

Initiation of Merger and Acquisition Negotiation with Two-Sided Private Information*

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Abstract

In a dynamic model of merger negotiation assuming two-sided private information and common knowledge of gains from trade, this paper investigates (1) what determines the delay in timing of M&A initiation, and (2) who initiates the M&A negotiation. The key driving force for the results is that the timing of initiation can reveal information about each firm's private signal. In addition, interpretation of the timing of initiation as a signal for private information depends crucially on whether the private information is about stand-alone value or about synergy. We conclude that if private information is about stand-alone value, the firm whose type is in the lower tail of its own population distribution compared to that of its opponent becomes the initiator. If private information is about synergy, then the firm whose type is in the upper tail of its own population distribution compared to that of its opponent becomes the initiator. In addition, discount rate, bargaining power, and cash constraint also affect who initiates first. The results obtained are broadly consistent with empirical evidence that emphasizing the role of private information in deal-initiation (Masulis and Simsir (2013)). Finally, we show that most results extend to an environment with market-wide uncertainty modeled as a Diffusion Process, in which the decision of Merger and Acquisition initiation is a real option.

Keywords: Merger and Acquisition, Endogenous Initiation of Bargaining, Two-Sided Private Information, Stand-Alone Value, Synergy, Real Option.

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There are large variations in who initiates a Merger and Acquisition (M&A hereafter) deal. For example, Boone and Mulherin (2007) document that during 1989-1999, 85% of M&As were initiated by the target, with the rest initiated by the bidders or a third party. In a more recent paper however, Masulis and Simsir (2013) show that during 1997-2006, only 40% of the deals are initiated by the target, while the rest are initiated by bidders. In addition, Masulis and Simsir (2013) show that a deal tends to be initiated by a target when the target is in financial distress, financially constrained, during industry or economic shock, or due to certain private information (e.g. assets in place are of low quality). In addition, they show that the private information about target's assets in place is the main reason why bid premium granted to target is lower for target-initiated deals.

Despite the evidence above, there is little in the theoretical literature that discusses initiation of M&A deals¹. To fill in the gap in the literature, we build a framework with two-sided private information to explain endogenous initiations of M&A negotiations. We will focus the following research questions: (1) what determines the timing of M&A initiation? (2) who becomes the initiator of M&A negotiation? In particular, we consider a dynamic game of negotiation initiation where we take an existing pair of merging firms as given. At any time, each firm can approach its opponent to initiate an M&A negotiation, during which they bargain over the splitting of shares of the new firm and monetary transfer. If the negotiation is successful, a new firm will be established, whose value amounts to the sum of the two firm's stand-alone values and their synergy. If the merger negotiation is unsuccessful, both firms return to their original stand-alone values before the merger negotiation. In addition, before the game starts, both merging parties privately observe a signal. The signals could be either about stand-alone values, or about the synergy. We can think of cross-industry M&As, such as vertical or conglomerate M&A as examples for the former. An example for the latter case is horizontal M&A within industry, where synergy lies in market power or reduction of costs. In this paper we focus on these two extreme cases and illustrate how results of initiation could be drastically different for the two scenarios. Finally, our focus is on cases where there is common knowledge of gains from trade, although there is some asymmetric information about stand-alone value or the exact value of synergy.

The key driving force is that the timing of initiation could reveal information about each firm's private signals. In addition, how to interpret the timing of initiation as a signal for private information depends crucially on whether the private information is about stand-alone value or about synergy. In particular, when private information is about stand-alone value, the type with lower stand-alone value initiates faster than higher-value types of the same firm. That is, we've found a unique separating equilibrium of initiation game, in which earlier initiation indicates lower stand-alone value, and not having initiated yet signals higher stand-alone value. The reason why

¹The existing works would be discussed in the section of literature review.

we have such equilibrium is as follows. Under the current bargaining protocol, the types who are believed to enjoy relatively higher stand-alone value are able to ask for higher compensation in the negotiation stage after initiation. Therefore lower stand-alone value types would like to mimic higher value types from the same firm, so as to obtain a better term of trade in the bargaining stage. Yet the higher value types are able to separate themselves from the lower types by postponing the initiation into a further future than the lower types. This is precisely because the marginal benefits of delaying initiation for higher types are higher.

However, when private information is about synergy, firms with higher synergy signals initiate faster than lower synergy types of the same firm. This is because opposite to the stand-alone value case, types with higher synergy would like to mimic lower types in order to get higher surplus in the bargaining stage, but lower synergy type is able to separate themselves from the higher types by delaying more because the marginal benefit of waiting is lower for types with higher synergy.

This paper makes the following predictions about the characteristics of an initiator who starts the M&A negotiation. The first feature of initiator is the relative position of a firm's true type in its own population distribution. When private information is about stand-alone value, under both the benchmark one-shot bargaining protocol and a two-stage alternative-offer game, the firm whose stand-alone value lies in the lower tail of its own population distribution compared to that of his opponent will be the initiator. When private information is about synergy, if assuming a two-stage alternative-offer bargaining protocol with the initiator making the first offer, we predict that firms with lower signals of synergy delay initiation to separate themselves from the higher synergy types, so as to obtain higher profit in the bargaining stage. Therefore, the firm with type in the upper tail of its own population compared to its opponent becomes the initiator. These results are broadly consistent with the empirical evidence in Masulis and Simsir (2013) that target private information affects who becomes the initiator.

However, for the synergy case and under the benchmark one-shot bargaining protocol, we predict that there will be less inefficient delay of initiation compared to that of the stand-alone value case. In particular, there would be no separating equilibrium in which firms inefficiently delay initiation. The intuition behind such results lies in the differences in conflicts of interests between these two cases. When private information is about stand-alone value, it brings about adverse selection. In contrast, private information about synergy creates positive selection. Since conflicts of interest are stronger in the case of stand-alone value than that of synergy, we would see more inefficient delay in the case of private stand-alone value.

The second factor that determines who becomes the initiator is the discount rate, which could be interpreted as the riskiness of the assets in place. If a firm's assets in place are riskier than its opponent, then the firm is more inclined to initiate. This result is again consistent with the evidence in Masulis and Simsir (2013) that a deal is target-initiated when the target is

financially distressed.

Moreover, cash constraints to the merging firms also affect incentives to initiate. For example, we show that if only one firm is completely cash constrained, then there is one equilibrium in which the constrained firm never initiates, while the unconstrained firm with higher expected synergy type initiates faster. This result indicates that cash constraint could possibly dampen the incentive to initiate early. In addition, bargaining power of each firm also plays a role.

Finally, we extend the model to consider an environment with merger costs and market-wide uncertainty. Under this extension, we obtain additional results. In this extension, we assume there is a fluctuating market-wide factor that affects the synergy, as well as both of the stand-alone values. This uncertain factor is modeled as an observable diffusion process with known parameters. Then we focus on Markov Perfect Bayesian Equilibrium with threshold-exercising strategies, where players initiate the first time the market-wide factor reach a certain threshold from below. Our results are as follows. Firstly, we find a unique separating equilibrium in both the stand-alone value case and the synergy case. Secondly, we characterize both separating equilibria. In the stand-alone value case, lower types initiate earlier, while in the synergy case, lower types initiate later. Finally, we find that as in the benchmark case, the firm with higher bargaining power initiates faster in the stand-alone value case. However, bargaining power has no effect on initiating strategies in the synergy case.

The remaining part of the paper is organized as follows. Section 1 cites related literature. Section 2 sets up the benchmark model. Section 3 solves the model in the benchmark case with one-shot bargaining protocol. Section 4 compares the results in benchmark with an two-stage alternative-offer bargaining protocol. Section 5 extends results to asymmetric cash constraints when private information is about synergy. Section 6 considers results with market-wide uncertainty and merger costs. Section 7 concludes.

1 Related Literature

Although there is a rich literature on M&A, both empirical and theoretical research on the initiation of M&A deal are very limited. One exception in empirical research is Masulis and Simsir (2013), which reveals that target financial or economic weakness, target financial constraints and economy wide shocks are important motives for target-initiated deals. They also find that average bid premiums, target cumulative abnormal returns (CAR) measured around the merger announcement dates and the deal value to EBITDA multiples of target-initiated deals is significantly lower than in bidder-initiated deals. However, this gap cannot be explained by weaker financial conditions of targets immediately prior to merger announcements. After adjusting for self-selection, they find evidence that the private information held by target firms is the main driver of the lower premiums

observed in target-initiated deals.

The exception in theoretical research is Gorbenko and Malenko (2014), which is the most related work to our paper. However, unlike our paper which focuses on the initiation of M&A *negotiation* potentially by both merging parties, their interest lies in the initiation of M&A *auction* by buyers only. Furthermore, in their paper all private information is only about synergy, and only buyers hold private information, while our paper allows for two-sided private information in both synergy and stand-alone value. Another related paper is Gorbenko and Malenko (2013), which allows for both buyer and seller to initiate an auction. However, the types of buyers are changing randomly over time, so agents do not signal fixed type by timing, unlike our model. Finally, our paper is related to Rhodes-Kropf and Viswanathan (2004). However, not only their paper is about auction, but the informational structure and initiation rule are different from our paper as well.

In reality, one-to-one negotiation is nearly as common as auction. Moreover, two-sided private information is often more realistic than one-sided private information under the M&A settings. Therefore, our paper complements the existing theory works by exploring a scenario which is different, but equally realistic.

Next, there is a literature on static mechanism design on merger and acquisition. According to Brusco, Lopomo, Robinson and Viswanathan (2007), one can always implement an efficient mechanism when private information is only about synergy. However, efficient mechanism is often not achievable when private information is about stand-alone value. Although stemming from a static mechanism design framework, this result is consistent with our game-theoretic result of inefficient delay. A similar paper is Gartner and Schmutzler (2009), which considers efficient mechanism design when terms of merger can depend on ex-post information.

In addition, it is related to a literature on merger timing that characterize merger decision as real option. A few examples are Lambrecht (2004), Morellec and Zhdanov (2005), and Leland (2007). However, none of these papers study the case where there is two-sided private information between merger parties.

Also, it is related to the literature on signaling game in real option. Grenadier and Malenko (2011) studies real option signaling games. Similar to our paper, the equilibrium strategy is a stationary stopping time problem. In addition, whether it is the higher type or the lower type to exercise at lower threshold level is subject to changes when the particular economic environment or payoff changes. However, their paper is very different from this paper, because there is only one-sided private information.

2 Model Setup: Benchmark Model

Preliminaries. Consider a pre-matched pair of firms that might potentially merge with each other. Both firms are risk neutral. Firm i 's discount rate is $r_i > 0$. There is no natural buyer or seller when the game starts. Instead, we allow the roles of buyer and seller to be endogenously determined in equilibrium. In addition, we focus on one-to-one *negotiation* of M&A.

There are two reasons for focusing on negotiation. First, it is a common practice in M&A transactions. According to Boone and Mulherin (2007) and Masulis and Simsir (2013), about half of M&As were settled in one-to-one negotiations without any third party involved, while the rest were settled in public or private auctions. Second, there might be some exogenous reasons for why a negotiation is chosen over an auction in an M&A deal. For example, in an auction with many bidders, the merging parties are concerned about confidential information leakage during the due diligence stage. In addition, although an auction facilitates searching for a better-matched merger partner, it is often more costly. Therefore, in cases where two firms are familiar with each other due to a pre-existing relationship, or where it is clear that the two parties constitute a good match, one-to-one negotiation is preferred to auction.

Firm Value. Firm value at any time t is the discounted value of its perpetual flow payment. For $i = 1, 2$, let firm i 's flow payment as a stand-alone firm be $r_i v_i$. Then firm i 's stand-alone value at any time before the merger is v_i . Suppose the synergy between the two firms after merger is a . Then the value of new firm after a successful merger is $v_1 + v_2 + a$, which is the sum of the stand-alone values of the two firms and their synergy. In addition, firm i 's value remains unchanged to its pre-merger stand-alone value v_i if the merger negotiation breaks down.

Private Information. Before the game starts at $t = 0$, there is two-sided private information between the two firms. We will explore the distinct results under the two cases about information structure below.

