

Foster-Hart Risk and the Too-Big-to-Fail Banks: An Empirical Investigation

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Abstract

The measurement of financial risk relies on two factors: determination of riskiness by use of an appropriate risk measure; and the distribution according to which returns are governed. Wrong estimates of either, severely compromise the accuracy of computed risk. We identify the too-big-to-fail banks with the set of “Global Systemically Important Banks” (G-SIBs) and analyze the equity risk of its equally weighted portfolio by means of the “Foster-Hart risk measure” — a new, reserve based measure of risk, extremely sensitive to tail events. We model banks’ stock returns as an ARMA-GARCH process with multivariate “Normal Tempered Stable” (NTS) innovations, to capture the skewed and leptokurtotic nature of stock returns. Our union of the Foster-Hart risk modeling with fat-tailed statistical modeling bears fruit, as we are able to measure the equity risk posed by the G-SIBs more accurately than is possible with current techniques. We also study the corresponding mean risk analysis problem and are able to show that an NTS distributed portfolio optimization strategy based on the Foster-Hart risk minimization with a general quadratic transaction cost function emphatically outperforms standard mean risk analysis based techniques.

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1 Introduction

While accurate measurement of financial risk is an important theoretical problem in its own right, the global financial meltdown in the late 2000s, its attendant financial turmoil and the Eurozone crisis have catapulted the subject into public limelight and highlighted its centrality in framing economic policy. Following the Lehman Brothers' bankruptcy in September 2008, the Basel Committee on Banking Supervision (BCBS) formulated a successor to the then Basel II regulatory framework. The new framework, Basel III, insisted on additional capital requirements for *Global Systemically Important Banks* (G-SIBs) whose failure or distress, it claimed, could trigger instability in the global financial markets.¹ In turn, the Financial Stability Board (FSB) was set up in April 2009 which drew up an initial list of 29 G-SIBs in November 2011.²

The Basel Committee on Banking Supervision is not alone in its calls for additional capital requirements. [Admati and Hellwig \(2013\)](#) make equally strident calls for increase in capital requirements for banks. Similar concerns have been echoed by Lord Adair Turner, the Chairman of the Financial Stability Authority (FSA) of the UK in numerous speeches and press briefings.³ All of these commentators make the point that benefits accruing from the long term financial stability of the systemically important institutions far exceed the modest risk of slow GDP growth in the near future. Moreover, the moral hazard generated by governments' intervention by means of taxpayer funded bailouts has detrimental consequences for corporate incentives and public finances. The recent nationalization and subsequent breakup and restructuring of Dexia, an erstwhile G-SIB, highlights the perils of inadequate capitalization, especially for those institutions that are exposed to positions whose riskiness becomes more pronounced during systemic market downturns.

The current determination of capital reserves relies on the Value at Risk estimates. While conventionally, such estimates are obtained from the profit and loss distribution (also known as the *payoff* distribution), it is often more convenient to compute risk estimates based on the distribution of (log) *returns*. The capital reserves are proportional to such normalized risk estimates, the proportionality constant being the market value (price) of the portfolio.

We carry out the exercise of normalized equity risk estimation by means of three different measures of risk — the currently in use Value at Risk

¹In Basel II, capital requirements for banks were uniform and not dependent on their systemic importance.

²For the latest list of G-SIBs, please visit http://www.financialstabilityboard.org/wp-content/uploads/r_141106b.pdf

³The FSA was dissolved from April 1st, 2013 and its duties were split between the Prudential Regulation Authority, the Financial Conduct Authority and the Bank of England.

(VaR) and Average Value at Risk (AVaR) (also referred to as ‘Expected Shortfall’ (ES) or Conditional VaR (CVaR)) — and the recently discovered *Foster-Hart risk*. [Foster and Hart \(2009\)](#) have proposed a new measure of risk which computes the minimal wealth needed to avoid bankruptcy for an agent who faces an unknown sequence of risky gambles in the future. In addition, it enjoys many other highly desirable properties that experts in the field deem “coherent” ([Artzner *et al.*, 1998](#)).

We postulate an underlying standard *Normal Tempered Stable* distribution for the returns of banking stocks and model their temporal dependence by means of an ARMA-GARCH stochastic process.⁴ In order to establish our skewed and fat-tailed distributional hypothesis’s contribution to statistical modeling, we compare it with the specification of stock returns being distributed as standard T and standard Normal random variables respectively.

