Social Movements in Democratic Regimes

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Abstract

We study how the threat of a social movement may influence the way in which a democratic government spends its budget. The democratic government has to choose a political program to maximize its expected payoff, which depends on (1) the political program itself and (2) the size of the social movement. Citizens have identical preferences and have to take a binary decision among joining the social movement or not. We show that there are equilibria in which all citizens join the social movement whenever the success of the social movement is beneficial; and there is a unique equilibrium strategy profile in which no citizen decides to join to the social movement. In the fist type of equilibria, the democratic government chooses the political program strategically, while in the second type of equilibrium the democratic government chooses its preferred political program.

1 Introduction

Inspired by the 2011 Chilean student movement, in this paper we examine to what extent can social movements influence the way a democratically elected government spends its budget.

Social movements in consolidated democracies tend to have moderate demands. Governments are elected to implement a political program by the majority of citizens, thus a social movement that seeks to make big changes in the political program might face great opposition from society. If instead of a big change, a social movement seeks to change only one policy in the entire political program, it can find that it could be possible to find support among citizens.

There are several reasons to explain why citizens would want to change a policy in the political program that a majority chose to implement. Revelation of information could change the citizens' preferences in a particular policy or reveal that the preferred policy of the government was different from that of the majority of voters. Also, a majority of voters could prefer the governments political program, but not a particular policy in the program.

Since a government has limited resources to implement the political program for which it was elected, even if it were to accept the citizens demands it would face a problem of resource reallocation. Thus, for the government, the problem of facing a social movement in a consolidated democracy encompasses the dilemma of modifying the way in which it spends its budget. For instance, during the 2011 Chilean student movement, students were demanding an increase in public spending on education and one of the ways they proposed to fund this increase was through a reduction in military spending.

To address our question, we develop a model in which a government threatened by the possibility of the rise of social movement, has to choose a political program. Depending on the government choice, citizens decide simultaneously and independently whether to join or not the social movement. The social movement succeeds if the mass of citizens who join it is large enough. Then, we are interested in how the government chooses its political program to discourage citizens to join the social movement. The outcome of the model provides insight about the process by which citizens make the decision to join the social movement or not, and the tools available to the government to prevent the rise of a social movement.

Different governments could react differently to the same social movement. To model this, we consider a parameter that represents the ability of the government to face a social movement. This parameter is known by the government but not by the citizens. A government can be "strong", meaning that no social movement can change its policy; weak, meaning that any social movement can change its policy or unstable, meaning that the success of the social movement depends on its size. A key feature of our model is that the government chooses strategically its political program to discourage citizens to join the social movement. Thus, this decision conveys information about the government's stability. Citizens' beliefs about the probability of achieving a change are shaped by this information.

1.1 Related literature

The literature on social movements is broad. Related to our work we may mention Shadmehr (2015), where there is a revolutionary entrepreneur that proposes an alternative to the status quo and citizens decide how much effort to devote to the social movement. He studies how the entrepreneur's optimal choice of alternative to the status quo is influenced by citizens preferences. In De Mesquita (2010) citizens have to decide whether to join or not a social movement that aims to overthrow the regime. Prior to this decision, a revolutionary institution may use violence to convince the citizens that antigovernment sentiment is high. Boix and Svolik (2013) study how the threat of a revolution forces a dictator to keep his promise to share power. Ellman and Wantchekon (2000) study how a threat of a coup can moderate the political agenda. Also, our results have some similarities to Acemoglu and Robinson (2001), where they study regimen change among two groups that are trying to implement different polices. They show that the implemented policy could be the one preferred by the ruling group or an intermediate policy that avoids a change of regime.

Our work is inserted into the global game literature. As citizens face a coordination game with incomplete information and strategic complements our formulation fits the settings of Morris and Shin (1998), Morris and Shin (2003) and Frankel et al. (2003), among others. This literature addresses the problem of multiple equilibria in simultaneous games with complete information. By incorporating incomplete information about a parameter, it can be possible to reduce the number of equilibria, even up to obtain uniqueness.

As our model has a stage prior to the citizens coordination game, the model is closely related to Angeletos et al. (2006) and Angeletos and Pavan (2013). In those papers the possibility of coordination in a global game is enlarged. The outcame of a stage added prior to the global game serves as a signal about the unknown parameter, conditionally on which players may change their behavior.

There are further applications of global games to political economy. In Shadmehr and Bernhardt (2011) there are two agents who can challenge the status quo by mounting a revolution, the innovation comes from the fact that the revolution payoff is uncertain, each agent receives a private signal about that payoff. Two more related papers are Angeletos et al. (2006) and Edmond (2013), in both citizens have to decide whether to join or not the social movement, the success of the social movement depends on the relation among the aggregate participation and the government strength, but citizens only have a private signal about the latter. Once again, in both papers there is a stage prior to the the decision of the citizens, in Angeletos et al. (2006) the government chooses the cost of joining to the social movement while in in Edmond (2013) the government can take a costly action to affect citizens' signal (information).

The rest of the paper is organized as follows. In section 2 we present the model and define our concept of equilibrium. In section 3 we develop an optimality analysis and present a characterization of the equilibrium. Section 4 presents the main results of the paper. In this section we highlight two types of equilibria: with or without the participation of a social movement. In section 5 we conclude. All the proofs are in Appendix A.

2 The model

Players and actions.– There is a government who has to choose a policy vector $(s_1, d_1, g_1) \in \mathbb{R}^3_+$, where s_1 is social expenditure, d_1 is expenditure in law enforcement and g_1 is expenditure in other items. The cost of a policy vector is determined by the strictly increasing continuous function $f : \mathbb{R}^3_+ \to \mathbb{R}$. A policy vector is affordable for the government if the cost is less than $m > 0$. There is a continuum of citizens of measure one, over the interval $[0, 1]$. Each citizen, indexed by $i \in [0, 1]$, has to choose an action $a_i \in \{0, 1\}$. The citizen joins the social movement if $a_i = 1$ and does not join the social movement if $a_i = 0$. The mass of citizens who join the social movement is $A \in [0, 1]$.

The social movement succeeds if $A > \theta$, where $\theta \in \mathbb{R}$ is the type of government (the "fundamentals"). If the social movement succeeds, the government has to replace the social expenditure s_1 for $\hat{s}$. To make this replacement the government has to pay a fixed cost of F. Then, the government has to choose a new policy vector $(\hat{s}, d_1, g_2) \in \mathbb{R}^3_+$, where defense spending is the same as in the first policy vector. The new policy vector is affordable for the government if the cost is less than $m - F$.

Payoffs.– The payoff of each citizen is given by

$$u_i(a_i, A, s_1, d_1, \theta) = \begin{cases} (|\hat{s} - s_1| - d_1)\, a_i & \text{if } A > \theta \\ -d_1 a_i & \text{if } A \leq \theta \end{cases}$$

Where each citizen has the same ideal social expenditure $\hat{s}$. The payoff for a citizen who does not join the social movement is zero, whereas a citizen who joins the social movement has to pay a cost of d_1 and the gain depends on the result of the social movement; if the social movement succeeds there is a gain produced by the change in the social spending. Thus, in case of a successful social movement, payoff of the citizen is greater the farther the social spending is from the citizens' ideal. There is a greater incentive to join the social movement when the governments policy is farther from the citizens' ideal.

