# Ignoring Experts' Honest Advice

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**Abstract**

This paper studies a cheap-talk game between a Decision-Maker, an Expert and an Observer. We argue that there are circumstances where an uniformed Decision-Maker elicits more information from a better informed Expert when the Decision-Maker ignores the Expert's honest advice with positive probability. Key to this result is that the Decision-Maker and Expert are concerned with the Observer's belief about the Expert's ability to be well informed, they have different prior beliefs about a payoff relevant state of nature and the communication between the Expert and the Decision-Maker is private. A direct consequence of this is that the Expert exerts at least as much information-acquisition effort when communication is private than when it is public. These results bear interesting implications for organizational design. Mainly, centralization with private communication often outperforms centralization with public communication and delegation.

**JEL:** D8, D2, C7

**Key Words:** Private Communication, Public Communication, Cheap-Talk, Non-common Priors, Reputation, Centralization, Delegation.

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# 1 Introduction

In this paper we ask the following questions. Are there circumstances where an uniformed Decision-Maker should ignore an Expert's informative advice? and Does it matter whether communication is private or public? These questions are asked in a setting where there are three players, a Decision-Maker, a privately informed Expert and an Observer, in a cheap-talk game with reputational concerns. We argue that when communication is private, despite the Expert's superior information, the Decision-Maker should sometimes ignore the Expert's informative advice since this strategy induces the Expert to truthfully reveal his private information more often. If communication is public, it is no longer possible to benefit from ignoring the expert's honest advice.

As an example lets consider a U.S. president in need of advice concerning a foreign affair issue such as to deploy more troops in Afghanistan or withdraw U.S. forces from Irak. Usually this advice comes from the secretary of states, national security adviser or other top aides. Lets assume that the president has to decide whether to deploy more troops or not. The success of this decision depends on the military and economic resources still available to the enemy's troops, which are unknown at the time the decision must be made. For concreteness suppose the president asks for advice to the secretary of states, who is better informed than the president about the resources at the enemy's disposal. The recommendation made by the secretary of state is private information (the public does not learn it). The quality of the secretary of states' information is determined by his ability and effort to understand the impact that previous bombing has had on the enemy's resources. The secretary of states not only cares about the war outcome, but also considers his reputation about being well informed in the eyes of the public since this will somehow affect the probability to be either kept in office next period or to be elected senator or president the future. The president also cares about the war outcome as well as the secretary of states' reputation, since both belong to the same political coalition. Furthermore, due to the very nature of the presidency, the president usually holds different beliefs about the likelihood of the different states occurring than the the secretary of states. This paper argues that the president can induce the secretary of states to truthfully reveal his private information for a lager set of priors when communication between the president and the secretary of states is kept private and the president sometimes ignores the advice. For instance, President Obama has been accused of ignoring the advice on Afghanistan troops level and also the advice about not withdrawing all U.S. forces from Iraq, thing he did boasting that he was finally bringing an end to "the long war in Iraq."

This article considers a three player game composed of a Decision-Maker, an Expert and an Observer.

The uninformed Decision-Maker have to choose a project whose outcome depends on an unknown state of nature. The Decision-Maker can consult a better informed Expert whose ability about being well informed is unknown to everyone including himself. The Observer sees the project chosen and the realized state, and forms a belief about the Expert's ability to be well informed. Hereinafter, the Expert's reputation. Both, the Decision-Maker and Expert care about the Expert's reputation about being well informed. The set of available projects has two projects, called status-quo project and new project, and the set of states also has two states, called time-to-be-cautious and time-to-act. The Decision-Maker and Expert have the same underlying preferences, but they differ in their prior beliefs about the occurrence of each state. The Expert receives a private signal about the state and the Expert's ability parameterizes the informativeness of the signal; the higher the Expert's ability, the more informative is his signal. Hence, the information about the state is necessarily intertwined with that about the Expert's ability. The Decision-Maker and the Expert's short-run payoff are both positive only when either the status-quo project is chosen and the time-to-be-cautious state is observed or the new project is chosen and the time-to-act state is realized, otherwise both payoffs are zero. The Decision-Maker and the Expert, both benefit from having a more reputed Expert about his ability to be well informed. The reputation of the Expert is evaluated by an external Observer based on the project chosen and the realized state. The conflict of interests is endogenous and arises when the Expert's prior about a given state occurring conditional on his signal is sufficiently different from the Expert's prior about the same state occurring. Hence, the model here is a reputational cheap-talk model where the Decision-Maker and the Expert agree to disagree, and both care about the Expert's reputation from the perspective of an external Observer.[1]

The timing of decisions is as follows. At Stage 1, the Expert gets a private signal about the state of the world. The private information of the Expert is soft and therefore the Expert cannot prove or certify his information.[2] Then, at Stage 2, the Expert privately communicates his information to the Decision-Maker. At Stage 3, the Decision-Maker, after observing the recommendation, chooses between the status-quo or the new project. Then at the final Stage, the state and the project chosen are publicly observed and belief updating about the Expert's ability to be well informed occurs.

When the Decision-Maker and Expert's priors are close to each other and around $1/2$, in the most informative Perfect Bayesian equilibrium, the Expert truthfully reveals his signal and the Decision-Maker

[1] In Section 2, we will discuss in detail the main assumptions of the model.

[2] Because the outcome and projects cannot be contracted upon, the Decision-Maker cannot use a standard mechanism to elicit the Expert's private information.

rubber-stamps the Expert's recommendation, since the Decision-Maker and Expert's ex-post preferences are aligned. They are aligned because the Expert's posterior is such that if he were to choose projects he would have chosen the same project that the Decision-Maker chooses when the Expert truthfully reveals his information.

When the Decision-Maker's prior is slightly biased towards one state and the Expert's prior is biased in favor of the other state, in the most informative Perfect Bayesian equilibrium, the Expert reports his signal truthfully and the Decision-Maker makes a decision that matches the Expert's advice when the Expert's recommendation confirms his prior and mixes between projects after a report contradicting his prior. By mixing after a recommendation that contradicts his prior, the Decision-Maker achieves two things: first, if the Observer sees that the Decision-Maker chooses the project consistent with his prior, but the realized state contradicts that, the Observer updates his belief about the Expert's ability only slightly negative, since the Decision-Maker might have not follow the Expert's advice; and second, if the Observer sees that the Decision-Maker chooses a project that contradicts his prior, but the realized state does not match the project, the Observer updates much more negatively, since the Decision-Maker will choose a project that contradicts his prior only if he was persuaded to do so. In short, by biasing the project choice against his prior, the Decision-Maker creates a reputation penalty that induces the Expert to fully reveal his information. More importantly, the Decision-Maker is willing to do so exactly because he is concerned with the Expert's reputation. Otherwise he would have rubber-stamped the Expert's recommendation. Technically, the Decision-Maker garbles the signal that the Observer gets in order to make the Observer's inference process harder, and in that way create a different belief updating after different project and state realizations. Under public communication this strategy cannot induce the Expert to reveal his private information since the Observer forms its beliefs based on the Expert's message, so garbling the Observer's signal is not feasible.

Furthermore, we show that when communication is private, the Expert's incentives to exert information-acquisition effort are higher than or equal to those arising from the most informative equilibrium when communication is public. The reason stands for the fact when messages are public information and the Expert's prior is sufficiently biased towards either state, he recommends the same project regardless of his information. Hence, information is worthless from the Expert's point of view and therefore he has no incentives to improve the quality of his information by undertaking costly effort.

The results show that there are parameterizations under which private communication not only improves, relative to public communication, the Expert's incentives to truthfully reveal his private information, but also increases the Expert's incentives to acquire information. The reason stands for the fact that from the Expert's point of view information revelation and information acquisition are strategic complements. A main economic insight arises from this result which is that Principals serve as filters that facilitate and control information transmission from informed Experts to the public. Under private communication, this filtering role that the Decision-Maker plays may induce the Expert to reveal his private information as well as to acquire more information in circumstances in which under public communication the Expert will not truthfully reveal his private information and therefore he will not exert a positive information-acquisition effort. Thus, under certain set of priors, transparency of communication may result in a loss of information.

The rest of the paper is as follows. In the next Section the basic setup is discussed. In Section 3, the equilibrium is derived ignoring information acquisition effort. In Section 4, we study the case in which the Expert's quality of information depends not only on his ability, but also on his information acquisition effort. In Section 5, we derive the implications for organizational design. The robustness of the result are discussed in the final section and this also presents some concluding remarks. Proofs can be found in the appendices.

## 2 The Model

**The Setup.** There are three players in the game, a Decision-Maker (DM), an Expert (E) and an Observer (O). A project must be adopted. The Expert privately receives a signal about the state of the world, and the Decision-Maker can benefit from this information by asking the Expert to recommend a project. The Observer learns the project chosen and the realized state and, based on this information, he forms a belief about the Expert's ability to be well-informed.

The are two non-contractible in an ex-ante and ex-post sense projects: the status-quo project ($d = -1$), and a new project ($d = 1$). We will call adopting the new project "acting". The project's return depends on the state of world. For the sake of simplicity, there are two states of the world called: time-to-act" ($x = 1$) and time-to-be-cautions ($x = -1$). If the DM acts when it is time to act, or if he chooses the status-quo when it is time to be cautions, the DM and Expert receive a non-verifiable payoff equal to 1.[3] In contrast, if the DM acts when it is time to be cautious or if he chooses the status-quo when it is time to act, then both receive a payoff equal to $0$. Hence, player $i$'s, $i \in \{E, DM\}$, preferences over projects are as follows: $u_i(x, d) = 1$ if $x = d$ and $u_i(x, d) = 0$ if $x \neq d$. This implies that there is no short-run conflict of interests

[3]This precludes the use mechanism design to elicit the Expert's information.

between the DM and Expert.

The payoff to the Expert and the DM depend on the consequence of the adopted project and on the posterior distribution of the Expert's ability to be well informed, denoted by $\theta$ and $\theta \in \Theta$, as follows: the Expert's payoff when state $x$ is observed and project $d$ is chosen is

$$U_E(x,d) = u_E(x,d) + \delta_E E(v_E(\theta)|\Omega), \tag{1}$$

where $\Omega$ represents the Observer's information set, $\delta_E \geq 0$ is the Expert's relative valuation of his reputational payoff and $v_E(\cdot)$ is an increasing function. When $\delta_E \leq 1$, he values the short-run payoff more than the long-run payoff, while when $\delta_E > 1$, the opposite happens. The payoff function $v_E(\cdot)$ is strictly increasing in the Expert's ability.

The DM's payoff function when state $x$ is observed and project $d$ is chosen is

$$U_{DM}(x,d) = u_{DM}(x,d) + \delta_{DM} E(v_P(\theta)|\Omega), \tag{2}$$

where $\delta_{DM} \geq 0$ is the DM's valuation of the Expert's reputational payoff relative to the valuation of his short-run payoff and $v_P(\cdot)$ is an increasing function. The Observer's information set contains the project chosen $d$ and the state $x$. Thus, $\Omega \equiv \{(x,d)|(x,d) \in \{-1,1\}^2\}$).

The Expert believes state $x$ occurs with probability $q_E(x) \in (0,1)$ and the DM believes that this occurs with probability $q_{DM}(x) \in (0,1)$. Prior beliefs are common knowledge and, therefore, they agree to disagree.

The DM as well as the Expert are uninformed about the state of the world, but the Expert privately observes a signal $s \in \{-1,1\}$ about it. This signal is drawn from the conditional probability density $g(s|x,\theta)$ specified below. Thus, the Expert's ability parameterizes the amount of information the state contains in the signal. The Expert's information is soft; that is, the Expert cannot certify or *prove* his information, and the random variables $\theta$ and $s$ are assumed to be mutually independent. The Expert's ability is a constant unknown to everyone (including the Expert himself), and all players have identical prior beliefs distributed $f(\theta)$. Furthermore, the ability and state are assumed to be statistically independent.

The signal's conditional probability density $g(s|x,\theta)$ is a mixture between an informative experiment with density $g(s|x)$ and uninformative experiment with density $h(s)$. The mixture puts weight $p(\theta)$ on the informative experiment and $1 - p(\theta)$ on the uninformative experiment. Hence, $p(\theta)$ is the probability that the Expert receives a signal from the informative experiment $g(s|x)$. In order to keep the analysis as simple as possible, we assume that the informative experiment gives rise to a density $g(s|x) = \frac{1+sx}{2}$ and the uninformative experiment to a density $h(s) = 1/2$. Hence, the Expert's signal has a conditional probability density function

$$g(s|x,\theta) = p(\theta)\frac{1+sx}{2} + (1-p(\theta))\frac{1}{2}.^{4} \tag{3}$$

When $p(\theta) = 1$, the Expert is fully informed about the state.

More able types are more likely to receive a signal drawn from the informative distribution $g(s|x)$ rather than from the uninformative distribution $h(s)$. Naturally, a more talented Expert receives better information in the sense of Blackwell. This structure is equivalent to the one in Ottaviani and Sorensen (2001), but for a discrete signal and state space. This experiment is a generalization to a continuos ability space of the standard binary model with two ability types presented for instance in Scharfstein and Stein (1990).

In Section 4, the case in which the signal has a conditional probability density function that depends on the Expert's unobservable effort is analyzed by assuming that the probability that the Expert receives a signal from the informative distribution is $p(\theta, e)$, where $e$ is the information-acquisition effort.

The timing of decisions, depicted in Figure 1, is as follows. At $t = 1$, the Expert gets a signal $s$ regarding the state of the world. At $t = 2$, the Expert sends a message to the DM. The Expert's strategy is a mapping from signals into messages, denoted by $m$, with $m \in \{-1, 1\}$.[5] The conditional probability that message $m$ is sent following signal $s$ is denoted by $h(m|s) \in [0, 1]$. After receiving message $m$, at Stage 3, the DM chooses project $d$ with conditional probability $z(d|m) \in [0, 1]$. At $t = 4$, the project's return is realized and everyone observes the project chosen and the realized state; that is, everyone observes the outcome $(x, d)$. The Observer forms beliefs based on this information. Notice that the Observer's information set does not include the message sent by the Expert to the DM and thus from the Expert's viewpoint his reputation is based only on $(x, d)$. In other words, communication between the DM and the Expert is private.

We study the perfect Bayesian equilibrium (PBE). Intuitively, we require that: (1) the DM's best response must be optimal given his beliefs; (2) the Expert's message must be optimal given his beliefs and his information; and (3) beliefs must be consistent with Bayes' rule whenever possible. For the sake of brevity, we will ignore any "mirror equilibrium", i.e., an equilibrium that takes an original equilibrium and

[4] This satisfies the monotone likelihood ratio property (MLRP) in $(s, x)$ for any $\theta \in \Theta$.

[5] Because the Expert randomizes between two distinct messages only if each of them yields the same expected payoff, without any loss of generality, we can restrict ourselves to a message space that contains only two messages. Call them 1 and $-1$, where 1 means signal 1 and -1 signal -1. Thus, $m \in \{-1, 1\}$.

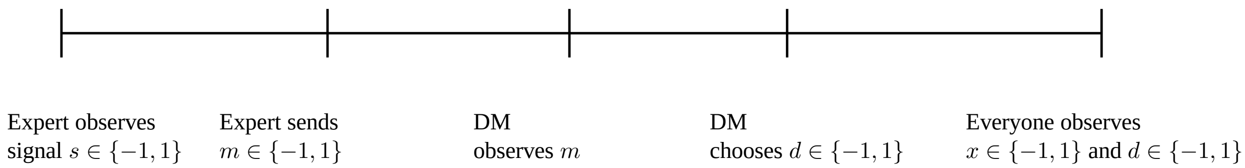


**Fig. 1.** The Timing

switches each project from 1 to -1 and vice-versa, and focus on non-perverse equilibrium. This means that the Expert's strategy is restricted to satisfy $h(m = s|s) \geq h(m \neq s|s)$; that is, the Expert sends the message time-to-act with higher or equal probability when his signal about the state says that it is time to act than when the signal says it is time to be cautious, and he chooses the time-to-be-cautious message with higher or equal probability when his signal about the state says that it is time to be cautious than when his signal says it time to act. Similarly, the DM's strategy satisfies $z(d = m|m) \geq z(d \neq m|m)$. In what follows we refer to a non-perverse perfect Bayesian equilibrium simply as an equilibrium.

