Asymptotic Properties of the Shapley Value of Patent Licensing Games

Naoki Watanabe*and Yair Tauman†

March 2002

Abstract

We study the asymptotic properties of the Shapley value of patent licensing games with the Cournot competition, shedding light on its relations to the nucleolus, core and bargaining set. The Shapley value of the outside patentee of a non-drastic cost-reducing innovation converges to $\epsilon(a - c)$, which coincides with the patentee's profit through non-cooperative licensing by means of upfront fee (or royalty) in Kamien and Tauman (1986). The distance between the asymptotic Shapley value and asymptotic nucleolus becomes larger as the magnitude ϵ of the cost reduction increases. The limit core is empty, and the asymptotic Shapley value is excluded from the limit bargaining set. All the results are based on a new way of deriving a v-function from n-person games in strategic form.

Keywords : licensing, cost-reducing innovation, Cournot competition coalition formation, Shapley value, nucleolus, core, bargaining set

JEL Classification Numbers : D45, D43, C71

---

*Department of Economics, SUNY at Stony Brook, Stony Brook, NY 11794-4384, USA. email: `nwatanab@ic.sunysb.edu` The first author wishes to thank Prof Pradeep Dubey and seminar participants at Kyoto University for helpful comments.

†The Leon Recanati Graduate School of Business Administration, Tel Aviv University, Ramat Aviv 69978, Tel Aviv, Israel; Center for Game Theory in Economics, Department of Economics, SUNY at Stony Brook, Stony Brook, NY 11794-4384, USA. email: `tauman@post.tau.ac.il`

1 Introduction

Patent licensing problems in the Cournot oligopolistic market have been studied as non-cooperative games, such as licensing by means of upfront fee and/or per-unit royalty in Kamien and Tauman (1984, 1986) and licensing by means of auction in Kats and Shapiro (1985, 1986). On the other hand, cooperative approach is much less common in the literature. Kats and Tauman (1986) showed asymptotic results of the Shapley value and core of a production economy in which only a limited number of agents have access to the production. Our aim is to derive asymptotic results in the Cournot competition, and compare them with the results through non-cooperative licensing studied by Kamien and Tauman.

We also propose a new way of representing strategic-form games in coalitional form, i.e. deriving v-function that represents the worth of each coalition of players. Cooperative game has provided many elegant and significant insights. In economics, nevertheless, its predictive power is not so deserved as that of non-cooperative games. One of the reasons, authors think, lies in the fact that there has been provided few "strategic" foundations of v-function which enable us to study the broader range of economic behaviors.

In the classical literature such as von Neumann and Morgenstern (1944) and Aumann (1959), the maxmin or minmax criterion is suggested as a way of deriving the worth of each coalition of players from games in strategic form. Underlying assumptions in both ways are as follows: (1) players are negotiating over how to divide the total fruits of such a cooperation that all the players are expected to support together. (2) if some players broke off the negotiations forming a coalition in order for them to cooperate with one another in S but without the others not in S, the members of coalition S should then anticipate that the other players left to the negotiations would minimize their payoffs as a punishment. (3) the players can jointly commit themselves to such a punishment for the unexpected break-off.

The worth of each coalition derived from strategic-form games under those three assumptions can be interpreted as the total amount of payoffs that members in each coalition at least guarantee to themselves even against the severest punishment for their breaking off the negotiations with the others. However, it would not be in the best interests of the players to punish the members of the deviating coalition and minimize their payoffs. Rather, they would be unwilling to abandon maximizing their own payoffs for such a severest punishment. If so, the members of the deviating coalition would not anticipate such a severe punishment. Instead, they might anticipate that players left to the negotiations would defensively respond to the deviation and play, as a result, Nash equilibrium strategies against their actions. We mitigate those three assumptions in succession to the spirit of the classical literature. The idea is to introduce the coalition formation with spin-offs of several subcoalitions, which is illustrated with an example in the next section.

With the v-function derived in our new way, we study some properties of the Shapley value. (1) How different is the Shapley value from the result obtained through licensing by means of up-front fee (per-unit royalty)? (2) How much discontent are the players with if the Shapley value is implemented? In many cases in cooperative games, the grand coalition is formed by players, which means that the market is monopolized by the patentee and firms and they simply negotiate on how to share that monopolist's profit. The consumer surplus is minimized in such a case. It is hence meaningful to ask the welfare issue only among players. The distance from the nucleolus is the measure we study here, since the nucleolus is derived in an iterative process of minimizing the maximal discontent of some coalition of players. (3) and (4) How is the Shapley value related to the core and bargaining set? If the core is empty, there exists no payoff distribution which can prevent some coalition of players from breaking off the negotiations, since they can obtain higher payoffs within the deviating coalition of themselves. The bargaining set is weaker solution concept than the core. The inclusion of the Shapley value in the core and bargaining set is questioned.

Our results are as follows. (1) The Shapley value payoff to the outside patentee of a non-drastic cost-reducing innovation converges to $\epsilon(a-c)$, which coincides with the patentee's profit through licensing by means of upfront fee (or royalty) in Kamien and Tauman (1986). (2) The distance between the asymptotic Shapley value and asymptotic nucleolus becomes larger as the magnitude of the cost reduction ϵ increases. (3) The limit core is empty. (4) The asymptotic Shapley value is excluded from the limit bargaining set.