The payoff of the government comes from social and other items expenditure, expenditure in law enforcement is only used to discourage citizens from joining the social movement. Therefore, the payoff of the government is determined by the twice continuously differentiable function $U : \mathbb{R}_+^2 \to \mathbb{R}$, which is increasing in both variables and concave.

Timing and information.– The game has three stages. In stage 1, the government privately learns θ and chooses a policy vector (s_1, d_1, g_1). In stage 2, after observing the policy vector and a private signal about θ, citizens decide simultaneously and independently whether to join the social movement or not. At the end of the stage 2, the result of the social movement is revealed. If the social movement fails, i.e. $A \leq \theta$, the game ends. In the other hand, if the social movement succeeds, the game has a third stage. In stage 3, the government has to replace g_1 by g_2, meeting the budget constraint which requires that the cost of the new policy vector $f(\hat{s}, d_1, g_2)$ is less than $m - F$.

Previous to stage 1, nature chooses θ from the improper uniform distribution on $\mathbb{R}$. Previous to stage 2, nature sends a private signal to each citizen. The signal citizen $i \in [0, 1]$ receives is $x_i = \theta + \sigma\varepsilon_i$, where $\sigma > 0$ and $\varepsilon_i \sim_{iid} N(0, 1)$.

Figure 1 shows a sketch of the timing of the game, where $\psi : [0, 1] \to \mathbb{R}$ is a function that assigns a signal to each citizen.

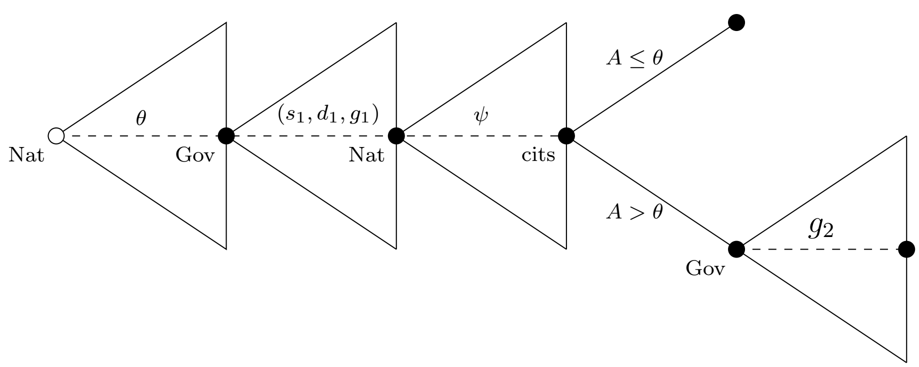

Figure 1: Sketch of the timing of the game.

Strategies and Equilibrium.– We consider symmetric Perfect Bayesian Equilibria. Symmetric in the sense that all citizens use the same strategy.

Rigorously, a strategy of the government consists of a choice of policy in the first stage as a function of it's type and a choice of expenditure in other items in the third stage.[1] The choice of the third stage should depend on the government's type, the mass of participants in the social movement and the amount spent on law enforcement. However, since the payoff of the government does not depend on it's type nor on the mass of participants (these only determine if there is or not a third stage), the choice on the third stage depends only on d_1, the expenditure on defense in the fist stage.

Thus, let $(s_1(\theta), d_1(\theta), g_1(\theta))$ denote the policy vector chosen in the first stage by a government of type θ and $g_2(d)$ denote the chosen expenditure on other items in the third stage by a government that spent d in law enforcement in the first stage.

We denote by $\mathbf{a}(x_i, s, d)$ the action chosen in the second stage by a citizen who observes (s, d) and receives a private signal x_i. The beliefs of a citizen $i \in [0, 1]$ are given by the posterior probability that θ is less than some threshold t, and is denoted by $\mu(t \mid x_i, s, d)$.

Given a realization of the government of type θ, a policy choice (s, d) and a symmetric strategy for the citizens $\mathbf{a}$, the expected mass of citizens who join the social movement is

$$A = \mathrm{A}(\theta, s, d) = \int_{-\infty}^{\infty} \mathbf{a}(x_i, s, d) \, \frac{1}{\sigma} \phi \left(\frac{x_i - \theta}{\sigma} \right) \mathrm{d}x_i, \tag{1}$$

where ϕ es the normal density function. Then, the expected payoff of the government when it chooses $(s_1, d_1, g_1, g_2) \in \mathbb{R}^4$ is given by

$$V(\theta, s_1, d_1, g_1, g_2) = \begin{cases} U(s_1, g_1) & \text{if } \mathrm{A}(\theta, s_1, d_1) \leq \theta \\ U(\hat{s}, g_2) & \text{if } \mathrm{A}(\theta, s_1, d_1) > \theta \end{cases} \tag{2}$$

Definition 1 (Democratic Equilibrium with Information Transmission)**.** *A* Democratic Equilibrium with Information Transmission (DEIT) *is a symmetric Perfect Bayesian Equilibrium of the game described above.*

That is, a DEIT *is a strategy profile* $(((s_1, d_1, g_1), g_2), \boldsymbol{a})$ *and a belief system* μ *such that*

[1] Expenditure in defense is fixed from the first stage and there is no choice of social expenditure in the third stage, since if the third stage is reached, it is because the social movement was successful and in that case the government is forced to implement the citizens' ideal expenditure.

(i) For all $d \in \mathbb{R}_+$, $g_2(d)$ is a solution of the following problem

$$\begin{aligned} \max_{g \in \mathbb{R}_+} \quad & U(\hat{s}, g) \\ \textit{subject to} \quad & f(\hat{s}, d, g) \leq m - F \end{aligned} \tag{3}$$

(ii) For all $\theta \in \mathbb{R}$, $(s_1(\theta), d_1(\theta), g_1(\theta))$ is a solution of the following problem

$$\begin{aligned} \max_{(s,d,g) \in \mathbb{R}^3} \quad & V(\theta, s, d, g, g_2(d)) \\ \textit{subject to} \quad & f(s, d, g) \leq m \end{aligned} \tag{4}$$

(iii) For all $(x_i, s, d) \in \mathbb{R} \times \mathbb{R}_+^2$

$$\boldsymbol{a}(x_i, s, d) \in \operatorname*{argmax}_{a \in \{0,1\}} \; a \left[|\hat{s} - s| \int_{-\infty}^{\infty} I_{[\mathrm{A}(\theta,s,d) > \theta]} \mathrm{d}\mu(\theta \mid x_i, s, d) - d \right]$$

(iv) $\mu(\theta \mid x_i, s, d)$ is obtained using Bayes' rule among all the preimages of (s, d) through (s_1, d_1).

Conditions (i) and (ii) ensure that the strategy of the government is sequentially rational. Condition (iii) ensures that the strategy of the citizens is sequentially rational according to their beliefs. Finally, condition (iv) ensures that in the equilibrium path the beliefs are correct.

3 Equilibrium characterization

Definition 1 is complex and difficult to work with, but through an analysis of the strategy of the government it is possible to provide a more tractable characterisation. Lemma 1 states that, in equilibrium, in the third stage the government spends all its budget.

Lemma 1. *In equilibrium, the constraint of the problem (3) is binding.*

The proof of the Lema 1 is straightforward: if the constraint is not binding, since f is a strictly increasing continuous function, it is possible to increase the payoff of the government by increasing g. Thus, in the third stage, the government chooses $g_2(d)$ that solves

$$f(\hat{s}, d, g_2) = m - F. \tag{5}$$

Since f a strictly increasing function, there is a unique solution. Therefore, g_2 is univocally determined by the government's expenditure in law enforcement in the first stage. Thus, in equilibrium, $g_2(d)$ is the continuous implicit function determined by the unique solution of (5).