## 2.1 Discussion of the Model Main Features

**Payoffs.** The Expert and DM's payoff function is consistent with that in most career concerns models dealing with information revelation issues such as Prat (2005) and Ottaviani and Sorensen (2006a).

When $v_P(\cdot)$ is convex, this model could be thought of as a reduced form of a two-period career concerns model in which the DM chooses between retaining the first-period Expert and hiring another one. Convexity arises from the fact that the DM's payoff is the upper envelope of a pair of expected payoffs: the payoff from the incumbent Expert and that from the challenger. Better information about the Expert is beneficial and therefore the DM's payoff rises with the Expert's reputation.

When $v_P(\cdot)$ is concave, this reduced-form formulation captures the case in which an Expert with better information tends to be more valuable to future Decision-Makers. This reputational payoff can be derived from the value of the services provided in a second and last period by the Expert, as in Holmström (1999). Since experts with higher ability have more informative signals, they provide information services of higher value in the second period and so should receive higher rewards. Again, this implies that the DM's payoff rises with the Expert's reputation.

**Common Beliefs.** According to Van den Steen (2010) the differing priors assumption not only implies... "that each player believes that he is right and others are wrong, but also that each player is aware that these others will often believe the opposite, i.e., that they are right and the focal player is wrong". He argues that this is the essence of subjective beliefs and of agreeing to disagree. In fact, when people learn they are wrong they change their opinions and casual empiricism indeed suggests that people tend to explain disagreement in terms of how they think others are wrong. This suggests that people act as if they have differing priors. We also know that Bayesian updating specifies how new information should be dealt with, but it does not say much about how priors should be or are actually formed. Hence, in the absence of a rationality-based model for selecting priors, the assumption that people cannot agree to disagree seem unfounded when heterogenous priors stem from insufficient data. Nor this hinder players' ability to process new information. Morris (1995) observes that introducing differing priors does not allow us to "explain anything" any more than does introducing differing utility functions, information sets, action, sets, etc.. Hence, we take the view that players are Bayes rational, but may initially openly disagree on the likelihood of the state. Typically, this disagreement can come from lack of experimental evidence or historical records that would allow players to otherwise reach a consensus on their prior views.

A crucial dimension of difference in beliefs –or open disagreement– is that it makes people (who care about the outcome) collect more information to persuade the other players. The intuition is that each player expects that, on average, the newly collected data will confirm his or his beliefs and thus convince the other player, i.e., move the belief of the other player closer to his own (see Van den Steen (2005)). This "persuasion" effect is unique to a situation with open disagreement or differing priors and is different from the effects that arise in a model with different preferences. In addition, difference in beliefs seems a fundamental feature of any advisor-advisee relationship.

A natural criticism of the non-common priors assumption is that one could argue that no DM will ever hire an Expert who has different priors. However, one could easily say the something about an Expert who is known to have different underlying preferences or his own agenda. In addition, Che and Kartik (2009) show that it is sometimes optimal to hire someone with differing priors because he exerts more effort to acquire information with the goal to persuade the DM.

**Conflicts of Interests.** Our assumptions imply that there is no short-run conflict of interests between the DM and Expert. Hence, in contrast to the well-known Crawford and Sobel's (1982) cheap talk model, the model here does not consider an explicit partisan dimension. It only considers a professional advice dimension as in Ottaviani and Sorensen (2001). However, as in Che and Kartik (2009), as long as signals are not perfectly informative about the state, differences in beliefs generate conflicts in preferred projects given any signal. In particular, when the Expert's belief about the state being $1$ conditional on signal $s$ being observed is greater than $1/2$ and the DM's belief about the probability that state $1$ conditional on message $m$ being received is lower than $1/2$, the Expert prefers project $1$, while the DM prefers project -1. When the opposite happens, preferences are reversed. While when both, the Expert and the DM's corresponding conditional beliefs belong to either the interval $[0, 1/2]$ or $(1/2, 1]$, the DM and the Expert's preferred projects coincide. Hence, non-common priors might result in an interim conflict in preferred projects, even though fundamental preferences agree.

## 3 Equilibrium Analysis

In any cheap-talk game there are equilibria in which talk is ignored. If the DM ignores all recommendations, then pooling is a best-response for the Expert (sender); because recommendations have no direct effect on the DM's payoff, and if the sender is pooling, then a best response for the DM is to ignore all recommendations. Thus, in a pooling (babbling) equilibrium, the DM behaves as a fully uninformed DM.

Let $\hat{h}(m|s) \in [0, 1]$ be the DM and Observer's conjecture about the strategy used by the Expert, and $\hat{z}(d|m) \in [0, 1]$ be the Expert and Observer's conjecture about the strategy used by the DM. Then, the Observer computes the chances of the evidence, $\hat{g}(d|x, \theta) = \sum_m \hat{z}(d|m) \sum_s \hat{h}(m|s)g(s|x, \theta)$ and $\hat{g}(d|x) = \int_\Theta \sum_m \hat{z}(d|m) \sum_s \hat{h}(m|s)g(s|x, \theta)f(\theta)d\theta$. The Expert's posterior reputation from the Observer's point of view when he observes $(x, d) \in \{-1, 1\}^2$ is then calculated by Baye's rule as $f(\theta|d, x) = f(\theta)\hat{g}(d|x, \theta)/\hat{g}(d|x)$.

The expected reputational payoff for an Expert who receives signal $s$ when the DM chooses project $d$ is given by

$$V_E(d|s) = \sum_x E(v_E(\theta)|d, x)q_E(x|s), \tag{4}$$

where his reputational payoff when $(d, x)$ is observed is

$$E(v_E(\theta)|d, x) = \int_\Theta v_E(\theta)f(\theta|d, x)d\theta. \tag{5}$$

Let $E(s|d)$ be the expected value of signal $s$ given project $d$, the Observer's conjecture about the Expert's strategy $\hat{h}(d|s)$ and the DM's strategy $\hat{z}(d|s)$. Hence,

$$E(s|d) = \frac{\sum_m \hat{z}(d|m) \sum_s \hat{h}(m|s)s}{\sum_m \hat{z}(d|m) \sum_s \hat{h}(m|s)}. \tag{6}$$

Observe that for all conjectures that the Observer may hold, $E(s|1) \in [0, 1]$ and $E(s|-1) \in [-1, 0]$.

We then have the following lemma.

**Lemma 1.**

*i)*

$$E(v_E(\theta)|d, x) = Ev_E(\theta) + \sigma_E^2 \frac{xE(s|d)}{1 + xE(s|d)Ep(\theta)}, \tag{7}$$

*where $\sigma_E^2$ is the covariance between $v_E(\theta)$ and $p(\theta)$.*

*ii)* $E(v_E(\theta)|d, x)$ *is supermodular in* $(E(s|d), x)$.

Observe that the Expert's reputational payoff from project $d$ and state $x = 1$ depends positively only on the Observer's belief about the average signal conditional on the project chosen (i.e.,$E(s|d)$) and negatively on that when the state is $x = -1$. Observe also that there are might be different strategies that give rise to the same average signal $E(s|d)$ and thus the Expert must be indifferent between these strategies with regard to the reputational payoff. In addition, these two strategies convey the same information not only about the Expert's ability, but also about the state $x$. Also as in Ottaviani and Sorensen (2006a), the Expert's reputational payoff depends on the prior reputation about his probability of being well informed only through the expected value of it, $Ep(\theta)$. This is a consequence of the fact that $g(s|x, \theta)$ is linear in $p(\theta)$.

It follows from equation (7) that if the Observer believes that project $d$ will be made more often after the Expert receives signal $s$ than after he gets signal $s'$, then the Expert's reputational payoff after $(d, x)$ rises when the realized state $x$ coincides with the signal $s$ for which the Observer conjetures that the DM will play $d$ more often, while that when the state does not coincide with the corresponding signal falls. In other words, the reputational payoff from playing project $d$ more often after signal $s$ is greater when the realized state confirms the Observer's conjecture about the DM playing $d$ more often after the Expert receives signal $s$. A direct consequence of this is that the Expert's expected reputational payoff when the DM plays $d$ rises as the Observer conjectures that $d$ will be played more often after the Expert receives signal $s$. This will give rise to multiple equilibria for some parameterizations.

The Expert wishes to induce the Observer to have the most favorable posterior beliefs about his ability to be well informed regardless of the signal received. The Expert's reputational payoff depends on the project chosen and the realized state, which is unknown by the Expert when he chooses a project. Different projects result in different lotteries over posterior reputations and thus an Expert (DM) can then find the lotteries differently appealing depending on his posterior beliefs about the state conditional on his privately observed signal (message). Because for any conjecture that the Observer may hold, the Expert's reputational payoff when the project chosen matches the state is at least as large as that when the project fails to match the state, the expected reputational payoff from playing $m = s$ is at least as large as that from playing $m \neq s$ when the Expert's posterior $q(x = s|s)$ is sufficiently large relative to $q(x \neq s|s)$ so that the lottery induced by message $m = s$ dominates the lottery induced by message $m \neq s$.

The Expert recommends project 1 when he receives signal $s$ if and only if

$$\hat{z}(1|1)\Big(q_E(1|s)1 + (1 - q_E(1|s))0 + \delta_E V_E(1|s)\Big) + \hat{z}(-1|1)\Big(q_E(1|s)0 + (1 - q_E(1|s))1 + \delta_E V_E(-1|s)\Big) \geq$$
$$\hat{z}(1|-1)\Big(q_E(1|s)1 + (1 - q_E(1|s))0 + \delta_E V_E(1|s)\Big) + \hat{z}(-1|-1)\Big(q_E(1|s)0 + (1 - q_E(1|s))1 + \delta_E V_E(-1|s)\Big).$$

Because we are focusing on non-perverse equilibrium $\hat{z}(1|1) \geq \hat{z}(1|-1)$, this can be written after a few steps of simple algebra in a more amenable form

$$q_E(1|s) \geq \frac{1}{2} - \frac{\delta_E}{2} \triangle REP_E(s), \tag{8}$$

where $\triangle REP_E(s)$ is the expected reputational payoff increase or decrease the Expert will receive if he decides to sent the message time-to-act instead of the message time-to-be-cautious after signal $s$. It follows from lemma 1 that

$$\triangle REP_E(s) = \sigma_E^2 \sum_x q(x|s)\Big(\frac{xE(s|1)}{1 + xE(s|1)Ep(\theta)} - \frac{xE(s|-1)}{1 + xE(s|-1)Ep(\theta)}\Big).$$

The term $\triangle REP_E(s)$ depends on three things: (i) the Expert's belief, conditional on signal $s$ being observed, about the probability that the state is time-to-act; (ii) how the Observer thinks the Expert will use his information, which is captured by the Observer's conjecture about the Expert's behavior; and (iii) how the Observer thinks the DM will use his information, which is captured by the Observer's conjecture about the DM's behavior. Points (ii) and (iii) are captured by the expected value of the signal conditional on project $d$

being chosen, $E(s|d)$ (see, equation (6)).

The DM's expected payoff from having an Expert whom, according to the Observer, has reputation $f(\theta|d,x)$ after state $x$ is revealed when the DM chooses project $d$ and receives message $m$ is

$$V_{DM}(d|m) = \sum_x E(v_{DM}(\theta)|d,x) q_{DM}(x|m), \tag{9}$$

where the DM's payoff from the Expert's reputation when $(d,x)$ is observed is given by

$$E(v_{DM}(\theta)|d,x) = \int_\Theta v_{DM}(\theta) f(\theta|d,x) d\theta, \tag{10}$$

and $q_{DM}(x|m)$ is the DM's belief about the probability that the state of the world is $x$ conditional on recommendation $m$ being received.

The DM's posterior belief about state $x$ conditional on receiving message $m$ is given by Baye's rule as follows $q_{DM}(x|m) = \hat{g}(m|x) q_{DM}(x)/\hat{g}(m)$, with $\hat{g}(m|x) = \sum_s \hat{h}(m|s) \int_\Theta g(s|x,\theta) f(\theta) d\theta$ and $\hat{g}(m) = \sum_x \sum_s \hat{h}(m|s) \big( \int_\Theta g(s|x,\theta) f(\theta) d\theta \big) q_{DM}(x)$.

Then we have the following lemma.

**Lemma 2.**

*i)*

$$E(v_{DM}(\theta)|d,x) = E v_{DM}(\theta) + \sigma^2_{DM} \frac{x E(s|d)}{1 + x E(s|d) E p(\theta)}, \tag{11}$$

*where $\sigma^2_{DM}$ is the covariance between $v_{DM}(\theta)$ and $p(\theta)$.*

*ii)* $E(v_{DM}(\theta)|d,x)$ *is supermodular in* $(E(s|d), x)$.

Thus, the DM's reputational payoff from project $d$ and state $x$ depends positively only on the Observer's belief about the average signal $E(s|d)$. The rationale for this is the same as the one given above and this payoff behaves as $E(v_E(\theta)|d,x)$ with respect to the Observer's conjectures.

Given the DM's belief about the probability that the actual state is $x$ conditional on message $m$ being received, the DM chooses project 1 after message $m$ is received if and only if

$$q_{DM}(1|m)1 + (1 - q_{DM}(1|m)0 + \delta_{DM} V_{DM}(1|m) \geq q_{DM}(1|m)0 + (1 - q_{DM}(1|m))1 + \delta_{DM} V_{DM}(-1|m). \tag{12}$$

This can be written in a more intuitive form as follows

$$q_{DM}(1|m) \geq \frac{1}{2} - \frac{\delta_{DM}}{2} \triangle REP_{DM}(m),$$

where $\triangle REP_{DM}(m)$ is the expected reputational payoff increase or decrease the DM will receive if he decides to act instead of not-acting after message $m$. It follows from lemma 2 that

$$\triangle REP_{DM}(m) = \sigma^2_{DM} \sum_x q(x|m) \Big( \frac{xE(s|1)}{1 + xE(s|1)Ep(\theta)} - \frac{xE(s|-1)}{1 + xE(s|-1)Ep(\theta)} \Big).$$

The term $\triangle REP_{DM}(m)$ depends on three things: (i) the DM's belief, conditional on message $m$ being observed, about the probability that the state is time-to-act ; (ii) how the Observer thinks the Expert will use his information, which is captured by the Observer's conjecture about the Expert's behavior; and (iii) how the Observer thinks the DM will use his information, which is captured by the Observer's conjecture about the DM's behavior. These last two points are captured by the expected value of the signal conditional on project $d$ being chosen, $E(s|d)$ (again, see equation (6)).

There are basically two cases to consider. First, if the Observer believes the Expert will recommend action whenever his signal suggests he should act and will recommend the status-quo when his signal recommends inaction, and also believes that the DM will follow the Expert's recommendation with probability 1. In this case the reputational penalty is the same after any mismatch between the state and the project chosen which implies that $\triangle REP_{DM}(m) = \triangle REP_E(m) = 0$ for $m \in \{-1, 1\}$, and therefore:[6] (i) the Expert will recommend acting if and only if he thinks acting is more appropriate; that is, $q(1|1) \geq 1/2$ and $q(1|-1) < 1/2$; and (ii) the DM will act if and only if the Expert recommends action. The Expert indeed recommends action whenever his signal suggests he should, and the DM indeed follows the Expert's recommendation if his prior is close enough to $1/2$. In this case the Expert's signal is informative enough so that his posterior, conditional on the observed signal, is greater than $1/2$ after the signal suggests it is time to act and lower than $1/2$ after the signal suggests it is not time to act. It is also the case that the DM's posterior, conditional on the observed message, is greater than $1/2$ after the message time-to-act and lower than $1/2$ after the message time-to-be-cautions.

The second case is more subtle, but it is at the crux of the model's main result. Suppose that the DM

[6]The fact that these are both zero is the result of the symmetry in terms of the payoff that each decision has when it matches the state. Imposing asymmetric payoffs will increase the algebraic burden without further gain in intuition.

believes that the probability that the state is time-to-act is close $1/2$ (i.e., $q_{DM}(1) \approx 1/2$), as in the previous case, but now suppose the Expert's prior about the state time-to-act is such that even after the signal suggests it is time to act, the Expert's posterior belief, conditional on this signal, is smaller than $1/2$ (this requires the prior to be lower than $1/2$). If he were allowed to choose projects, he would choose not to act, and therefore his actions would not reveal any information. However, because the DM chooses the project, he may nevertheless recommend acting, but only if $\triangle REP_E(1) > 0$, so that the Expert's posterior after $s = 1$ is between $\frac{1}{2} - \frac{\delta_E}{2}\triangle REP_E(1)$ and $1/2$.