- Case 1.* **Stand-Alone Value Case:** The private information is only about stand-alone value v_i , while synergy a is common knowledge with $a > 0$. The prior of v_i has support $[l_i, h_i]$, c.d.f. $F_i(\cdot)$ and p.d.f. $f_i(\cdot)$.
- Case 2.* **Synergy Case:** The private information is only about synergy a , but stand-alone value v_i is common knowledge. Each of the two firms privately obtains a signal a_i about the synergy a , with $a = a_1 + a_2$. The prior of a_i has support $[l_i, h_i]$, c.d.f. $F_i(\cdot)$ and p.d.f. $f_i(\cdot)$. Assume $l_i > 0$, implying common knowledge of gains from trade.

For both cases we allow asymmetric distribution for the prior of the two firms; hence we permit both mergers of equals and mergers between a small and large firm².

The Initiation Game and The Continuation Game of Bargaining. From $t = 0$ onwards, both firms can initiate a M&A bargaining at any time. A bargaining starts instantly as long as any firm approaches its opponent. In the benchmark model, we assume the bargaining protocol in the continuation game after initiation as a *reduced-form, one-shot* game. That is, with probability θ_1 , firm 1 gives a Take-It-Or-Leave-It (TIOLI) offer $\{(S_{1,j}, T_{1,j})\}_{j=1,2}$ to firm 2; with probability θ_2 , firm 2 gives a TIOLI offer $\{(S_{2,j}, T_{2,j})\}_{j=1,2}$ to firm 1. The two probabilities sum up to 1, that is, $\theta_1 + \theta_2 = 1$. Here firm i 's offer $\{(S_{i,j}, T_{i,j})\}_{j=1,2}$ specifies $S_{i,j}$ as the fraction of merged firm allocated to j , and $T_{i,j}$ as the monetary transfer to j . If the offer $\{(S_{i,j}, T_{i,j})\}_{j=1,2}$ is accepted, the payoff to firm i is $S_{i,j} [v_1 + v_2 + a] + T_{i,j}$. Otherwise, both firms return to their pre-merger stand-alone values. In Section 4, we will consider an alternative bargaining protocol and compare the results under the two protocols.

The reasons we assume the particular one-shot bargaining protocol in the benchmark model are as follows. First, the negotiation is assumed to be a one-shot game, while bargaining is often a dynamic process in reality. This is because to have a better understanding of who becomes an initiator, we focus on the dynamics of initiation by abstracting away the dynamics in the bargaining stage. Second, as will be seen later, the probability θ_i of firm i making a TIOLI offer captures the share of total surplus attributed to firm i if the merger is successful. Therefore, we interpret θ_i as the intrinsic bargaining power of firm i . Finally, initiation does not necessarily imply making an offer, because the firm that initiates faster does not have to enjoy stronger bargaining power.

Cash Constraints. We assume that there are no cash constraints for either firms. That is, there is no upper bound for $|T_{i,j}|$, $i, j = 1, 2$.

Equilibrium Concept. The equilibrium concept we apply here is Perfect Bayesian Nash Equilibrium (PBE).

Equilibrium Refinement. As will be shown later, this bargaining protocol allows a firm with private information to offer a mechanism. Therefore, it leads to multiple equilibria in the continuation game. We select the equilibrium that achieves the maximum outcomes for every type. This is equivalent to Pareto Dominance Criterion when we only consider the utility of informed proposers;

²We use the same notation for the prior for v_i in the first case and that of a_i in the second. This only serves to simplify notation and does not imply the two distributions are identical. Since the two cases are completely separate, such a notation will not cause any confusion.

therefore we name it *One-Sided Pareto Dominance Criterion*. In addition, in the initiation game, we impose standard Pareto Dominance Criterion to pin down the unique separating equilibrium.

Reputation Cost for Off-Scheduled Deviation. Given a Perfect Bayesian Nash Equilibrium (PBE), we assume that firms will suffer from reputation cost $K > a$ if they conduct *off-Scheduled Deviation* from the equilibrium.

Following Athey and Bagwell (2001), and Athey, Bagwell and Sanchirico (2004), we separate off-equilibrium-path deviations into two categories, i.e., “off-scheduled” and “on-scheduled”. In accordance with their definition, we define an “off-scheduled” deviation as an action that is not assigned to any types according to the equilibrium belief, and thus represents a clear deviation which is observable to its opponent. We then define an “on-scheduled” deviation as an action to misrepresent one’s private information and choose an equilibrium action intended for a different type. Such an “on-schedule” deviation is not observable, as a deviation, to the opponent firm.

Therefore, the cost K on “off-scheduled” deviation is a reduced-form punishment that a firm can exert on its opponent during some future interactions after the M&A deal ends, if it observes a clear deviation by its opponent. Since $K > a$, the reputation cost is higher than the maximum profit a firm can get from the M&A deal, “off-scheduled” deviation is not profitable. With this in mind, we only need rule out “on-scheduled” deviations in order to justify an PBE, because a PBE requires both “off-scheduled” and “on-scheduled” deviations to be unprofitable. Note that although the reputation cost resembles off-EQ-Path punishing beliefs, it is not an equilibrium refinement.

Time-Line. We wrap up the settings above with the succeeding time-line.

- As time goes by, one firm initiates first at t_i ;
- Then both firms enter an one-shot continuation game of bargaining:
 - with probability θ_i , i offers $\{(S_{i,j}, T_{i,j})\}_{j=1,2}$; with probability θ_{-i} , i decides whether to accept an offer made by its opponent.

Therefore, the action for firm i is in the form of $\{t_i, x_i, d_i\}$, where $t_i \in [0, +\infty)$ is the time for initiation, $x_i = \{(S_{i,j}, T_{i,j})\}_{j=1,2} \in ([0, 1] \times R)^2$ for $i = 1, 2$ is the cash-stock combination assigned to both parties if i makes the TIOLI offer, and $d_i \in \{0, 1\}$ is the decision for acceptance if i ’s opponent makes the offer.

3 Benchmark: Separating Equilibrium

We would like to focus on separating PBE, and we pin down the separating equilibrium with the condition that no “on-scheduled” deviation is profitable.

We solve the equilibrium backwards by two steps.

Step 1: given one firm has initiated at t , we solve for the bargaining stage;

Step 2: given the terminal payoff in the bargaining stage, we solve for the initiation timing in equilibrium.

3.1 Negotiation Stage: Continuation Game after Initiation

We start with Step 1 and study the continuation game in the bargaining stage. Suppose in equilibrium, for $i = 1, 2$, firm $-i$ believes that firm i 's type is $x_i(t)$ if it observes firm i initiating at t . In a separating equilibrium where initiation timing reveals all the private information about type, $x_i(\cdot)$ is strictly monotonic.

The information structure of the continuation game of a separating equilibrium is as follows. The initiator reveals the value of his type by the timing of initiation. From the initiator's perspective, the posterior for the non-initiator's type becomes a truncated distribution of the prior, given that the non-initiator has not initiated yet, while the exact value of the non-initiator's type is still unknown to the initiator. Therefore, the bargaining game is basically a screening game when the uninformed initiator offers, and a signaling game when the informed non-initiator offers. With that in mind, we investigate the continuation game for both the stand-alone value case and the synergy case.

Proposition 1. (*Continuation Game for the Stand-Alone Value Case*)

Suppose private information is about stand-alone value. Suppose firm $-i$ believes firm i 's type to be $x_i(t)$ if i initiates at t , $i = 1, 2$, and that $x_i(t)$ is strictly monotonic. Then we have the following descriptions about the outcome in the continuation game:

(i) *If it is the initiator i who makes the ultimatum offer, it offers to sell all its shares to the non-initiator $-i$ at*

$$\underbrace{a}_{\text{total surplus}} + \underbrace{x_i(t)}_{i's \text{ revealed stand-alone value}}$$

and trade occurs with probability one.

(ii) *If it is the non-initiator $-i$ who makes the ultimatum offer, it offers to buy all shares of the initiator's firm at $x_i(t)$, and trade occurs with probability one.*

In both scenarios, all types of non-initiator trade, and the offer-er gets all the surplus.

Here is a sketch of proof. To start with, since we only allow “on-scheduled” deviation from

the separating equilibrium, the initiator i 's action in the continuation game should be consistent with type $x_i(t)$, which is the type it mimics by initiating at t . With this in mind, we analyze the following two cases.

(1) When initiator i proposes, it designs the optimal contract to minimize the information rent extracted by the informed initiator. Using the standard techniques by Myerson (1981), we show that the optimal direct mechanism requires all types of non-initiator to own the entire new firm, and to pay $a + x_i(t)$ to the initiator. This direct mechanism is implemented by (i) in the proposition.

(2) When non-initiator offers, one can verify that (ii) is indeed a pooling equilibrium. The off-EQ-path belief supporting this pooling equilibrium is that if the non-initiator offers to get only a fraction of the entire firm, the initiator believes that its opponent must be of the lowest stand-alone value. Although there are multiple equilibria, the pooling equilibrium as described in (ii) survives the criterion of *One-Sided Pareto Dominance*; hence it becomes the natural equilibrium we select. This is because the pooling equilibrium grants the informed proposer with the entire total surplus whatever its type is, therefore it achieves the maximum payoff for proposers of all types. In addition, this equilibrium also survives intuitive criterion in Cho and Kreps (1987) and D-1 criterion in Banks and Sobel (1987), indicating that the pooling equilibrium we select could be reasonable.

The intuition for the selling-out and buying-out result is as follows. When the uninformed initiator offers, the optimal screening contract makes private information irrelevant for the informed non-initiator's acceptance decision. Hence, for the stand-alone value case, when informed non-initiator privately knows her stand-alone value, optimal mechanism requires her to retain zero share of her own firm value. When the informed non-initiator offers, it proves its type to be of high stand-alone value by retaining as much shares of the merged firm as possible. Since any types not offering to own the entire new firm is regarded as the worst type, there is a pooling equilibrium where all types of non-initiator offer to get the entire new firm.

With a very similar reasoning, we also obtain the outcome of the continuation game for the synergy case.

Proposition 2. (*Continuation Game for the Synergy Case*)

Suppose private information is about signals on synergy. Suppose firm $-i$ believes firm i 's type to be $x_i(t)$ if i initiates at t , $i = 1, 2$, and that $x_i(t)$ is strictly monotonic. Then we have the following descriptions about the outcome in the continuation game:

(i) *When the initiator i makes the TIOLI offer, it offers to buy all shares of the non-initiator at non-initiator's stand-alone value, v_{-i} .*

(ii) *When the non-initiator $-i$ makes the TIOLI offer, it offers to buy all shares of the initiator at initiator's stand-alone value, v_i , and trade occurs with probability 1.*

In both scenarios, all types of non-initiator trade, and the offer-er gets all the surplus.

Compare the Continuation Games in the Two Cases. The results in the two cases shared some features in common. First, when the informed non-initiator offers, there is a pooling equilibrium where all types of non-initiator offer to get the entire new firm so as to avoid adverse selection in both cases. Second, all gains from trade is realized, and the offer-er gets all the surplus. This result is somewhat surprising because the uninformed initiator still obtains all the surplus when it makes the offer, although by initiating early it reveals its private type and faces informational disadvantage. This is because when there are no cash constraints, the uninformed initiator leaves the informed non-initiator zero surplus by optimally choosing the screening contract. In contrast, when there are cash constraints and the optimal contract can no longer to be achieved, we would expect that the informed non-initiator to enjoy some information rent. Finally, in both cases, the roles of buyer and seller are endogenously determined. That is, it is an equilibrium result who ends up buying out whom.

However, the two cases are also very different. To start with, the allocation of shares are very distinct when the uninformed initiator makes the offer. In the stand-alone value case, when the informed non-initiator privately knows its stand-alone value, optimal mechanism requires it to retain zero share of its own firm value. But in the synergy case, when the informed non-initiator privately knows its contribution to synergy, optimal mechanism requires it to get zero share of synergy from new firm. Although the optimal mechanisms in both cases make the private information of the offer acceptor irrelevant for its acceptance decision, the resulting mechanism differs due to the distinct information structures.

Also, the monetary transfer from non-initiator $-i$ to initiator i is different. For the stand-alone value case, the transfer is increasing in the perceived stand-alone value $x_i(t)$. However, for the synergy case, the transfer does not depend on perceived type.