If a government of type θ chooses in the first stage a certain policy vector (s, d, g), it knows that at the end of the game there are only two policy vectors that could be implemented: either the social movement fails and the policy vector remains (s, d, g), or the social movement succeeds and the government is forced to change its' policy to $(\hat{s}, d, g_2(d))$. Lemma 2 says that if the policy vector implemented at the end of the game is the one chosen at the first stage, then the government spent its entire budget in stage 1.

Lemma 2. *In equilibrium, if for some $\theta \in \mathbb{R}$,*

$$(s_1(\theta), d_1(\theta), g_1(\theta)) = (s, d, g) \qquad \textit{and} \qquad \mathrm{A}(\theta, s, d) \leq \theta,$$

then $f(s, d, g) = m$.

The proof of Lemma 2 is analogous to that of Lema 1: if $f(s, d, g) < m$, since neither g_2 nor A are affected by g, the government can increase g improving its payoff.

We cannot ensure that, in equilibrium, all the policy vectors chosen by the government have a cost of m. The problem arises when the social movement succeeds. Since in this case the government has to replace its choice of social spending, in the first stage there is no incetive to spend all the budget.

Lemma 3. *Fix $d \in \mathbb{R}_+$. A government of type $\theta \in \mathbb{R}$ is indifferent in the first stage among all the policy vectors (s', d, g') such that $\mathrm{A}(\theta, s', d) > \theta$.*

Lemma 3 says that if the social movement succeeds in the continuation history, any affordable policy vector that induces the success of the social movement and that has the same spending in law enforcement is equally preferred by the government. This comes from the fact that social spending and expenditure in other items will be replaced in the third stage. Then, the choice in the first stage of these two variables is irrelevant as long as the social movement succeeds in the continuation path.

Moreover, Lemma 3 says that if in equilibrium a government of type θ choses a policy vector (s, d, g) such that $\mathrm{A}(\theta, s, d) > \theta$, then the government could choose g such that the budget constraint is binding. Assumption 1 below simplifies the analysis by requiring that when the social movement succeeds in equilibrium, the government chooses a policy vector with a cost of m.

Assumption 1. *In equilibrium, if for some* $\theta \in \mathbb{R}$

$$(s_1(\theta), d_1(\theta), g_1(\theta)) = (s, d, g) \qquad \text{and} \qquad \mathrm{A}(\theta, s, d) > \theta,$$

then $f(s, d, g) = m$.

Assumption 1 together with the Lemma 2 ensure that the constraint in problem (4) is binding. Therefore, in the first stage in equilibrium, the problem of a government of type θ is reduced to choosing a policy vector $(s_1(\theta), d_1(\theta))$, and $g_1(\theta)$ is determined by the implicit function $h_1 : \mathbb{R}_+^2 \to \mathbb{R}$, where $h_1(s, d)$ is given by the unique solution in g of $f(s, d, g) = m$. This is, $g_1(\theta) = h_1(s_1(\theta), d_1(\theta))$.

Thus, for a given strategy of the citizens (that determines $\mathrm{A}(\theta, s, d)$), the first stage expected payoff for the government of type θ that chooses (s, d) is given by

$$v(\theta, s, d) = \begin{cases} U(s, h_1(s, d)) & \text{if } \mathrm{A}(\theta, s, d) \leq \theta \\ U(\hat{s}, g_2(d)) & \text{if } \mathrm{A}(\theta, s, d) > \theta \end{cases} \tag{6}$$

We can finally state our characterization of equilibrium

Lemma 4 (DEIT characterization)**.** *A* DEIT *is a strategy profile* $(((s_1, d_1, g_1), g_2), \boldsymbol{a})$ *and a belief system* μ *such that*

(i) For all $\theta \in \mathbb{R}$, $g_1(\theta) = h_1(s_1(\theta), d_1(\theta))$ *where* $h_1(s, d)$ *is given by the unique solution in* g *of* $f(s, d, g) = m$ *and*

$$(s_1(\theta), d_1(\theta)) \in \underset{(s,d) \in \mathbb{R}^3}{\operatorname{argmax}}\ v(\theta, s, d)$$

(ii) For all $d \in \mathbb{R}_+$, $g_2(d)$ *is the unique solution of* (5).

(iii) For all $(x_i, s, d) \in \mathbb{R} \times \mathbb{R}_+^2$

$$\boldsymbol{a}(x_i, s, d) \in \underset{a \in \{0,1\}}{\operatorname{argmax}}\ a \left[|\hat{s} - s| \int_{-\infty}^{\infty} I_{[\mathrm{A}(\theta, s, d) > \theta]} \mathrm{d}\mu(\theta \mid x_i, s, d) - d \right]$$

(iv) $\mu(\theta \mid x_i, s, d)$ *is obtained using Bayes' rule among all the preimages of* (s, d) *through* (s_1, d_1).

4 Democratic Equilibrium with Information Transmission

The outcome of the social movement depends on the relation between $\theta \in \mathbb{R}$ and $A \in [0, 1]$. Then, for some types of government, the mass of citizens who join the social movement is irrelevant to determine the outcome of the social movement. We say that a government is strong if its type is $\theta \geq 1$, weak if its type is $\theta < 0$ and unstable if its type is $\theta \in [0, 1)$. No matter the mass of citizens who join the social movement, a social movement in a strong government will fail and a social movement in a weak government will succeed.

Since a social movement in a strong government will always fail, a strong government chooses the policy vector that solves the following problem

$$\max_{(s,d)\in\mathbb{R}^2_+} \quad U(s, h_1(s, d)). \tag{7}$$

The expenditure in law enforcement is only used to discourage citizens from joining the social movement, therefore a strong government chooses $d = 0$.

Lemma 5. *A strong government chooses* $d = 0$*.*

We denote the social spending that solves problem (7) as s^*. This social expenditure is the one a government would choose if it was not threatened by a social movement. As it is common for social movements to seek an increase in social spending, we assume that $s^* < \hat{s}$.

Lemma 6. *In equilibrium, if for some* $\theta \in \mathbb{R}$

$$(s_1(\theta), d_1(\theta), g_1(\theta)) = (s, d, g) \qquad \textit{and} \qquad \mathrm{A}(\theta, s, d) > \theta,$$

then $d = 0$*.*

Lemma 6 says that, in equilibrium, if the social movement succeeds, then the optimal expenditure in law enforcement is zero. Since law enforcement spending is only useful to end the social movement, if the social movement succeeds, law enforcement spending is wasted.

Since a social movement in a weak government will always succeed, following Lemma 3, Assumption 1 and Lemma 6, a weak government will choose any policy vector that spends the entire budget and has zero expenditure in law enforcement.

Since the result of a social movement in an unstable government is not predetermined, the optimal choice of the government depends on the strategy of the citizens. We now turn the analysis to the decision of the citizens, in which the key feature is the belief system.

4.1 DEIT with social activity.

If for some reason all the citizens believe that the government is unstable, in equilibrium, there are only two possible outcomes: all the citizens join the social movement or no citizen joins the social movement.