The key question then becomes: When will it be the case that there exists an equilibrium where $\triangle REP_E(1) > 0$? The answer is that $\triangle REP_E(1) > 0$ when the Observer thinks that acting when it is time to act is more positive news about the Expert's expertise than it is not acting when it is time to be cautious. This requires that the Observer believes that (i) the Expert will recommend action whenever his signal suggests he should; (ii) the DM will act whenever the Expert recommends he should do so; i.e., the DM's posterior belief given a message saying it is time to act is higher than or equal to $\frac{1}{2} - \frac{\delta_{DM}}{2}\triangle REP_{DM}(1)$; and (iii) the DM will sometimes act when the Expert recommends him that it is time not to act; i.e., the DM's posterior belief given a message saying it is time not to act is equal to $\frac{1}{2} - \frac{\delta_{DM}}{2}\triangle REP_{DM}(-1)$. In this case, if the Observer sees that the DM acted when it is time to be cautious (i.e., $x = -1$), the Observer only updates slightly negatively about the Expert's expertise, since even if the Expert recommended that it is time to act, the DM might not have followed the Expert's advice. In contrast, if the Observer sees that the DM failed to act when it is time to act, the Observer updates much more negatively about the Expert's expertise, since the DM would only fail to act if he was persuaded through a recommendation against acting.

By biasing the project rule in favor of acting, the DM therefore can create a reputation penalty for the Expert that is large enough to get him to truthfully reveal his information. And, more importantly, the DM is willing to bias the project rule in favor of acting only if he also cares about the Expert's reputation–if he did not, then he would rubber stamp the Expert's recommendation if he though the Expert was fully revealing his information. Proposition 1 shows that in the most informative equilibrium either the first case holds, or the second case (or its mirror opposite) holds, or the Expert reveals no information. Figure 2 below depicts the most-informative equilibrium in the $(q_{DM}(1), q_E(1))$-space

Notice also that the Expert's reputation $f(\theta|d, x)$ is equal to the prior $f(\theta)$ when the Observer conjectures that Expert's message does not reveal any information regardless of his conjecture about the DM's strategy. The reason is that the Observer anticipates that the DM's project was made entirely based on his

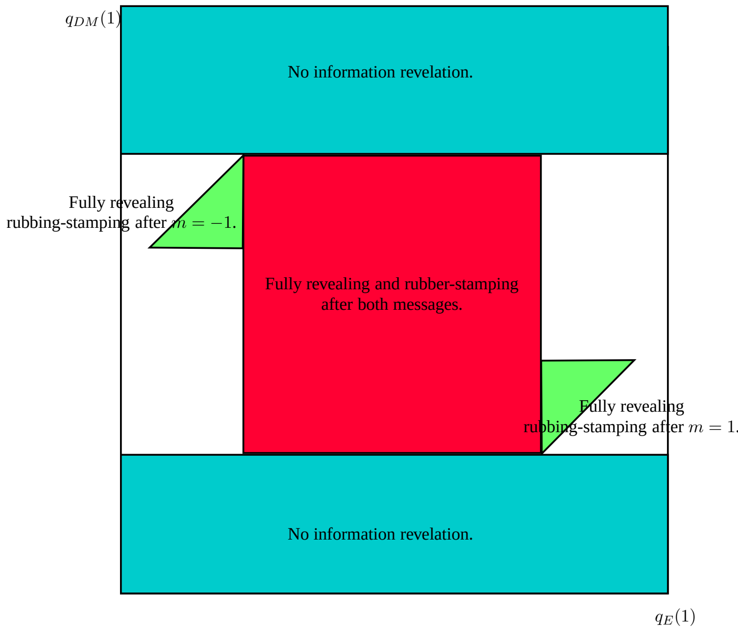


**Fig. 2.** Most Informative Equilibrium

prior belief and therefore, according to the Observer, the DM's project conveys no information regarding the Expert's privately observed signal.

In the Appendix it is shown that the Expert sends a message saying that is time to act if $q_E(1|s) \geq \triangle_E(E(s|1), E(s|{-1}))$ and the Expert chooses acting after message $m$ if $q_{DM}(1|m) \geq \triangle_{DM}(E(s|1), E(s|{-}1))$. A crucial property of this function (see the Appendix) is that it is increasing in $(E(s|1), E(s|-1))$ for all $(E(s|1), E(s|-1)) \in [0,1] \times [-1,0]$. The next proposition focuses on the most informative equilibrium and a full characterization of the equilibrium is presented in Appendix A. Mainly, there it is shown that that there are priors under which a fully revealing and partially revealing equilibrium coexist.

**Proposition 1.** *Suppose communication is private. Then,*

*i) There exists a fully revealing equilibrium in which the DM rubber stamps the Expert's recommendation if and only if for all $i \in \{E, DM\}$,*

$$\frac{(1 - Ep(\theta))\triangle_i(1,-1)}{1 + Ep(\theta) - 2Ep(\theta)\triangle_i(1,-1)} \leq q_i(1) \leq \frac{(1 + Ep(\theta))\triangle_i(1,-1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_i(1,-1)}.$$

*ii) A fully revealing equilibrium in which the DM rubber-stamps the Expert's recommendation after a* *not-time-to-act recommendation and randomizes after a time-to-act recommendation exists if and only if*

$$\frac{(1+Ep(\theta))\triangle_E(1,E(s|-1)^*)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,E(s|-1)^*)} \geq q_E(1) > \frac{(1-Ep(\theta))\triangle_E(1,E(s|-1)^*)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(1,E(s|-1)^*)}$$

*and*

$$\frac{(1+Ep(\theta))\triangle_{DM}(1,0)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,0)} > q_{DM}(1) > \frac{(1-Ep(\theta))\triangle_{DM}(1,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_{DM}(1,-1)},$$

*where $E(s|-1)^*$ is the unique solution to*

$$q_{DM}(1) = \frac{(1-Ep(\theta))\triangle_{DM}(1,E(s|-1)^*)}{1+Ep(\theta)-2Ep(\theta)\triangle_{DM}(1,E(s|-1)^*)}.$$

*iii) A fully revealing equilibrium in which the DM rubber-stamps the Expert's recommendation after a time-to-act recommendation and randomizes after a not-time-to-act recommendation exists if and only if*

$$\frac{(1+Ep(\theta))\triangle_E(E(s|1)^*,-1))}{1-Ep(\theta)+2Ep(\theta)\triangle_E(E(s|1)^*,-1)} \geq q_E(1) > \frac{(1-Ep(\theta))\triangle_E(E(s|1)^*,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(E(s|1)^*,-1)}$$

*and*

$$\frac{(1+Ep(\theta))\triangle_{DM}(1,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,-1)} > q_{DM}(1) > \frac{(1+Ep(\theta))\triangle_{DM}(0,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,-1)},$$

*where $E(s|1)^*$ is the unique solution to*

$$q_{DM}(1) = \frac{(1+Ep(\theta))\triangle_{DM}(E(s|1)^*,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(E(s|1)^*,-1)}.$$

*iv) There exists a partially revealing equilibrium only if there exists a fully revealing equilibrium.*

*v) Else, no revealing equilibrium exists.*

This proposition shows that when communication is private and the Expert and the DM' priors are slightly biased in either direction, in the most informative equilibrium the DM rubber stamps the Expert's recommendation and the latter recommends the project that matches his signal for each signal. In short, the Expert's private information is truthfully transmitted to the public (Observer). When the Expert's prior is biased even further towards acting, but the DM's prior remain slightly biased, the DM rubber stamps a non-acting recommendation, and mixes after an acting recommendation. The same happens when the

Expert's prior is biased even further towards non-acting and the DM's prior remains slightly biased. Hence, when the Expert's prior is sufficiently biased towards a given state and the DM is slightly biased, the DM, by ignoring the Expert's advice with positive probability, can induce the Expert to truthfully reveal his private information. In other words, the DM, by filtering the information revealed by the Expert that the Observer can see, is able to create reputational gain and loses that induce the Expert to truthfully reveal his information. This shows that keeping the Expert's advice secret provides the Expert with stronger incentives to truthfully reveal his private information. In fact we have the following corollary.

**Corollary 1.** *Suppose communication is public. Then,*

*i) There exists a fully revealing equilibrium if and only if for all* $i \in \{E, DM\}$*,*

$$\frac{(1 - Ep(\theta))\triangle_i(1,-1)}{1 + Ep(\theta) - 2Ep(\theta)\triangle_i(1,-1)} \leq q_i(1) \leq \frac{(1 + Ep(\theta))\triangle_i(1,-1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_i(1,-1)}. \tag{13}$$

*In this equilibrium the DM rubber stamps the Expert's recommendation.*

*ii) There exists a partially revealing equilibrium only if there exists a fully revealing equilibrium.*

*iii) Else, no revealing equilibrium exists.*

To understand this, it is worthwhile to notice that the main difference with private communication is that under the latter $E(s|d)$ must be replaced by $E(s|m)$ in the function $\triangle_{DM}(E(s|1), E(s|-1))$, where

$$E(s|m) = \frac{\sum_s \hat{h}(m|s)s}{\sum_s \hat{h}(m|s)}. \tag{14}$$

Hence, the DM cannot affect the Expert's reputation by ignoring the Expert's advice. The reason is that the Expert's reputation is independent of the DM's project choice since messages do not add more information to the Observer's inference process. This means that the DM chooses project 1 after message $m$ if and only if $q_{DM}(1|m) \geq 1/2$ and project $-1$ when $q_{DM}(1|m) < 1/2$; he chooses project 1 regardless of the message when $q_{DM}(1|-1) \geq 1/2$; and he chooses project -1 regardless of the message when $q_{DM}(1|1) < 1/2$. This implies that the DM rubber stamps the Expert's message when the inequality in equation (13) holds for $q_{DM}(1)$.

If the Observer conjectures that the Expert will send a message equal to his signal regardless of the signal received, then the Expert will do so if and only if $q_E(1|1) \geq 1/2$ and $q_E(1|-1) < 1/2$, since the reputation penalty after a mismatch between the state and project 1 is the same as that after a mismatch between the state and project -1. Thus, a fully-revealing equilibrium occurs in this case only when the inequality in equation (13) holds also for $q_E(1)$.

If the Observer conjectures that the Expert will send a message equal to his signal regardless of the signal received and the inequality in equation (13) holds for $q_{DM}(1)$, but it does not hold for $q_E(1)$, then the Expert chooses to ignore his signal because he is convinced that he is more likely to be right ex-post by sending the same message after either signal; mainly, the Expert will choose message 1 if and only if $q_E(1|-1) \geq 1/2$, which occurs when the Expert's prior belief about the state being time-to-act is sufficiently large and message -1 if and only if $q_E(1|1) < 1/2$, which occurs when the Expert's prior belief about the state being time-to-be-cautious is sufficiently large. Hence, whenever messages are public information and the inequality in equation (13) does not hold, the Expert does not have any incentives to reveal his private information.

This together with the result in proposition 1 gives rise to the following result.

**Proposition 2.** *Under private communication between the Expert and the DM and in the most informative equilibrium, the set of priors* $(q_E(1), q_{DM}(1))$ *under which information revelation takes place is greater than that under public communication.*

Here it is a good place to compare our results to those in Ottaviani and Sorensen (2006b). They study a reputational cheap-talk model in which an Expert, concerned about appearing to be well informed in the eyes of the DM, is asked to provide advice to an Expert. The Expert, as in here, is assumed to observe a private signal with a simple and particularly tractable (multiplicative linear) structure. The Expert's reputation is based on the message he sends and the realized state of the world. The main difference stands for the fact that the reputation here is based on the project chosen and realized state and the DM and Expert have non-common priors. Under private communication, these two things makes the equilibrium different since in their most informative equilibrium either there is full revelation or nor information revelation at all.[7] Hence, their result is equivalent to the one under public communication since in that case the DM cannot garble the signal that the Observer gets and therefore it is as if the Observer plays no role.

In the next two propositions we do comparative statics of the most informative equilibrium for the case in which messages are private information.

[7] In Ottaviani and Sorensen (2006a) the project and state space are continuous. There, they show that the most informative equilibrium never entails full information revelation. The same will occur here with a continuous project and state space.

**Proposition 3.** *Suppose communication is private. Then*

*i) If the most informative equilibrium is such that the DM randomizes after message $m = 1$ and rubber stamps the Expert's recommendation after $m = -1$. Then, the set of priors under which the DM rubber stamps the Expert's advice rises with $(\delta_{DM}, \sigma^2_{DM})$ and falls with $q_{DM}(1)$.*

*ii) If the most informative equilibrium is such that the DM randomizes after message $m = -1$ and rubber stamps the Expert's recommendation after $m = 1$. Then, the set of priors under which the DM rubber stamps the Expert's advice rises with $(\delta_{DM}, \sigma^2_{DM})$ and falls with $q_{DM}(-1)$.*

The intuition for part (i) is as follows. As either $\delta_{DM}$ or $\sigma^2_{DM}$ or both increase, the DM cares more about the Expert's reputation about being well-informed. In this scenario, biasing less the decision rule towards non-acting (i.e., towards $d = -1$), decreases the reputation penalty that the Expert will receive in the case the outcome $(d, x) = (1, -1)$ is observed, which is consistent with the DM's concern with the Expert's reputational payoff. Less intuitive is the fact that as the DM's prior is less biased towards state -1, ceteris-paribus, he is less likely to follow the Expert's advice when this goes against his prior. Recall that the DM's prior is biased towards state -1 (i.e., $q_{DM}(1|1) < 1/2$) and as $q_{DM}(1)$ rises, according to the DM the worst outcome $(d, x) = (1, -1)$ is less likely to take place. In this case biasing more the decision rule towards non-acting (i.e., towards $d = -1$), increases the reputation penalty that the Expert will receive in the case the outcome $(d, x) = (1, -1)$ is observed. Because the DM believes this is less likely to take place and he is concerned with the Expert's payoff, he must rubber-stamp the Expert's advice less often in order to induce him to reveal his information.

**Proposition 4.** *Suppose communication is private. Then, the set of the DM's prior beliefs under which the most informative equilibrium in which the Expert truthfully reveals his information and the DM randomizes after one of the two messages and rubber stamps the Expert's message after the other rises with $(\delta_{DM}, \sigma^2_{DM})$ and is independent of $(\delta_E, \sigma^2_E)$. While that set for the Expert rises with $(\delta_E, \sigma^2_E)$ and falls with $(\delta_{DM}, \sigma^2_{DM})$*

The former is due to the envelope theorem and the fact that an increase in $(\sigma^2_{DM}, \delta_{DM})$ implies that the DM cares more about the Expert's reputation. Therefore when choosing a project, the DM takes more into account, according to his prior, the impact that his decision* have on the Expert's reputation. This means that the DM puts relatively more weight to the Expert's reputational loss/gain from any given project. The Expert's weight on his reputational payoff does not affect the DM's set of prior beliefs under which he randomizes since the Expert truthfully reveals his information. In contrast, the Expert's set of priors under which he truthfully reveals his information when the DM randomizes after one of the messages and rubber stamps the Expert's recommendation after the other rises with $(\delta_E, \sigma_E^2)$ since this means that the Expert is more concerned with his reputational gain/loss from any given message. This set falls with $(\delta_{DM}, \sigma_{DM}^2)$ because the DM's randomizes after a message that contradicts his prior with a lower probability, which means that the Expert's expect reputational loss from the outcome $d \neq x$ is lower.

# 4 Information Acquisition Effort

So far we have assumed that the Expert's quality of information is exogenously given and have shown that information is truthfully revealed for a larger set of priors when communication is private than when it is public. Two questions arise naturally: Does the Expert have an incentive to invest costly effort to improve the quality of his information, and Are the incentives to improve the quality of information stronger under private or public communication?.

In order to answer these questions we assume that the Expert can, before getting his private signal, exert a non-observble effort in order to increase the probability that the signal acquired comes from the informative distribution $g(s|x)$. In particular, the Expert's signal has a conditional probability density function

$$g(s|x, \theta, e) = p(\theta, e)g(s|x) + (1 - p(\theta, e))h(s) = p(\theta, e)\frac{1 + sx}{2} + (1 - p(\theta, e))\frac{1}{2}, \tag{15}$$

where $e \in \mathcal{E} \equiv [0, \bar{e}]$ is the Expert's unobservable effort and $p(\theta, e)$ satisfies the following properties.