Finally, we look at the allocation of shares and monetary transfer together. The overall terms of trade to initiator i includes both the fraction of merged firm and the monetary transfer. As we've already seen, the former does not depend on the perceived type in both cases, while the latter depends on perceived type only for stand-alone value case. This idea is further illustrated when we look at the terminal payoff in the continuation game for both cases.

Proposition 3. *(Terminal Payoff for the Stand-Alone Value Case)*

Suppose private information is about stand-alone value. Suppose firm $-i$ believes firm i 's type to be $x_i(t)$ if i initiates at t , $i = 1, 2$, and that $x_i(t)$ is strictly monotonic. Also assume without loss of generality that the bargaining stage is initiated by firm i at t_i . Then payoffs in the bargaining stage to both firms are as follows.

(i) firm i of type v_i gets in addition to its stand-alone value the amount

$$G_i^{SV}(v_i, t_i, v_{-i}) = \theta_i \cdot \underbrace{a}_{\text{total surplus}} + \underbrace{x_i(t_i) - v_i}_{\text{benefit of mimicking}},$$

(ii) firm $-i$ of type v_{-i} gets in addition to its stand-alone value the amount

$$H_{-i}^{SV}(v_{-i}, t_i, v_i) = \theta_{-i} \cdot \underbrace{a}_{\text{total surplus}} - [x_i(t_i) - v_i].$$

The proposition is a straightforward implication of Proposition 1. As have been discussed earlier, the payoff $G_i^{SV}(v_i, t_i, v_{-i})$ to initiator is increasing in perceived type $x_i(t_i)$, so it is beneficial for lower types to mimic higher types.

The next proposition summarizes the terminal payoff for the synergy case.

Proposition 4. (*Terminal Payoff for the Synergy Case*)

Suppose private information is about signals on synergy. Suppose firm $-i$ believes firm i 's type to be $x_i(t)$ if i initiates at t , $i = 1, 2$, and that $x_i(t)$ is strictly monotonic. Also assume without loss of generality that the bargaining stage is initiated by firm i at t_i . Then payoffs in the bargaining stage to both firms are as follows.

(i) firm i of type a_i gets in addition to its stand-alone value the amount

$$G_i^{SY}(a_i, t_i, a_{-i}) = \theta_i \cdot \underbrace{(a_i + a_{-i})}_{\text{total surplus}},$$

(ii) firm $-i$ of type a_{-i} gets in addition to its stand-alone value the amount

$$H_{-i}^{SY}(a_{-i}, t_i, a_i) = \theta_{-i} \cdot \underbrace{(a_i + a_{-i})}_{\text{total surplus}}.$$

Unlike in the stand-alone value case, the payoff to initiator does not depend on the non-initiator's belief on the initiator's type. Therefore, in this case, there would be no mimicking incentive, hence no separating equilibria and no delay.

Despite the differences between the terminal payoffs in the two cases, they both include the term $\theta_i \cdot \text{total surplus}$. This shows that θ_i indeed captures the fraction of total surplus that goes to i , which justify our reduced-form assumption on bargaining protocol.

Given the terminal payoff in the continuation game, we proceed to the initiation game.

3.2 Initiation Game

At any time t , if its opponent has not initiated yet, firm i decide between initiating right away and waiting until $t + \varepsilon$. If it chooses to wait, it collects all flow payment between t to $t + \varepsilon$, and gets the continuation value at $t + \varepsilon$. With the following Lemma, we convert the original problem into a equivalent but simpler problem.

Lemma 5. *When time is continuous, the following two stopping time problems are equivalent.*

- (i) *Getting flow payment with rate rv if not stopped yet, and getting G_t if stopped at t .*
- (ii) *Getting no flow payment until stopped, and getting $G_t - v$ when stopped at t .*

Proof. At time t , the tradeoff in problem (i) is whether to stop and obtain G_t today, or to wait until $t + \varepsilon$ to get $rv\varepsilon + e^{-r\varepsilon}U_{t+\varepsilon}$, where $U_{t+\varepsilon}$ is the continuation value at $t + \varepsilon$. Since $\varepsilon \rightarrow 0$ when time is continuous, by Taylor's Expansion the total payoff if waiting is $rv\varepsilon + e^{-r\varepsilon}U_{t+\varepsilon} \approx rv\varepsilon + (1 - r\varepsilon)U_{t+\varepsilon} = rv\varepsilon + (1 - r\varepsilon)U_{t+\varepsilon}$. That is to say, the tradeoff in problem (i) is whether to stop and obtain G_t today, or to wait until $t + \varepsilon$ to get $rv\varepsilon + (1 - r\varepsilon)U_{t+\varepsilon}$. This is equivalent to the tradeoff in problem (ii) between stop and obtaining $G_t - v$ today, or to wait until $t + \varepsilon$ to get $(1 - r\varepsilon)(U_{t+\varepsilon} - v) \approx e^{-r\varepsilon}(U_{t+\varepsilon} - v)$. $\square$

With Lemma 5 in mind, we can ignore the flow payment and consider only terminal payoff net of stand-alone value.

3.2.1 The Stand-Alone Value Case

Proposition 6. *(Initiation Timing for the Stand-Alone Value Case)*

If the stand-alone values are private information, then there exists a unique separating equilibrium with continuous differentiable strategy. In this equilibrium, firm i with type v_i initiates at $t_i(v_i)$, such that

$$\begin{aligned} t'_i(v) &= \frac{1}{ar_i\theta_i} > 0, \\ t_i(l_i) &= 0. \end{aligned}$$

Moreover, the equilibrium is invariant to changes in the prior distribution of types.

To understand the proposition, we go through the basic idea of the proof. Denoting $t_i(v)$ as the equilibrium time to initiate for type v of firm i . We assume $t_i(v)$ to be strictly monotonic and continuously differentiable to solve for $t_i(\cdot)$, and we verify later. Denote $x_i(t) = t_i^{-1}(t)$ as the inverse function. Then we can characterize the decision of timing for firm i in a Perfect Bayesian Nash Equilibrium.

Consider firm i 's decision. Since the problem is time consistent, we consider firm i 's decision of initiating time $\hat{t}$ at $t = 0$, given firm $-i$'s equilibrium strategy $t_{-i}(\cdot)$ or its inverse function $x_{-i}(\cdot)$, and given firm $-i$'s belief on firm i 's strategy $t_i(\cdot)$ or its inverse function $x_i(\cdot)$. According to Proposition 3, if firm i initiates first at time $\hat{t}$, firm i 's terminal payoff net of its stand-alone value in the bargaining stage is $G_i^{SV}(v_i, \hat{t}, \tilde{v}_{-i}) = \theta_i a + x_i(\hat{t}) - v_i$. If firm $-i$ initiates first at time $t_{-i}(\tilde{v}_{-i})$, firm i 's terminal payoff net of its stand-alone value in the bargaining stage is $H_i^{SV}(v_i, t_{-i}(\tilde{v}_{-i}), \tilde{v}_{-i}) = \theta_i a - [x_{-i}(t_{-i}(\tilde{v}_{-i})) - v_{-i}] = \theta_i a$. That is, for a $\hat{t}$ at time 0, there are two cases that could happen depending on the value of $\tilde{v}_{-i}$.

Case 1. If $t_{-i}(\tilde{v}_{-i}) \leq \hat{t}$, then $-i$ initiates first at $t_{-i}(\tilde{v}_{-i})$, and firm i gets

$$e^{-r_i t_{-i}(\tilde{v}_{-i})} H_i^{SV}(v_i, t_{-i}(\tilde{v}_{-i}), \tilde{v}_{-i}) = e^{-r_i t_{-i}(\tilde{v}_{-i})} \theta_i a.$$

Case 2. If $t_{-i}(\tilde{v}_{-i}) > \hat{t}$, then i initiates first at $\hat{t}$, and firm i expects to get

$$e^{-r_i \hat{t}} \mathbb{E} \left[G_i^{SV}(v_i, \hat{t}, \tilde{v}_{-i}) \mid t_{-i}(\tilde{v}_{-i}) > \hat{t} \right] = e^{-r_i \hat{t}} \left[\theta_i a + x_i(\hat{t}) - v_i \right].$$

Furthermore, If denoting the p.d.f. of $t_{-i}(\tilde{v}_{-i})$ as $f_{t_{-i}}(\cdot)$, then $f_{t_{-i}}(t) = f_{-i}(x_{-i}(t)) \cdot x'_{-i}(t) \cdot \text{Sign}(x'_{-i}(t))$, where $f_{-i}(\cdot)$ is the p.d.f. for $\tilde{v}_{-i}$, and $x_{-i}(t) = t_{-i}^{-1}(t)$. Then firm i 's problem at $t = 0$ is as below:

$$\begin{aligned} \max_{\hat{t}} \quad & \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) \cdot e^{-r_i t_{-i}} H_i^{SV}(v_i, t_{-i}, x_{-i}(t_{-i})) dt_{-i} \\ & + \left[1 - \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) dt_{-i} \right] \cdot e^{-r_i \hat{t}} \mathbb{E} \left[G_i^{SV}(v_i, \hat{t}, \tilde{v}_{-i}) \mid t_{-i}(\tilde{v}_{-i}) > \hat{t} \right] \end{aligned}$$

In equilibrium, with consistent belief, we should have

$$x_i(\hat{t}) = v_i.$$

F.O.C. to $\hat{t}$ and the belief consistency give the simple Ordinal Differential Equation

$$x'_i(\hat{t}) = \theta_i a r_i.$$

Then we have

$$t'_i(v) = \frac{1}{a r_i \theta_i} > 0,$$

$$t_i(l_i) = 0.$$

where the initial condition is required by efficiency concern. That is, this equilibrium Pareto Dominates all others, hence is the unique separating equilibrium that survives the Pareto Dominance Criterion. Finally, we show in the Appendix that the solution derived from F.O.C is indeed the global maximum.

Through the derivation above, we've already shown that any "on-scheduled" deviation with actions in the continuation game consistent with the type revealed by initiation timing is not profitable. Since no "off-scheduled" deviation is profitable due to reputation cost $K > a$, the above strategy profile and corresponding belief indeed constitute a PBE.

Discussion on Proposition 6

Proposition 6 has several implications.

The first is that $t'_i(v) > 0$, which implies that the type with lower stand-alone value initiates faster than higher-value types of the same firm. That is, we've found a unique separating equilibrium of the initiation game, in which an earlier initiation signals lower stand-alone value, and not having initiated yet signals higher stand-alone value. The underlying reasoning for the equilibrium is as follows. Under the current bargaining protocol, we show that terminal payoff of i if i initiates at $\hat{t}$ is:

$$G_i^{SV}(v_i, \hat{t}, \tilde{v}_{-i}) = \theta_i a + x_i(\hat{t}) - v_i$$

That is, the type who is believed to enjoy relatively higher stand-alone value is able to ask for higher compensation in the negotiation stage after initiation. Hence lower stand-alone value types would like to mimic higher value types from the same firm, so as to obtain a better term of trade in the bargaining stage. However, the higher value types are able to separate themselves from the lower types by postponing the initiation into a further future than the lower types. The key driving force for this successful separation lies in the fact that marginal benefits of delaying initiation for distinct types are different. Specifically, firm has to lose their stand-alone value in a successful merger after initiation. By waiting for an additional unit of time, the firm will benefit from delaying the date on which the loss of stand-alone value materializes, because expected loss gets to be lower when it is discounted further. Moreover, such reduction in expected loss is proportional to the stand-alone value. Therefore, types with higher stand-alone value salvage more from the loss by waiting longer. Such a difference in the marginal benefits of delaying initiation generates a Single-Crossing-Property, and enables the higher types to distinguish themselves from the lower mimicking types by waiting longer.

The second implication from Proposition 6 is about who becomes the initiator.

The first feature of initiator has to do with the relative position of firm's true type in its own population distribution. Specifically, the firm whose stand-alone value lies in the lower tail of

its own population distribution compared to that of his opponent will be the initiator. In this equilibrium, $t'_i(v) > 0$ implies that lower types of a particular firm initiate faster than higher types of the same firm. In addition, which firm becomes the initiator depends on whether the firm initiate faster than its opponent firm. Therefore, it is not the firm with higher absolute stand-alone value that initiate the negotiation. On the contrary, the initiator will be the firm with type in a relatively lower position of its own population distribution compared to its opponent. This result is consistent with the empirical evidence in Masulis and Simsir (2013) that target becomes the initiator when its assets in place are of unobservable low quality.