Lemma 7. *Let (s, d, g) be the policy choice of the government in the first stage. In equilibrium, if all citizens believe that $\theta \in [0, 1)$, then*

$$|\hat{s} - s| - d \leq 0 \qquad \Longrightarrow \qquad A = 0$$

and

$$|\hat{s} - s| - d > 0 \qquad \Longrightarrow \qquad A = 0 \text{ or } A = 1$$

If in the first stage the government chooses a policy vector (s, d) such that $|\hat{s} - s| - d \leq 0$, the payoff of a citizen who joins the social movement is less or equal to zero. Therefore, in equilibrium, no citizen joins the social movement.[2]

On the other hand, if in the first stage the government chooses a policy vector (s, d) such that $|\hat{s} - s| - d > 0$, the payoff of a citizen who joins the social movement is greater than zero if the social movement succeeds and less or equal to zero if it fails. Therefore, in equilibrium, all citizens choose the action with the highest payoff: this is, all citizens join a social movement that succeeds and no citizens join a social movement that fails. In this case, citizens face a self-fulfilling prophecy. If citizens believe that the social movement will succeed, then they choose to join the social movement and the social movement succeeds. If citizens believe that the social movement will fail, then they choose not to join the social movement and the social movement fails.

As established in Lemma 7, when the citizen's beliefs are that the government is unstable, one of the alternatives of an unstable government to ensure that the social movement fails is to choose a policy vector such that $|\hat{s} - s| - d \leq 0$. We denote by $U^{\sharp}$ the optimal value of the following problem

$$\begin{aligned} \max_{(s,d)\in\mathbb{R}^2_+} \quad & U(s, h_1(s, d)) \\ \text{subject to} \quad & |\hat{s} - s| - d \leq 0. \end{aligned} \tag{8}$$

[2] We are assuming that if a citizen is indifferent between joining and not joining the social movement, he chooses not to join.

This is, among all policy vectors such that $|\hat{s}-s|-d \leq 0$, problem (8) selects one that is preferred by the government. Thus, $U^{\sharp}$ is the highest payoff that the government can attain by ensuring that citizens do not join the social movement.

We also make an assumption about the government's payoff. In Assumption 2 we impose that when s approaches $\hat{s}$ from below the government's payoff decreases.[3]

Assumption 2. *For each $s > s^*$, the payoff of the government $U(s, h_1(s,d))$ is decreasing in s.*

As s increases, there are two effects in the government's payoff: first, payoff increases because $U_s > 0$; and second, payoff decreases because $\frac{\partial h_1}{\partial s} < 0$ and $U_g > 0$. Therefore Assumption 2 implies that for all $s > s^*$

$$\frac{\mathrm{d}U}{\mathrm{d}s}(s, h_1(s,d)) = \underbrace{\frac{\partial U}{\partial s}(s, h_1(s,d))}_{>0} + \overbrace{\frac{\partial h_1}{\partial s}(s,d)}^{<0} \underbrace{\frac{\partial U}{\partial g}(s, h_1(s,d))}_{>0} < 0.$$

The following proposition establishes that there are multiple DEIT in which citizens may choose to join the social movement. We refer to this type of equilibria as DEIT *with social activity.*

Proposition 1. *For each policy vector $\left(s^{\dagger}, d^{\dagger}\right) >> (s^*, 0)$ such that*

$$U\left(s^{\dagger}, h_1\left(s^{\dagger}, d^{\dagger}\right)\right) > U^{\sharp}$$

There is a DEIT *where*

$$(s_1(\theta), d_1(\theta)) = \begin{cases} \left(s^{\dagger}, d^{\dagger}\right) & \text{if } \theta \in [0,1) \\ (s^*, 0) & \text{if not} \end{cases}$$

$$\boldsymbol{a}(x_i, s, d) = \begin{cases} 1 & \text{if } |\hat{s}-s| > d \text{ and } \left(s < s^{\dagger} \text{ or } d < d^{\dagger}\right) \\ 0 & \text{if not} \end{cases}$$

Let's notice that in a DEIT with social activity, the government's preferred policy vector $(s^*, 0)$ is chosen by both the strong and weak governments. It is clear why the strong government chooses this policy vector. On the other hand, we may also restrict the choice

[3]We use Assumption 2 only in subsection 4.1.

of the weak government to $(s^*, 0)$ because, since the social movement will succeed, then the choice of s is irrelevant and from Lemma 6 we know that $d = 0$. More importantly, the policy vector $(s^*, 0)$ is useful to construct the beliefs of the citizens according to Bayes' rule.

The unstable government has to choose its policy vector according to the strategy of the citizens. We begin the analysis showing that the unstable government prefers $\left(s^\dagger, d^\dagger\right)$ over $(s^*, 0)$. Recall that given a policy choice of (s, d), if the social movement succeeds, the government receives a payoff equal to $U(\hat{s}, g_2(d))$ and if it fails then the government obtains a payoff of $U(s, h_1(s, d))$. Figure 2 shows the payoffs of choosing the policy vectors $(s^*, 0)$ and $\left(s^\dagger, d^\dagger\right)$, as a function of the government type. According to the strategy of the citizens, if the government chooses the policy vector $(s^*, 0)$, all citizens will join the social movement and the social movement succeeds in any government of type $\theta < 1$. If instead the government chooses the policy vector $\left(s^\dagger, d^\dagger\right)$, no citizens join the social movement and thus the social movement will succeed only if the government type θ is less than 0. Therefore, the social movement succeeds if the unstable government chooses the policy vector $(s^*, 0)$, providing the government with payoff $U(\hat{s}, g_2(0))$, and fails if the unstable government chooses the policy vector $\left(s^\dagger, d^\dagger\right)$, providing the government with payoff $U\left(s^\dagger, h_1\left(s^\dagger, d^\dagger\right)\right)$. Finally, we have

$$U(\hat{s}, g_2(0)) < U(\hat{s}, h_1(\hat{s}, 0)) \leq U^\sharp < U\left(s^\dagger, h_1\left(s^\dagger, d^\dagger\right)\right). \tag{9}$$

The first inequality in (9) is true because $g_2(0)$ and $h_1(\hat{s}, 0)$ are (respectively) solutions of

$$f(\hat{s}, 0, g) = m - F \qquad \text{and} \qquad f(\hat{s}, 0, g) = m$$

and f and U are increasing in g. As $(\hat{s}, 0)$ is feasible in problem (8), we get the second inequality.

Therefore, the unstable government prefers $\left(s^\dagger, d^\dagger\right)$ over $(s^*, 0)$.

