A COMPENSATION RULE FOR PROJECT-ALLOCATION GAMES

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ABSTRACT. By introducing the notions of project and proportion, we develop a new game theoretical framework for studying a class of economic environments in which agents may work together in a joint program with one or a number of projects. Using this framework, in addition to studying the effects of the structures of coalitions, projects and proportions on surplus sharing, we mainly focus on the problem of compensation in cooperation and propose a general compensation rule by which the compensator, compensatee and amount of compensation can be easily specified in a just and mutually acceptable way. Moreover, we characterize it as the unique compensation rule satisfying efficiency, symmetry, dummy player, additivity and equity. As an application of the framework, we construct a core-peripheral project-allocation model to investigate the relevant problems in the cases of public sector reforms and show that the competitiveness/bargaining power (specialities and proportions) comparison among players determines the compensation. We further show the richness of the framework and its broad applications in various fields of economics such as ownership and control, organization of firms, contract design, network economics, law economics, and so forth by making some extensions and refining the compensation rule.

Keywords: proportion; Shapley value; compensation

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EXTENDED ABSTRACT

This paper has two aims. Firstly, from a cooperative game theoretic point of view, it develops a general framework for studying a class of economic environments in which agents may work together in a joint program with one or a number of projects. Secondly, using this framework, we analyze the problem of compensation and design a general compensation rule by which the transferring direction and the amount of compensation can be easily determined in a "just" and mutually acceptable way.

By introducing the notions of *project* and *proportion*, we construct a simple TU (transferable utility) game theoretic model (the so-called *project-allocation games*) for extending the study of cooperation to a more real life setting: player-project-proportion framework. Among other things, like the standard cooperative game

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theoretic models, the central problem of such economic situations is still how to allocate the surplus that the agents jointly created among them justly. Thus, the effects of the relations of the players, the structures of a program and the shares of the projects that players initially have on surplus sharing can be studied in a unified framework. While at the same time, our model exposes some companion problems of the allocation of gains, such as *compensation*, service pricing, outsourcing and so on (actually, all of them can be treated as variations of compensation in some sense, which will be discussed later in the paper), and makes it feasible to study them. It is true that the problems of damage compensation and worker's compensation have been studied substantially from the legal point of view. However, no much work has been done on that topic in a more general sense, i.e. compensation for a service or losses. Especially from an economic perspective, the problem of compensation, as a key aspect in cooperation, happens quite often (e.g. especially when regulatory reforms take place in public sectors). But unfortunately, it has been overlooked for a long time in the academic world of economics and lacks systematic study. Thus, in addition to discussing the features of the corresponding games, classifying different situations and studying the effects of the structures of coalitions, projects and proportions in cooperation, based on this player-project-proportion framework, our paper mainly focuses on the problems of surplus sharing and compensation. In that sense, the model further develops surplus sharing games by taking compensation into account. Here, the natural and necessary concern in respect of compensation is how to design a compensation rule that is mutually acceptable among the players and helps to form a cooperation as an efficient outcome for greater collective interests (or to facilitate the reforms and realize the greater social welfare, if thinking about the above example). Based on the clarification of different payoffs in the project-allocation environments and taking the Shapley value as the benchmark, we propose a general compensation rule for telling compensatee and compensator from players and determining the transferring direction and the amount of compensation justly. It is the unique compensation rule that satisfies efficiency, symmetry, dummy player, additivity and equity over such games. Moreover, as an application of the framework, we construct a core-peripheral project-allocation model to investigate the relevant problems in the cases of public sector reforms and show that the competitiveness/bargaining power (specialities and proportions) comparison among players determines the compensation. The richness of the model is further shown by some extensions.

Cooperative game theory is of much usefulness in analyzing the environments and problems mentioned above that are characterized by transferable utility. The most well-known solution concepts satisfying several desirable normative properties are the Core and the Shapley value, which provide us a reasonable benchmark to make further discussions about surplus sharing problem. Especially, the Shapley value (Shapley (1953)), as the unique function that satisfies the properties of efficiency, symmetry, dummy player, and additivity over games, and the Myerson value (Myerson (1977)), help to find a unique point solution in those environments: the former is for a more general setting while the latter is for the games with graph-restricted communications. Another useful field is Linear Production (LP) games, defined by Owen (1975), which help to find a part of the Core elements by using shadow prices for the resources. But our paper puts much emphasis on players' specialities and proportions and adopts an alternative way to study the

player-project-proportion setting so that the model can be used very generally. In respect of economic theory, the properties of private goods and public goods and the allocation mechanisms (see Ruys (1974)) are stressed in this paper. Except for those seminal works, there is no other relevant literature which are directly related to our work as the problem of compensation has been ignored in theoretic research. However, there are still some indirect related work that are helpful for our research in several specific aspects. Moulin (1987) and Tijs et. al. (2002) studied the cost sharing games. Although those work deal with a dual problem of profit sharing and especially Tijs and Branzei (2002) considers cost sharing in compound joint project, their emphases are still on the forms of coalitions or of chosen project but do not touch the problems of proportions and compensation. As for the non-cooperative bargaining foundation of the Shapley value, we refer to Gul (1989) and Perez-Castrillo and Wettstein (2001). The latter uses a non-cooperative approach to prove the Shapley value as the subgame perfect equilibrium outcome when sharing surplus through a well-designed bidding mechanism. The applied side of our model is benefitted much from Grossman and Hart (1986) and Hart and Moore (1990).

The paper is structured as follows. The first section introduces the problem and reviews the literature briefly. In the second section, we present the main analytical framework by defining project-allocation games and addressing the problems of compensation. Based on the discussion about the solution concept for sharing surplus in project-allocation games and differentiating the payoffs involving in such environments, section 3 studies the rule of compensation and shows its uniqueness. In section 4, by constructing a simple core-peripheral project-allocation model, we focus on a special case of project-allocation games, which is particularly useful for investigating the problem of compensation in public sector reforms. Some extensions of the model on the topics like ownership and control, organization of firms, etc. and the refinement of the compensation rule follow in the fifth section. The final section outlines the further research.