In **Figure 3.1**, the first picture on the left together with the picture in the middle illustrate this result. In the first picture on the left, we assume the two firms having the same support of priors for the stand-alone values, as well as equal bargaining power and discount rate. Then we see that firm 1 with lower stand-alone value initiates faster and hence becomes the initiator. However, the picture in the middle shows that it is not the absolute value that determines who is the initiator. Instead, the relative position in one's own population is the key determinant. We consider the case where the support of firm 1's stand-alone value is far below the support of firm 2's stand-alone value but with the same bargaining power and discount rate. That is, it is common knowledge that firm 1 is smaller. Consider v_1 close to h_1 , and v_2 close to l_2 . That is, although $v_1 < v_2$, v_1 lies in the upper tail of its own population distribution compared to that of v_2 . As predicted, $t_1(v_1) > t_2(v_2)$.

The second factor at play is discount rate r_i . The prediction is that firms with higher discount rate initiate faster. This result is again consistent with Masulis and Simsir (2013) that target becomes the initiator when it is financially distressed. Indeed, the third picture in **Figure 3.1** demonstrates that if we assume identical support of priors across two firms but different discount rates, the firm with higher bargaining power (higher r) initiates faster (smaller r) and becomes the initiator.

Finally, we predict that the firm with stronger bargaining power (higher θ_i) initiates faster, since it would like to capture the larger fraction of total surplus faster.

Next, we consider the synergy case.

3.2.2 The Synergy Case

Proposition 7. (*Initiation Timing for Synergy Case*)

If the signals of synergy is private information. Then

(i) *If assuming i 's opponent follows a separating timing strategy, then firm i 's optimal strategy is to initiate immediately at $t = 0$.*

(ii) *There is no separating equilibrium.*

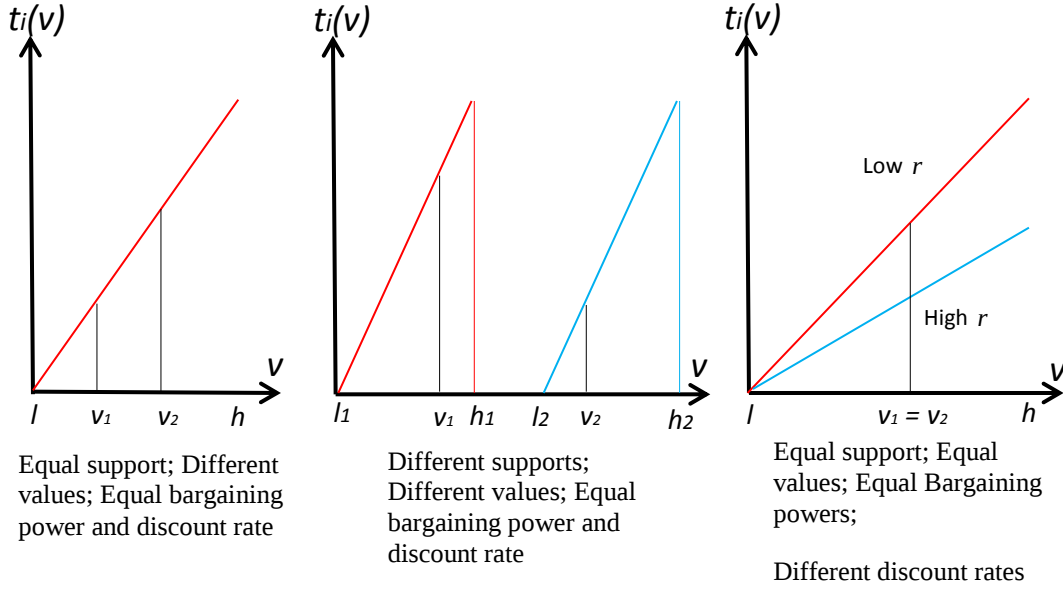


Figure 3.1: Comparative Statics of Initiation Time, Stand-Alone Value case

The procedure of the proof is basically the same as the stand-alone value case, so we omit it here.

Discussion on Proposition 7

When private information is about synergy, it turns out that there is no fully separating equilibrium where different types initiate at different dates. More specifically, if one firm expects to be approached at distinct times when faced with different types of its opponent, it will initiate immediately at time 0 whatever its own type is. This result is in sharp contrast with the stand-alone value case, where in the unique separating equilibrium under that scenario, higher types delays initiation to separate themselves from lower types so as to capture higher surplus in the bargaining stage. The source of such distinction lies in the bargaining stage. From Proposition 4 and the fact that $-i$ has not initiated yet, we know that the expected payoff to i if i initiates at $\hat{t}$ does not depend on perceived type. That is, the term

$$\begin{aligned}
 \mathbb{E} \left[G_i^{SY} \left(a_i, \hat{t}, \tilde{a}_{-i} \right) | t_{-i}(\tilde{a}_{-i}) > \hat{t} \right] &= \mathbb{E} \left[\theta_i (a_1 + a_2) | t_{-i}(\tilde{a}_{-i}) > \hat{t} \right] \\
 &= \theta_i \cdot \underbrace{\left[a_i + \mathbb{E} \left(a_{-i} | -i \text{ has not initiated at } \hat{t} \right) \right]}_{\text{expected total surplus}}
 \end{aligned}$$

is not a function of perceived type $x_i(\hat{t})$. As a result, no one would try to trick its opponent into a false belief in order to obtain a better term of trade in the bargaining stage. That is to

say, without the threat from a mimicking type, no type will find it necessary to separate itself by delaying. Therefore, there exists no separating equilibrium of initiation game.

Comparing Proposition 6 to Proposition 7 gives implications on inefficient delay in the timing of M&A initiation. We predict that there will be more delay when private information is about stand-alone value than when it is about synergy. Since we assume there are gains from trade for sure, any delay of initiation is inefficient. Therefore we can also conclude that more inefficiency due to delay arises in the stand-alone value case than in the synergy case. As we have discussed earlier, this result derives from the fact that higher types try to separate from lower types by delaying in the stand-alone value case, while there is no mimicking incentive and therefore no need to separate from a mimicking type by delaying when private information is about synergy.

This result is comparable to some existing literature on M&A mechanism design. According to Brusco, Lopomo, Robinson and Viswanathan (2007), one can always implement an efficient mechanism when private information is only about synergy. However, efficient mechanism is often not achievable when private information is about stand-alone value. Although stemming from a static mechanism design framework, this result is consistent with our game-theoretic result of inefficient delay. The common intuition behind such results lies in the differences in conflicts of interests between these two cases. When private information is about stand-alone value, it brings about adverse selection. That is, if one firm's stand-alone value is higher, the firm is more reluctant to merge, but its opponent is more willing to merge. In contrast, private information about synergy creates positive selection. That is, the firm with higher synergy is more inclined to merge, while its opponent is also more eager to merge. Since conflicts of interest are stronger in the case of stand-alone value than that of synergy, we would see more inefficient delay in the case of private stand-alone value.

4 Alternative Bargaining Protocol

In this section, we replace the reduced-form one-shot bargaining protocol with bargaining with an explicit form and compare results of the two. In particular, we assume a two-staged bargaining game with the initiator offering first and the non-initiator making a counter offer if rejecting the first one. We see that for the stand-alone value case, the terminal payoff (and hence the initiation game) of the new bargaining become a sub-case of the reduced-form one-shot bargaining protocol game. However, for the synergy case, the terminal payoff to the initiator depends on the non-initiator's belief on the initiator's type, hence there is a unique separating equilibrium in which the firm with higher signal of synergy initiates faster than the lower types of the same firm.

For a formal analysis, suppose when approaching its opponent, the initiator comes up with an offer that could be of any stock and cash combination. If the non-initiator accepts the offer, game

ends. If not, after time $\Delta > 0$, the non-initiator makes an ultimatum counteroffer. Assume Δ is small.

Denote the discount factor for firm i as $\delta_i = 1 - e^{-r_i \Delta} \approx r_i \Delta$, $i = 1, 2$, where the second equality is based on Taylor Expansion. Then following Lemma 5, we can ignore the flow payment and replace actual payoff with payoff net of stand-alone value.

4.1 Stand-alone Value Case

Consider the case where private information is about stand-alone value. Suppose the initiator i signals its type to be $x_i(t)$ by initiating at time t .

In round 2, with similar logic in the one-shot bargaining game, the informed non-initiator $-i$ offers to buy all the shares of the uninformed initiator i at $x_i(t)$. So in the second round, the uninformed i gets in net $\max(x_i(t) - v_i, 0)$, and the informed $-i$ believes $-i$ gets in net $x_i(t) + v_{-i} + a - x_i(t) - v_{-i} = a$.

In round 1, with similar logic in the one-shot bargaining game, the uninformed i offers to sell all its shares to $-i$. Suppose the price is P_1 . If the game ends in round 1, the uninformed i gets in net $P_1 - v_i$, and the informed $-i$ believes $-i$ gets in net $x_i(t) + v_{-i} + a - P_1 - v_{-i} = x_i(t) + a - P_1$.

If the initiator i would like to induce $-i$ to accept the offer at round 1, we must have $x_i(t) + a - P_1 \geq \delta_{-i}a$, so the best price for i is $P_1^* = x_i(t) + a(1 - \delta_{-i})$. So if ending in round 1, i gets at the most $P_1^* - v_i = a(1 - \delta_{-i}) + x_i(t) - v_i$, while $-i$ believes itself to get $\delta_{-i}a$ (but actually gets $v_i + v_{-i} + a - P_1^* - v_{-i} = v_i + a - (x_i(t) + a(1 - \delta_{-i})) = \delta_{-i}a - (x_i(t) - v_i)$). If i would like to end in round 2, i gets $\delta_i \max(x_i(t) - v_i, 0)$, and $-i$ believes itself to get $\delta_{-i}a$.

Hence the game will end in round 1 with offer $P_1 = P_1^*$ iff

$$a(1 - \delta_{-i}) + x_i(t) - v_i > \delta_i \max(x_i(t) - v_i, 0).$$

Due to the assumption of reputation cost $K > a$ for off-scheduled deviations, i must act as if $v_i = x_i(t)$ no matter what the true value of v_i is. Then the inequality above is equivalent to

$$a(1 - \delta_{-i}) > 0,$$

which holds under our assumptions that $a > 0$ and $\delta_{-i} < 1$.

Therefore, in equilibrium, the initiator i offers to sell all its shares to the non-initiator $-i$ at

$$P_1^* = x_i(t) + a(1 - \delta_{-i}) \tag{4.1}$$

in round 1, and the non-initiator $-i$ accepts the offer. Then the terminal payoffs are as follows:

$$\begin{aligned} G_i^{SV}(v_i, t, v_{-i}) &= (1 - \delta_{-i})a + x_i(t) - v_i; \\ H_{-i}^{SV}(v_{-i}, t, v_i) &= \delta_{-i}a - (x_i(t) - v_i). \end{aligned}$$

This result is a sub-case of the reduced-form bargaining protocol in Proposition 3, with $\theta_i = 1 - \delta_{-i}$, and $\theta_{-i} = \delta_{-i}$.

4.2 Synergy Case

Consider the case where private information is about synergy. Suppose the initiator i signals its type to be $x_i(t)$ by initiating at time t .

In round 2, with similar logic in the one-shot bargaining game, the informed non-initiator $-i$ offers to buy all shares of the uninformed initiator i at v_i . So in the second round, the uninformed i gets in net $v_i - v_i = 0$, while the informed $-i$ believes he gets in net $v_i + v_{-i} + x_i(t) + a_{-i} - v_i - v_{-i} = x_i(t) + a_{-i}$.

In round 1, with similar logic in the one-shot bargaining game, the uninformed i offers to buy all shares of $-i$. Suppose the price is P_1 . If the game ends in round 1, the uninformed i gets in net $v_i + v_{-i} + a_i + a_{-i} - P_1 - v_i = v_{-i} + a_i + a_{-i} - P_1$, while the informed $-i$ believes it gets in net $P_1 - v_{-i}$.