**Assumption 1.**

*i)* $\forall(\theta, e) \in (\Theta, \mathcal{E}),\ \ p(\theta, e) \in (0, 1)$ *and* $p_\theta(\theta, e) > 0$.

*ii)* $\forall(\theta, e) \in (\Theta, \mathcal{E}),\ \ p_e(\theta, e) > 0$ *and* $p_{ee}(\theta, e) < 0$.

*iii)* $\forall\theta \in \Theta,\ \ \lim_{e\to\bar{e}} p_e(\theta, e) \to 0$ *and* $\lim_{e\to 0} p_e(\theta, e) > 1$.

The first part establishes that the Expert's probability to get a signal from the informative distribution is positive and lower than 1 for all effort levels, and it is increasing in the Expert's type regardless of the effort level chosen. The second part imposes that the probability to get a signal from the informative distribution is a strictly concave function of effort. The third part asserts that this probability satisfies standard Inada's conditions. These are meant to guarantee existence of an interior equilibrium.

Effort entails a private cost to the Expert equals to the strictly increasing and convex function $c(e)$, with $c(0) = c_e(0) = 0$.

Let $\hat{e}$ be the DM and the Observer's conjecture about the Expert's effort, $\sigma_i^2(\hat{e})$ be the covariance between $v_i(\theta)$ and $p(\theta, \hat{e})$, $\mathbb{I}_{d=x}$ an indicator function that takes the value 1 when $d = x$ and 0 otherwise and $(h^*(m|s), z^*(d|m))$ be the equilibrium strategy profile in the most informative equilibrium in the continuation game. Then, the Expert will choose effort $e \in \mathcal{E}$ to maximize his expected utility given by:

$$V_E(e, \hat{e}) \equiv \sum_s p(s|e) \sum_d \sum_m h^*(m|s) z^*(d|m) \big( \sum_x q_E(x|s, e) \mathbb{I}_{d=x} + \delta_E V_E(d|s, e, \hat{e}) \big) - c(e), \tag{16}$$

where

$$p(s|e) = \frac{1}{2} \big( 1 + \sum_x sxEp(\theta, e) q(x) \big),$$

$$V_E(d|s, e, \hat{e}) = \sum_x q_E(x|s, e) E(v_E(\theta)|d, x, \hat{e}),$$

$$E(v_E(\theta)|d, x, \hat{e}) = Ev_E(\theta) + \sigma_E^2(\hat{e}) \frac{xE(s|d)}{1 + xE(s|d)Ep(\theta, \hat{e})},$$

and

$$q_E(x|s, e) = \frac{\big(1 + sxEp(\theta, e)\big) q(x)}{1 + \sum_x sxEp(\theta, e) q(x)}.$$

The continuation game after the Expert chooses his information-acquisition effort is exactly the same as the one already derived in the last section and therefore for any conjecture about the Expert's effort, the equilibrium in the continuation game is given by that in Proposition 1, but with $\triangle_i(E(s|1), E(s|-1))$ redefined to incorporate the conjectured effort. We denote this new function by $\triangle_i(E(s|1), E(s|-1), \hat{e})$.

The first-order condition is given by

$$\begin{aligned} &\sum_s \sum_d \sum_m h^*(m|s) z^*(d|m) \Big( \frac{\partial p(s|e)}{\partial e} \big( \sum_x q_E(x|s, e) \mathbb{I}_{d=x} + \delta_E V_E(d|s, e, \hat{e}) \big) + \\ &p(s|e) \Big( \sum_x \frac{\partial q_E(x|s, e)}{\partial e} \mathbb{I}_{d=x} + \delta_E \frac{\partial V_E(d|s, e, \hat{e})}{\partial e} \Big) \Big) - c_e(e) = 0 \end{aligned} \tag{17}$$

where

$$\frac{\partial V_E(d|s,e,\hat{e})}{\partial e} = \sum_x \frac{\partial q_E(x|s,e)}{\partial e} E(v_E(\theta)|d,x,\hat{e}).$$

The first term is the change in the probability that the signal received comes from the informative distribution $g(s|x)$ when effort increases times the Expert's expected payoff for any given signal $s$ and this added over all possible signals; the second term is the probability that the signal is $s$ conditional on effort $e$ times the change in the short-run payoff and the expected reputational payoff for any given signal $s$ added over all possible signals. Since the effort $e$ used to calculate the reputational payoffs is the equilibrium belief about the Expert's choice of effort $\hat{e}$, the Expert cannot influence these reputational payoffs. However, the Expert's choice of $e$ does influence the relative likelihoods of the potential reputational payoffs; higher effort makes reputational payoffs in which the project matches the state more likely to take place; and the third term is the marginal cost of effort.

First, suppose that a fully revealing equilibrium in which the DM rubber-stamps the Expert recommendation will be played in the continuation game; that is, $h^*(1|1) = h^*(-1|-1) = z^*(1|1) = z^*(-1|-1) = 1$. Then, the first-order condition in equation (17) re-writes as follows:

$$\Psi(e,\hat{e}) \equiv \frac{1}{2} Ep_e(\theta,e)\Big(1 + \delta_E \frac{2}{1 - Ep(\theta,\hat{e})^2}\sigma_\theta^A(\hat{e})\Big) - c_e(e) = 0. \tag{18}$$

Because $Ep(\theta,e)$ is concave in $e$ and $c(e)$ is convex in it and the term in parenthesis in equation (18) depends only on the DM and Observer's belief about the Expert's effort (i.e., $\hat{e}$), for any belief that they may hold, the Expert's effort is uniquely determined by the first-order condition in equation (18). Given this and the Inada's condition, we know the solution belongs to the interior.

An equilibrium then is a fixed point of $\Psi(\cdot)$; i.e., $\Psi(e^*,e^*) = 0$. The existence of an equilibrium is guaranteed by the Inada's conditions and continuity of $\Psi(e,e)$ with respect to $e$. This leads to the following result.

**Lemma 3.** *Suppose assumption 1 holds and the equilibrium in the continuation game is fully revealing and the DM rubber-stamps the Expert's recommendation. Then, there exists at least one positive equilibrium choice of effort, denoted by $e^*$, that represents the Expert's privately optimal choice of effort satisfying $\Psi(e^*,e^*) = 0$.*

This establishes that a reputation-conscious Expert (i.e., $\delta_E > 0$) invests more in information acquisition than an Expert lacking this concern. The intuition is the following. The expected value of the Expert's reputation must be $Ev_E(\theta)$ in equilibrium. So if the Observer could observe $\theta$, the Expert's long-run expected utility would simply be $\delta_E Ev_E(\theta)$. This result follows from the fact that the Expert's objective function is a martingale with respect to beliefs because the payoffs are linear functions of the posterior beliefs (see Holmström (1999)). In this case, the first-order condition would be $\frac{1}{2}Ep_e(\theta,e) - c_e(e) = 0$, yielding the optimal effort in the absence of reputational concerns. The reason why $\frac{1}{2}Ep_e(\theta,e) - c_e(e) = 0$ does not give the Expert's choice of $e$ is that effort affects the probabilities of the Observer's relevant conditional expectations about the Expert's ability, without affecting the equilibrium belief $\hat{e}$. We know that the Expert's reputation increases– relative to the Observer's prior beliefs about the Expert's ability– only when the project matches the realized state, and the probability of this outcome occurring in an informative equilibrium is increasing in the Expert's effort $e$. Hence, the second term in (18)–which relates to the marginal impact of $e$ on the probabilities of more favorable reputational states–is strictly positive, which leads to $\frac{1}{2}Ep_e(\theta,e) - c_e(e) < 0$, implying that the effort level is higher than that in the absence of reputational concerns.

Observe also that this model admits multiple equilibria. To see the intuition behind the multiplicity of equilibria, note that the Expert's optimal choice of effort trades-off his private cost of effort against the increase in the probability of better reputation and higher expected short-run payoff as a result of choosing a project more likely to match the state. We have shown that in any equilibrium, the marginal cost of effort is positive, which means a positive marginal reputational benefit to the Expert of increasing his choice of effort. This reputational gain is affected by the effort anticipated by the Observer and the DM, which is what creates the possibility of multiple equilibria. Whether multiple equilibria occurs depends on the whether the function $\Psi(e^*,e^*)$, that determines $e^*$ is monotonic.

Since $p(\theta,e)$ is strictly concave and $c(e)$ is strictly convex, $\Psi(e^*,e^*)$ decreases monotonically with $e^*$ when the term in parenthesis in equation (18) falls with $e^*$ and may either rise or fall with it when that term rises with $e^*$. When $\Psi(\cdot)$ is monotonic, the equilibrium effort is unique, while when $\Psi(\cdot)$ is non-monotonic, there are multiple equilibria.[8] The economic intuition behind multiple equilibria here can be further illuminated by examining how the DM and the Observer's expectations lead to a self-fulfilling

[8] It is easy to check that $\Psi(\cdot)$ could be non-monotonic in $e^*$. Differentiating the term in parenthesis in $\Psi(\cdot)$ one gets that it is non-decreasing in $e^*$ if and only if

$$\frac{\partial\sigma_\theta^A(e)}{\partial e}|_{e=e^*} + \frac{2}{1-Ep(\theta,e^*)^2}\sigma_\theta^A(e^*)Ep(\theta,e^*)Ep_e(\theta,e^*) \geq 0.$$

prophecy. Suppose the DM and the Observer expect high effort. Then the reputational gain in equation (18), which is assumed increasing in effort, will be large, reflecting the fact that the Expert is willing to exercise more effort in order to pursue a reputation of being well informed. By the convexity of $c(\cdot)$ and strict concavity of $p(\theta, e)$, this is consistent with a high effort, and the fixed point needed for (18) to hold will be the high effort conjectured by the DM and the Observer. But a fixed point may also exist for a relatively low effort. At a lower effort, the reputational gain in equation (18) is smaller, reflecting a smaller benefit of pursuing reputation. Thus, the Expert is now less willing to exercise effort, which is consistent with a smaller effort, as conjectured by the DM and the Observer. Effort $e^*$ is an equilibrium of the whole game if the condition in part (i) in proposition 1 holds when $\triangle_i(E(s|1), E(s|-1))$ is substitute for $\triangle_i(E(s|1), E(s|-1), \hat{e})$.

Next consider the case in which a fully revealing equilibrium in which the DM follows the Expert advice after one of the two messages is sent and randomizes when the other is used; that is, $h^*(1|1) = h^*(-1|-1) = 1$ and either $z^*(-1|-1) = 1$ and $z^*(-1|1) \in (0,1)$ or $z^*(1|1) = 1$ and $z^*(1|-1) \in (0,1)$. The Expert's incentives with respect to his choice of effort in these two cases are identical and thus, for the sake of brevity, we will study the case in which $z^*(1|-1) = 1$ and $z^*(1|1) \in (0,1)$. This implies that $E(s|1) = 1$ and $E(s|-1) \in (-1,0)$. Then, the first-order condition in equation (17) re-writes as follows:

$$\begin{aligned}\dot{\Psi}(e, \hat{e}) \equiv \frac{1}{2} Ep_e(\theta, e) z^*(1|1) \Bigg( 1 + \delta_E \frac{2}{1 - Ep(\theta, \hat{e})^2} \Big( 1 - \frac{1 + E(s|-1)^*}{2} - \\ \frac{1}{2} \frac{(1 - (E(s|-1)^*)^2) Ep(\theta, \hat{e})(Ep(\theta, \hat{e}) E(s|-1)^* + 2q_E(1) - 1)}{1 - Ep(\theta, \hat{e})^2 (E(s|-1)^*)^2} \Big) \sigma_\theta^A(\hat{e}) \Bigg) - c_e(e) = 0.\end{aligned} \tag{19}$$

Because $Ep(\theta, e)$ is concave in $e$ and $c(e)$ is convex and the term in parenthesis in equation (19) depends only on the Observer's belief about the Expert's effort $\hat{e}$, the Expert's effort is uniquely determined by the first-order condition. Given this and the Inada's condition, we know the solution belongs to the interior. As before, an equilibrium then is a fixed point of $\Psi$; i.e., $\dot{\Psi}(\dot{e}, \dot{e}) = 0$. The existence of an equilibrium is guaranteed by the Inada's conditions and continuity of $\Psi(e, e)$ with respect to $e$.

An important difference with the case in which the DM rubber stamps the Expert's recommendation is that the short- and long-run benefit of being informed are smaller since the DM is less likely to act upon the information provided by the Expert. This stands for the fact that the DM does not always follows the Expert advice after one of the two messages is sent and therefore the Expert, anticipating this, realizes that the marginal value of being better informed is lower. As the probability that the DM follows the Expert's advice increases, the Expert is willing to exert more information-acquisition effort.

The first-order condition establishes that a reputation-conscious Expert ($\delta_E > 0$) invests more in information relative to an Expert lacking this concern. The intuition is the same as the one given in the case of a fully separating equilibrium and thus omitted

**Lemma 4.** *Suppose assumption 1 holds and the equilibrium in the continuation game entails truthful revelation of information and mixing by the DM. Then, there exists at least one positive equilibrium choice of effort, $\dot{e}$, that represents the Expert's privately optimal choice of effort satisfying equation* (19).

In this case also there could be multiple equilibria. To see the intuition behind the multiplicity of equilibria, note that the Expert's optimal choice of effort trades-off his private cost of effort against the increase in the probability of a better reputation due to that the project is more likely to match the state and higher expected short-run payoff from the project chosen. We argue that in any equilibrium, the marginal cost of effort is positive, which means a positive marginal reputational benefit to the Expert of increasing his choice of effort. This reputational gain is affected by the effort anticipated by the DM, which is what creates the possibility of multiple equilibria. Whether multiple equilibria occur depends on the whether the function $\dot{\Psi}(\cdot) = constant$, that determines $\dot{e}$ is monotonic. There is a unique equilibrium when the marginal reputational gain given by the term multiplied by $\delta_E$ inside the parenthesis falls as $\dot{e}$ rises, and the model might have multiple equilibria when the opposite occurs.

Since $p(\theta, e)$ is strictly concave and $c(e)$ is strictly convex, $\Psi(\dot{e}, \dot{e})$ decreases monotonically with $\dot{e}$ when the term in parenthesis falls with $\dot{e}$ and may either increase or decrease with $\dot{e}$ when the opposite holds. When $\Psi(\dot{e}, \dot{e})$ falls monotonically, the equilibrium is unique, while when the term in parenthesis raises, $\dot{\Psi}(\cdot)$ may be non-monotonic and we can obtain multiple equilibria.[9] The economic intuition is the same as the one already given above.

Effort $\dot{e}$ is an equilibrium of the whole game if the conditions in part (ii) and (iii) in proposition 1 holds when $\triangle_i(E(s|1), E(s|-1))$ is substitute for $\triangle_i(E(s|1), E(s|-1), \dot{e})$.

Finally, it is easy to show that when the equilibrium is babbling the Expert has no incentive to choose a positive information acquisition effort.

**Proposition 5.** *Suppose that the equilibrium effort is unique. Then, in the most informative equilibrium, the*

[9] It is easy to check that $\dot{\Psi}(\cdot)$ could be non-monotonic in $\dot{e}$.

*Expert's information acquisition effort under private communication is higher than or equal to that under public communication.*

This proposition is the result of the fact that the quality of information is relevant only when the Expert wants to reveal some information. When messages are publicly observed and the most informative equilibrium does not entail truthful revelation of information, the Expert chooses to exert no information acquisition effort. The reason is that he will send the same message regardless of the signal realized and therefore information is, from his point of view, irrelevant. In other words, from the Expert's point of view, information revelation and information acquisition are strategic complements. In contrast, when message are private, the Expert invests a positive effort in information acquisition since he intends to truthfully reveal his information and the DM will follow the Expert's advice with positive probability and thus information is valuable.

Mainly the result here suggests two things: first, it is better from the DM's point of view to keep messages private since this induces the Expert to acquire more information and to truthfully reveal his information for a larger set of priors; and second, Experts' incentives to acquire information are greater, the more likely is the DM to follow the Expert's advice.

