible ordering, $C = (\{\theta_{1:N}, ..., \theta_{N:N}\})$ or $c_1 = \{1, ..., N\}$, corresponds to an uninformative babbling strategy. It is well known that there always exists an equilibrium where the sender uses the babbling strategy.

For any candidate equilibrium ordering C, let $F_{c_j:N}$ denote the distribution of θ_k given that the sender has announced that it belongs to category j. Note that for the special case of a complete ordering, this corresponds to standard notation for the distributions of order statistics. Clearly, for any ordinal cheap talk strategy C, the possible equilibrium beliefs of the receiver with respect to θ_k are summarized by the collection $\{F_{c_j:N}\}_{j=1}^{|C|}$ with corresponding densities $\{f_{c_j:N}\}_{j=1}^{|C|}$.[11] Furthermore, due to the assumed additivity of payoffs, note that if an action profile $a = (a_1, ..., a_N)$ maximizes R's expected total payoff given a message that the issue k belongs to category j, it must be that

$$a_k \in \arg\max_{a'_k} \int u^R(\theta_k, a'_k) dF_{c_j:N}(\theta_k) \tag{1}$$

for each issue k and category j. Note that our assumptions on $u^R(\theta_k, a_k)$ and A imply that the maximization in (1) has a solution for all j and C. Let $a_{c_j:N}$ denote this solution.

Our first result provides sufficient conditions for ordinal cheap talk to be an equilibrium. These complementarity conditions take the form of a single-crossing condition and a supermodularity condition. We adapt from Athey (2002) the definitions of these two concepts that are adequate for our purposes.

Definition 2 *For $i \in \{S, R\}$,*

[11] As is standard in cheap talk games, given any candidate equilibrium ordering C, the possibility of out–of–equilibrium messages is ruled out by assuming that to *each* message $m \in M$ the receiver ascribes a meaning corresponding to one element of the partition of Θ^N that is generated by C, and forms his beliefs $\phi(m)$ accordingly.

1. *u^i satisfies the single-crossing property if, for all $a_k > a'_k$, the difference $u^i(\theta_k, a_k) - u^i(\theta_k, a'_k)$ as a function of θ_k crosses zero at most once and from below.*

2. *u^i satisfies supermodularity if, for all $a_k > a'_k$, the difference $u^i(\theta_k, a_k) - u^i(\theta_k, a'_k)$ as a function of θ_k is non-decreasing in θ_k.*

When u^R satisfies the single-crossing property the receiver will take higher actions for issues announced to be in higher categories. And when u^S satisfies the supermodularity property the sender has no incentive to misreport the order. Ordinal cheap talk is therefore an equilibrium. Note that the single-crossing property is a weaker restriction than supermodularity,[12] implying that supermodularity of both payoff functions is sufficient for ordinal cheap talk to be an equilibrium.

Theorem 1 *If u^R satisfies the single-crossing property and u^S is supermodular then every ordering C is an equilibrium ordering.*

Proof. Consider any ordering C and note that for any message that puts issue k in category C_j, a best-response for R is to choose $a_k = a_{c_j:N}$, the solution to (1). More generally, for any announced message corresponding to the ordering C, it is a best-response for R to choose the action $a_{c_1:N}$ for the $|c_1|$ issues with the lowest value, the action $a_{c_2:N}$ for the next set of $|c_2|$ issues and so on, finally choosing the action $a_{c_{|C|}:N}$ for the $\left|c_{|C|}\right|$ issues that have the highest value.

We show now that $a_{c_j:N}$ is non-decreasing in j. Since u^R satisfies the single-crossing property this follows from Athey (2002) as long as the collection $\{f_{c_j:N}\}_{j=1}^{|C|}$ satisfies the monotone likelihood ratio property in j. We prove next that this property holds.

Standard derivations for order statistics imply that, for all $h = 1, ..., N$ and all θ_k,

$$f_{h:N}(\theta_k) = \frac{N!}{(N-h)!(h-1)!} F(\theta_k)^{h-1} (1 - F(\theta_k))^{N-h} f(\theta_k). \tag{2}$$

Since F is strictly increasing, for $h > h'$ and $\theta_k > \theta'_k$,

$$\begin{aligned} \frac{f_{h:N}(\theta_k)}{f_{h':N}(\theta_k)} &= \frac{(N-h')!(h'-1)!}{(N-h)!(h-1)!} [\frac{F(\theta_k)}{1-F(\theta_k)}]^{h-h'} \\ &> \frac{(N-h')!(h'-1)!}{(N-h)!(h-1)!} [\frac{F(\theta'_k)}{1-F(\theta'_k)}]^{h-h'} = \frac{f_{h:N}(\theta'_k)}{f_{h':N}(\theta'_k)}. \end{aligned}$$

[12] For instance, the function u^R could be locally submodular and still satisfy the single-crossing property.