If the initiator i would like to end in round 1, we must have $P_1 - v_{-i} \geq \delta_{-i}(x_i(t) + a_{-i})$, so the best price is $\hat{P}_1 = v_{-i} + \delta_{-i}(x_i(t) + a_{-i})$. So i gets at the most $v_{-i} + a_i + a_{-i} - \hat{P}_1 = v_{-i} + a_i + a_{-i} - (v_{-i} + \delta_{-i}(x_i(t) + a_{-i})) = a_i + a_{-i} - \delta_{-i}(x_i(t) + a_{-i})$, while $-i$ believes to (and actually) get $\hat{P}_1 - v_{-i} = v_{-i} + \delta_{-i}(x_i(t) + a_{-i}) - v_{-i} = \delta_{-i}(x_i(t) + a_{-i})$. If i would like to end in round 2, i gets $\delta_i \cdot 0 = 0$. So the payoff to initiator i is $\max\{a_i + a_{-i} - \delta_{-i}(x_i(t) + a_{-i}), 0\}$. Since $a_i + a_{-i} - \delta_{-i}(x_i(t) + a_{-i}) < 0$ only if $x_i(t) \neq a_i$, offering a price that forces the non-initiator $-i$ to the second round is a clear deviation (“off-scheduled deviation”). The reputation cost on i following the such “off-scheduled deviation” $K > a$ rules it out, hence initiator offer price $\hat{P}_1 = v_{-i} + \delta_{-i}(x_i(t) + a_{-i})$, and the bargaining ends in the first round. The actual payoffs to both firms are as follows:

$$\begin{aligned} G_i^{SY}(a_i, t, a_{-i}) &= a_i + a_{-i} - \delta_{-i}(x_i(t) + a_{-i}); \\ H_{-i}^{SY}(a_{-i}, t, a_i) &= \delta_{-i}(x_i(t) + a_{-i}). \end{aligned} \tag{4.2}$$

Note that the terminal payoffs here are actually very different from those in Proposition 4. In

particular, the payoff to initiator i , $G_i^{SY}(a_i, t, a_{-i})$, is a decreasing function of the perceived type $x_i(t)$! That is, agent i of higher synergy would like to mimic agent i of lower synergy, and lower synergy type would separate themselves from higher synergy types by delaying initiation more.

More formally, consider firm i 's decision of initiating time $\hat{t}$ at $t = 0$, given firm $-i$'s equilibrium strategy $t_{-i}(\cdot)$ or its inverse function $x_{-i}(\cdot)$, and given firm $-i$'s belief on firm i 's strategy $t_i(\cdot)$ or its inverse function $x_i(\cdot)$. According to (4.2), if firm i initiates first at time $\hat{t}$, firm i 's terminal payoff net of its stand-alone value in the bargaining stage is $G_i^{SY}(a_i, \hat{t}, \tilde{a}_{-i}) = a_i + \tilde{a}_{-i} - \delta_{-i}(x_i(\hat{t}) + \tilde{a}_{-i})$; . If firm $-i$ initiates first at time $t_{-i}(\tilde{a}_{-i})$, firm i 's terminal payoff net of its stand-alone value in the bargaining stage is $H_i^{SY}(a_i, t_{-i}(\tilde{a}_{-i}), \tilde{a}_{-i}) = \delta_i(x_{-i}(t_{-i}(\tilde{a}_{-i})) + a_i) = \delta_i(\tilde{a}_{-i} + a_i)$. That is, for a $\hat{t}$ at time 0, there are two cases that could happen depending on the value of $\tilde{a}_{-i}$.

Case 1. If $t_{-i}(\tilde{a}_{-i}) \leq \hat{t}$, then $-i$ initiates first at $t_{-i}(\tilde{a}_{-i})$, and firm i gets

$$e^{-r_i t_{-i}(\tilde{a}_{-i})} H_i^{SY}(a_i, t_{-i}(\tilde{a}_{-i}), \tilde{a}_{-i}) = e^{-r_i t_{-i}(\tilde{a}_{-i})} \delta_i(\tilde{a}_{-i} + a_i).$$

Case 2. If $t_{-i}(\tilde{a}_{-i}) > \hat{t}$, then i initiates first at $\hat{t}$, and firm i expects to get

$$\begin{aligned} e^{-r_i \hat{t}} \mathbb{E} [G_i^{SY}(a_i, \hat{t}, \tilde{a}_{-i}) | t_{-i}(\tilde{a}_{-i}) > \hat{t}] &= e^{-r_i \hat{t}} \mathbb{E} [a_i + \tilde{a}_{-i} - \delta_{-i}(x_i(\hat{t}) + \tilde{a}_{-i}) | t_{-i}(\tilde{a}_{-i}) > \hat{t}] \\ &= e^{-r_i \hat{t}} \left\{ a_i - \delta_{-i} x_i(\hat{t}) + (1 - \delta_{-i}) \mathbb{E} [\tilde{a}_{-i} | t_{-i}(\tilde{a}_{-i}) > \hat{t}] \right\} \end{aligned}$$

Furthermore, If denoting the p.d.f. of $t_{-i}(\tilde{a}_{-i})$ as $f_{t_{-i}}(\cdot)$, then $f_{t_{-i}}(t) = f_{-i}(x_{-i}(t)) \cdot x'_{-i}(t) \cdot \text{Sign}(x'_{-i}(t))$, where $f_{-i}(\cdot)$ is the p.d.f. for $\tilde{a}_{-i}$, and $x_{-i}(t) = t_{-i}^{-1}(t)$. Then firm i 's problem at $t = 0$ is as below:

$$\begin{aligned} \max_{\hat{t}} \quad & \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) \cdot e^{-r_i t_{-i}} H_i^{SY}(a_i, t_{-i}, x_{-i}(t_{-i})) dt_{-i} \\ & + \left[1 - \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) dt_{-i} \right] \cdot e^{-r_i \hat{t}} \mathbb{E} [G_i^{SY}(a_i, \hat{t}, \tilde{a}_{-i}) | t_{-i}(\tilde{a}_{-i}) > \hat{t}] \end{aligned}$$

In equilibrium, with consistent belief, we should have

$$x_i(\hat{t}) = a_i.$$

F.O.C. to $\hat{t}$ and the belief consistency would give the solution for $x_i(\cdot)$.

In order to obtain a simple analytical solution, assume a_i follows $U[l_i, h_i]$. Assume $t_{-i}(\cdot)$ is strictly decreasing and verify afterward. Then firm i 's problem becomes

$$\begin{aligned}
& \max_{\hat{t}} \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) \cdot e^{-r_i t_{-i}} \delta_i(x_{-i}(t_{-i}) + a_i) dt_{-i} \\
& + \left[1 - \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) dt_{-i} \right] \cdot e^{-r_i \hat{t}} \left\{ a_i - \delta_{-i} x_i(\hat{t}) + (1 - \delta_{-i}) \mathbb{E}[\tilde{a}_{-i} | t_{-i}(\tilde{a}_{-i}) > \hat{t}] \right\} \\
\Leftrightarrow & \max_{\hat{t}} \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) \cdot e^{-r_i t_{-i}} \delta_i(x_{-i}(t_{-i}) + a_i) dt_{-i} \\
& + \left[1 - \int_0^{\hat{t}} f_{t_{-i}}(t_{-i}) dt_{-i} \right] \cdot e^{-r_i \hat{t}} \left\{ a_i - \delta_{-i} x_i(\hat{t}) + (1 - \delta_{-i}) \frac{l_i + x_{-i}(\hat{t})}{2} \right\}
\end{aligned}$$

F.O.C to $\hat{t}$ and let

$$x_i(\hat{t}) = a_i,$$

we find the solution. Therefore, we have the following proposition.

Proposition 8. (*Synergy Case with Two-Stage Bargaining*)

Suppose the private information is about synergy, and the negotiation after initiation is a two-stage alternative-offer game with the initiator making the first offer. If a_i follows $U[l_i, h_i]$, then there exists a unique separating equilibrium in which $x'_i(\cdot) < 0$.

Example 9. Figure 4.1 shows a numerical Solution to $x_i(\cdot)$ when $l_i = 1$, $h_i = 2$, $\delta_1 = 0.7$, $\delta_2 = 0.9$, $r_1 = 1$, $r_2 \approx \frac{(1-\delta_2)}{(1-\delta_1)} r_1$. The last line comes from the approximation that $\delta_i = e^{-r_i \Delta} \approx 1 - r_i \Delta$. $x_i(t)$ is on the y-axis, while t is on the x-axis. As can be seen, $x_i(\cdot)$ is strictly decreasing and so is $t_i(\cdot)$, meaning that types with lower synergy initiates later. Also, $x_1(t) < x_2(t)$, or $t_1(a) > t_2(a)$, meaning that the firm with higher discount rate initiates faster.

5 Extension I: Cash Constraints in Synergy Case

In this section, we introduce cash constraints into the benchmark model. We would like to focus on the case where private information is about synergy, while we left the case for stand-alone value for future research.

Assumption. Firm i can pay at most M_i in cash, where M_i is positive number.

As long as some stock payment is involved, the problem become much less tractable. Still, we can get a sense of the impact of cash constraints by looking at a particular example, where the cash constraint is binding for firm 1, but not for firm 2. The examples implies that for the synergy case, the constrained firm 1 is less eager to initiate.

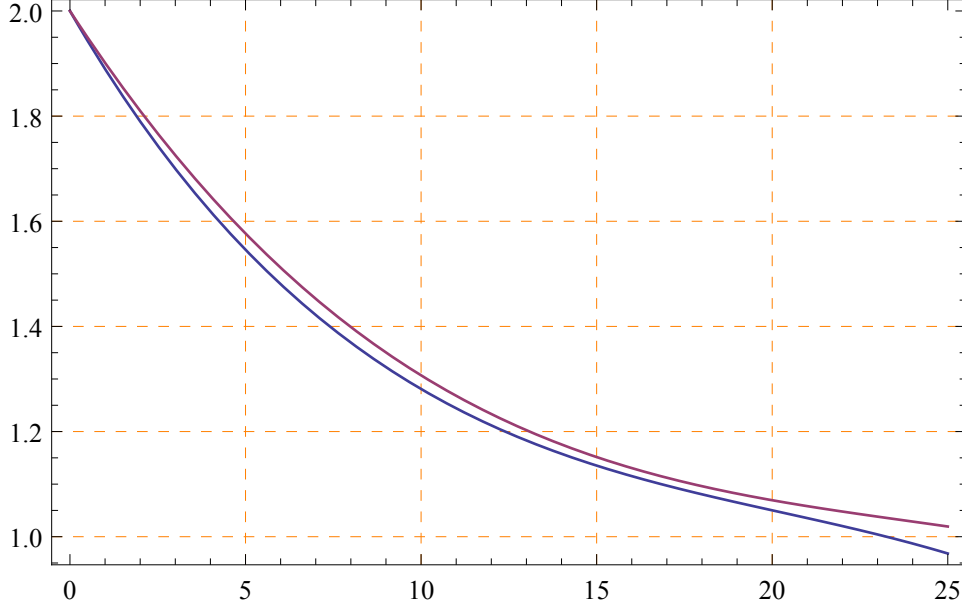


Figure 4.1: $x_i(t)$ in Two-Stage Bargaining and Synergy Case

This figure is an numerical example for the perceived type $x_i(t)$ when $l_i = 1$, $h_i = 2$, $\delta_1 = 0.7$, $\delta_2 = 0.9$, $r_1 = 1$, $r_2 \approx \frac{(1-\delta_2)}{(1-\delta_1)}r_1$. $x_i(t)$ is on the y-axis, while t is on the x-axis. As can be seen, $x_i(\cdot)$ is strictly decreasing and so is $t_i(\cdot)$, meaning that types with lower synergy initiates later. Also, $x_1(t) < x_2(t)$, or $t_1(a) > t_2(a)$, meaning that the firm with higher discount rate initiates faster.

Example 10. Assume $M_1 < v_2$ and $M_2 \geq v_1$. That is, the cash constraint is binding for firm 1, but not for firm 2. Then

- (i) there is no separating equilibrium.
- (ii) there exists an equilibrium where the constrained firm 1 never initiates. For the unconstrained firm 2, those with lower synergy delay initiation. That is, $t'_2(v_2) < 0$.

Figure 5.1 illustrates the equilibrium of initiation.