To see that the unstable government is worse off with any policy vector other than $\left(s^\dagger, d^\dagger\right)$, note that according to the citizens' strategy, if an unstable government chooses any policy vector $(s, d) \ngeq \left(s^\dagger, d^\dagger\right)$, then the social movement succeeds and the government's payoff is $U(\hat{s}, g_2(d))$. Recall that g_2 is decreasing in d and thus $U(\hat{s}, g_2(d)) \leq U(\hat{s}, g_2(0))$. Moreover, from (9), $U(\hat{s}, g_2(0)) < U\left(s^\dagger, d^\dagger\right)$ and thus the policy vector $\left(s^\dagger, d^\dagger\right)$ is preferred to (s, d) by the unstable government. Now, if an unstable government chooses a policy vector $(s, d) \geq \left(s^\dagger, d^\dagger\right)$, then the social movement will fail, and the government's payoff is $U(s, h_1(s, d))$. From Assumption 2, since $s^\dagger > s^*$, $U(s, h_1(s, d))$ is decreasing in s for $s > s^\dagger$

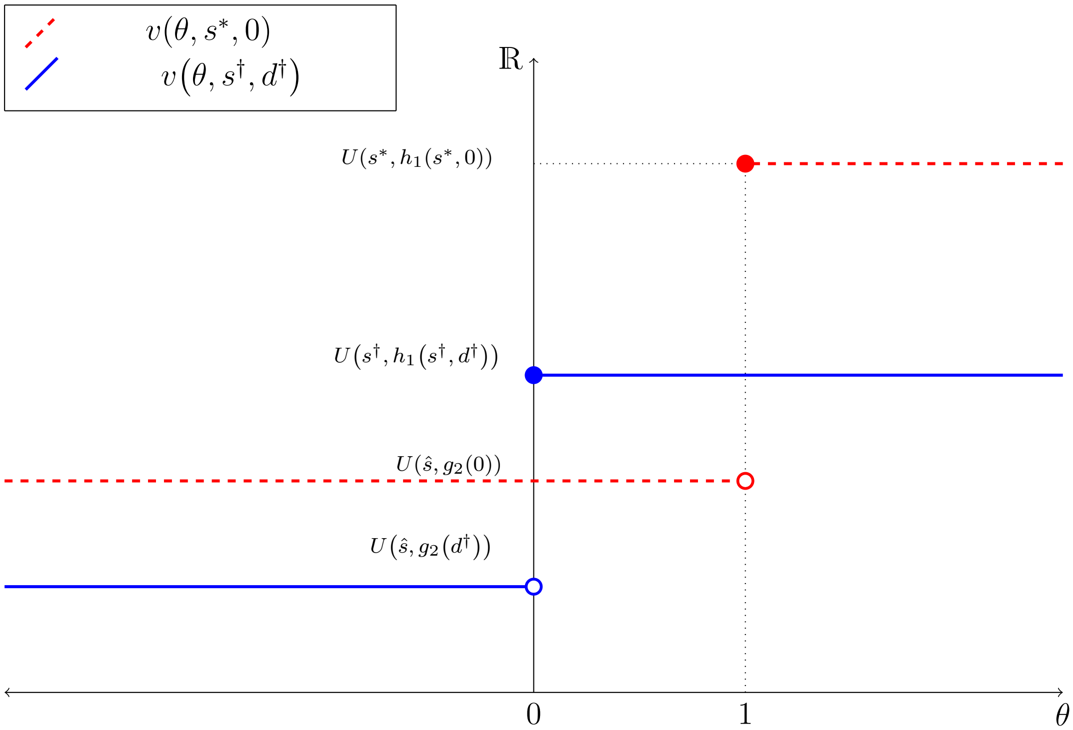

Figure 2: Government payoff for the policy vectors $(s^*, 0)$ and $\left(s^\dagger, d^\dagger\right)$.

and so the government has incentive to lower social expenditure. Thus, it cannot be that $(s, d) \geq \left(s^\dagger, d^\dagger\right)$ and $s > s^\dagger$. Finally, we know that $h_1\left(s^\dagger, d\right)$ is decreasing in d, so, since U is increasing in g, it cannot be the case that $d > d^\dagger$. Therefore, given the strategy of the citizens, any policy vector (s, d) is less preferred by the unstable government, than the policy vector $\left(s^\dagger, d^\dagger\right)$.

To construct the citizens' strategy, we focus on citizens' beliefs. From Definition 1, beliefs in the equilibrium path follow the Bayes' rule. If a citizen observes the policy vector $(s^*, 0)$, he knows that the government is strong or weak, thus he knows that the social movement would succeed only if $\theta < 0$.[4] As the expenditure in law enforcement is zero, the expected payoff of a citizen who joins the social movement is $|\hat{s} - s^*|\mu(0 \mid x_i, s^*, 0)$ which is strictly greater than 0. Thus, it is optimal to join the social movement. If citizens observe the policy vector $\left(s^\dagger, d^\dagger\right)$, they know that no citizen will join the social movement and the government is unstable. As no citizen joins the social movement, the latter succeeds only if $\theta < 0$, but

[4]To be more accurate, he knows that the social movement would succeed only if $\theta < 1$, but as the government is not unstable $\theta \notin [0, 1)$.

since the citizen knows that the government is unstable, we must have $\mu\big(0 \mid x_i, s^\dagger, d^\dagger\big) = 0$. Therefore, the certain payoff of joining the social movement is $-d^\dagger < 0$ and thus it is optimal not to join the social movement. The only restriction on beliefs out of the equilibrium path is that the strategy of the citizens be optimal according to them. Here, we restrict attention to beliefs that meet the intuitive criteria of Cho and Kreps (1987).[5] Let's notice that for every policy vector other than $(s^*, 0)$, citizens face a coordination problem. Since the optimal choice for the strong and weak government is an expenditure of zero in law enforcement, according to the intuitive criteria, if citizens observe a policy vector with $d > 0$ they know that the government is unstable, so citizens use the policy vector to coordinate among the possible results established in Lemma 7. Since the strong government always chooses the policy vector $(s^*, 0)$, according to the intuitive criteria, when citizens observe any other policy vector $(s, 0)$ we have that $\mu(1 \mid x_i, s, 0) = 1$, i.e. the government is weak or unstable, so citizens are coordinating on joining the social movement if $s \neq \hat{s}$.

4.2 DEIT without social activity.

If for some reason all citizens believe that the government is not weak and the government chooses an expenditure in law enforcement of zero, then in equilibrium, there are only two possible outcomes: all citizens join the social movement or no citizen joins the social movement.

Lemma 8. *Let $(s, 0, g)$ be the policy vector chosen by the government in the first stage. If citizens believe that $\theta \geq 0$, then*

$$s = \hat{s} \qquad \Longrightarrow \qquad A = 0$$

and

$$s \neq \hat{s} \qquad \Longrightarrow \qquad A = 0 \text{ or } A = 1$$

If in the first stage the government chooses a policy vector $(s, 0, g)$ such that $s = \hat{s}$, the payoff of a citizen who joins the social movement is zero. Therefore, in equilibrium, no citizen joins the social movement.

If in the first stage the government chooses a policy vector $(s, 0, g)$ such that $s \neq \hat{s}$, the payoff of a citizen who joins the social movement is greater than zero if the social movement

[5] For details, see the proof of Proposition 1.

succeeds and zero if the social movement fails. Therefore, in equilibrium, all citizens join a social movement that succeeds and no citizen join a social movement that fails. So, once again citizens face a self-fulfilling prophecy, as we claim in Lemma 7.

Proposition 2 below establishes that there is a unique strategy profile that supports a DEIT in which all citizens choose not to join the social movement regardless of the policy vector chosen by the government in the first stage and regardless of the signal. We refer to this type of equilibria as DEIT without social activity.