## 5 Organizational Design

Our results have important implications for the issue of optimal allocation of authority within organizations. Consider the problem of allocating decision rights within an organization when the Expert have private information and information acquisition is costly. Mainly in our setting there are three ubiquitous organizational forms:

- *Centralization with Private Communication*: the DM keeps for himself the right to choose projects after receiving the Expert's advice and the Expert's advice is kept secret.
- *Centralization with Public Communication*: the DM keeps for himself the right to choose projects after receiving the Expert's advice and the Expert's advice is made public.
- *Delegation*: the Expert is endowed with the right to choose projects.

The question asked here is pervasive in all types of organizations. In fact, politicians as well as management experts generally advance the view that authority should be exercised by the better informed individuals since this is the superior organizational form to rip the benefits of information. This is clearly expressed by Alexander Hamilton in The Federalist No. 23's examination of defense policy and management. There he puts forward a rationale for transferring decision-making authority from less informed to more informed individuals, anticipating that the more knowledgeable ones would make better choices. Hamilton's view has persisted to the present day and is pervasive in the study of bureaucracy and management. For instance, Wilson (1989) in his famous book *Bureaucracy* argues that "In general, authority should be placed at the lowest level at which all essential elements of information are available" and, Andrew Carnegie, the founder of Carnegie Mellon among many other things, one said "No person will make a great business who wants to do it all himself or get all the credit".

The economics literature, based on models where differing preferences play a crucial role, also asserts the superiority of letting better-informed individuals (experts) to exercise real authority. Mainly, Aghion and Tirole (1997) argue that delegation understood as real authority provides stronger incentives for information acquisition, Dessein (2002) shows that delegation favors information revelation and Aghion et al. (2004) contend that transferable control creates more incentives for information revelation with respect to the Expert's ability through the way in which the Expert exercises control. In these papers the trade-off is one of a loss of control under delegation against an information loss under centralization. They all provide different mechanisms under which the payoff consequences for the DM of a loss of control are lower than those of a loss of information.

The first thing to have in mind is that the equilibrium under delegation is basically the same as that under public communication. The reason stands for the fact that under delegation the Observer gets to see the Expert's choice of projects and therefore the Expert's reputation is based on his project choice and the realized state and not on the message sent and the realized state. It is straightforward to show the following

**Corollary 2.** *Suppose that the Expert is endowed with decision rights. Then,*

*i) There exists a fully revealing equilibrium if and only if,*

$$\frac{(1-Ep(\theta))\triangle_E(1,-1,e^*)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(1,-1,e^*)} \leq q_E(1) \leq \frac{(1+Ep(\theta))\triangle_E(1,-1,e^*)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,-1,e^*)}. \tag{20}$$

*ii) There exists a partially revealing equilibrium only if there exists a fully revealing equilibrium.*

*iii) Else, no revealing equilibrium exists.*

In this case, for any effort $e$, the Expert chooses project 1 when he receives signal $s$ if and only if

$$q_E(1|s)1 + (1 - q_E(1|s))0 + \delta_E V_E(1|s) \geq q_E(1|s)0 + (1 - q_E(1|s))1 + \delta_E V_E(-1|s).$$

This can be written after a few steps of simple algebra in the more amenable form

$$q_E(1|s) \geq \frac{1}{2} - \frac{\delta_E}{2} \triangle REP_E(s), \tag{21}$$

where $\triangle REP_E(s)$ is the expected reputational payoff increase or decrease the Expert will receive if he decides to choose the new project instead of the status-quo project after signal $s$. As done before from this equation we can derive $\triangle_E(1, -1, e^*)$, which is identical to that given in equation (3), but her $E(s|d)$ is given by

$$E(s|d) = \frac{\sum_s \hat{r}(d|s)s}{\sum_s \hat{r}(d|s)}. \tag{22}$$

where $\hat{r}(d|s) \in [0, 1]$ is the DM and Observer's conjecture about the strategy used by the Expert, denoted by $r(d|s) \in [0, 1]$.

Hence, $\triangle REP_E(s)$ depends only on how the Observe thinks the Expert will use his information. If the Observer believes the Expert will act only when it is time to act and he will not only when it is time to be cautious $\triangle REP_E(s) = 0$, and therefore the Expert will act if and only if he thinks acting is more appropriate. He indeed will act whenever his signal suggests he should if his prior $q_E(1)$ is close enough to $1/2$ and his posterior after $s = 1$ is greater than 1 (i.e., $q_E(1|1) \geq 1/2$) and his posterior after $s = -1$ is lower than $1/2$. This condition, as shown in the corollary above, is therefore sufficient for there to be an equilibrium in which the Expert fully reveals his information through his project choice.[10] When this condition does not hold, the Expert chooses the same project regardless of the signal received because he wants to maximize his change of being right ex-post since his prior is sufficiently biased so that either $q_E(1|-1) \geq 1/2$ or $q_E(1|1) < 1/2$. This is reminiscent of Prat's (2005) Proposition 2, which is discussed in the last section in more detail.

Observe that this equilibrium is different from that under centralization with public communication since the Expert chooses his preferred project when information revelation does not occur which might be different from the DM' preferred project. In fact, if the DM's prior $q_{DM}(1)$ is sufficiently biased towards

[10] It is easy to show that this is also necessary. The proof is available upon request.

state 1 and the Expert's prior $q_E(1)$ is sufficiently biased towards state -1, under public communication the DM will choose project 1 and under delegation the Expert will choose project -1 and if the opposite holds, under public communication the DM will choose project -1 and under delegation the Expert will choose project 1.

The next proposition readily follows from Proposition 1, corollaries 1 and 2.

**Proposition 6.** *In the most informative equilibrium under each organizational design, the DM (weakly) prefers centralization with private communication to both centralization with public communication and delegation.*

The reason is straightforward, centralization with private communication elicits at least as much information revelation and acquisition effort than centralization with public communication and delegation, and there is no loss of control. Hence, the usual trade-off between loss of control and loss communication does not arise in our setting.

This seems the right place to compare my results to Dessein (2002).[11] The result in Proposition 1 is different from the equivalent one in Dessein (2002). He shows that the DM prefers delegation to communication when the Expert's bias is such that informative communication is feasible and delegation dominates uninformed centralization when the bias is not too large. The reason stands for the fact that here the Expert's incentives are fully aligned with those of the DM regardless of the reputational concerns, and the agency conflict arise from the Expert's imperfect information and non-common priors, while in Dessein (2002) the conflicts of interest arise from divergence of preferences or objectives. Thus, Dessein (2002) focuses on conflicts of interest from divergence of objectives with informed agents, while here the conflict of interest arises from the lack of relevant information and the fact that the DM and Expert have different *mental models* of how the world works despite of their rationality. Thus, Bayesian persuasion, understood as changing the DM's behavior through changing his beliefs, plays no role in Dessein (2002). Our results do not argue against the tenant of Dessein (2002), our results do, however, caution against the conclusion that delegation is the superior organizational form when there is information revelation since this depends crucially on the details of the model studied; in Dessein's (2002) model delegation trades-off the loss of control from del-

[11] Harris and Raviv (2005) study optimal communication and allocation of authority in an organization where both the DM and the Expert are privately (and costlessly) informed. Under centralization, the Expert sends a noisy message to the DM who then decides, whereas under delegation, the DM sends a noisy message to the Expert who decides. Harris and Raviv (2005) show that the probability of delegation increases with the importance of the Expert's information and decreases with the importance of the DM's information. Hvide and Kaplan (2003) develop a similar idea in a slightly different model.

egation against loss of information under centralization, while here delegation may result in both a loss of control and loss of information.

The result here is also different from that in Che and Kartik (2009). The result in their paper hinges on the fact that centralization induces the Expert to exert more information-acquisition effort in order to persuade the DM and avoid prejudice. The former means that information reduces the DM's interim bias and the later means that the DM's incentives to choose an action closer to his preferred action is higher when the Expert claims to be uninformed. Under delegation the extra incentives to acquire information due to the persuasion and prejudice effect vanished, yet an initiative effect as the one documented by Aghion and Tirole (1997) arises. However, the loss of information due to the vanishing persuasion and prejudice effect more than compensate the gain in information due to the initiative effect. In sum, they find that non-common priors entail a loss of information through strategic communication, but result in more powerful incentives for information acquisition, while here centralization result in an information gain through strategic communication. Thus, we provide a different mechanism by which non-common priors favor the optimal retention of control rights by the DM.

# 6 Discussion and Conclusions

Before concluding it is useful to discuss in more detail some aspects of the model.

We have considered so far the case of partisan ally (i.e., $\delta_{DM} > 0$) and here we briefly discuss partisan rivalry (i.e., $\delta_{DM} \leq 0$).[12] This is like having a partisan dimension. Partisan rivalry is common in politics when the DM and Expert belong to different factions of their corresponding political party or to different political coalitions. For instance, congressman are sometimes accused of being overly acquiescent to executive demands and others for being needlessly obstructionist. The political science literature argues that these inefficiencies may be the result of the existence of partisanship of different kinds for different issues. One may correctly conjecture that the most informative equilibrium in which the Expert truthfully reveals his information and the DM rubber-stamps the Expert's recommendation is still the most informative equilibrium for the same parameterization. The reason is that these strategies are mutually best responses when priors belong to an interval around 1/2 that is independent of $\delta_i$ for $i \in \{E, DM\}$. When the Expert's prior belief belongs to the interval considered above, but the DM's prior is slightly outside of it, it readily follows

[12] For the analysis carried out here, we require that $\delta_{DM}$ is such that $2 + \delta_{DM}\big(E(v_{DM}(\theta)|1,1) + E(v_{DM}(\theta)|-1,-1) - E(v_{DM}(\theta)|1,-1) - E(v_{DM}(\theta)|-1,1)\big) \geq 0$.

from lemma 9 in Appendix 1 that the equilibrium in which the DM rubber-stamps the Expert's recommendation and the Expert randomizes after a signal that contradicts his prior is observed and truthfully reveals his signal when this supports his prior is the most informative equilibrium when $\delta_{DM} \leq 0$ for a given set of priors. In this case the DM rubber-stamps the Expert's recommendation after one of the messages since it provides supports to his prior and he rubber-stamps the Expert's recommendation after a message that contradicts his prior. The latter occurs because the DM believes that the Expert's recommendation is likely to be mistaken ex-post and therefore following the Expert's advice harms the Expert's reputation, which increases the DM's payoff. The Expert, aware of this, reveals his signal when this supports his prior and randomizes after a signal that contradicts his prior in order to avoid getting the reputation penalty due to the Observer believing that he is not well-informed due to the DM's rubber-stamping behavior. However, he reveals some information because that persuades the DM to choose the project that the Expert believes is more likely to be optimal. When the DM's prior is too extreme, he is not willing to follow the Expert's advice because of his short-run concern. This induces the DM to choose the project that he believes is more likely to be correct ex-post. Hence, the DM's partisan rivalry rather than inducing him to contradict the Expert, counter intuitively it provides him with incentives to conform with the Expert to make him to look, in the eyes of the Observer, poorly informed.

Other assumptions in the model that can be modified are: (i) the Expert knows his ability; (ii) there are not only differing priors, but also different underlying preferences; (iii) the Observer neither observe the state $x$ nor the project chosen $d$, but he observes the consequences; that is, $u_i(x, d)$ for $i \in \{E, DM\}$; and (iv) there are also non-common priors regarding the distribution of the Expert's ability to be well informed, $g(\theta)$. Dealing with all these will require another paper in its own, yet we will briefly comment on these based on what we have learned from the literature. From Ottaviani and Sorensen (2006a), we know that when the Expert knows his own ability, he has an incentive to send more extreme messages in order to signal ability, but in equilibrium the more informed Experts are forced to be more often biased towards the expected. This suggests that the Expert, being aware of his ability, wants to reveal his information so that the Observer can have a better appraisal of his ability, and a well-informed Expert have more extreme priors and therefore he wants to persuade the DM to chooses his preferred action. With regard to the second point is enlightening to note that when the underlying preferences differ, but there are common priors, Bayesian persuasion no longer plays a role, which is crucial for our results. Hence, incorporating differing underlying preferences in our setting combines Bayesian persuasion with the standard incentives problems arising from cheap-talk with differing underlying preferences. We conjecture that this will result in a smaller set of priors under which the equilibrium is fully revealing and may also result in that the most informative equilibrium entails the Expert partially revealing his information for certain set of priors. With regard to the third point, we know from Prat (2005) that a DM can be hurt from observing more information about the Expert. Mainly, he shows that when there is not only information about the consequences of an Expert's actions, but also about the actions themselves, the Expert faces an incentive to disregard useful private signals and act according to how an able Expert is expected to act a priori. This conformist behavior hurts the DM in two ways: the project chosen by the Expert is less likely to be the right one (discipline) and ex post it is more difficult to evaluate the Expert's ability (sorting). We conjecture from this result that if only the consequences are observed, Bayesian persuasion will still play a role as done here, yet it is likely that the set of priors under which the most informative equilibrium entails full revelation will be smaller since the reputation penalties for a mismatch between the project and the state must be the same regardless of the project chosen and realized state. This limits the DM's ability to elicit information by ignoring sometimes the Expert's advice. Finally, it is easy to see that having non-common priors with respect to the Expert's ability does not change the results qualitatively.

This paper provides two main lessons: first, a DM who wishes to make informed decisions must sometimes ignore the advice of a better informed Expert even when the Expert provides an honest advice; second, private communication improves, with regard to public communication, the Expert's incentives for information revelation as well as information acquisition. For instance, in many countries legal prosecution mandates that the communication between the prosecutor and the judge is kept public, while in others this must be kept private. The latter is usually considered a vice of the system. Here, we argue that the prosecutor may have better incentives to acquire and reveal his information when his communication with the judge is kept secret. Thus, making communication between prosecutors and judges public may harm the quality of the ruling by limiting the amount of information available to the Judge.

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# A Proof of Proposition1

Let $h(m|s) \in [0,1]$ be the probability that the Expert makes recommendation $m \in \{-1,1\}$ when he observes signal $s \in \{-1,1\}$. After receiving message $m$, the DM chooses project $d$ with probability $z(d|m) \in [-1,1]$. Let $\hat{h}(m|s) \in [0,1]$ be the DM's and Observer's conjecture about the strategy used by the Expert and $\hat{z}(d|s) \in [0,1]$ be the Expert and Observer's conjecture about the strategy used by the DM. Then, the Observer computes the chances of the evidence, $\hat{g}(d|x,\theta) = \sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)g(s|x,\theta)$ and $\hat{g}(d|x) = \int_\Theta \sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)g(s|x,\theta)f(\theta)d\theta$. The Expert's posterior reputation from the Observer's point of view when the Observer observes $(x,d) \in \{-1,1\}^2$ is then calculated by Baye's rule as $f(\theta|d,x) = f(\theta)\hat{g}(d|x,\theta)/\hat{g}(d|x)$. Thus, the Expert's reputation is given by

$$f(\theta|d,x) = \frac{\sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)\big(1+sxp(\theta)\big)}{\sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)\big(1+sxEp(\theta)\big)} f(\theta). \tag{A1}$$

These are well defined only if the denominator is non-zero. When this is not the case, Bayesian updating is not possible and the equilibrium concept imposes no restrictions on beliefs. If the denominator is zero, we assume that the posterior is equal to the prior. Thus, $f(\theta|d,x) = f(\theta)$.

The expected reputational payoff for an Expert who receives signal $s$ when the DM chooses project $d$ is

$$V_E(d|s) = \sum_x \Big(\int_\Theta v_E(\theta)f(\theta|d,x)d\theta\Big) q_E(x|s). \tag{A2}$$

It follows from equations (A1) and (A2) and a few steps of algebra that

$$V_E(d|s) = Ev_E(\theta) + \sigma_E^2 \sum_x q_E(x|s)\frac{\sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)sx}{\sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)\big(1+sxEp(\theta)\big)}. \tag{A3}$$

Let the expected value of $s$ given project $d$, the Observer's conjectures about the Expert's strategy $\hat{h}(m|s)$ and the DM's strategy $\hat{z}(d|m)$ be

$$E(s|d) = \frac{\sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)s}{\sum_m \hat{z}(d|m)\sum_s \hat{h}(m|s)}.$$

Then, equation (A3) can be written as follows

$$V_E(d|s) = Ev_E(\theta) + \sigma_E^2 \sum_x q_E(x|s) \frac{xE(s|d)}{1 + xE(s|d)Ep(\theta)}. \tag{A4}$$

**Lemma 5.**

*i)* $E(v_E(\theta)|d,x)$ *is supermodular in* $(d,x)$.