There are a few implications from the example. First, it indicates that the more constrained party could be less eager to initiate when private information is about synergy. The intuition is as follows. If the constrained firm 1 initiates, with probability θ_1 firm 1 wants to buy out firm 2 but cannot due to cash constraint, so 1 is worse off compared to that of the benchmark model; with probability θ_2 firm 1 still breaks even and remains to the same with the benchmark model. If the unconstrained firm 2 initiates, with probability θ_2 firm 2 buys out firm 1, so there is no change compared to that of the benchmark model; with probability θ_1 firm 1 wants to buy all shares of firm 2 but cannot due to cash constraint, so firm 2 is better off compared with the benchmark. Overall, compared to benchmark, the constrained firm 1 is worse off if it initiates, the unconstrained firm 2 is better off if it initiates. so unconstrained firm is more willing to initiate.

Second, in this synergy case, those with lower synergy initiation later. That is, $t'_2(v_2) < 0$.

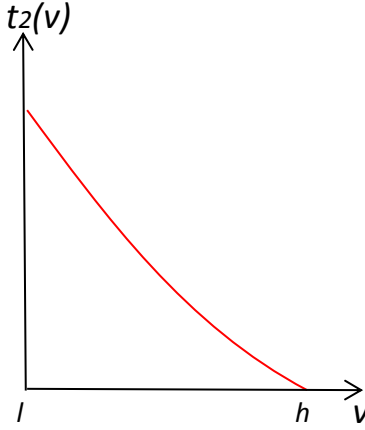


Figure 5.1: Timing of Initiation when $M_1 < v_2$ and $M_2 \geq v_1$, Synergy Case

This figure illustrates one equilibrium in Example 10. In this equilibrium, $t_1 = +\infty$. That is, firm 1 never initiates. The initiation timing of firm 2, $t_2(v)$, is decreasing in v .

This is in contrast with the stand-alone value case in the benchmark, where $t'_i(v_i) > 0$. Although we observe the fact from an example, this should be a robust phenomenon due to the nature of single-crossing property in this case. More specifically, when the private information is about synergy and there is cash constraints, the terminal payoff for initiator should be decreasing in the perceived type, since lower synergy type indicates stronger bargaining position. Then higher type wants to mimic lower type. However, the marginal cost of delay is higher for higher synergy types since they are more eager to materialize the synergy. Then in equilibrium, lower synergy types delay to separate from higher type so as to signal their stronger bargaining position in the continuation game.

6 Extension II: Results with Market-Wide Uncertainty and Merger Cost

In this section, we introduce market-wide uncertainty and merger cost. We view the initiation decision as a real option, and apply the techniques in real option signaling game to study the problem.

The basic settings are as follows. We assume that firm i 's stand-alone flow payoff is $q_t \cdot v_i$, and the merged firm's flow payoff to be $q_t(v_1 + v_2 + a)$. That is, $q_t \sim GBM(\mu, \sigma^2)$ stands for the market-wide uncertainty that affects both the existing firms and the new firm after merger. Note that since q_t is time-varying, unlike the benchmark setting we specify the flow payoff instead of present value of the firm cash flow. Also, we assume the merger cost for firm i to be $c_i > 0$.

All other assumptions about private information, initiation rule and bargaining protocol remain unchanged.

We adopt the typical equilibrium concept in real option signaling game and consider *Markov Perfect Bayesian Equilibrium*. We consider threshold-exercising strategy only, which means the strategy of type x is to initiate when q_t first reaches $\bar{q}(x)$ from below. That is, it follows a stopping time $\tau = \inf \{t \geq 0 : q_t \geq \bar{q}(x)\}$. So threshold $\bar{q}(x)$ characterize the equilibrium strategy. The proposition below summarizes the unique separating equilibrium for the stand-alone value case and for the synergy case.

Proposition 11. *Assume private types are subject to Uniform Distribution on $[l, h]$. Denote threshold of initiation for firm i with type x is $\bar{q}_i(x)$.*

(a) *In the bargaining continuation game, the allocation of shares remains the same as that of the benchmark model. For the monetary transfer, replacing the total surplus “ a ” with the new total surplus “ $a \cdot \frac{\bar{q}_i(x)}{r-\mu} - (c_1 + c_2)$ ” in the benchmark generates the result in this case.*

(b) *The initiation threshold $\bar{q}_i(x)$ is characterized below:*

(i) *Stand-alone value case:*

$$\begin{aligned} \dot{\bar{q}}_i &= \frac{\bar{q}_i^2}{\theta_i [(\beta - 1)(a \cdot \bar{q}_i) - \beta(c_1 + c_2)(r - \mu)]}, \\ \bar{q}_i(l) &= \frac{\beta(c_1 + c_2)(r - \mu)}{a(\beta - 1)} \end{aligned}$$

where β is the positive root for the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$.

Moreover, we have $\dot{\bar{q}}_i > 0$. Also, if $\theta_1 \neq \theta_2$, we have $\bar{q}_1(\cdot) \neq \bar{q}_2(\cdot)$.

(ii) *Synergy case:*

$$\bar{q}_i(x) = \frac{2\beta(c_1 + c_2)(r - \mu)}{(\beta - 1)(l + 3x)}.$$

Moreover, we have $\dot{\bar{q}}_i < 0$. Also, if $\theta_1 \neq \theta_2$, we have $\bar{q}_1(\cdot) = \bar{q}_2(\cdot)$.

Discussion on Proposition 11

Under the new settings, we not only extend most results from the benchmark model, but also obtain some additional ones. Firstly, we are able to find a unique separating equilibrium in both the stand-alone value case and the synergy case. The reason for the existence of separating equilibrium in the synergy case is that unlike the benchmark model where initiating immediately is the universal optimal choice for all types, in this extension each type enjoys a different optimal threshold absent any private information. Secondly, we characterize both separating equilibria. In the stand-alone value case, lower types initiate earlier, at lower threshold, while in the synergy case, lower types initiate later or at a higher threshold. That is, $\dot{\bar{q}}_i > 0$ for stand-alone value case, while $\dot{\bar{q}}_i < 0$ with

synergy case. The reason behind such a difference lies in the how differently marginal benefit of waiting varies when type varies. In particular, the marginal benefit of waiting is higher for higher types in the stand-alone value case, but is lower for higher types in the synergy case. Finally, we find that as in the benchmark case, the one with higher bargaining power initiates faster in the stand-alone value case. However, bargaining power has no effect on initiating strategies in the synergy case.

7 Conclusion

Assuming one-to-one M&A negotiation, two-sided private information and common knowledge of gains from trade, this paper investigates the following research questions: (1) what determines the delay in timing of M&A initiation; (2) who initiates the M&A negotiation.

We show that there are three main driving forces of results in this paper, (1) whether private information is more about synergy or stand-alone value, (2) discount rates, (3) cash constraints, (4) relative bargaining power of the two firms.

This paper makes the following predictions. The first set of result is about who initiates the M&A negotiation. The first factor has to do with firm’s relative position in its own population distribution. When private information is about stand-alone values, the party whose stand-alone value lies in the lower tail of its own population distribution compared to that of his opponent initiates faster. When private information is about synergy, the party whose synergy type lies in the upper tail of its own population distribution compared to that of his opponent initiates faster. The second and third factor is discount rate. If a firm’s discount rate is higher, it would be more inclined to initiate. These results are broadly consistent with the empirical evidence in Masulis and Simsir (2013). In addition, we show that bargaining power and cash constraints also determines who becomes the initiator. Finally, we show that the results without cash constraints extend to an environment with market-wide uncertainty and merger costs.

The second set of results are about delay in timing of M&A initiation. We predict that there will be more delay in horizontal M&A (private information on stand-alone value) than vertical M&A (private information on synergy). This is because there are stronger conflicts of interest between two merging parties in the former than in the latter.

This paper provides a simple benchmark to allow more complicated future research. For example, we focus on ownership only, without considering control rights and the related incentive problem. So an natural extension is to include control rights. Another extension is to consider explicitly the latent competition behind the simple one-to-one bargaining. For example, we can include a potential auction if the bargaining fails.

Appendix

Proof of Proposition 1

Proof. For ease of exposition, assume without loss of generality that firm 1 initiates. In equilibrium, firm 2 observes the initiation timing and hence forms a belief on path that $\hat{v}_1 = x_1(t)$, and firm 1, on the other hand, takes into account the fact that firm 2 has not initiated yet, so that $v_2 \in [\underline{v}, \bar{v}]$ with truncated distribution.

Case (i)

This case obtains with probability θ_1 where firm 1 makes a TIOLI offer to firm 2. Since firm 1 has no private information on path while firm 2 does, the former faces a screening problem. As a temporary mechanism designer, firm 1 optimally proposes a menu of triples $\{p(\hat{v}_2), s(\hat{v}_2), T(\hat{v}_2)\}$, which is a direct mechanism thanks to Revelation Principle (although later we will provide a simple indirect mechanism too). $p(\hat{v}_2)$ is the probability of trade, $s(\hat{v}_2)$ is the share of merged firm assigned to firm 1 if trade occurs, and $T(\hat{v}_2)$ is the expected monetary transfer from firm 2 to firm 1. Then, in a truthful direct mechanism, firm 1's problem is

$$\begin{aligned} \max_{\{p(\cdot), s(\cdot), T(\cdot)\}} \quad & \int_{\underline{v}}^{\bar{v}} \{p(v_2) [s(v_2) (a + v_2 + v_1) - v_1] + t(v_2)\} dG(v_2) \\ \text{s.t.} \quad & (IC) \ v_2 \in \arg \max_{\hat{v}_2} p(\hat{v}_2) [(1 - s(\hat{v}_2)) (a + v_2 + \hat{v}_1) - v_2] - T(\hat{v}_2) \\ & (IR) \ p(v_2) [(1 - s(v_2)) (a + v_2 + \hat{v}_1) - v_2] - T(v_2) \geq 0 \\ & \text{for } \forall v_2 \in [\underline{v}, \bar{v}] \end{aligned}$$

Denote firm 2's indirect utility function $U(v_2)$ as

$$U(v_2) = \max_{\hat{v}_2} p(\hat{v}_2) [(1 - s(\hat{v}_2)) (a + v_2 + \hat{v}_1) - v_2] - T(\hat{v}_2).$$

Applying Envelop Theorem to (IC) yields

$$\begin{aligned} U'(v_2) &= \left. \frac{\partial \{p(\hat{v}_2) [(1 - s(\hat{v}_2)) (a + v_2 + \hat{v}_1) - v_2] - T(\hat{v}_2)\}}{\partial v_2} \right|_{\hat{v}_2=v_2} \\ &= p(v_2) [1 - s(v_2) - 1] \\ &= -p(v_2) s(v_2) \leq 0. \end{aligned}$$

Since $-p(v_2) s(v_2)$ is uniformly bounded, we can see that $U(v_2)$ is Lipschitz continuous, hence absolutely continuous and differentiable a.e., and can be written as the integral of its derivatives:

$$\begin{aligned} U(v_2) &= U(\bar{v}) - \int_{v_2}^{\bar{v}} [-p(v) s(v)] dv \\ &= U(\bar{v}) + \int_{v_2}^{\bar{v}} p(v) s(v) dv \end{aligned} \tag{7.1}$$

In addition, the above implies that (IR) is equivalent to

$$U(\bar{v}) \geq 0 \tag{7.2}$$

Recall the definition of indirect utility:

$$U(v_2) = P(v_2) [(1 - S(v_2)) (a + v_2 + \hat{v}_1) - v_2] - T(v_2),$$

Plugging Equation (7.1) into the equation above, we have

$$T(v_2) = -U(\bar{v}) - \int_{v_2}^{\bar{v}} p(v) s(v) dv + p(v_2) [(1 - s(v_2)) (a + v_2 + \hat{v}_1) - v_2] \quad (7.3)$$