Proposition 2. *If in equilibrium citizens never join the social movement, that is* $\forall\ (x_i, s, d)$*,*

$$\boldsymbol{a}(x_i, s, d) = 0,$$

then the government strategy is

$$(s_1(\theta)\,, d_1(\theta)) = \begin{cases} (s^*, 0) & \text{if } \theta \geq 0 \\ (\hat{s}, 0) & \text{if not} \end{cases}. \tag{10}$$

Indeed, by the analysis made so far, it is clear that the government's strategy is optimal: a weak government chooses a policy vector with zero expenditure in law enforcement and any other type of government chooses its preferred policy vector. Citizens' strategy has to be optimal according to their beliefs and these beliefs have to follow Bayes' rule in the equilibrium path. Let's notice that in this equilibrium the choice of the policy vector $(\hat{s}, 0)$ conveys the information that $\theta < 0$, but citizens do not have an incentive to join the social movement because the certain payoff of joining is zero. On the other hand, the choice of the policy vector $(s^*, 0)$ conveys the information that $\theta \geq 0$ and citizens know, according to their strategy, that no citizen will join the social movement. As no citizen joins the social movement , the latter succeeds only if $\theta < 0$, but since the government is not weak, we have $\mu(0 \mid x_i, s^*, 0) = 0$. Therefore, the certain payoff of joining the social movement is zero. Once again, beliefs off the equilibrium path are chosen in order to make citizens' strategy optimal and we consider only the beliefs that meet the intuitive criteria of Cho and Kreps (1987).

The only element that can change in any DEIT without social activity is the beliefs off the equilibrium path. The outcome in any DEIT without social activity is the same: if the government is weak, then it chooses the policy preferred by the citizens; and if it is not weak it chooses its' preferred policy.

Let us notice that in the DEIT without social activity, the government does not allocate resources to law enforcement and implements its preferred social expenditure when it is not weak. Thus, the government is reaching its first best. So, a pertinent question is whether the government can choose the DEIT without social activity as the equilibrium to be played. As in the second stage citizens only observe a point in the government's strategy, if for example citizens observe the policy vector $(s^*, 0)$, they can not distinguish if the government is using the strategy of the DEIT without social activity or the strategy of the DEIT with social activity. Therefore, in the first stage the government can not determine that DEIT without social activity will be played.

Note that beside the DEIT with and without social activity, there are other DEIT. For example, in the DEIT without social activity, if citizens change their action to "join" to the social movement if they see the policy vector $(s, 0)$ with $s \neq \hat{s}$ and $s \neq s^*$, there is a belief system that sustains this strategy profile as a DEIT. Thus, even considering the intuitive criteria of Cho and Kreps (1987) multiplicity of equilibria may rise due to the choice of beliefs out of the equilibrium path.

5 Conclusion and Discussion

In this paper we study two characteristics of social movements: the interaction among citizens and the interaction among the government and the social movement. Our model endogenizes the government's policy choice prior to a social movement. We found that the government's policy choice could be influenced by the threat of a social movement, depending on the coordination of citizens. Citizens could coordinate on different thresholds when facing the same government's policy choice. This behavior is consistent with different interpretations of the same government's policy choice.

A key feature of our model is that the government conveys information with its policy choice and citizens use this information to update their beliefs about the likelihood with which the social movement succeeds. This information transmission is what differentiates our model from the classical global game framework. Since citizens can coordinate in a variety of strategies depending on their beliefs, we found multiple equilibria. In those equilibria, we found that the possible outcomes of the game range from the policy preferred by the government to the policy preferred by the citizens, passing through each intermediate policy.

A Proofs

A.1 Proof of Proposition 1

For the sake of completeness we re-state Proposition 1:

Proposition 1. *For each policy vector* $\left(s^{\dagger}, d^{\dagger}\right) >> (s^{*}, 0)$ *such that*

$$U\left(s^{\dagger}, h_1\left(s^{\dagger}, d^{\dagger}\right)\right) > U^{\sharp}$$

There is a DEIT *where*

$$(s_1(\theta), d_1(\theta)) = \begin{cases} \left(s^{\dagger}, d^{\dagger}\right) & \textit{if } \theta \in [0, 1) \\ (s^{*}, 0) & \textit{if not} \end{cases}$$

$$\boldsymbol{a}(x_i, s, d) = \begin{cases} 1 & \textit{if } |\hat{s} - s| > d \textit{ and } \left(s < s^{\dagger} \textit{ or } d < d^{\dagger}\right) \\ 0 & \textit{if not} \end{cases}$$

Proof. We will first prove that if the citizens' strategy is of the form

$$a(x_i, s, d) = \begin{cases} 1 & \text{if } |\hat{s} - s| > d \text{ and } \left((x_i, s) << \left(\bar{k}, s^{\dagger}\right) \text{ or } (x_i, d) << (\bar{k}, d^{\dagger})\right) \\ 0 & \text{if not} \end{cases} \tag{11}$$

then an optimal strategy of the government is $e : \mathbb{R} \to \mathbb{R}^2_+$, given by

$$e(\theta) = \begin{cases} \left(s^{\dagger}, d^{\dagger}\right) & \text{if } \theta \in \left[0, \breve{\theta}\right) \\ (s^{*}, 0) & \text{if not} \end{cases} \tag{12}$$

for some $\breve{\theta} > 0$.

We begin by proving three claims:

Claim 1. *If the citizens' use a strategy as in (11), the policy vector* $(s^{*}, 0)$ *is weakly preferred to any policy vector,* (s, d)*, such that* $|\hat{s} - s| > d$ *and* $(s, d) \ngeq \left(s^{\dagger}, d^{\dagger}\right)$.

Proof. If a government of type θ chooses any policy vector (s, d) such that $|\hat{s} - s| > d$ and $(s, d) \ngeq \left(s^{\dagger}, d^{\dagger}\right)$, then, given the citizens' strategy, each citizen joins the social movement if his signal is less than $\bar{k}$. The mass of citizens who join the social movement is $\mathrm{A}(\theta, s, d) = \Phi\left(\frac{\bar{k}-\theta}{\sigma}\right)$. This implies that the mass of citizens who join the social movement is the same for any policy vector such that $|\hat{s} - s| > d$ and $(s, d) \ngeq \left(s^{\dagger}, d^{\dagger}\right)$. Moreover, the policy vector

$(s^*, 0)$ belongs to this class of policy vectors. We conclude that there is no policy vector, among this class of policy vectors, with a payoff greater than that provided by $(s^*, 0)$.

$\square$

Claim 2. *If the citizens' use a strategy as in* (11), *the policy vector* $\left(s^\dagger, d^\dagger\right)$ *is weakly preferred to any policy vector,* (s, d)*, such that* $|\hat{s} - s| > d$ *and* $(s, d) \geq \left(s^\dagger, d^\dagger\right)$.

Proof. If the government of type θ chooses any policy vector (s, d) such that $|\hat{s} - s| > d$ and $(s, d) \geq \left(s^\dagger, d^\dagger\right)$, the citizens' strategy dictates that citizens do not join the social movement, regardless of the signal. Thus, the mass of citizens who join the social movement is $\mathrm{A}(\theta, s, d) = 0$. Therefore, for any of those policy vectors the social movement fails if the government is not weak and succeeds if the government is weak. As $U(s, h_1(s, d))$ is decreasing in d and, following Assumption 2, $U(s, h_1(s, d))$ is decreasing in s for all $s \geq s^\dagger$, for a government of type $\theta \geq 0$ the optimal choice, among this class of policy vectors, is $\left(s^\dagger, d^\dagger\right)$. On the other hand, as $U(\hat{s}, g_2(d))$ is decreasing in d, for a government of type $\theta < 0$ there is no policy vector that provides a payoff greater than $\left(s^\dagger, d^\dagger\right)$. $\square$

Claim 3. *For each* $\theta \in \mathbb{R}$ *the policy vector* $\left(s^\sharp, d^\sharp\right)$*, that induces the payoff* $U^\sharp$*, is less preferred than* $(s^*, 0)$ *or than* $\left(s^\dagger, d^\dagger\right)$.