Thus, the likelihood ratio $\frac{f_{h:N}(\theta_k)}{f_{h:N}(\theta'_k)}$ is increasing in h for all $\theta_k > \theta'_k$. Consider now any two categories C_j and $C_{j'}$ with $j > j'$, $c_j = \{a, ..., b\}$ and $c_{j'} = \{a', ..., b'\}$ so that $a' \leq b' < a \leq b$. Since $f_{c_j:N}(\theta_k) = \frac{1}{|c_j|}\sum_{h=a}^{b} f_{h:N}(\theta_k)$ and $f_{c_{j'}:N}(\theta_k) = \frac{1}{|c_{j'}|}\sum_{h=a'}^{b'} f_{h:N}(\theta_k)$, it is immediate that for $\theta_k > \theta'_k$,

$$\frac{f_{c_j:N}(\theta_k)}{f_{c_j:N}(\theta'_k)} = \frac{\sum_{h=a}^{b} f_{h:N}(\theta_k)}{\sum_{h=a}^{b} f_{h:N}(\theta'_k)} \geq \frac{f_{a:N}(\theta_k)}{f_{a:N}(\theta'_k)} > \frac{f_{b':N}(\theta_k)}{f_{b':N}(\theta'_k)} \geq \frac{\sum_{h=a'}^{b'} f_{h:N}(\theta_k)}{\sum_{h=a'}^{b'} f_{h:N}(\theta'_k)} = \frac{f_{c_{j'}:N}(\theta_k)}{f_{c_{j'}:N}(\theta'_k)}. \tag{3}$$

Hence, the collection $\{f_{c_j:N}\}_{j=1}^{|C|}$ satisfies the monotone likelihood ratio property in j.

Finally, since u^S is supermodular and $a_{c_j:N}$ is non-decreasing in j, the seller has no incentive to misreport the correct ordering. To see this, note that for any realization of θ if the sender announces a lower category for issue θ_k than for issue $\theta_{k'}$ with $\theta_k > \theta_{k'}$, he can do at least weakly better by switching the announced categories for these two issues, keeping the announced categories for the other issues fixed. Therefore for every realization of θ the seller can do no better than to announce that the $|c_1|$ issues with the lowest values belong to category 1, the next $|c_2|$ issues to category 2 and so on.[13] ■

When ordinal cheap talk is an equilibrium the sender can credibly reveal comparative information. Even if interests are so divergent that the sender always wants the receiver to take a particular action in each dimension, comparative information is credible as long as the complementarity conditions are satisfied. Such information can be surprisingly informative. We find that as the number of issues N becomes large, the per-issue expected payoffs under the complete ordering converge to the expected payoffs in the full information case.

Theorem 2 *Under the complete ordering $C = (\theta_{1:N}, ..., \theta_{N:N})$, expected sender and receiver payoffs asymptotically approach the full information case as the number of issues N increases.*

Proof. For each $q \in (0, 1)$, by the Glivenko-Cantelli Theorem (see, e.g., Billingsley (1995)),

$$\lim_{N\to\infty} \theta_{\lceil qN\rceil:N} = F^{-1}(q) \text{ a.s.} \tag{4}$$

where $\lceil x\rceil$ denotes the smallest integer at least as large as x. It follows that

$$\lim_{N\to\infty} a_{\lceil qN\rceil:N} = a(F^{-1}(q)). \tag{5}$$

[13]Note that the function u^S need only be supermodular over the restricted domain of actions actually chosen. Note also that the identical result holds when u^S is submodular (i.e., $-u^S$ is supermodular) and $-u^R$ satisfies the single-crossing condition. In such a case $a_{c_j:N}$ is non-increasing in j.

To see this, suppose that (5) does not hold. Fix q. Since the sequence $a_{\lceil qN\rceil:N}$ is a sequence in a closed set A, it has a convergent subsequence converging to a point in A, say, $a' \neq F^{-1}(q)$. Since $a(F^{-1}(q))$ is the unique maximand of $u^R(a, F^{-1}(q))$, it follows that $u^R(a(F^{-1}(q)), F^{-1}(q)) > u^R(a', F^{-1}(q))$. Then for N large enough, by continuity of u^R, $E[u^R(a(F^{-1}(q)), \theta_{j:N})] > E[u^R(a(\theta_{j:N}), \theta_{j:N})]$ a contradiction with the definition of $a_{j:N}$. By continuity of u^R and u^S (5) implies

$$\lim_{N\to\infty} E[u^i(a_{\lceil qN\rceil:N}, \theta_{\lceil qN\rceil:N})] = u^i(a(F^{-1}(q)), F^{-1}(q)) \text{ for all } i \in \{S, R\}. \tag{6}$$

Under the complete ordering, the limit of the ex-ante expected average payoff for $i \in \{S, R\}$ is

$$\begin{aligned}
\lim_{N\to\infty} \frac{1}{N}\sum_{j=1}^{N} E[u^i(a_{j:N}, \theta_{j:N})] &= \lim_{N\to\infty} \int_0^1 E[u^i(a_{\lceil qN\rceil:N}, \theta_{\lceil qN\rceil:N})]dq \\
&= \int_0^1 \lim_{N\to\infty} E[u^i(a_{\lceil qN\rceil:N}, \theta_{\lceil qN\rceil:N})]dq \\
&= \int_0^1 u^i(a(F^{-1}(q)), F^{-1}(q))dq \\
&= E[u^i(a(\theta_k), \theta_k)]
\end{aligned}$$

where the first equality is definitional, the second follows from the boundedness of a continuous function on a compact domain, the third follows from (6), and the last is again definitional. ■

Note that since the receiver can always choose the same actions even when he has more information, he is necessarily better off the more information that is revealed. Thus, the receiver's ex-ante expected payoff is the highest under the full ordering (and, more generally, higher the finer is the ordering). The result above shows that asymptotically under the complete ordering, the receiver is as well off as he would be under full information.