With (7.3), firm 1's problem becomes

$$\begin{aligned} \max_{\{p(\cdot), s(\cdot)\}} \quad & \int_{\underline{v}}^{\bar{v}} \{p(v_2) [a + (1 - s(v_2)) (\hat{v}_1 - v_1)]\} dG(v_2) \\ & - \int_{\underline{v}}^{\bar{v}} \left[\int_{v_2}^{\bar{v}} p(v) s(v) dv \right] dG(v_2) - U(\bar{v}) \\ \text{s.t.} \quad & (IR) U(\bar{v}) \geq 0 \end{aligned}$$

where by integration by parts,

$$\begin{aligned} & \int_{\underline{v}}^{\bar{v}} \left[\int_{v_2}^{\bar{v}} p(v) s(v) dv \right] dG(v_2) \\ = & G(v_2) \left[\int_{v_2}^{\bar{v}} p(v) s(v) dv \right] \Big|_{\underline{v}}^{\bar{v}} + \int_{\underline{v}}^{\bar{v}} G(v_2) p(v_2) s(v_2) dv_2 \\ = & \int_{\underline{v}}^{\bar{v}} G(v_2) p(v_2) s(v_2) dv_2 \end{aligned}$$

So firm 1's problem is further simplified to

$$\begin{aligned} \max_{\{p(\cdot), s(\cdot)\}} \quad & \int_{\underline{v}}^{\bar{v}} p(v_2) \left\{ [a + (1 - s(v_2)) (\hat{v}_1 - v_1)] - \frac{G(v_2)}{g(v_2)} s(v_2) \right\} dG(v_2) - U(\bar{v}) \\ \text{s.t.} \quad & (IR) U(\bar{v}) \geq 0 \end{aligned}$$

Recall that we only consider “on-scheduled” deviations in which actions in the continuation game is consistent with the type revealed by initiation timing. That is, if i signaled his type to be $\hat{v}_1 = x_1(t)$ by initiating at time t , his behavior in subsequent bargaining game should be consistent with type $\hat{v}_1$, i.e., player i should offer an optimal menu as if it were type $\hat{v}_1$. Therefore, we need to replace v_1 with $\hat{v}_1$ in the objective function:

$$\begin{aligned} \max_{\{p(\cdot), s(\cdot)\}} \quad & \int_{\underline{v}}^{\bar{v}} p(v_2) \left\{ a - \frac{G(v_2)}{g(v_2)} s(v_2) \right\} dG(v_2) - U(\bar{v}) \\ \text{s.t.} \quad & (IR) U(\bar{v}) \geq 0 \end{aligned}$$

Point-wise maximization gives

$$\begin{aligned} U(\bar{v}) &= 0, \\ s(v_2) &= 0, \\ p(v_2) &= 1. \end{aligned}$$

and plugging back in Equation (7.3) to get expected transfer:

$$\begin{aligned}
T(v_2) &= -U(\bar{v}) - \int_{v_2}^{\bar{v}} p(v) s(v) dv + p(v_2) [(1 - s(v_2)) (a + v_2 + \hat{v}_1) - v_2] \\
&= a + \hat{v}_1 \\
&= a + x_1(t).
\end{aligned}$$

In this way, we've computed the optimal truth-telling direct mechanism, which turns out to be pooling. To implement it, we simply need a post-price mechanism. That is, firm 1 as the initiator always posts a price $a + x_1(t)$ to sell all its shares to firm 2, and the offer is TIOLI.

Case (ii)

This case obtains with probability θ_2 where firm 2 makes a TIOLI offer to firm 1. Since firm 1 has no private information on path while firm 2 does, the latter faces a signaling game. In this game, firm 2 offers a duple (s, T) as signal, so the game features multiple equilibria as usual. Fortunately, a pooling equilibrium exists which payoff-dominates all other equilibria for all types of firm 2, and that is the one we choose as the prediction of outcome.

First, we verify that it is a pooling equilibrium for all types of firm 2 to offer to buy all shares of firm 1 at a single share-price combination $(s, T) = (0, x_1(t))$, along with certain belief. Furthermore, we will show that this equilibrium survives D-1 criterion.

To see this, consider a pooling equilibrium where $s = 0$ and $T = x_1(t)$. The belief of firm 1 upon seeing an offer is such that no updating takes place for the equilibrium pooling offer, while any other offer leads to the pessimistic belief that $v_2 = \underline{v}$.

We first show that it is indeed a PBE. Recall that we consider "on-scheduled" deviations, so firm 1, who revealed himself as type $x_1(t)$, must act exactly as if it is of this type, regardless of his true type³. Then faced with the pooling offer, type $x_1(t)$ of firm 1 should be marginally willing to accept, receiving net payoff of $s(x_1(t) + a + \mathbb{E}_1(v_2)) + T - x_1(t) = 0$. On the other hand, all types of firm 2 that have not initiated so far find it profitable to make the offer because $(1 - s)(x_1(t) + a + v_2) - T - v_2 = a > 0$. Finally, firm 2 of type v_2 does not want to deviate from $s = 0$ and $T = x_1(t)$ to any other acceptable offer (s', T') . Suppose $s' = s = 0$, then if firm 2 offers $T' < T = x_1(t)$, firm 1 would reject since $s'(x_1(t) + a + \underline{v}) + T' - x_1(t) = T' - x_1(t) < 0$. Since firm 2 gets 0 if firm 1 rejects, firm 2 would rather stay with the equilibrium payoff $a > 0$. If firm 2 offers $T' > T = x_1(t)$, then firm 1 would accept, but firm 2 gets $(1 - s')(x_1(t) + a + v_2) - T' - v_2 = x_1(t) + a - T' < a$. Therefore, firm 2 would not deviate to $s' = s = 0$, $T' \neq T$. Now suppose $s' > s = 0$. The best (lowest) T' to firm 2 that guarantees acceptance at the same time is $T' = -s'(x_1(t) + a + \underline{v}) + x_1(t)$. So firm 2 gets at most

$$\begin{aligned}
(1 - s')(x_1(t) + a + v_2) - T' - v_2 &= (1 - s')(x_1(t) + a + v_2) - (-s'(x_1(t) + a + \underline{v}) + x_1(t)) - v_2 \\
&= x_1(t) + a - s'(v_2 - \underline{v}) - x_1(t) \\
&= a - s'(v_2 - \underline{v}) \\
&< a.
\end{aligned}$$

Therefore, firm 2 does not want to deviate to $s' \neq s$. Hence $s = 0$ and $T = x_1(t)$ is indeed a pooling PBE.

Furthermore, the above PBE survives Intuitive Criterion trivially, and furthermore, it also survives D-1 Crite-

³We do need this assumptions here. First, facing offer $(s = 0, T = -x_1(t))$, only firm 1 with $v_1 \leq x_1(t)$ accepts the offer, while $v_1 > x_1(t)$ rejects the offer since the price out of selling out cannot cover their stand-alone value. Second, off-EQ-belief cannot restrict firm 1's acceptance behavior.

tion. To see this, consider deviation of firm 2 to an arbitrary offer (s', T') . If $T' \geq (1 - s')x_1(t) - s'(\underline{v} + a)$, then such deviation is equilibrium dominated for all types of firm 2. If $T' < (1 - s')x_1(t) - s'(\underline{v} + a)$, then deviation is profitable for the lowest type $\underline{v}$ if the probability of acceptance p is not too small. Specifically, for type v_2 to benefit from deviation, we must have

$$\begin{aligned} & p((1 - s')(x_1(t) + v_2 + a) - T') \geq v_2 + a \\ \Rightarrow & p \in D(v_2) \equiv \left[\frac{v_2 + a}{(1 - s')(x_1(t) + v_2 + a) - T'}, \infty \right) \cap [0, 1] \end{aligned}$$

Note that $D(\underline{v}) \supset D(v_2)$ for all $v_2 > \underline{v}$. According to D-1, the deviation is perceived to be from type $\underline{v}$. Consequently, acceptance only occurs when $s'(x_1(t) + \underline{v} + a) + T' \geq x_1(t)$, but then deviation is unprofitable because

$$\begin{aligned} & (1 - s')(x_1(t) + v_2 + a) - T' - v_2 \\ \leq & (1 - s')(x_1(t) + v_2 + a) - x_1(t) + s'(x_1(t) + \underline{v} + a) - v_2 \\ = & a + (1 - s')v_2 + s'\underline{v} - v_2 \\ \leq & a \end{aligned}$$

Next, we show that the pooling equilibrium with $s_0 = 0$, $T_0 = x_1(t)$ is the best equilibrium for all types of firm 2. As we have seen, the equilibrium net payoff to firm 2 of any type v_2 is $a > 0$. However the maximum total surplus available is a , then all types of v_2 has already obtained the most they can get. Hence is the best equilibrium for the offer-er firm 2, hence survives the *One-Sided Pareto Dominance Criterion*.

Therefore, combining Case (i) and (ii), we conclude that when firm 2 (non-initiator) makes offer, the equilibrium where it offer to buy all shares of firm 1 with price $x_1(t)$ is a natural equilibrium to select.

□

Proof for Proposition 2

Proof. The proof is basically to follow the same procedure as that of stand-alone value case, so we omit it here.

□

Proof of Global Optimality for Proposition 6

Proof. Given firm $-i$'s strategy $t_{-i}(\cdot)$ and firm $-i$'s belief $t_i^{-1}(\cdot)$ on firm i , a necessary condition for equilibrium of the initiation game is that

$$\hat{x}_i = v_i, \quad i = 1, 2$$

for the following reframed problem :

$$\begin{aligned} \max_{\hat{x}_i} & \int_l^{t_{-i}^{-1}(t_i(\hat{x}_i))} e^{-r_i t_{-i}(w)} H_i^{SV}(v_i, t_{-i}(w)) f_{-i}(w) dw \\ & + \left[1 - \int_l^{t_{-i}^{-1}(t_i(\hat{x}_i))} f_{-i}(w) dw \right] e^{-r_i t_i(\hat{x}_i)} G_i^{SV}(v_i, \hat{x}_i) \end{aligned}$$

From FOC of the original problem of choosing $\hat{t}$, we obtained the following solution:

$$t_1(v) = \frac{v-l}{\theta_1 a r_i} \quad (7.4)$$

$$t_2(v) = \frac{v-l}{\theta_2 a r_i}. \quad (7.5)$$

Plugging into the rewritten objective function for firm i , and take the first derivative w.r.t. $\hat{x}_i$, then the first derivative FD_i

$$FD_i = (\hat{x}_i - v_i) \cdot \frac{e^{-\frac{\hat{x}_i-l}{a\theta_i}}}{a\theta_i} \cdot \left[-a(1-\theta_i) f_{-i} \left(l + \frac{(\hat{x}_i-l)(1-\theta_i)}{\theta_i} \right) - \left[1 - \int_l^{l+\frac{(\hat{x}_i-l)(1-\theta_i)}{\theta_i}} f_{-i}(w) dw \right] \right].$$

Since $f_{-i}(\cdot)$ is the p.d.f. for v_{-i} , $1 - \int_l^{l+\frac{(\hat{x}_i-l)(1-\theta_i)}{\theta_i}} f_{-i}(w) dw < 0$.

Therefore, the third term $-a(1-\theta_i) f_{-i} \left(l + \frac{(\hat{x}_i-l)(1-\theta_i)}{\theta_i} \right) - \left[1 - \int_l^{l+\frac{(\hat{x}_i-l)(1-\theta_i)}{\theta_i}} f_{-i}(w) dw \right] < 0$. In addition,

the second term $\frac{e^{-\frac{\hat{x}_i-l}{a\theta_i}}}{a\theta_i} > 0$. Therefore,

$$FD_i < 0 \text{ when } \hat{x}_i > v_i;$$

$$FD_i > 0 \text{ when } \hat{x}_i < v_i;$$

$$FD_i = 0 \text{ when } \hat{x}_i = v_i.$$

Then we've verified that under the $t_1(\cdot)$ and $t_2(\cdot)$ given by (7.4) and (7.5), truth-telling ($\hat{x}_i = v_i$) is the maximum.

Therefore, we've shown that in the original problem, $t_i(v_i)$ is firm i 's optimal choice of the choice set $\hat{t} \in [t_i(l), t_i(h)]$. Finally, we show that any $\hat{t}$ outside $[t_i(l), t_i(h)]$ is not optimal. First, since $t_i(l) = 0$, one cannot choose $\hat{t} < t_i(l)$. In addition, for any $\hat{t} > t_i(h)$, the opponent will initiate before firm i for sure. Therefore, firm i obtained the same payoff as initiating at $\hat{t} = t_i(h)$. Since $\hat{t} = t_i(h)$ is dominated by $\hat{t} = t_i(v_i)$, this is not optimal either.