Proof. If the government type is $\theta \geq 0$, with both policy vectors $\left(s^\sharp, d^\sharp\right)$ and $\left(s^\dagger, d^\dagger\right)$ the social movement fails and consequently for both vectros payoff is given by $U(s, h_1(s, d))$. Since, $U\left(s^\dagger, h_1\left(s^\dagger, d^\dagger\right)\right) > U\left(s^\sharp, h_1\left(s^\sharp, d^\sharp\right)\right) = U^\sharp$, $\left(s^\dagger, d^\dagger\right)$ is preferred to $\left(s^\sharp, d^\sharp\right)$. If the government type is $\theta < 0$, the social movement will succed and thus payoff of the government is given by $U(\hat{s}, g_2(d))$. Since g_2 is decreasing in d and U is increasing in g, the function $d \to U(\hat{s}, g_2(d))$ is decreasing in d. Thus, $U(\hat{s}, g_2(0)) \geq U\left(\hat{s}, g_2\left(d^\sharp\right)\right)$, with equality only in the case that $d^\sharp = 0$.

$\square$

Therefore, if the government's strategy assigns to each type of government the policy vector that maximizes the government's payoff among the policy vectors $(s^*, 0)$ and $\left(s^\dagger, d^\dagger\right)$, there is no other strategy that delivers a higher payoff to the government. Below we provide the optimal policy vector as a function of θ,

- If $\theta < 0$, the social movement succeeds. As $U(\hat{s}, g_2(d))$ is decreasing in d and $d^\dagger > 0$, the optimal choice for the government is $(s^*, 0)$.

- We previously established that with the policy vector $(s^*, 0)$, the mass of citizens who join to the social movement is $\Phi\left(\frac{\bar{k}-\theta}{\sigma}\right)$. As $\Phi\left(\frac{\bar{k}-\theta}{\sigma}\right)$ is decreasing in θ, the social movement succeeds if $\theta < \breve{\theta}$, where $\breve{\theta}$ solves $\breve{\theta} = \Phi\left(\frac{\bar{k}-\breve{\theta}}{\sigma}\right)$.[6] This is, if $\theta < \breve{\theta}$ and the government choses $(s^*, 0)$, it's payoff is $U(\hat{s}, g_2(0))$. Moreover, according to the citizens' strategy if the policy vector is $\left(s^\dagger, d^\dagger\right)$ no citizens join the social movement and thus it fails and the government's payoff is $U\left(s^\dagger, h_1\left(s^\dagger, d^\dagger\right)\right)$. Therefore, if $\theta \in [0, \breve{\theta})$, the optimal choice for the government is $\left(s^\dagger, d^\dagger\right)$ because (see (9))

$$U(\hat{s}, g_2(0)) < U(\hat{s}, h_1(\hat{s}, 0)) \leq U^\sharp < U\left(s^\dagger, h_1\left(s^\dagger, d^\dagger\right)\right).$$

- Finally, if $\theta > \breve{\theta}$, the optimal choice for the government is $(s^*, 0)$ because with this policy vector the social movement fails.

We conclude that the government strategy given by (12) is optimal.

Now we turn the attention to the citizens' strategy.

If the government chooses the policy vector $(s, d) \geq \left(s^\dagger, d^\dagger\right)$ with $|\hat{s} - s| > d$ and citizens follow the strategy given in (11), no citizen joins the social movement. Then, all citizens expect that the social movement succeeds only if $\theta < 0$.

On the other hand, If the government chooses the policy vector $(s, d) \ngeq \left(s^\dagger, d^\dagger\right)$ with $|\hat{s} - s| > d$ and citizens follow the strategy given in (11), each citizen joins the social movement if his signal is less than $\bar{k}$. Then, all citizens expect that the social movement succeeds if $\theta < \breve{\theta}$.

Finally, if the government chooses the policy vector (s, d) with $|\hat{s} - s| \leq d$ and citizens follow the equilibrium strategy, no citizen joins the social movement. Then, all citizens expect that the social movement succeeds only if $\theta < 0$.

Therefore, condition (iii) of Definition 1 changes to

[6] Recall that the social movement fails if $A(\theta, s, d) = \Phi\left(\frac{\bar{k}-\theta}{\sigma}\right) \leq \theta$.

$$\mathbf{a}_i(x_i,s,d) \in \underset{a\in\{0,1\}}{\operatorname{argmax}}\ a\left[|\hat{s}-s|\mu(0 \mid x_i,s,d)-d\right] \qquad \text{if } (s,d) \geq \left(s^\dagger,d^\dagger\right) \text{ and } |\hat{s}-s|>d; \tag{13}$$

$$\mathbf{a}_i(x_i,s,d) \in \underset{a\in\{0,1\}}{\operatorname{argmax}}\ a\left[|\hat{s}-s|\mu\Big(\breve{\theta} \mid x_i,s,d\Big)-d\right] \qquad \text{if } (s,d) \ngeq \left(s^\dagger,d^\dagger\right) \text{ and } |\hat{s}-s|>d; \tag{14}$$

$$\mathbf{a}_i(x_i,s,d) \in \underset{a\in\{0,1\}}{\operatorname{argmax}}\ a\left[|\hat{s}-s|\mu(0 \mid x_i,s,d)-d\right] \qquad \text{if } |\hat{s}-s| \leq d. \tag{15}$$

It is clear that in (15) the optimal choice is always not to join the social movement. Therefore, citizens' equilibrium strategy is optimal if conditions (13) and (14) meet respectively

$$\mu(0 \mid x_i,s,d) < \frac{d}{|\hat{s}-s|} \qquad\qquad \text{when } (s,d) \geq \left(s^\dagger,d^\dagger\right) \text{ and } |\hat{s}-s|>d \tag{16}$$

$$\mu\Big(\breve{\theta} \mid x_i,s,d\Big) > \frac{d}{|\hat{s}-s|} \iff x_i < \bar{k} \qquad \text{when } (s,d) \ngeq \left(s^\dagger,d^\dagger\right) \text{ and } |\hat{s}-s|>d \tag{17}$$

Condition (16) establishes that the expected payoff of joining the social movement when the policy vector is $(s,d) \geq \left(s^\dagger,d^\dagger\right)$ with $|\hat{s}-s|>d$, is not grater than zero for any signal. Condition (17) establishes that the expected payoff of joining the social movement when the policy vector is $(s,d) \ngeq \left(s^\dagger,d^\dagger\right)$ with $|\hat{s}-s|>d$, which is equal to $|\hat{s}-s|\mu\Big(\breve{\theta} \mid x_i,s,d\Big)-d$, is greater than zero only if the signal is less than $\bar{k}$. Now we show that there is a system of beliefs that meets conditions (17), (16) and the intuitive criteria.