*ii) If* $\hat{h}(d|1) + \hat{h}(d|-1) > 0$, *then* $E(v_E(\theta)|d = x, x) \geq E(v_{DM}(\theta)|d \neq x, x)$.

*Proof.* Observe that

$$E(v_E(\theta)|d,x) = \frac{xE(s|d)}{1 + xE(s|d)Ep(\theta)} \int_\Theta v_{DM}(\theta)p(\theta)f(\theta)d\theta$$

Then, supermodularity requires that

$$E(v_E(\theta)|1,1) + E(v_E(\theta)|-1,-1) \geq E(v_E(\theta)|1,-1) + E(v_E(\theta)|-1,1).$$

Suppose that $\hat{h}(d|1) + \hat{h}(d|-1) > 0$ for $d \in \{-1, 1\}$ and $\hat{z}(d|1) + \hat{z}(d|-1) > 0$ , then supermodularity requires the following

$$\frac{E(s|1)}{1 + E(s|1)Ep(\theta)} + \frac{-E(s|-1)}{1 - E(s|-1)Ep(\theta)} \geq \frac{-E(s|1)}{1 - E(s|1)Ep(\theta)} + \frac{E(s|-1)}{1 + E(s|-1)Ep(\theta)}, \tag{A5}$$

where

$$E(s|d) = \frac{\hat{z}(d|1)(\hat{h}(1|1) - \hat{h}(1|-1)) + \hat{z}(d|-1)(\hat{h}(-1|1) - \hat{h}(-1|-1))}{\hat{z}(d|1)(\hat{h}(1|1) + \hat{h}(1|-1)) + \hat{z}(d|-1)(\hat{h}(-1|1) + \hat{h}(-1|-1))}.$$

Equation (A5) re-writes as follows

$$2\frac{E(s|1)}{1 - (E(s|1)Ep(\theta))^2} \geq 2\frac{E(s|-1)}{1 - (E(s|-1)Ep(\theta))^2}$$

Because the LHS rises with $E(s|1)$ and the RHS rises with $E(s|-1)$, one can show that the inequality holds if and only if $E(s|1) \geq E(s|-1)$, which entails

$$\frac{\hat{z}(1|1)(\hat{h}(1|1)-\hat{h}(1|-1))+\hat{z}(1|-1)(\hat{h}(-1|1)-\hat{h}(-1|-1))}{\hat{z}(1|1)(\hat{h}(1|1)+\hat{h}(1|-1))+\hat{z}(1|-1)(\hat{h}(-1|1)+\hat{h}(-1|-1))} \geq \frac{\hat{z}(-1|1)(\hat{h}(1|1)-\hat{h}(1|-1))+\hat{z}(-1|-1)(\hat{h}(-1|1)-\hat{h}(-1|-1))}{\hat{z}(-1|1)(\hat{h}(1|1)+\hat{h}(1|-1))+\hat{z}(-1|-1)(\hat{h}(-1|1)+\hat{h}(-1|-1))}$$

This holds because $h(d=s|s) \geq h(d \neq s|s)$ and $z(d=s|s) \geq z(d \neq s|s)$.

Suppose that $\hat{h}(d|1)+\hat{h}(d|-1)=0$ or $\hat{z}(d|1)+\hat{z}(d|-1)=0$ for $d=1$ or both, then $E(v_{DM}(\theta)|1,x) = Ev_{DM}(\theta)$ for all $x$. Then, equation (A5) re-writes as follows

$$\frac{-E(s|-1)}{1-E(s|-1)Ep(\theta)} \geq \frac{E(s|-1)}{1+E(s|-1)Ep(\theta)}.$$

This holds if and only if $E(s|-1) \leq 0$. If $\hat{h}(-1|1)+\hat{h}(-1|-1)=1$, this entails $\hat{h}(-1|1) \leq \hat{h}(-1|-1)$ and this holds true since we focus on non-pervasive equilibria. IF $\hat{z}(-1|1)+\hat{z}(-1|-1)=1$, this entails $\hat{z}(-1|1) \leq \hat{z}(-1|-1)$ and and this holds true since we focus on non-pervasive equilibria. Similarly, for $d=-1$. $\square$

It readily follows from the incentive constraint in equation (21), that the Expert's best response is as follows: if $\hat{z}(1|1) \geq \hat{z}(1|0)$, then the Expert makes recommendation $m=1$ after signal $s \in \{-1,1\}$ with probability

$$h(1|s) = \begin{cases} 1 & \text{if } q_E(1|s) \geq \triangle_E, \\ [-1,1] & \text{if } q_E(1|s) = \triangle_E, \\ 0 & \text{if } q_E(1|s) \leq \triangle_E, \end{cases} \tag{A6}$$

where

$$\triangle_E \equiv \frac{1+\delta_E(E(v_E(\theta)|-1,-1)-E(v_E(\theta)|1,-1))}{2+\delta_E(E(v_E(\theta)|1,1)+E(v_E(\theta)|-1,-1)-E(v_E(\theta)|1,-1)-E(v_E(\theta)|-1,1))}.$$

After substituting into for the values of $E(v_E(\theta)|d,x)$, this can be written as follows

$$\triangle_E(E(s|1),E(s|-1)) \equiv \frac{1}{2}\frac{1+\delta_E\sigma_E^2\frac{\left(E(s|1)-E(s|-1)\right)\left(1+E(s|1)Ep(\theta)\right)\left(1+E(s|-1)Ep(\theta)\right)}{\left(1-Ep(\theta)^2E(s|1)^2\right)\left(1-Ep(\theta)^2E(s|-1)^2\right)}}{1+\delta_E\sigma_E^2\frac{\left(E(s|1)-E(s|-1)\right)\left(1+Ep(\theta)^2E(s|1)E(s|-1)\right)}{\left(1-Ep(\theta)^2E(s|1)^2\right)\left(1-Ep(\theta)^2E(s|-1)^2\right)}}.$$

Observe that $\triangle_E(E(s|1), E(s|-1))$ rises with $E(s|1)$ and $E(s|-1)$. This follows from the fact that $2E(s|1)(1-(Ep(\theta)E(s|-1))^2)+\delta_E\sigma_E^2(E(s|1)-E(s|-1))^2 > 0$, $-2E(s|-1)(1-(Ep(\theta)E(s|1))^2)+\delta_E\sigma_E^2(E(s|1)-E(s|-1))^2 > 0$, $E(s|-1) \leq 0$ and $E(s|-1) \geq 0$.

The DM's expected payoff from the Expert's reputation according to the Observer when the DM chooses project $d$ and receives message $m$ is

$$V_{DM}(d|m) = \sum_x E(v_{DM}(\theta)|d,x)q_{DM}(x|m), \tag{A7}$$

where

$$E(v_{DM}(\theta)|d,x) = \int_\Theta v_{DM}(\theta)f(\theta|d,x)d\theta, \tag{A8}$$

and $q_{DM}(x|m)$ is the DM's belief about the probability that the state of the world is $x$ conditional on recommendation $m$ being received.

It is easy to show after a few steps of simple algebra that equation (A8) is given by

$$V_{DM}(d|m) = Ev_{DM}(\theta) + \sigma_{DM}^2 \sum_x q_{DM}(x|m)\frac{xE(s|d)}{1+xE(s|d)Ep(\theta)}, \tag{A9}$$

where $\sigma_{DM}^2$ is the covariance between $v_{DM}(\theta)$ and $p(\theta)$.

The DM's posterior belief about state $x$ conditional on receiving message $m$ is given by Baye's rule as follows $q_{DM}(x|m) = \hat{g}(m|x)q_{DM}(x)/\hat{g}(m)$, with $\hat{g}(m|x) = \sum_s \hat{h}(m|s)\int_\Theta g(s|x,\theta)f(\theta)d\theta$ and $\hat{g}(m) = \sum_x\sum_s \hat{h}(m|s)\big(\int_\Theta \hat{g}(s|x,\theta)f(\theta)d\theta\big)q_{DM}(x)$. Hence,

$$q_{DM}(x|m) = \frac{\sum_s \hat{h}(m|s)\big(1+xsEp(\theta)\big)q_{DM}(x)}{\sum_s \hat{h}(m|s)\sum_x\big(1+xsEp(\theta)\big)q_{DM}(x)}. \tag{A10}$$

This is well defined as long as the denominator is non-zero.

**Lemma 6.**

*i)* $E(v_{DM}(\theta)|d,x)$ *is supermodular in* $(d,x)$.

*ii) If* $\hat{h}(d|1)+\hat{h}(d|-1) > 0$*, then* $E(v_{DM}(\theta)|d=x,x) \geq E(v_{DM}(\theta)|d \neq x,x)$.

*Proof.* The proof is identical to the proof of lemma 5 and thus omitted. □

It readily follows from the incentive constraint in equation (12), that the DM's best response is as follows: the DM chooses project $d = 1$ after message $m \in \{-1, 1\}$ with probability

$$z(1|m) = \begin{cases} 1 & \text{if } q_{DM}(1|m) \geq \triangle_{DM}, \\ [-1,1] & \text{if } q_{DM}(1|m) = \triangle_{DM}, \\ 0 & \text{if } q_{DM}(1|m) \leq \triangle_{DM}, \end{cases} \tag{A11}$$

where

$$\triangle_{DM} \equiv \frac{1 + \delta_{DM}\big(E(v_{DM}(\theta)|-1,-1) - E(v_{DM}(\theta)|1,-1)\big)}{2 + \delta_{DM}\big(E(v_{DM}(\theta)|1,1) + E(v_{DM}(\theta)|-1,-1) - E(v_{DM}(\theta)|1,-1) - E(v_{DM}(\theta)|-1,1)\big)}.$$

After substituting into for the values of $E(v_{DM}(\theta)|d,x)$, this can be written as follows

$$\triangle_{DM}(E(s|1), E(s|-1)) \equiv \frac{1}{2}\frac{1 + \delta_{DM}\sigma^2_{DM}\frac{\left(E(s|1)-E(s|-1)\right)\left(1+E(s|1)Ep(\theta)\right)\left(1+E(s|-1)Ep(\theta)\right)}{\left(1-Ep(\theta)^2E(s|1)^2\right)\left(1-Ep(\theta)^2E(s|-1)^2\right)}}{1 + \delta_{DM}\sigma^2_{DM}\frac{\left(E(s|1)-E(s|-1)\right)\left(1+Ep(\theta)^2E(s|1)E(s|-1)\right)}{\left(1-Ep(\theta)^2E(s|1)^2\right)\left(1-Ep(\theta)^2E(s|-1)^2\right)}}.$$

Observe that $\triangle_{DM}(E(s|1), E(s|-1))$ rises with $E(s|1)$ and $E(s|-1)$. This follows from the fact that $2E(s|1)(1-(Ep(\theta)E(s|-1))^2)+\delta_{DM}\sigma^2_{DM}(E(s|1)-E(s|-1))^2 > 0$, $-2E(s|-1)(1-(Ep(\theta)E(s|1))^2)+\delta_{DM}\sigma^2_{DM}(E(s|1) - E(s|-1))^2 > 0$, $E(s|-1) \leq 0$ and $E(s|-1) \geq 0$.

**Lemma 7.** *There exists a fully reveling PBE (that is, $h(m = s|s) = 1,\ \forall s \in \{-1, 1\}$) in which the DM rubber-stamps the Expert's recommendations (that is, $z(d = m|m) = 1,\ \forall m \in \{-1, 1\}$) if and only if for $i \in \{E, DM\}$*

$$\frac{(1 - Ep(\theta))\triangle_i(1, -1)}{1 + Ep(\theta) - 2Ep(\theta)\triangle_i(1, -1)} \leq q_i(1) \leq \frac{(1 + Ep(\theta))\triangle_i(1, -1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_i(1, -1)}. \tag{A12}$$

*Proof.* The proof follows directly from substituting into equations (A6) and (A11) the fact that under a fully revealing strategy in which the DM rubber-stamps the Expert's recommendation $E(1|1) = 1$ and $E(-1|-1) = -1$. Because player $i$ follows his private information if and only if $q_i(1|1) \geq \triangle_i(1, -1) > q_i(1|-1)$ and because $q_i(1|1) = \frac{q_i(1)(1+Ep(\theta))}{1+(2q_i(1)-1)Ep(\theta)}$ and $q_i(1|-1) = \frac{q_i(1)(1-Ep(\theta))}{1+(1-2q_i(1))Ep(\theta)}$, full revelation and rubber stamping take place if and only if

$$\frac{(1-Ep(\theta))\triangle_i(1,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_i(1,-1)} \leq q_i(1) \leq \frac{(1+Ep(\theta))\triangle_i(1,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_i(1,-1)}.$$

□

**Lemma 8.**

*i) There exists a fully revealing PBE in which the DM rubber-stamps the Expert's recommendation after recommendation -1 and randomizes after recommendation 1 if and only if*

$$\frac{(1+Ep(\theta))\triangle_{DM}(1,0)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,0)} > q_{DM}(1) > \frac{(1-Ep(\theta))\triangle_{DM}(1,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_{DM}(1,-1)},$$

*and*

$$\frac{(1+Ep(\theta))\triangle_E(1,E(s|-1)^*)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,E(s|-1)^*)} \geq q_E(1) > \frac{(1-Ep(\theta))\triangle_E(1,E(s|-1)^*)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(1,E(s|-1)^*)}.$$

*ii) There exists a fully revealing PBE in which the DM rubber-stamps the Expert's recommendation after recommendation 1 and randomizes after recommendation -1 if and only if*

$$\frac{(1+Ep(\theta))\triangle_{DM}(1,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,-1)} > q_{DM}(1) > \frac{(1+Ep(\theta))\triangle_{DM}(0,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(0,-1)},$$

*and*

$$\frac{(1-Ep(\theta))\triangle_E(E(s|1)^*,-1))}{1-Ep(\theta)-2Ep(\theta)\triangle_E(E(s|1)^*,-1)} \geq q_E(1) > \frac{(1-Ep(\theta))\triangle_E(E(s|1)^*,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(E(s|1)^*,-1)}.$$

*Proof.* It follows from the DM's best response in equation (A11) that he chooses project $d = m$ after receiving message $m$ if and only if the following holds

$$q_{DM}(1|1) \geq \frac{1}{2} - \frac{\delta_{DM}}{2}\triangle REP_{DM}(1) \text{ and } q_{DM}(1|-1) < \frac{1}{2} - \frac{\delta_{DM}}{2}\triangle REP_{DM}(-1).$$

It readily follows from the best-response function in equation (A11) that this entails the following

$$q_{DM}(1|1) \geq \triangle_{DM}(E(s|1),E(s|-1)) \geq q_{DM}(1|-1). \tag{A13}$$

Because $q_{DM}(1|1) > q_{DM}(1|-1)$ for all $q_{DM}(1) < 1$, it follows from equation (A13) that the two incentive compatibility constraints cannot bind at the same time and therefore there are two different types of hybrid equilibrium: (i) $z(1|1) = 1$ and $z(-1|-1) \in (0,1)$ and (ii) $z(-1|-1) = 1$ and $z(1|1) \in (0,1)$.

Consider first the case in which $z(-1|-1) = 1$ and $z(1|1) \in (0,1)$ and $\hat{h}(1|1) = \hat{h}(-1|-1) = 1$. Then,

$$E(s|d) = \frac{\hat{z}(d|1) - \hat{z}(d|-1)}{\hat{z}(d|1) + \hat{z}(d|-1)}.$$

and therefore $E(s|1) = 1$ and $E(s|-1) = -z(1|1)/(2 - z(1|1))$

It follows from the Expert's best response in equation (12) that the Expert chooses project $d = s$ after receiving signal $s$ if and only if the following holds

$$q_E(1|1) \geq \frac{1}{2} - \frac{\delta_E}{2}\triangle REP_E(1) \text{ and } q_E(1|-1) < \frac{1}{2} - \frac{\delta_E}{2}\triangle REP_E(-1).$$

It readily follows from the best-response function in equation (A6) that this entails the following

$$q_E(1|1) \geq \triangle_E(E(s|1), E(s|-1)) \geq q_E(1|-1). \tag{A14}$$

The DM, after observing message $m = 1$, is willing to randomize between decisions 1 and -1 if and only if there exists a $E(s|-1) \in [-1, 0]$ such that $q_{DM}(1|1) = \triangle_{DM}(1, E(s|-1))$, and he is willing to make project -1 with probability 1 after message $m = -1$ is observed if and only if $q_{DM}(1|-1) \leq \triangle_E(1, E(s|-1))$.