That completes that proof. □

Proof of Proposition 11: We build the proof with a few claims and lemmas. We will focus on the proof for the stand-alone value case, since the proof for the synergy case is very similar.

Claim 12. Suppose $q_s \sim GBM(\mu, \sigma^2)$ with $q_t > 0$ and $s \geq t$. Denote $\mathbb{F}$ as the filtration generated by q_s , and denote $\mathcal{F}_t$ as the corresponding σ -algebra at time t . Then

$$\mathbb{E} \left[\int_t^\infty e^{-rs} q_s ds | \mathcal{F}_t \right] = \frac{q_t}{r - \mu}.$$

Proof. By Fubini Theorem,

$$\begin{aligned}
\mathbb{E} \left[\int_t^\infty e^{-rs} q_s ds | \mathcal{F}_t \right] &= \int_t^\infty e^{-rs} \mathbb{E} [q_s | \mathcal{F}_t] ds \\
(\text{strong markov property}) &= \int_t^\infty e^{-rs} \mathbb{E} [q_s | q_t] ds \\
(\text{mean of GBM}) &= \int_t^\infty e^{-rs} q_t e^{\mu s} ds \\
&= \frac{q_t}{r - \mu}.
\end{aligned}$$

□

Claim 13. Suppose $q_t \sim GBM(\mu, \sigma^2)$ with $0 < q_0 \leq \bar{q}$. Suppose $G(q)$ is intrinsic value of exercising an option if exercising at $q_t = q$. Then the present value of the option with exercising stopping time strategy $\tau = \inf \{t \geq 0 : q_t \geq \bar{q}\}$ is

$$V(q_0) = \left(\frac{q_0}{\bar{q}} \right)^\beta G(\bar{q})$$

where β is the positive root for $r = \mu x + \frac{1}{2}\sigma^2 x(x-1)$, and $\beta > 1$.

Proof. For q such that $q_0 \leq q \leq \bar{q}$, we have

$$rV(q) = \mu q V'(q) + \frac{1}{2} \sigma^2 q^2 V''(q).$$

The solution for the differential equation is the linear combination of two general solutions:

$$V(q) = A_1 q^\beta + A_2 q^\lambda.$$

To find β and λ , plug in $V(q) = q^\alpha$ into the ODE, we get

$$r = \mu \alpha + \frac{1}{2} \sigma^2 \alpha (\alpha - 1).$$

There are two solutions $\alpha_1 = \beta$, $\alpha_2 = \lambda$ to the equation, with $\beta > 1 > 0 > \lambda$.

Since $V(0) = \max(G(0), 0) < \infty$, $A_2 = 0$. Apply value-matching, i.e. $V(\bar{q}) = G(\bar{q})$, we have $A_1 = \frac{G(\bar{q})}{(\bar{q})^\beta}$. Then

$$\begin{aligned}
V(q) &= A_1 q^\beta \\
&= \left(\frac{q}{\bar{q}} \right)^\beta G(\bar{q}).
\end{aligned}$$

Let $q = q_0$, we prove the claim.

□

We can look at the initiation game at time 0, because this problem is time consistent (no time-varying private information). The objective function of type $v_i = x$ is characterized by the following lemma.

Denote p_i^j as the monetary transfer from firm i to firm $-i$ when firm i initiates and firm j offers, and s_i^j as the share allocating to firm i when firm i initiates and firm j offers. Denote $p_i = \sum_{j=1}^2 \theta_j p_i^j$ as the expected monetary transfer from firm i to firm $-i$ when firm i initiates, and $s_i = \sum_{j=1}^2 \theta_j s_i^j$ as the expected share allocating to firm i when firm i initiates.

Suppose firm 1 initiates threshold q . Then in a separating equilibrium with $\dot{q}_i(\cdot) > 0$ or $\dot{x}_i(\cdot) > 0$ for $i = 1, 2$, firm 1's type is revealed to be $x_1(q)$, while firm 2's type is subject to $U[x_2(q), h]$. Then we follow similar argument in Proposition 1. If with probability θ_1 the initiator firm 1 offers, he will let type h breakeven while giving type h the entire new firm. That is,

$$\begin{aligned} s_1^1 &= 0, \\ h \cdot \frac{q}{r-\mu} &= (1-s_1^1)(x_1(q) + h + a) \frac{q}{r-\mu} + p_1^1 - c_2. \end{aligned}$$

So

$$p_1^1 = -\frac{q}{\mu-r}(a + x_1(q)) - c_2.$$

Also, when with probability θ_2 the non-initiator firm 2 makes the offer, he retains all the shares and let firm 1 breakeven. That is, in this case $s_1^2 = 0$, $x_1(q) \frac{q}{r-\mu} = s_1^2 \left(a + x_1(q) + \frac{h+x_2(q)}{2} \right) \frac{q}{r-\mu} - p_1^2 - c_1$, implying $p_1^2 = -\frac{q}{r-\mu}x_1(q) - c_1$. Then the average monetary transfer and share becomes

$$\begin{aligned} s_1 &= \theta_1 \cdot s_1^1 + (1-\theta_1) s_1^2 = 0 \\ p_1 &= \theta_1 \cdot p_1^1 + (1-\theta_1) p_1^2 \\ &= -\frac{q}{r-\mu}(\theta_1 a + x_1(q)) - \theta_1 c_1 - (1-\theta_1) c_2. \end{aligned}$$

Similarly, we have

$$\begin{aligned} s_2 &= 0 \\ p_2 &= -\frac{q}{r-\mu}((1-\theta_1)a + x_2(q)) - (1-\theta_1)c_2 - \theta_1 c_1. \end{aligned}$$

Then firm 1's net payoff if firm 1 with true value v_1 initiates at q is

$$G_1^{SV}(v_1, q) = \left\{ s_1(q) \left[v_1 + a + \frac{x_2(q) + h}{2} \right] - v_1 \right\} \frac{q}{r-\mu} - p_1(q) - c_1,$$

and firm 1's net payoff if firm 1's true value is v_1 , and firm 2 initiates at q is

$$H_1^{SV}(v_1, q) = \{(1-s_2(q))[v_1 + a + x_2(q)] - v_1\} \frac{q}{r-\mu} + p_2(q) - c_1.$$

Then we also have

$$\begin{aligned} G_2^{SV}(v_2, q) &= \left\{ s_2(q) \left[v_2 + a + \frac{x_1(q) + h}{2} \right] - v_2 \right\} \frac{q}{r-\mu} - p_2(q) - c_2, \\ H_2^{SV}(v_2, q) &= \{(1-s_2(q))[v_2 + a + x_1(q)] - v_2\} \frac{q}{r-\mu} + p_1(q) - c_2. \end{aligned}$$

Then we now turn to the initiation game, given the terminal payoffs in the continuation game.

Lemma 14. *Suppose private information is about stand-alone value. Then in the separating equilibrium with $\dot{q}(\cdot) > 0$ and $\dot{x}(\cdot) > 0$,*

(i) firm i with type v_i chooses the optimal threshold $\hat{q}$ of initiation, to solve the following problem:

$$\max_{\hat{q}} \underbrace{\left(\frac{q_0}{\hat{q}}\right)^\beta G_i^{SV}(v_i, \hat{q}) \frac{h - x_{-i}(\hat{q})}{h - l}}_{\text{if } v_{-i} \geq x_{-i}(\hat{q}), i \text{ initiates at } \hat{q}} + \underbrace{\int_l^{x_{-i}(\hat{q})} \left(\frac{q_0}{q}\right)^\beta H_i^{SV}(v_i, q) \frac{x'_{-i}(q)}{h - l} dq}_{\text{if } v_{-i} < x_{-i}(\hat{q}), -i \text{ initiates at } q}$$

(ii) Belief should be consistent with the outcome.

$$v_i = x_i(q^*),$$

where q^* is the optimal threshold determined in problem of (i).

Proof. (i) Denote $U(v_i, \hat{q})$ as the objective function, then

$$\begin{aligned} U(v_i, \hat{q}) &= \mathbb{E}[\mathbb{E}[\text{PV of subgame payoff to } i \mid v_{-i}]] \\ &= \Pr(v_{-i} > x_{-i}(\hat{q})) \mathbb{E}[\text{PV of subgame payoff to } i \mid v_{-i} > x_{-i}(\hat{q})] \\ &\quad + \mathbb{E}[\text{PV of subgame payoff to } i \mid v_{-i} \leq x_{-i}(\hat{q})] \\ (\text{Claim 13}) &= \left(\frac{q_0}{\hat{q}}\right)^\beta G_i^{SV}(v_i, \hat{q}) \frac{h - x_{-i}(\hat{q})}{h - l} \\ &\quad + \int_l^{x(\hat{q})} \left(\frac{q_0}{q_{-i}(v)}\right)^\beta H_i^{SV}(v_i, q) \frac{1}{h - l} dv \\ (x'_{-i}(\cdot) > 0) &= \left(\frac{q_0}{\hat{q}}\right)^\beta G_i^{SV}(v_i, \hat{q}) \frac{h - x_{-i}(\hat{q})}{h - l} \\ &\quad + \int_l^{x_{-i}(\hat{q})} \left(\frac{q_0}{q}\right)^\beta H_i^{SV}(v_i, q) \frac{x'_{-i}(q)}{h - l} dq. \end{aligned}$$

(ii) is obvious. □

Based on the lemma above, we solve the problem in four steps.

Step 1. Find F.O.C. for the individual maximization problem.

FOC on firm 1's objective function w.r.t. $\hat{q}$, we have

$$\begin{aligned} &\frac{1}{(h - l)\hat{q}(\mu - r)} \{ (h - x_2(\hat{q})) [\beta(a\hat{q} + (c_1 + c_2)(\mu - r))\theta_1 - \beta\hat{q}v_1 + \hat{q}(-a\theta_1 + v_1) - \hat{q}^2x'_1(\hat{q})] \\ &- \hat{q}^2v_1x'_2(\hat{q}) + \hat{q}x_1(\hat{q})[(\beta - 1)(h - x_2(\hat{q})) + \hat{q}x'_2(\hat{q})] \} \\ &= 0 \end{aligned}$$

FOC on firm 2's objective function w.r.t. $\hat{q}$, we have

$$\begin{aligned} &\frac{1}{(h - l)\hat{q}(\mu - r)} \{ (h - x_1(\hat{q})) [\beta(a\hat{q} + (c_1 + c_2)(\mu - r))(1 - \theta_1) - \beta\hat{q}v_2 + \hat{q}(-a(1 - \theta_1) + v_2) - \hat{q}^2x'_2(\hat{q})] \\ &- \hat{q}^2v_2x'_1(\hat{q}) + \hat{q}x_2(\hat{q})[(\beta - 1)(h - x_1(\hat{q})) + \hat{q}x'_1(\hat{q})] \} \\ &= 0 \end{aligned}$$

Step 2. Replace v_i with $x_i(q)$, and derive for the Ordinal Differential Equations of $x_1(q)$ and $x_2(q)$.

Replace v_i with $x_i(q)$ in the two FOCs above, we have the ODEs:

$$\begin{aligned} x'_1(q) &= \frac{\theta_1 [-aq + \beta (aq + (c_1 + c_2)(\mu - r))]}{q^2} \\ x'_2(q) &= \frac{(1 - \theta_1) [-aq + \beta (aq + (c_1 + c_2)(\mu - r))]}{q^2} \end{aligned}$$

Step 3. To pin down the initial values at $v_i = l$, we consider type l 's decision. Since it will initiate before everyone else, it's problem becomes

$$\max_{\hat{q}} \left(\frac{q_0}{\hat{q}} \right)^\beta G_i^{SV}(l, \hat{q})$$

FOC to $\hat{q}$ gives

$$q_i(l) = \frac{\beta (c_1 + c_2) (r - \mu)}{a(\beta - 1)}$$

or

$$x_i \left(\frac{\beta (c_1 + c_2) (r - \mu)}{a(\beta - 1)} \right) = l, \quad i = 1, 2.$$

Step 4. Verify that the F.O.C gives global maximum. Since the problem is a super-modular problem ($U_{12}(\hat{q}, q) \cdot \hat{q}'(q) \geq 0$), the F.O.C is sufficient for global maximum.

Then we arrive at the result in the proposition.

The proof for the synergy case is very similar, therefore we omit it here.

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