In the equilibrium path, i.e. if the government chooses the policy vectors $(s^*,0)$ or $\left(s^\dagger,d^\dagger\right)$, beliefs must follow Bayes' rule. If the government chooses the policy vector $(s^*,0)$, the posterior beliefs about the success of the social movement is

$$\mu\Big(\breve{\theta} \mid x_i,s^*,0\Big) = \mu(0 \mid x_i,s^*,0) = \frac{1-\Phi\left(\frac{x_i-0}{\sigma}\right)}{1-\Phi\left(\frac{x_i-0}{\sigma}\right)+\Phi\left(\frac{x_i-\breve{\theta}}{\sigma}\right)} \tag{18}$$

Where the fist equality comes from the fact that the government is not of type $\theta \in \Big[0,\breve{\theta}\Big)$. As (18) is decreasing in x_i, to meet condition (17) it is enough that $\bar{k}$ solves $\mu\big(0 \mid \bar{k},s^*,0\big)=0$.[7] If the government chooses the policy vector $\left(s^\dagger,d^\dagger\right)$, the posterior beliefs about the success of the social movement is

$$\mu\big(0 \mid x_i,s^\dagger,d^\dagger\big)=0$$

[7]The equality that $\bar{k}$ must solve is $\mu\big(0 \mid \bar{k},s^*,0\big) = \frac{d}{\hat{s}-s}$, we use the equality $\mu(0 \mid \bar{k},s^*,0)=0$ because $d=0$ and $s \neq \hat{s}$.

because this policy vector is chosen only by the government of type $\theta \in \left[0, \breve{\theta}\right)$. As $d^\dagger > 0$ and $s^\dagger \neq \hat{s}$, the condition (16) is met.

Let us notice that choosing $\bar{k}$ that solves $1 - \Phi\left(\frac{\bar{k}-0}{\sigma}\right) = 0$ is the same as choosing $\bar{k}$ that solves $\mu\left(0 \mid \bar{k}, s^*, 0\right) = 0$. Then, it is clear that $\bar{k} = +\infty$ and $\breve{\theta} = 1$, the latter comes from the equality $\breve{\theta} = \Phi\left(\frac{\bar{k}-\breve{\theta}}{\sigma}\right)$.

Out of the equilibrium path, we focus our attention in beliefs that meet the intuitive criteria (Cho and Kreps, 1987):

- If the government chooses a policy vector with an expenditure in law enforcement greater than zero and $|\hat{s} - s| > d$, as for the weak and strong government is optimal to choose $d = 0$, the only type of government that would choose such a policy vector is $\theta \in [0, 1)$. Then, $\mu(0 \mid x_i, s, d) = 0$ and $\mu\left(\breve{\theta} \mid x_i, s, d\right) = \mu(1 \mid x_i, s, d) = 1$. Therefore conditions (16), (17) are met.

- If the government chooses a policy vector with an expenditure in law enforcement of $d = 0$ and a social expenditure $s \neq s^*$, with $|\hat{s} - s| > d = 0$, since for the strong government it is optimal to choose $(s^*, 0)$, the only type of government that would choose such a policy vector is $\theta < 1$ and thus $\mu\left(\breve{\theta} \mid x_i, s, 0\right) = \mu(1 \mid x_i, s, 0) = 1$. Therefore, condition (17) is met. In this case, we do not care about condition (16) because $d^\dagger > 0$.

Finally, we conclude that the citizens' strategy (11) is optimal along with $\breve{\theta} = 1$ and $\bar{k} = +\infty$.

We have proven that given a strategy of the citizens of the form (11), the strategy given in (12) is optimal for the government. Furthermore, we have provided beliefs that make the strategy (11) optimal for the citizens given the strategy (12) of the government. In the way we concluded that $\breve{\theta} = 1$ and $\bar{k} = +\infty$. This closes the proof of Proposition 1.

□

A.2 Proof of Proposition 2

Proof. It is clear that the government is using an optimal strategy, so we focus our attention in the citizens' strategy.

If citizen $i \in [0, 1]$ expects that all others citizens use the equilibrium strategy, then for all policy vector he expects that the social movement succeeds if $\theta < 0$. Then Condition (iii) in Definition 1 simplifies to

$$\mathbf{a}(x_i, s, d) \in \text{argmax}_{a\in\{0,1\}}\, a\, \{|\hat{s} - s|\mu(0 \mid x_i, s, d) - d\}$$

Therefore, citizens' strategy is optimal if for each policy vector the expected payoff of citizens for joining the social movement is less or equal to zero, that is

$$\mu(0 \mid x_i, s, d) = 0 \qquad \text{or} \qquad s = \hat{s} \qquad \text{or} \qquad \mu(0 \mid x_i, s, d) < \frac{d}{|\hat{s} - s|}.$$

In the equilibrium path, citizens' beliefs are uniquely determined by Bayes' rule. If in the first stage the government chooses the policy vector $(s^*, 0)$, citizens know that the government is not weak, thus $\mu(0 \mid x_i, s, d) = 0$. If in the first stage the government chooses the policy vector $(\hat{s}, 0)$, citizens know that the government is weak, hence $\mu(0 \mid x_i, s, d) = 1$. Therefore, in the former case it is optimal not to join the social movement because $\mu(0 \mid x_i, s, d) = 0$ and in the latter case it is optimal not to join the social movement because $s = \hat{s}$.

Off the equilibrium path, we focus our attention in beliefs that meet the intuitive criteria of Cho and Kreps (1987):

- Since $d = 0$ is the optimal choice of expenditure in law enforcement for both, the strong and weak government; if in the first stage the government chooses a policy vector with $d > 0$, citizens know that the government is unstable, thus $\mu(0 \mid x_i, s, d) = 0$. Indeed, any belief with support in $[0, 1[$ makes not joining the social movement optimal.

- Since the optimal choice of the strong government is $(s^*, 0)$. If in the first stage the government chooses a policy vector $(s, 0)$ with $s \neq s^*$,[8] citizens know that the government is not strong, hence $\mu(1 \mid x_i, s, d) = 1$. In particular, a belief that satisfies $\mu(0 \mid x_i, s, 0) = 0$ makes not joining the social movement optimal.

Consequently, we conclude that citizens' strategy is optimal and thus the profile is an equilibrium.

To see that there is no other equilibrium strategy profile, note first that if the government is strong or unstable, it is choosing its first best policy. Thus, in any equilibrium with $\mathbf{a}(x_i, s, d) \equiv 0$, if $\theta \geq 0$, $s_1(\theta) = s^*$.

[8]Recall that we are out of the equilibrium path, so $s \neq \hat{s}$.

Second, note that for a weak government ($\theta < 0$) it is optimal to choose $d_1(\theta) = 0$ (see Lemma 6).

Now, if a weak government chooses s^*, then by the use of Bayes' rule the equilibrium belief will give positive probability to the government being weak. Therefore, the expected payoff of joining the social movement given that the other citizens do not join is:

$$|\hat{s} - s^*|\mu(0 \mid x_i, s^*, 0) - 0 > 0$$

which si not compatible with $\mathbf{a}(x_i, s, d) \equiv 0$.

On the other hand, if citizens observe a chosen policy $s \neq s^*$ they will know that the government is weak: $\mu(0 \mid x_i, s, d) = 1$.

Consequently, in any equilibrium in which the other citizens join the social movement and the weak government chooses $s \neq s^*$, the expected payoff of joining the social movement is

$$|\hat{s} - s| * 1 - 0 > 0 \qquad \text{for } s \neq \hat{s}.$$

which si also not compatible with $\mathbf{a}(x_i, s, d) \equiv 0$.

Finally, we conclude that the only equilibrium strategy for the weak government that is compatible with $\mathbf{a}(x_i, s, d) \equiv 0$ is $s_1(\theta) = \hat{s}$.

□

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