First, notice that

$$\triangle_{DM}(1, -1) = \frac{1}{2}$$

and

$$\triangle_{DM}(1, 0) = \frac{1}{2}\frac{1 - Ep(\theta)^2 + \delta_{DM}\sigma_{DM}^2(1 + Ep(\theta))}{1 - Ep(\theta)^2 + \delta_{DM}\sigma_{DM}^2}.$$

Notice that $\triangle_{DM}(1, 0) \geq \triangle_{DM}(1, -1)$, and recall that $\triangle_{DM}(E(s|1), E(s|-1))$ rises with $E(s|-1)$. Hence, if $\triangle_{DM}(1, 0) > q_{DM}(1|1) > \triangle_{DM}(1, -1)$, it readily follows from the Intermediate Value Theorem that there exists a unique $E(s|-1)$, denoted by $E(s|-1)^*$, such that the DM is willing to randomize after message $m = 1$ and chooses project -1 after message $m = -1$.

Because $q_{DM}(1|1) = \frac{q_{DM}(1)(1+Ep(\theta))}{1+(2q_{DM}(1)-1)Ep(\theta)}$, one can show after a few steps of simple algebra that

$\triangle_{DM}(1,0) > q_{DM}(1|1) > \triangle_{DM}(1,-1)$ entails the following

$$\frac{(1+Ep(\theta))\triangle_{DM}(1,0)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,0)} > q_{DM}(1) > \frac{(1-Ep(\theta))\triangle_{DM}(1,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_{DM}(1,-1)},$$

Recall that the Expert follows a fully revealing strategy if and only if $q_E(1|1) > \triangle_E(1, E^*(s|-1)) > q_E(1|-1)$. Because $q_E(1|1) = \frac{q_E(1)(1+Ep(\theta))}{1+(2q_E(1)-1)Ep(\theta)}$ and $q_E(1|-1) = \frac{q_E(1)(1-Ep(\theta))}{1+(1-2q_E(1))Ep(\theta)}$, this entails the following

$$\frac{(1+Ep(\theta))\triangle_E(1,E(s|-1)^*)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,E(s|-1)^*)} \geq q_E(1) > \frac{(1-Ep(\theta))\triangle_E(1,E(s|-1)^*)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(1,E(s|-1)^*)}.$$

Next consider the following strategy profile: $z(-1|-1) \in (0,1)$ and $z(1|1) = 1$ and $\hat{h}(1|1) = \hat{h}(-1|-1) = 1$. Then, $E(s|-1) = -1$ and $E(s|1) = z(1|1)/(2-z(1|1))$. Then, we can proceed as before. Substituting the strategies into equation (A14), the DM, after observing $m = -1$, is willing to randomize between project 1 and project -1 if and only if $q_{DM}(1|-1) = \triangle_{DM}(E(s|1), -1)$, and choose project 1 after message $m = 1$ if and only if $q_{DM}(1|1) \geq \triangle_{DM}(E(s|1), -1)$. Notice that $\triangle_{DM}(1,-1) = 1/2$ and

$$\triangle_{DM}(0,-1) = \frac{1}{2}\frac{1-Ep(\theta)^2+\delta_{DM}\sigma^2_{DM}(1-Ep(\theta))}{1-Ep(\theta)^2+\delta_{DM}\sigma^2_{DM}}.$$

Notice that $\triangle_{DM}(1,-1) \geq \triangle_{DM}(0,-1)$, and recall that $\triangle_{DM}(E(s|1), E(s|-1))$ rises with $E(s|-1)$. Hence, if $\triangle_{DM}(1,-1) > q_{DM}(1|-1) > \triangle_{DM}(0,-1)$, it readily follows from the Intermediate Value Theorem that there exists a unique $E(s|1)$, denoted by $E(s|1)^*$, such that the DM is willing to randomize after message $m = -1$ and choose project -1 after message $m = -1$ with probability 1.

Because $q_{DM}(1|-1) = \frac{q_{DM}(1)(1-Ep(\theta))}{1+(1-2q_{DM}(1))Ep(\theta)}$, it is easy to show that $\triangle_{DM}(1,-1) > q_{DM}(1|-1) > \triangle_{DM}(0,-1)$ entails the following

$$\frac{(1+Ep(\theta))\triangle_{DM}(1,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(1,-1)} > q_{DM}(1) > \frac{(1+Ep(\theta))\triangle_{DM}(0,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(0,-1)},$$

Recall that the Expert follows a fully revealing strategy if and only if $q_E(1|1) > \triangle_E(E(s|1)^*, -1) >$

$q_E(1|-1)$. This entails the following

$$\frac{(1+Ep(\theta))\triangle_E(E(s|1)^*,-1))}{1-Ep(\theta)+2Ep(\theta)\triangle_E(E(s|1)^*,-1)} \geq q_E(1) > \frac{(1-Ep(\theta))\triangle_E(E(s|1)^*,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(E(s|1)^*,-1)}.$$

$\square$

**Lemma 9.**

*i) There exists an equilibrium in which $h(1|1) \in (0,1)$ and $h(-1|-1)=1$ and the DM rubber stamps the Expert's recommendation if and only if*

$$\frac{(1-Ep(\theta))\triangle_E(1,0)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(1,0)} > q_E(1) > \frac{(1-Ep(\theta))\triangle_E(1,-1)}{1+Ep(\theta)-2Ep(\theta)\triangle_E(1,-1)}.$$

*and*

$$\frac{(1+Ep(\theta)E^*(s|-1))\triangle_{DM}(1,E^*(s|-1))}{1-Ep(\theta)E^*(s|-1)+2Ep(\theta)\triangle_{DM}(1,E^*(s|-1))E^*(s|-1)} > q_{DM}(1) >$$
$$\frac{(1-Ep(\theta))\triangle_{DM}(1,E^*(s|-1))}{1+Ep(\theta)-2Ep(\theta)\triangle_{DM}(1,E^*(s|-1))}.$$

*where $E^*(s|-1)$ is the unique solution to $q_E(1|1) = \triangle_E(1,E^*(s|-1))$.*

*ii) There exists an equilibrium in which $h(-1|-1) \in (0,1)$ and $h(1|1)=1$ and the DM rubber stamps the Expert's recommendation if and only if*

$$\frac{(1+Ep(\theta))\triangle_E(1,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,-1)} \geq q_E(1) > \frac{(1+Ep(\theta))\triangle_E(0,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(0,-1)}.$$

*and*

$$\frac{(1+Ep(\theta))\triangle_{DM}(E^*(s|1),-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(E^*(s|1),-1)} > q_{DM}(1) >$$
$$\frac{(1-Ep(\theta)E^*(s|1))\triangle_{DM}(E^*(s|1),-1)}{1+Ep(\theta)E^*(s|1)-2Ep(\theta)\triangle_{DM}(E^*(s|1),-1)E^*(s|1)}.$$

*where $E^*(s|1)$ is the unique solution to $q_E(1|-1) = \triangle_E(E^*(s|1),-1)$.*

*Proof.* It follows from the Expert's best response in equation (A15) that the Expert chooses project $d=s$ after receiving signal $s$ if and only if the following holds

$$q_E(1|1) \geq \frac{1}{2} - \frac{\delta_E}{2} \triangle REP_E(1) \text{ and } q_E(1|-1) < \frac{1}{2} - \frac{\delta_E}{2} \triangle REP_E(-1).$$

It readily follows from the best-response function in equation (A11) that this entails the following

$$q_E(1|1) \geq \triangle_E(E(s|1), E(s|-1)) \geq q_E(1|-1). \tag{A15}$$

Because $q_E(1|1) > q_E(1|-1)$ for all $q_E(1) < 1$, it follows from equation (A15) that the two incentive compatibility constraints cannot bind at the same time and therefore there are two different types of hybrid equilibrium: (i) $h(1|1) = 1$ and $h(-1|-1) \in (0, 1)$ and (ii) $h(-1|-1) = 1$ and $h(1|1) \in (0, 1)$.

It follows from the DM's best response in equation (A11) that he chooses project $d = m$ after receiving message $m$ if and only if the following holds

$$q_{DM}(1|1) \geq \frac{1}{2} - \frac{\delta_{DM}}{2} \triangle REP_{DM}(1) \text{ and } q_{DM}(1|-1) < \frac{1}{2} - \frac{\delta_{DM}}{2} \triangle REP_{DM}(-1).$$

It readily follows from the best-response function in equation (A11) that this entails the following

$$q_{DM}(1|1) \geq \triangle_{DM}(E(s|1), E(s|-1)) \geq q_{DM}(1|-1). \tag{A16}$$

Consider first the case in which $h(-1|-1) = 1$ and $h(1|1) \in (0, 1)$ and $\hat{z}(1|1) = \hat{z}(-1|-1) = 1$. Then,

$$E(s|d) = \frac{\hat{h}(d|1) - \hat{h}(d|-1)}{\hat{h}(d|1) + \hat{h}(d|-1)}.$$

and therefore $E(s|1) = 1$ and $E(s|-1) = -h(1|1)/(2 - h(1|1))$

The Expert, after observing signal $s = 1$, is willing to randomize between messages 1 and -1 if and only if there exists a $E(s|-1) \in (0, 1)$ such that $q_E(1|1) = \triangle_E(1, E(s|-1))$ and he is willing to send message -1 with probability 1 after signal $s = -1$ is observed if and only if $q_E(1|-1) \leq \triangle_E(1, E(s|-1))$. This is implied by the fact that $q_E(1|1) > q_E(1|-1)$ and $q_E(1|1) = \triangle_E(1, E(s|-1))$.

First, notice that

$$\triangle_E(1, -1) = \frac{1}{2}$$

and

$$\triangle_E(1, 0) = \frac{1}{2} \frac{1 - Ep(\theta)^2 + \delta_E \sigma_E^2 (1 + Ep(\theta))}{1 - Ep(\theta)^2 + \delta_E \sigma_E^2}.$$

Notice that $\triangle_E(1,0) \geq \triangle_E(1,-1)$, and recall that $\triangle_E(E(s|1), E(s|-1))$ rises with $E(s|-1)$. Hence if $\triangle_E(1,0) \geq q_E(1|1) > \triangle_E(1,-1)$, it readily follows from the Intermediate Value Theorem that there exists a unique $E(s|-1)$, denoted by $E(s|-1)^*$ such that the Expert is willing to randomize after signal $s = 1$ and chooses project -1 after signal $s = -1$ with probability 1.

Substituting for the value of $q_E(1|1)$ into $\triangle_E(1,0) \geq q_E(1|1) > \triangle_E(1,-1)$, one gets that this inequality holds if and only if

$$\frac{(1 - Ep(\theta))\triangle_E(1,0)}{1 + Ep(\theta) - 2Ep(\theta)\triangle_E(1,0)} > q_E(1) > \frac{(1 - Ep(\theta))\triangle_E(1,-1)}{1 + Ep(\theta) - 2Ep(\theta)\triangle_E(1,-1)}.$$

The DM rubber stamps the Expert's recommendation if and only if $q_{DM}(1|1) > \triangle_{DM}(1, E^*(s|-1)) > q_{DM}(1|-1)$. Because $h(-1|-1) = 1$ and $h(1|1) \in (0,1)$,

$$q_{DM}(1|1) = \frac{q_{DM}(1)(1 + Ep(\theta))}{1 + (2q_{DM}(1) - 1)Ep(\theta)}$$

and

$$q_{DM}(1|-1) = \frac{q_{DM}(1)(1 - E^*(s|-1)Ep(\theta))}{1 - Ep(\theta)(2q_{DM}(1) - 1)E^*(s|-1)},$$

this entails the following

$$\frac{(1 + Ep(\theta)E^*(s|-1))\triangle_{DM}(1, E^*(s|-1))}{1 - Ep(\theta)E^*(s|-1) + 2Ep(\theta)\triangle_{DM}(1, E^*(s|-1))E^*(s|-1)} > q_{DM}(1) >$$
$$\frac{(1 - Ep(\theta))\triangle_{DM}(1, E^*(s|-1))}{1 + Ep(\theta) - 2Ep(\theta)\triangle_{DM}(1, E^*(s|-1))}.$$

Consider next the case in which $h(1|1) = 1$ and $h(-1|-1) \in (0,1)$ and $\hat{z}(1|1) = \hat{z}(-1|-1) = 1$. Then,

$$E(s|d) = \frac{\hat{h}(d|1) - \hat{h}(d|-1)}{\hat{h}(d|1) + \hat{h}(d|-1)}.$$

and therefore $E(s|1) = h(-1|-1)/(2 - h(-1|-1))$ and $E(s|-1) = -1$. Then, we can follow the same steps as before. Substituting the strategies into equation (A14), the Expert, after observing $s = -1$, is willing to randomize between project 1 and project -1 if and only if $q_E(1|-1) = \triangle_E(E(s|1), -1)$.

Notice that $\triangle_E(1,-1) = 1/2$ and

$$\triangle_E(0,-1) = \frac{1}{2}\frac{1 - Ep(\theta)^2 + \delta_E\sigma_E^2(1 - Ep(\theta))}{1 - Ep(\theta)^2 + \delta_E\sigma_E^2}.$$

Note that $\triangle_E(1,-1) > \triangle_E(0,-1)$, and recall that $\triangle_E(E(s|1), E(s|-1))$ rises with $E(s|1)$. Hence, if $\triangle_E(1,-1) > q_E(1|-1) > \triangle_E(0,-1)$, it readily follows from the Intermediate Value Theorem that there exists a unique $E(s|1)$, denoted by $E(s|1)^*$, such that the Expert is willing to randomize after signal $s=-1$ and chooses project 1 after signal $s=1$ with probability 1.