More formally, we form an equally weighted portfolio of G-SIBs, assume its constituents’ returns to be governed by an ARMA(1,1)-GARCH(1,1) stochastic process with Normal Tempered Stable innovations; and track its normalized equity risk by means of VaR, AVaR and Foster-Hart risk on the basis of the constituent banks’ stock market returns from January 4th, 2000 to February 28th, 2014. We observe that the equity risks estimated according to Foster-Hart (FH) are higher than those suggested by more conventional VaR or AVaR. This difference is accentuated in times of crises as the Foster-Hart risk is more sensitive to tail events than the aforementioned conventional risk measures.

Such an exercise, we emphasize, does not measure the equity risk *of* G-SIBs but in fact measures the equity risk *posed by* them on a hypothetical investor who holds the equally weighted G-SIB portfolio. We interpret this hypothetical investor to be in fact, the common public, whose financial fortunes are closely tied to those of the systemically important banks. A rise in the levels of portfolio risk comprising the too-big-to-fail banks implies a rise in the financial risk faced by ordinary citizens.⁵ Although they hold the portfolio only symbolically, they face the quite real consequences of savings lost due to financial panic, which in turn is often fueled by risky G-SIB behavior. In this sense, the analysis in our paper is helpful in measuring the extent to which the common public is exposed to equity risks. We also note

⁴The ARCH stochastic process was introduced in the now classic [Engle \(1982\)](#), while [Bollerslev \(1986\)](#) extended it to GARCH.

⁵Since the G-SIB portfolio is emblematic of the equity risk that the too-big-to-fail banks pose on the representative investor, we choose to weigh all constituent banks equally. While this is not the only set of weights consistent with such an objective, (other candidates include weights being proportional to the market capitalizations of individual banks; or to their assets under management, among others) it is the simplest one. Moreover for the purposes of comparison between estimates of different risk measures, the results remain qualitatively similar and do not depend too much on the details of the weight distribution between constituent banks.

that in order to measure the equity risk *of* banks, one must have access to banks' internal portfolios. However, our methodology is flexible enough for banks' in-house risk management teams to adapt and thus, our approach can be used to compute equity risks *of* banks as well.

We demonstrate the superiority of the Foster-Hart risk as a tool in portfolio optimization as well. Mean risk analysis using FH risk shows robust outperformance of the classic mean variance analysis or those based on mean VaR or mean AVaR. Hence from this point of view, FH is useful for portfolio managers as well.

The rest of the paper is organized as follows. In section 2 we introduce the theory behind risk measurement and statistical modeling. In section 3 we describe the data used while the 4th section outlines the methodology employed to estimate the statistical model and the portfolio risks. Section 5 describes the results and interprets them. Section 6 introduces the portfolio optimization program and the results of mean risk analysis under different risk measures. We outline our conclusions in section 7 and end the paper by including additional results and tables in the Appendices.

2 Preliminaries

Our research work is primarily based on two parallel strands of literature — one from the theory of risk measurement and the other from the theory of time series econometrics.

The following section presents a broad outline of the two branches of literature we draw upon.

2.1 Risk Assessment

Two of the most popular notions of financial risk are the *Value at Risk* and the *Average Value at Risk*.

The Value at Risk is defined as a specified quantile of the return distribution. Its use was popularized by JP Morgan in the late 1980s and remains the most widely used risk measure in the industry. In the mid 1990s, it was endorsed by the Basel Committee of Banking Supervision (BCBS) to be used to compute capital reserves of financial institutions.

Mathematically:

$$\text{VaR}_\alpha(G) := -\inf\{g : G(g) \geq \alpha\} = -G^{-1}(\alpha)$$

where g is the return, $G(\cdot)$ is its distribution and $100 \cdot (1 - \alpha)$ is the user-specified (percentage) confidence level.⁶

⁶The second right hand side follows if we assume that G is continuous.

A more sophisticated measure of risk is the Average Value at Risk, which as the name suggests, averages the different VaR values beyond the confidence level. Mathematically, it is defined to be the following:

$$\text{AVaR}_\alpha(g) := \frac{1}{\alpha} \int_0^\alpha \text{VaR}_p(g) dp$$

The Average Value at Risk is much more informative about the losses beyond the stipulated confidence level and enjoys highly desirable properties of “coherence” (Artzner *et al.*, 1998) that VaR lacks.

Foster-Hart Risk Foster and Hart (2009) proposed a new measure that was based on considerations of how much wealth an agent should possess, irrespective of her utility function, to render a gamble “not risky”. They describe “gamble” to be any *bounded* random variable with a positive expectation and a positive probability of losses:

$$\mathbb{E}(g) > 0 \text{ and } \mathbb{P}(g < 0) > 0$$

The main result of Foster and Hart (2009) is that for each gamble g , there is a function $R(g)$ which is the unique positive solution to the following equation:

$$\mathbb{E} \left(\log \left[1 + \frac{g}{R(g)} \right] \right) = 0$$

This function $R(g)$ — the Foster-Hart risk measure — may be interpreted as the *minimal reserve* needed to play the gamble g .