In a variety of contexts, such as the Crawford-Sobel expert game, the sender is also better off from more information being revealed. But fully revealing information is not desirable in all games. We will consider a game with many applications where the receiver's incentive to take a binary action – such as hiring a student or not based on a professor's recommendation – induces the sender to prefer a partial ordering. But first we consider the multi-dimensional Crawford-Sobel expert game in more detail.

2.1 Application to Crawford-Sobel expert game

We now show how this model applies to a multi-dimensional version of the Crawford-Sobel expert game between an informed but biased expert (the sender S) and an uninformed decision-maker (the receiver R). In many situations, such as cheap talk about how tight a monetary policy will be pursued, the assumption of a single dimension is appropriate. But in other situations multiple dimensions of uncertainty may be important. For instance, as Battaglini (2002) argues, a Congressional committee reports on different aspects of a bill to the full Congress, and it also reports on different bills.

We consider a multi-dimensional model which satisfies the Crawford-Sobel conditions in each dimension. For $i = R, S$ and all θ_k, the utility functions satisfy $u_1(a_k, \theta_k) = 0$ for some a_k and $u^i_{11}(a_k, \theta_k) \leq 0$ for all a_k and θ_k, which assures that the expert and the decision-maker each prefer a unique action for each realization of θ_k, and $u^i_{12}(a_k, \theta_k) > 0$, which assures that the action is increasing in θ_k.

We will follow most of the literature in assuming that the payoff functions take the particular form of quadratic loss functions,

$$u^S(a_k, \theta_k, b) = -(a_k - (\theta_k + b))^2 \tag{7}$$

and

$$u^R(a_k, \theta_k) = -(a_k - \theta_k)^2. \tag{8}$$

The bias factor $b > 0$ in the expert's utility function captures the expert's preference for a larger action than an informed decision-maker would take. In particular, note that an informed decision-maker would take action $a_k(\theta_k) = \theta_k$ but the expert's payoff is maximized when the decision-maker takes action $a_k(\theta_k) = \theta_k + b$.

As a reference point, consider the babbling equilibrium where the decision-maker refuses to ascribe any meaning to the expert's message. The decision-maker then chooses a_k such that

$$a_k = \arg\max_{a'_k} - \int_0^1 (a'_k - \theta_k)^2 f(\theta_k) d\theta_k. \tag{9}$$

Solving,

$$a_k = E[\theta_k]. \tag{10}$$

The expert's expected payoff from babbling is therefore

$$E[u^S] = \int_0^1 - \left(E[\theta_k] - (\theta_k + b)\right)^2 f(\theta_k) d\theta_k = -Var[\theta_k] - b^2 \tag{11}$$

where $Var[\theta_k]$ is the variance of each θ_k.

Note that uncertainty over the realization of θ_k decreases the expert's payoff. Before considering how partition cheap talk can help reduce this problem, we first consider ordinal cheap talk. Since revealing more information is always in the expert's interest, we restrict attention to the complete ordering of the different θ_k. If R believes that variable θ_k is ranked as jth smallest then, since $c_j = \{j\}$ in the complete ordering, the decision-maker chooses a_k to be

$$a_{j:N} = \arg\max_{a'_k} \left(-\int_0^1 (a'_k - \theta_k)^2 dF_{j:N}(\theta_k) \right). \tag{12}$$

Solving,

$$a_{j:N} = E[\theta_{j:N}]. \tag{13}$$

Notice from (7) and (8) that the cross-partials of u^R and u^S with respect to a_k and θ_k are positive so both functions are supermodular (Topkis, 1978). From Theorem 1, this is sufficient for revealing the complete ordering to be an equilibrium. The quadratic loss function implies that the expert would like to minimize the distance between the expert's ideal point and the decision maker's action on each dimension. Because $E[\theta_{j:N}]$ is increasing in j by the properties of rank order statistics, from (13) the decision-maker's actions are strictly increasing in j. Therefore, if the expert follows the ordering, the decision-maker's actions are increasing along with the value of θ_k and the difference $a_k - (\theta_k + b)$ is not too large in any dimension. If the expert deviated by, for instance, inaccurately ranking one θ_k higher than another, the difference could be reduced in one dimension but the gain would not compensate for the larger difference in the other dimension.

Regarding expected payoffs, from (11) the expert's payoff is hurt both by the bias factor b and by the uncertainty as represented by the variance $Var[\theta_k]$. Ordinal cheap talk therefore raises the expert's payoff by reducing uncertainty. In the full information case, the decision-maker knows the exact value of θ_k and therefore chooses $a_k = \theta_k$, implying that the expert's expected utility rises from $E[u^S] = -Var[\theta_k] - b^2$ without any information to $E[u^S] = -b^2$ with full information. For a large number of issues, ordinal cheap talk approximates this full information solution in accordance with Theorem 2. Under ordinal cheap talk the per-issue payoff to the expert is $E[u^S] = \frac{1}{N}\sum_{j=1}^{N}(-Var[\theta_{j:N}] - b^2)$. As N goes to infinity, $\frac{1}{N}\sum_{j=1}^{N}(-Var[\theta_{j:N}])$ goes to zero so $E[u^S]$ goes to $-b^2$.