Substituting $q_E(1|-1)$ into $\triangle_E(1,-1) > q_E(1|-1) > \triangle_E(0,-1)$, one can show that this holds if and only if

$$\frac{(1+Ep(\theta))\triangle_E(1,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,-1)} \geq q_E(1) > \frac{(1+Ep(\theta))\triangle_E(0,-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_E(0,-1)}.$$

One can show that the DM rubber stamps the Expert's recommendation if and only if $q_{DM}(1|1) > \triangle_{DM}(E(s|1)^*, -1) > q_{DM}(1|-1)$. Because $h(1|1)=1$ and $h(-1|-1) \in (0,1)$,

$$q_{DM}(1|1) = \frac{q_{DM}(1)(1+E(s|1)Ep(\theta))}{1+Ep(\theta)(2q_{DM}(1)-1)E(s|1)}$$

and

$$q_{DM}(1|-1) = \frac{q_{DM}(1)(1-Ep(\theta))}{1-(2q_{DM}(1)-1)Ep(\theta)},$$

this entails the following

$$\frac{(1+Ep(\theta))\triangle_{DM}(E^*(s|1),-1)}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(E^*(s|1),-1)} > q_{DM}(1) >$$
$$\frac{(1-Ep(\theta)E^*(s|1))\triangle_{DM}(E^*(s|1),-1)}{1+Ep(\theta)E^*(s|1)-2Ep(\theta)\triangle_{DM}(E^*(s|1),-1)E^*(s|1)}.$$

$\square$

**Lemma 10.**

*i) There is no open set of parameters under which* $h(1|1)=1$ *and* $h(-1|-1) \in (0,1)$ *and* $z(1|1)=1$ *and* $z(-1|-1) \in (0,1)$ *is a PBE.*

*ii) There is no open set of parameters under which* $h(1|1) \in (0,1)$ *and* $h(-1|-1)=1$ *and* $z(1|1) \in (0,1)$ *and* $z(-1|-1)=1$ *is a PBE.*

*iii) There exists a PBE in which* $h(1|1) \in (0,1)$ *and* $h(-1|-1)=1$ *and* $z(1|1)=1$ *and* $z(0|0) \in (0,1)$

*if and only if*

$$\triangle_{DM}(0,0) \geq q_{DM}(1) \geq \frac{\triangle_{DM}(1, E(s|-1)(1))(1-Ep(\theta))}{1+Ep(\theta)-2Ep(\theta)\triangle_{DM}(1, E(s|-1)(1))}$$

*and*

$$\frac{\triangle_E(E(s|1)^*,0)(1+Ep(\theta))}{1-Ep(\theta)+2Ep(\theta)\triangle_E(E(s|1)^*,0)} > q_E(1) > \frac{\triangle_E(E(s|1)^*,-1)(1+Ep(\theta))}{1-Ep(\theta)+2Ep(\theta)\triangle_E(E(s|1)^*,-1)}$$

*where where $E(s|1)^*$ is the unique solution to $q_E(1|1) = \triangle_E(E(s|1)^*, E(s|-1)^*)$ and $E(s|-1)(E(s|1))$ is a solution to $q_{DM}(1|1)(E(s|1)) = \triangle_{DM}(E(s|1), E(s|-1))$.*

*iv) There exists a PBE in which $h(0|0) \in (0,1)$ and $h(1|1) = 1$ and $z(0|0) = 1$ and $z(1|1) \in (0,1)$ if and only if*

$$\frac{\triangle_{DM}(E(s|1)(-1),-1)(1+Ep(\theta))}{1-Ep(\theta)+2Ep(\theta)\triangle_{DM}(E(s|1)(-1),-1)} \geq q_{DM}(1) \geq \triangle_{DM}(0,0).$$

*and*

$$\frac{\triangle_E(1,E(s|-1)^*)(1+Ep(\theta))}{1-Ep(\theta)+2Ep(\theta)\triangle_E(1,E(s|-1)^*))} > q_E(1) > \frac{\triangle_E(0,E(s|-1))(1+Ep(\theta))}{1-Ep(\theta)+2Ep(\theta)\triangle_E(0,(E(s|-1)^*)}$$

*where $E(s|-1)^*$ is the unique solution to $q_E(1|-1) = \triangle_E(E(s|1)^*, E(s|-1)^*)$ and and $E(s|1)(E(s|-1))$ is a solution to $q_{DM}(1|-1)(E(s|-1)) = \triangle_{DM}(E(s|1), E(s|-1))$.*

*Proof.* First, lets consider the case in which $h(1|1) \in (0,1)$ and $h(-1|-1) = 1$ and $z(1|1) \in (0,1)$ and $z(-1|-1) = 1$. Then $E(s|-1) = -z(1|1)h(1|1)/(2 - z(1|1)h(1|1))$ and $E(s|1) = 1$.

Then after substituting these values into equations (A6) and (A11), the DM is willing to randomize between project 1 and -1 after message $m = 1$ if and only if there exists a $E(s|-1) \in (0,1)$ such that $q_{DM}(1|1) = \triangle_{DM}(1, E(s|-1))$, and the Expert is willing to randomize between messages 1 and 0 after signal $s = 1$ if and only if there exists a $E(s|-1) \in (0,1)$ such that $q_E(1|1) = \triangle_E(1, E(s|-1))$. The Expert is willing to send message -1 with probability 1 after signal $s = -1$ is observed if and only if $q_E(1|-1) \leq \triangle_E(1, E(s|-1))$ and the DM chooses project $-1$ after $m = -1$ with probability 1 if and only if $q_{DM}(1|-1) \leq \triangle_{DM}(1, E(s|-1))$. These two inequalities are implied by the fact that $q_i(1|1) > q_i(1|-1)$ and in any equilibrium $q_i(1|1) = \triangle_i(1, E(s|-1))$. Because $\triangle_i(1, E(s|-1))$ rises with $E(s|-1)$, there is no open set of parameters such that a mixed strategy equilibrium of this kind exists.

The same can be shown for the case in which $h(1|1) = 1$ and $h(-1|-1) \in (-1,1)$ and $z(1|1) = 1$ and $z(-1|-1) \in (-1,1)$. It is enough to notice that $E(s|-1) = -1$ and $E(s|1) = z(-1|-1)h(-1|-1)/(2 - z(-1|-1)h(-1|-1))$ and $\triangle_i(E(s|1), -1)$ rises with $E(s|1)$.

Next lets consider the case in which $h(1|1) = 1$ and $h(-1|-1) \in (0,1)$ and $z(1|1) \in (0,1)$ and $z(-1|-1) = 1$. Then, $E(s|1) = h(-1|-1)/(2-h(-1|-1))$ and $E(s|-1) = -z(1|1)h(-1|-1)/(2(1-z(1|1)) + z(1|1)h(-1|-1)) = -z(1|1)E(s|1)/(1 - z(1|1) + E(1|s))$. This is an equilibrium if and only if there exists $h(-1|-1) \in (0,1)$ and $z(1|1) \in (0,1)$ such that $q_{DM}(1|1) = \triangle_{DM}(E(s|1), E(s|-1))$ and $q_E(1|-1) = \triangle_E(E(s|1), E(s|-1))$.

Because $h(1|1) = 1$ and $h(-1|-1) \in (-1,1)$,

$$q_{DM}(1|1)(E(s|1)) = \frac{q_{DM}(1)(1 + E(s|1)Ep(\theta))}{1 + Ep(\theta)(2q_{DM}(1) - 1)E(s|1)}$$

and

$$q_E(1|-1) = \frac{q_E(1)(1 - Ep(\theta))}{1 - (2q_E(1) - 1)Ep(\theta)},$$

and therefore an equilibrium exists if and only if there exists $(E(s|-1), E(s|1)) \in [-1,0] \times [0,1]$ satisfying

$$\begin{aligned} q_{DM}(1|1)(E(s|1)) &= \triangle_{DM}(E(s|1), E(s|-1)) \\ q_E(1|-1) &= \triangle_E(E(s|1), E(s|-1)) \\ E(s|-1) &= \frac{-z(1|1)E(s|1)}{1 - z(1|1) + E(s|1)} \end{aligned}$$

Because $\triangle_E(E(s|1), E(s|-1))$ is continuous and rises with $E(s|-1)$, the Intermediate Value Theorem ensures that a solution to the second equation exists if and only if for all $E(s|1) \in [0,1]$, $q_E(1|-1) > \triangle_E(E(s|1), -1)$ and $q_E(1|-1) < \triangle_E(E(s|1)), 0)$. It is easy to check that this requires the following to hold

$$\frac{\triangle_E(E(s|1), 0)(1 + Ep(\theta))}{1 - Ep(\theta) + 2Ep(\theta)\triangle_E(E(s|1), 0)} > q_E(1) > \frac{\triangle_E(E(s|1), -1)(1 + Ep(\theta))}{1 - Ep(\theta) + 2Ep(\theta)\triangle_E(E(s|1), -1)}$$

Lets denote this solution by $E(s|-1)(E(s|1))$. Because $q_{DM}(1|1)$ increases with $E(s|1)$, the first equation can be written as follows

$$E(s|1) = \frac{\triangle_{DM}(E(s|1), E(s|-1)(E(s|1))) - q_{DM}(1)}{Ep(\theta)(q_{DM}(1) - \triangle_{DM}(E(s|1), E(s|-1)(E(s|1)))(2q_{DM}(1) - 1))} \tag{A17}$$

Because $\triangle_{DM}(E(s|1), E(s|-1)(E(s|1))) \in [0,1]$ for all $(E(s|1), E(s|-1)) \in [-1,0] \times [0,1]$, it is easy to check that the RHS belongs to the set $[0,1]$ if and only if

$$\triangle_{DM}(0, E(s|-1)(0)) \geq q_{DM}(1) \geq \frac{\triangle_{DM}(1, E(s|-1)(1))(1 - Ep(\theta))}{1 + Ep(\theta) - 2Ep(\theta)\triangle_{DM}(1, E(s|-1)(1))}.$$

where $E(s|-1)(0) = 0$ and $E(s|-1)(1) = -z(1|1)/(2-z(1|1))$. Hence, if this holds, by Brouwer's Fixed Point Theorem, a fixed point in $[0,1]$ exists. Lets denote this fixed point by $E(s|1)^*$ and $E(s|-1)(E(s|1)^*)$ by $E(s|-1)^*$.

Finally, in order for this to be an equilibrium it must be the case that there exists $z(1|1) \in (0,1)$ such that the following holds

$$z(1|1) = \frac{E(s|-1)^*(1 + E(s|1)^*)}{E(s|-1)^* - E(s|1)^*} \in (0,1).$$

It is straightforward to check that this holds for all $(E(s|1)^*, E(s|-1)^*) \in (-1,0) \times (0,1)$.

Next lets consider the case in which $h(1|1) \in (-1,1)$ and $h(-1|-1) = 1$ and $z(1|1) = 1$ and $z(-1|-1) \in (-1,1)$. Then $E(s|-1) = -h(1|1)/(2-h(1|1))$ and $E(s|1) = -z(-1|-1)h(1|1)/(2(1-z(-1|-1)) + z(-1|-1)h(1|1)) = -z(-1|-1)E(s|-1)/(-E(s|-1) + 1 - z(-1|-1)))$. This is an equilibrium if and only if there exists $h(1|1) \in (0,1)$ and $z(-1|-1) \in (0,1)$ such that $q_{DM}(1|-1) = \triangle_{DM}(E(s|1), E(s|-1))$ and $q_E(1|1) = \triangle_E(E(s|1), E(s|-1))$.

Because $h(-1|-1) = 1$ and $h(1|1) \in (0,1)$,

$$q_E(1|1) = \frac{q_E(1)(1 + Ep(\theta))}{1 + Ep(\theta)(2q_E(1) - 1)}$$

and

$$q_{DM}(1|-1)(E(s|-1)) = \frac{q_{DM}(1)(1 + E(s|-1)Ep(\theta))}{1 + Ep(\theta)(2q_{DM}(1) - 1)E(s|-1)}.$$

An equilibrium exists if and only if there exists $(E(s|-1), E(s|1)) \in [-1, 0] \times [0, 1]$ satisfying

$$\begin{aligned} q_{DM}(1|-1)(E(s|-1)) &= \triangle_{DM}(E(s|1), E(s|-1)) \\ q_E(1|1) &= \triangle_E(E(s|1), E(s|-1)) \\ E(s|1) &= \frac{-z(-1|-1)E(s|-1)}{1 - z(-1|-1) - E(s|-1)} \end{aligned}$$

Because $\triangle_E(E(s|1), E(s|-1))$ is continuous and rises with $E(s|1)$, the Intermediate Value Theorem ensures that a solution to the second equation exists if and only if $q_E(1|1) > \triangle_E(0, E(s|-1))$ and $q_E(1|1) < \triangle_E(1, E(s|1))$. It is easy to check that this requires the following to hold

$$\frac{\triangle_E(1, E(s|-1))(1 + Ep(\theta))}{1 - Ep(\theta) + 2Ep(\theta)\triangle_E(1, E(s|-1))} > q_E(1) > \frac{\triangle_E(0, E(s|-1))(1 + Ep(\theta))}{1 - Ep(\theta) + 2Ep(\theta)\triangle_E(0, (E(s|-1))}$$

Lets denote the solution by $E(s|1)(E(s|-1))$.

The first equation can be written as follows

$$E(s|-1) = \frac{\triangle_{DM}(E(s|1)(E(s|-1)), E(s|-1)) - q_{DM}(1)}{Ep(\theta)(q_{DM}(1) - \triangle_{DM}(E(s|1)(E(s|-1)), E(s|-1))(2q_{DM}(1) - 1))}.$$

Furthermore, $\triangle_{DM}(E(s|1), E(s|-1)) \in [0, 1]$ for all $(E(s|1), E(s|-1)) \in [-1, 0] \times [0, 1]$ and therefore it is easy to check that the RHS belongs to the set $[-1, 0]$ if and only if

$$\frac{\triangle_{DM}(E(s|1)(-1), -1)(1 + Ep(\theta))}{1 - Ep(\theta) + 2Ep(\theta)\triangle_{DM}(E(s|1)(-1), -1)} \geq q_{DM}(1) \geq \triangle_{DM}(E(s|1)(0), 0)).$$

where $E(s|1)(0) = 0$ and $E(s|1)(-1) = z(-1|-1)/(2 - z(-1|-1)))$. Hence, if this holds, by Brouwer's fixed point theorem a fixed point in $[-1, 0]$ exists. Lets denote this fixed point by $E(s|-1)^*$ and $E(s|1)((E(s|-1)^*)$ by $E(s|1)^*$.

Finally, in order for this to be an equilibrium it must be the case that there exists $z(-1|-1) \in (0, 1)$ such that the following holds

$$z(-1|-1) = \frac{E(s|1)^*(1 - E(s|-1)^*)}{E(s|1)^* - E(s|-1)^*} \in (0, 1).$$

It is straightforward to check that this holds for all $(E(s|1)^*, E(s|-1)^*) \in (-1, 0) \times (0, 1)$.

□

**Lemma 11.**

*i) There exists a pooling PBE in which the Expert makes recommendation 1 and the DM chooses project 1 if and only if for $i \in \{E, DM\}$*

$$q_i(1) > \frac{(1 + Ep(\theta))\triangle_i(1, -1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_i(1, -1)}$$

*and the DM chooses project -1 if and only if*

$$q_{DM}(1) < \frac{(1 - Ep(\theta))\triangle_{DM}(1, -1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_{DM}(1, -1)}.$$

*ii) There exists a pooling PBE in which the Expert makes recommendation -1 and the DM chooses project -1 if and only if for $i \in \{E, DM\}$*

$$q_i(1) < \frac{(1 - Ep(\theta))\triangle_i(1, -1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_i(1, -1)}$$

*and the DM chooses project 1 if and only if*

$$q_{DM}(1) > \frac{(1 + Ep(\theta))\triangle_{DM}(1, -1)}{1 - Ep(\theta) + 2Ep(\theta)\triangle_{DM}(1, -1)}.$$

*Proof.* **Proof of Proposition 3**

Lets define $y = \{\delta_{DM}, \sigma^2_{DM}, q_{DM}(1)\}$. First, lets consider the case in which $E(s|1)^* = 1$ and $E(s|-1)^* \in (0, 1)$; that is, the DM rubber stamps the Expert's recommendation after $m = -1$ and randomizes after $m = 1$. It readily follows from Lemma 8 and that

$$\frac{\partial \triangle_{DM}(1, E(s|-1)^*)}{\partial E(s|-1)^*} \frac{\partial E(s|-1)^*}{\partial z(1|1)^*} \frac{z(1|1)^*}{\partial y} = \frac{\partial q_{DM}(1|1)}{\partial y} - \frac{\partial \triangle_{DM}(1, E(s|-1)^*)}{\partial y}$$

In Lemma 8, we showed that the first partial derivative of the LHS is positive and the second is negative. Because the first partial derivative of the RHS with respect to $q_{DM}(1)$ is positive and the second is zero, $z(1|1)^*$ falls with $q_{DM}(1)$. Because the first partial derivative of the RHS with respect to $\delta_{DM}\sigma^2_{DM}$ is zero and the second is positive, $z(1|1)^*$ rises with $\delta_{DM}\sigma^2_{DM}$.

Lets define $y = \{\delta_{DM}, \sigma^2_{DM}, q_{DM}(-1)\}$.Second, lets consider the case in which $E(s|-1)^* = 1$ and

$E(s|1)^* \in (0,1)$; that is, the DM rubber stamps the Expert's recommendation after $m = 1$ and randomizes after $m = -1$. It readily follows from Lemma 8 that

$$\frac{\partial \triangle_{DM}(E(s|1)^*, -1)}{\partial E(s|1)^*} \frac{\partial E(s|1)^*}{\partial z(1|1)^*} \frac{\partial z(1|1)^*}{\partial y} = \frac{\partial q_{DM}(1|-1)}{\partial y} - \frac{\partial \triangle_{DM}(E(s|1)^*, -1)}{\partial y}$$

In Lemma 8, we showed that the first partial derivative of the LHS is positive and the second is negative. Because the first partial derivative of the RHS with respect to $q_{DM}(-1)$ is negative and the second is zero, $z(1|1)^*$ falls with $q_{DM}(-1)$. Because the first partial derivative of the RHS with respect to $\delta_{DM}\sigma^2_{DM}$ is zero and the second is negative, $z(1|1)^*$ rises with $\delta_{DM}\sigma^2_{DM}$. $\square$