In the next section, our idea on the v-function and the intuition of our main result are illustrated with examples. The patent licensing game, the definition of the v-function and solution concepts are formally described in section 3. Theorems are shown in section 4. The derivation of the v-function and proofs are given mainly in Appendix. The comparison with the classical derivation of v-function is briefly reviewed in the discussion. Some extension is also suggested there.

2 Examples

Example 1: v-function

To illustrate the idea, consider the Cournot competition among 7 firms, each of which produces q_i units of an identical commodity with the same unit cost c. The market price p is determined by $p = a - Q$, where $a(> c)$ is a constant and $Q = q_1 + \cdots + q_7$ is the total sum of quantities of the commodity supplied to the market. Suppose that 3 coalitions are initially formed, each of which consists of 4 firms, 2 firms, and 1 firm. The aim of firms in the same coalition is to maximize their joint profit.

If firms in each coalition S cooperated simply to maximize the total sum of individual profits in S, $\sum_{i \in S} pq_i - cq_i$, the equilibrium joint profit for each coalition would then be $((a - c)/4)^2$. This amount is obtained also when firms in each coalition behave as a single entity in the market.

Each coalition can now decide whether to remain as a single entity or to spin off several subcoalitions. Once the coalition spins off subcoalitions, each subcoalition competes not only with all the "opponent" subcoalitions but also with the other "colleague" subcoalitions, while firms in the same subcoalition cooperate to maximize the total sum of individual profits in the subcoalition. The joint profit of each coalition is naturally defined as the total sum of profits of its subcoalitions.

The coalition S^* of 4 firms is now in question. Given that the other coalitions behaved as a single entity, it would be best for the coalition to spin off 3 subcoalitions, since it could receive the larger joint profit $3((a - c)/6)^2$. At that time, the coalition of 2 firms would receive $((a - c)/6)^2$. It may then spin off 2 subcoalitions to receive the larger joint profit of $2((a - c)/7)^2$. Anticipating such a counter spin-off by the rival coalition, it is best for coalition S^* to spin off 4 subcoalitions and receive the joint profit of $4((a - c)/8)^2$.

The coalition S^* can receive at least $4((a - c)/8)^2$ even in the worst anticipation that 2 rival coalitions spin off 3 subcoalitions in total so as to minimize the joint profit of coalition S^*. Hence, $4((a - c)/8)^2$ can be considered as a guaranteed worth of the coalition S^* available through spinning off subcoalitions.

Both the Shapley value and nucleolus of this example assigns to each firm the payoff of $(1/7)((a - c)/2)^2$, since they take the equal division of the total worth of the grand coalition in the games of symmetric players.

Example 2: Patent Licensing Game

Let us introduce into example 1 a patentee who has a patent of a new technology that reduces the unit cost of production from c to $c - \epsilon > 0$. Without any facilities and equipments of production, the patentee would like to earn the profit by licensing to firms the patent of the new technology.

The patentee and the licensee firms form a coalition S^0 in response to some possible coalitions of non-licensee firms against them. Suppose that coalition S^0 does not spin off subcoalitions, and so the only subcoalition in S^0 is S^0 itself. When S^0 produces $a - c$, any non-licensee firms will stop the production, since the price of the commodity is less than $p = a - (a - c) = c$ while the unit cost for the non-licensees is c. At that time the profit of coalition S^0 is $(p - c + \epsilon)(a - c) = \epsilon(a - c)$.

It is easy to calculate the Shapley value for 7 firms exactly, but the expression would be quite complicated for more firms. Many entrants will come to the market, as far as the incumbent firms can earn some positive amounts of profit. We would then like to estimate the value of the patent in such a large industry. We show in Theorem 1 that the Shapley value of the patentee converges to $\epsilon(a - c)$, as the number of firms tends to infinity.

The intuition is as follows: (1) the more the number of rival firms in the market, the smaller the guaranteed worth (joint profit) of coalition S^0 of the patentee and licensee firms[1]. (2) Even if the patentee licenses to many firms in order to mitigate the severe competition with the rival firms, the contribution of the new technology to the total profits of coalition S^0 does not become so large. (3) The profit of $\epsilon(a - c)$ is guaranteed to coalition S^0 even when the patentee licenses only to one firm.

If we took the limit in the sense that the number n of firms tends to infinity, a problem would arise. Since the Shapley value $\mathrm{Sh_i}$ of each firm i is the total worth $v(N')$ of the grand coalition N' minus Shapley value of the patentee, we have that $n\mathrm{Sh_i} = \mathrm{n(1/n)(v(N') - Sh_0)}$. However, $\lim_{n\to\infty} \mathrm{Sh_i} = \mathrm{\lim_{n\to\infty}(1/n)(v(N') - \epsilon(a - c)) = 0}$. To facilitate the approximation of the Shapley value in a large industry, each firm of the original game is considered as a type of firms, and each type is replicated k times. We take a limit of the value in the sense that k tends to infinity. The payoff to each type is the total sum of k identical firms of the type.

[1] This is the implication of the well-known Cournot's limit theorem