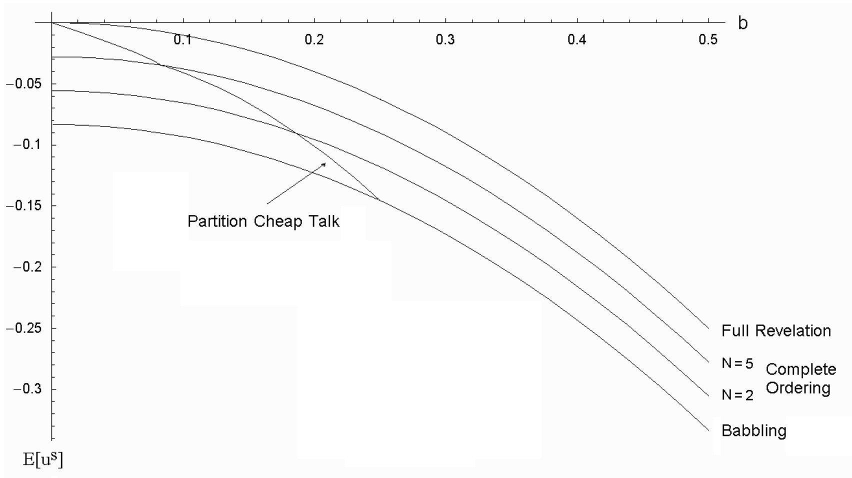

Figure 1: Expected sender per-issue payoffs in expert game

These results can be compared with partition cheap talk in each dimension. In partition cheap talk the expert states what interval θ_k lies in among different partitions. For instance if $\Theta = [0,1]$ it may be credible for the expert to state that $\theta_k \in [0, \frac{1}{3}]$ or $\theta_k \in (\frac{1}{3}, 1]$. In general as the bias b becomes smaller the set of possible partitions increases and as $b \to 0$ the partitions are so fine that the expert can fully reveal her information. Therefore, for any given number of issues, the expert's per-issue expected payoff is higher under partition cheap talk than under ordinal cheap talk if b is sufficiently close to 0. But, by Theorem 2, for any given b the expert's per-issue expected payoff is higher under ordinal cheap talk than under partition cheap talk for sufficiently large N.

To explicitly compare partition cheap talk with ordinal cheap talk, consider the case where F is the uniform distribution on $[0,1]$. In this case Crawford and Sobel show that the possible partitions can be directly solved for as a function of b. For instance, for two partitions, which is possible for $b \leq \frac{1}{4}$, the expert can indicate whether θ_k is in the range $[0, \frac{1}{2} - 2b]$ or $(\frac{1}{2} - 2b, 1]$, and for three partitions, which is possible for $b \leq \frac{1}{12}$, the expert can indicate whether θ_k is in the range $[0, \frac{1}{3} - 4b]$, $(\frac{1}{3} - 4b, \frac{2}{3} - 4b]$, or $(\frac{2}{3} - 4b, 1]$.

Figure 1 compares the maximum per-issue expert payoffs from partition cheap talk, ordinal cheap talk, and full revelation. First considering full revelation, if the bias factor is $b = 0$ then expert and decision-maker interests are fully aligned and the informed decision-maker's choice of $a_k = \theta_k$ also maximizes expert payoffs. In this case it is clearly an equilibrium for the expert to inform the decision-maker of the exact value of θ_k. As b increases for any given θ_k the decision-maker's choice of a_k diverges from the expert's preferred choice so full revelation is no longer credible. Nevertheless, for small b expert and decision-maker interests are still partly aligned in that for some realizations of θ_k the expert prefers a higher choice of a_k than an uninformed decision-maker would choose, and for some realizations the expert prefers a lower realization. Figure 1 shows that for very small b, partition cheap talk allows the seller to nearly replicate the gains from full revelation. However as b increases the expert can only make coarser and coarser statements about the range of θ_k. For $b > \frac{1}{4}$ the bias is so strong that the expert is never credible and no partition equilibria exist. In this case the decision-maker chooses $a_k = E[\theta_k]$ so expert payoffs are the same as they are under babbling.

Whereas partition cheap talk is only an equilibrium for $b \leq \frac{1}{4}$, ordinal cheap talk is always an equilibrium for any b. For instance, even if $b > 1$, meaning that for any realization of θ_k the expert wants the decision-maker to believe $\theta_k = 1$, the expert can still state the relative sizes of the different θ_k. The gains from a complete ordering are seen in Figure 1 for the two-issue and five-issue cases. Taking the payoff gap between babbling and full revelation as the basis, about a third of the gap is eliminated for two issues and most of it is eliminated for five issues. By Theorem 2, the gap goes to zero asymptotically as the number of issues increases.

An attractive feature of ordinal cheap talk is its simplicity. Regardless of the bias factor, the distribution of θ_k, or the number of issues, the expert simply ranks the sizes of θ_k. For partition cheap talk the problem is more complex since, even for the uniform distribution, both the number of partitions and their ranges depend on b.

2.2 Application to recommendation game

We now consider a recommendation game where interests diverge more strongly than in the Crawford-Sobel expert game. The game could apply to many situations but we will refer to the case of a professor who recommends students for jobs. Student quality θ_k is known only by the professor and is distributed uniformly over $[0, 1]$. The employer's payoff is the net quality of the student above a cutoff c if the student is hired and zero otherwise, i.e., $u^R = (\theta_k - c)a_k$ where $c \in [0, 1]$ and a_k is the probability the student is hired.[14,15] The professor's payoff is

[14] Because of the cutoff it is always a best response of the employer to choose $a_k \in \{0, 1\}$.

[15] Note that each student can be hired or not independently of the other students - there is no competition for limited positions. Also note that if student quality is multidimensional there may be room for ordinal cheap

$u^S = \theta_k a_k$ so the professor receives θ_k if the student is hired and 0 if the student is not hired.

Since the professor prefers that a student is hired regardless of the student's quality, the incentive to exaggerate implies that fully revealing the quality of the students is not credible.[16] Similarly, in any one dimension, partition cheap talk is not credible. However, this example satisfies the complementarity conditions of Theorem 1, so ordinal cheap talk is still possible with multiple students.

If there is only one student the employer hires her if $E[\theta_k] \geq c$. For more students, the decision can depend on cheap talk recommendations. First consider a complete ordering. If there are multiple students and the professor completely ranks them then a student is hired if

$$E[\theta_{j:N}] \geq c \tag{14}$$

where $E[\theta_{j:N}] = \frac{j}{N+1}$ given our assumption that θ_k is distributed uniformly on $[0, 1]$.

For instance, if $c = \frac{2}{3}$ then if there is only one student she is not hired since $E[\theta_k] = \frac{1}{2}$ but if there are two students then $E[\theta_{1:2}] = \frac{1}{3}$ and $E[\theta_{2:2}] = \frac{2}{3}$ so the best one is hired. If there are enough students it is possible for more of them to make the cutoff. For instance, with five students the best student has expected quality of $E[\theta_{5:5}] = \frac{5}{6}$ and the second best student has expected quality of $E[\theta_{4:5}] = \frac{4}{6}$, so both make the cutoff. As the number of students increases, the professor's payoffs from disclosing the ranking approach the full revelation case.

While in the expert game full revelation is the best the sender can do, in the recommendation game the sender can do better. To see this consider a partial ordering where students are divided into an upper and lower category. Since the employer cannot differentiate between the students, the employer will hire all those in the upper category if the expected quality exceeds c, even though the marginal student in the category might not be hired if the employer could identify her. The professor will then put as many students into the top category as possible subject to the condition that their average quality is high enough for employment. If m students are in the bottom category and $N - m$ in the top category then the expected quality of

talk regarding the different attributes of a single student. This model does not directly apply since there are not independent actions for each dimension of uncertainty.

[16] Also because interests are so divergent, the receiver does not benefit from delegating authority to the sender as in the expert game (Crawford and Sobel, 1982; Dessein 2002). Note that even when delegation is preferable in the expert game it may not be feasible, e.g., when the sender is a hypochondriac who knows his ailment(s) and the receiver is a doctor providing treatment.

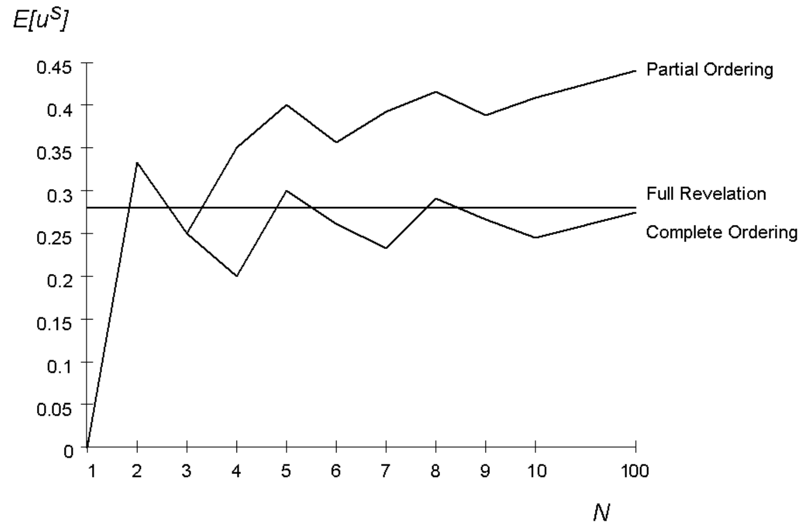

Figure 2: Expected sender per-issue payoffs in recommendation game

the top category is

$$E[\theta_{\{m+1,..,N\}:N}] = \frac{1}{N-m}\sum_{i=m+1}^{N} E[\theta_{i:N}]. \tag{15}$$

Again consider the case where $c = \frac{2}{3}$. If there are three students and two are placed in the top category $E[\theta_{\{2,3\}:3}] = (\frac{2}{4} + \frac{3}{4})/2 = \frac{5}{8}$ which is not quite sufficient for the employer to hire them. However as the number of students increases the ability to place more students in the top category increases. For instance, for four students with two in the top category, $E[\theta_{\{3,4\}:4}] = (\frac{3}{5} + \frac{4}{5})/2 = \frac{7}{10}$ so both are hired, and for five students with three in the top category, $E[\theta_{\{3,4,5\}:5}] = (\frac{3}{6} + \frac{4}{6} + \frac{5}{6})/3 = \frac{2}{3}$ so all three are hired. Since $E[\theta_k|\theta_k \geq \frac{1}{3}] = \frac{2}{3}$, it is not difficult to verify that in the limit, as the number of students increases, two-thirds of the students can be placed in the top category and so will obtain a job. In contrast, under full information, in the same limit just one-third of the students will obtain a job. Thus a partial disclosure of student quality is better for the professor than either revealing no information, revealing a complete order or, if it were possible, full revelation.[17] Figure 2 shows the professor's payoffs from the payoff-maximizing partial ordering, from the complete

[17]Ostrovsky and Schwarz (2003) consider the strategic use of transcripts by universities and find that transcripts might be designed to hide information even when there is competition between universities.

ordering, and from full revelation (which is not an equilibrium) as a function of the number of students. The asymptotic payoffs per issue are 0 from no information, $\frac{5}{18}$ from the complete ordering and from full revelation, and $\frac{4}{9}$ from the optimal partial ordering.

2.3 Application to analyst game

We now consider another example where the expert is extremely biased, i.e., prefers the same action for every state. Let

$$u^S = w(a_k) \tag{16}$$

for some increasing function w, and

$$u^R = -(\theta_k - a_k)^2. \tag{17}$$

One interpretation of this model is the following. The sender is an analyst with private information θ_k, the fundamental value of stock k. The variable a_k is the market price of stock k. The receiver's payoff function is chosen to represent in reduced form the efficient market assumption that the stock price will equal its expected fundamental value, given the analyst's message. However, the analyst is biased and prefers each stock price to be as high as possible, regardless of θ.

It is easy to see in this example that there is no possibility for cheap talk in any one dimension. However, since u^S and u^R are both supermodular, Theorem 1 implies every possible ordering is an equilibrium in spite of the extreme analyst bias. The analyst is indifferent between lying and telling the truth in any such equilibrium.

From Theorem 2, for the complete ordering, both the sender and receiver payoffs will converge to that under full disclosure as the number of stocks N grows. As discussed above, the receiver's ex-ante expected payoff is the highest for the complete ordering. What about the sender? The ex-ante expected price $E[a_k]$, for any stock k, in *any* ordinal cheap talk equilibrium, is equal to $E[\theta_k]$. Thus, from the sender's perspective, the ex-ante expected payoff from the different orderings differ only in terms of their effect on the distribution of the stock price a_k, with finer orderings corresponding to mean-preserving increases in the spread of the price distribution. For example, for the babbling equilibrium, a_k equals $E[\theta_k]$ with probability 1. On the other hand, for the complete ordering a_k equals one of $E[\theta_{j:N}]$ for $j = 1, ..., N$, each with probability $\frac{1}{N}$. More generally, for any ordering C, a_k equals one of $E[\theta_{c_j:N}]$ for $j = 1, ..., |C|$, each with probability $\frac{1}{|C|}$.

It follows that if w is convex, the analyst obtains a weakly higher payoff from finer orderings compared to coarser ones, implying that the complete ordering has the weakly highest payoffs, in ex-ante terms. On the other hand if w is concave, the analyst obtains a weakly higher payoff from coarser orderings compared to finer ones, implying that the babbling equilibrium has the weakly highest payoffs, in ex-ante terms. However, if w is neither concave nor convex then the analyst may obtain a strictly higher ex-ante expected payoff from a partial ordering compared to both a complete ordering and babbling.

3 Reputation

Reputation is perhaps the most important consideration in any real world communication game. If the sender is sufficiently patient in a game with a sufficiently high probability of continuing, and information is fully revealed between periods, reputation can support full revelation (Sobel 1985; Stocken, 2000). The main point of our paper is that ordinal cheap talk can be credible even without reputational considerations. Nevertheless, it is interesting to consider whether a reputation for honest ordinal cheap talk is easier to sustain than a reputation for fully revealing information through cardinal statements.[18]

There is good reason to suspect that for a large number of issues it is easier to sustain a reputation for honest ordinal cheap talk. As the number of issues increases, the gains from a complete ordering are asymptotically equivalent to those of full revelation. Hence the value of a reputation for honestly ranking the issues is asymptotically equivalent to that for honestly revealing the absolute values. However, comparing the short term gains of lying, the gains are less with ordinal cheap talk. The only way to lie is to invert the ranking of the issues which helps on some dimensions but also hurts on others. Hence there is less incentive to lie than if the sender could make exaggerated claims about the values for each issue.

To see this formally we consider a slightly modified version of the recommendation game. We suppose that the employer's payoff function is the same,

$$u^R(\theta_k, a_k) = (\theta_k - c)a_k \tag{18}$$

where $\theta_k, a_k \in [0, 1]$ and $1 > c > E[\theta_k]$, but that the professor's payoff function no longer satisfies the supermodularity condition for existence from Theorem 1. When the condition is

[18]Note that we will only consider the incentives to reveal ordinal and cardinal information. It may also be true that it is easier to verify ordinal information.

satisfied ordinal cheap talk is always an equilibrium even in the one-shot game so reputation is only interesting when it is not satisfied. We therefore assume that the professor's payoff function is submodular in that the marginal benefit to the professor of the employer's action is decreasing in student quality θ_k. In particular, we assume

$$u^S(\theta_k, a_k) = (1 - \theta_k)a_k. \tag{19}$$

For instance, the professor prefers that the worst student gets a job because the other students have other opportunities. Or, in a different context, a salesperson prefers to unload the worst good on a customer so as to clear out inventory.

We suppose further that this sender-receiver game is played every period without limit. The time between periods is identical and the professor has some discount factor $\delta \in [0, 1]$ across periods according to a standard exponential discounting function.[19] The professor's signals are independent across periods and, as previously, across issues. In between periods the professor's actual signals are revealed with certainty but it is not possible to contract on this information. The employer follows a trigger strategy of acting as if the professor is telling the truth as long as the professor has not previously lied and acting as if the professor is babbling if the professor has ever lied previously. Such a trigger strategy by the employer is most favorable for supporting an equilibrium in which the professor tells the truth.[20] Note that since $c > E[\theta_k]$ the expected payoff of the professor under babbling is equal to 0 as the employer will set $a_k = 0$ for all k.

First consider full revelation. For a realization of signals $(\theta_{1,...,}\theta_N)$, the professor can either report accurately the realization or deviate from it. The maximum gain from deviating occurs when $\theta_k = 0$ for all k and the professor reports that all students have quality above c. The gain from deviating then equals N. On the other hand, the per-period benefit from maintaining a reputation for truthfulness is

$$N(1 - F(c))(1 - E[\theta_k|\theta_k \geq c]), \tag{20}$$

i.e., the number of students times the probability that a student has quality above c (and is therefore hired) times the expected surplus for the professor given that the student is hired.

[19]This discount factor can also be interpreted as the probability of the game continuing.

[20]Note that the sender can lie about cardinal information while still being truthful about ordinal information. In such a situation the receiver might then believe that the sender will continue to tell the truth about ordinal information. Such a possibility increases the incentive to deviate from honest revelation of cardinal information.

Absolute statements are credible if and only if the discounted value of maintaining a reputation is higher than the one-shot gain from deviation:

$$\frac{\delta}{1-\delta} N\{(1-F(c))(1-E[\theta_k|\theta_k \geq c])\} \geq N. \tag{21}$$

Now consider a complete ordering. Let $x_c(N) < N$ be the number of students who are employed when the professor discloses the complete ordering:

$$x_c(N) = N - \min\{j|E[\theta_{j:N}] \geq c\} + 1. \tag{22}$$

For any realization of signals $(\theta_{1,...,}\theta_N)$, the professor can either report the complete ordering accurately or deviate from it. Due to the submodularity condition, the most profitable deviation is to reverse the ranking of the N students. The maximum gain from such a deviation is when the worst x_c students have quality 0 and the best x_c students have quality 1. This results in a gain of x_c when $x_c \leq \frac{N}{2}$ and of $N - x_c$ when $x_c > \frac{N}{2}$. On the other hand, the per-period gain from maintaining a reputation for disclosing the complete ordering is

$$x_c(N)(1 - E[\theta_{j:N}|j \geq N - x_c(N) + 1]). \tag{23}$$

Thus, providing a complete ordering is credible iff

$$\frac{\delta}{1-\delta} x_c(N)\left(1 - E[\theta_{j:N}|j \geq N - x_c(N) + 1]\right) \geq \min\{x_c(N), N - x_c(N)\}. \tag{24}$$

Finally compare the optimal partial ordering for each N. Such an ordering divides the students into two groups, with $x_p(N)$ students in the top group such that all students in the top group get hired, where

$$x_p(N) = \max\{x|\frac{1}{x}\sum_{j=N-x+1}^{N} E[\theta_{j:N}] \geq c\}. \tag{25}$$

Note that $x_p(N) \geq x_c(N)$ for each N. As for the complete ordering, the maximum gain from deviation is $\min\{x_p, N - x_p\}$. However, the one-period gain from maintaining a reputation for a partial order is

$$x_p(N)(1 - E[\theta_{j:N}|j \geq N - x_p(N) + 1]). \tag{26}$$

Thus, providing the optimal partial ordering is credible iff

$$\frac{\delta}{1-\delta} x_p(N)\left(1 - E[\theta_{j:N}|j \geq N - x_p(N) + 1]\right) \geq \min\{x_p(N), N - x_p(N)\}. \tag{27}$$

For each N, let δ_f^N, δ_c^N and δ_p^N be the lowest values of δ for which, respectively, (21), (24) and (27) hold. We have the following result.

Theorem 3 *For the recommendation game with submodular payoffs, (i) it is easier to sustain a reputation for the optimal partial ordering than for the complete ordering:* $\delta_p^N \leq \delta_c^N$ *for all* N *and (ii) for a sufficiently large number of issues* N*, it is easier to sustain a reputation for a complete ordering than for full revelation: there exists* $\overline{N}$ *such that for all* $N > \overline{N}$, $\delta_c^N < \delta_f^N$.

Proof. (i) Note that (24) and (27) can be rewritten as

$$\frac{\delta}{1-\delta}\left(1 - E[\theta_{j:N} | j \geq N - x_c(N) + 1]\right) \geq \frac{\min\{x_c(N), N - x_c(N)\}}{x_c(N)} \tag{28}$$

and

$$\frac{\delta}{1-\delta}\left(1 - E[\theta_{j:N} | j \geq N - x_p(N) + 1]\right) \geq \frac{\min\{x_p(N), N - x_p(N)\}}{x_p(N)} \tag{29}$$

respectively. Since $x_p(N) \geq x_c(N)$ for all N the right-hand side (left-hand side) of the second inequality above is lower (higher) than the corresponding side of the first inequality, so the result follows.

(ii) Compare (21) and (24). From Theorem 2, in the limit as $N \to \infty$ the sender payoffs from a complete ordering are the same from full revelation, so that the term inside the braces on the left-hand side of (24) converges to the corresponding term on the left-hand side of (21). Since $N > x_c(N)$ for all N the result follows. ■

Consider now the uniform distribution for θ_k so that $c > \frac{1}{2}$. If the professor deviates from the truth then the employer ignores any future statements by the professor and the professor's payoff is 0 thereafter. Therefore, for full revelation, the per-period, expected per-student value of a reputation for truthfulness is $\int_c^1 (1-\theta_k) d\theta_k$. The gains from deviating are greatest when $\theta = 0$ in which case the professor can report each $\theta = 1$ and receive a per-issue gain of 1. So full revelation is an equilibrium if

$$\frac{\delta}{1-\delta} \int_c^1 (1-\theta_k) d\theta_k \geq 1 \tag{30}$$

or

$$\delta \geq \frac{2}{3 - 2c + c^2}. \tag{31}$$

For example if $c = \frac{2}{3}$ then full revelation is an equilibrium for approximately $\delta \geq 0.947$.

Consider the complete ordering for the limiting case limiting case where the number of students goes to infinity. In the limit for the complete ordering $1-c$ proportion of the students are employed so the most tempting case to lie is when $1-c$ proportion of the students are

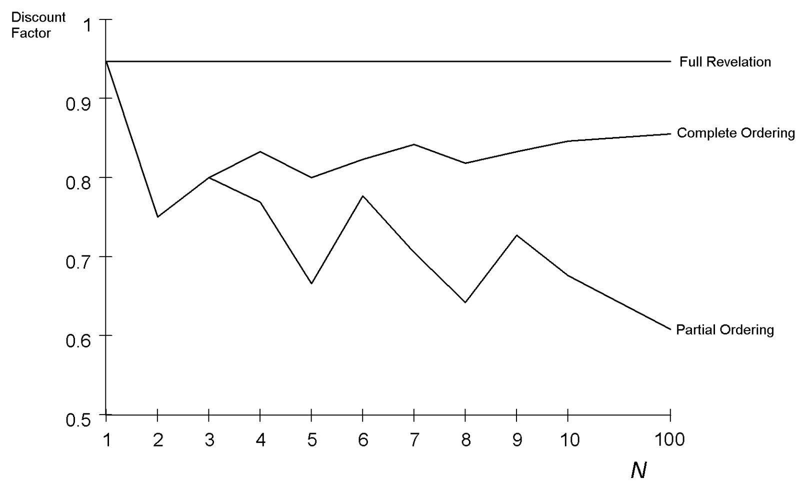

Figure 3: Minimum discount factors supporting cheap talk

$\theta_k = 0$ and another $1-c$ proportion are $\theta_k = 1$, so that the short term per-student gain from lying is $1-c$. The ordering is an equilibrium in the repeated game if

$$\frac{\delta}{1-\delta}\int_c^1 (1-\theta_k)d\theta_k > 1-c \tag{32}$$

or

$$\delta > \frac{2}{3-c}. \tag{33}$$

Comparing (31) and (33) the latter is smaller for $c < 1$ so, consistent with Theorem 3(ii), ordinal cheap talk with a complete ordering is supported by a lower discount rate than full revelation. In particular, a complete ordering is an equilibrium for $\delta > \frac{6}{7}$ for the example where $c = \frac{2}{3}$.

Finally consider the limiting case when there is a partial ordering with two categories. If the professor puts a fraction of the students into the top category that is arbitrarily close to $2-2c$, the expected quality of any student in the top category is greater than c, so that they are hired. Since $c > \frac{1}{2}$, the most tempting case to lie is when $\min\{2-2c, 2c-1\}$ proportion of the students are $\theta_k = 0$ and another $\min\{2-2c, 2c-1\}$ proportion are $\theta_k = 1$, so that the short term per-student gain from lying is $\min\{2-2c, 2c-1\}$. This partial ordering is therefore an equilibrium if

$$\frac{\delta}{1-\delta}\int_{2c-1}^{1}(1-\theta_k)d\theta_k > \min\{2-2c, 2c-1\} \tag{34}$$

or

$$\delta < \begin{cases} \frac{1}{2-c} & \text{if} \quad c \geq \frac{3}{4} \\ \frac{2c-1}{2c^2-2c+1} & \text{if} \quad c < \frac{3}{4} \end{cases} \tag{35}$$

So for the example with $c = \frac{2}{3}$, a partial ordering is an equilibrium in the limit for $\delta < \frac{3}{5}$.

Figure 3 shows the minimum discount factors supporting ordinal cheap talk and full revelation for $c = \frac{2}{3}$. Even for two students it is much easier to sustain ordinal cheap talk than full revelation and, consistent with Theorem 3(i), the partial ordering is easier to sustain than the complete ordering for any number of students. As implied by Theorem 3(i) and 3(ii), in the limit the partial ordering is the easiest to sustain of all three possibilities. It also has the highest ex-ante average payoffs for the sender.

4 Conclusion

Natural language is full of simple, comparative statements. Nevertheless, most models of cheap talk involve relatively complex statements regarding ranges of real numbers or functions of real numbers. We investigate when the simplest form of comparative statements – ordinal statements – can be credible and how much information they can reveal. We find that simple complementarity conditions are sufficient for ordinal statements to be credible and that these conditions are satisfied in standard cheap talk models even when interests are too opposed to support cheap talk in a single dimension. We also find that the amount of information revealed by ordinal statements can be considerable.

These results broaden our understanding of how individuals and institutions use cheap talk. For instance, cheap talk models have been applied extensively to model communication between the President and Congress over how much spending the President believes to be appropriate. This model indicates that even if it might be difficult for the President to credibly communicate the desired overall amount of spending, the President will often be able to credibly communicate spending priorities for different issues. Similarly, cheap talk models have been used to model communication between Congressional committees and the general Congress. Even if the committees represent special interests that conflict with the interests of the larger Congress, in many cases the committees will still be able to credibly compare and rank different proposals.

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