The Foster-Hart risk is more sophisticated than VaR and AVaR since it is free from arbitrary confidence levels and time horizons; and because it guarantees non-bankruptcy in the face of infinite, unknown sequences of arbitrary gambles.

In addition, the Foster-Hart risk enjoys many other highly desirable properties:

1. *Homogeneity*: $\forall \lambda > 0, R(\lambda g) = \lambda R(g)$ — doubling gambles doubles wealth requirement.
2. *Subadditivity*: $R(g + h) \leq R(g) + R(h)$ — a diversified portfolio has lesser risk than that of the sum of its components.
3. *First Order Monotonicity*: If gamble g first order stochastically dominates gamble h , $R(g) < R(h)$.
4. *Second Order Monotonicity*: If gamble g second order stochastically dominates gamble h , $R(g) < R(h)$.

5. *Fat-Tail Consistency*⁷: For each gamble, Foster-Hart riskiness is at least as high as the *maximum loss* L of that gamble.
6. *Continuity*: If a sequence of gambles converges and their maximum losses converge, their Foster-Hart risks converge.

General Foster-Hart Risk: In the Foster-Hart setup, gambles faced by the agent are assumed to have discrete probability mass functions. As shown in [Riedel and Hellmann \(2014\)](#), the defining equation for the Foster-Hart riskiness has no solution for many common continuous distributions. However, the Foster-Hart risk may be consistently extended to such continuous random variables, in which case it coincides with *maximum loss* L , i.e.,

$$R(g) = \begin{cases} r^* \text{ such that } \mathbb{E} \left(\log \left[1 + \frac{g}{r^*} \right] \right) = 0 & \text{if } \mathbb{E} \left(\log \left[1 + \frac{g}{L} \right] \right) < 0 \\ L & \text{if } \mathbb{E} \left(\log \left[1 + \frac{g}{L} \right] \right) \geq 0 \end{cases} \quad (1)$$

The Weak Foster-Hart Risk Heuristic: In order for the Foster-Hart risk to be defined, the gamble under consideration must be a random variable whose expectation is positive ($X : \mathbb{E}(X) > 0$). However, during severe economic downturns, portfolios may exhibit mean *negative* returns, thereby making the Foster-Hart risk undefined.

Hence we define the notion of the weak Foster-Hart risk heuristic in which, just as above, we identify the Foster-Hart risk with the maximum loss L for scenarios in which the gamble’s mean is negative.

2.2 Statistical Modeling

The solution of (1) presupposes that one has the “correct” distribution according to which gambles are governed. After many decades of investigation, researchers now agree that return distributions of various financial instruments are *skewed and leptokurtotic* — asymmetric, and more peaked around the mean, with fat, Pareto-type tails.⁸

The use of the Stable distribution in fitting stock-index and exchange rate data, as well as in option pricing is studied in detail in [Rachev and Mitnik \(2000\)](#). More recently, [Rachev et al. \(2003\)](#) and [Rachev et al. \(2005\)](#)

⁷Consistent with asymptotic power law decay of stock returns, the Foster-Hart risk incorporates rare events’ potential for causing extreme losses.

⁸There is a substantially large body of literature, starting from the ’60s (see in particular, [Alexander \(1961\)](#), [Mandelbrot \(1963a\)](#), [Mandelbrot \(1963b\)](#), [Fama \(1963\)](#), [Fama \(1965\)](#), [Mandelbrot \(1967\)](#) etc.) that provides extensive evidence for this assertion. For a comprehensive literature review, see [Haas and Pigorsch \(2009\)](#).

argue in favor of the Stable distribution for fitting US treasury and corporate bond return and US stock return data respectively.

However, the lack of existence of moments presents difficulties in dealing with Stable distributions. *Tempered* Stable distributions present a possible solution.⁹ Indeed, [Kim *et al.* \(2008\)](#) and [Kim *et al.* \(2010\)](#) pursue the Tempered Stable approach. In addition, even more recent findings from [Kim *et al.* \(2012\)](#) propel us to choose a specific distribution — the “*multivariate Normal Tempered Stable*” distribution — to model asymmetric and interdependent natures of stock returns.¹⁰

The Normal Tempered Stable distribution fits the stylized facts of skewness and leptokurtosis much better than the standard Normal or standard T distributions. The standard Normal suffers from symmetry and exponentially decaying tails while the T distribution, although possessing fatter than Normal tails, cannot accommodate skewness. We establish the superiority of NTS over Normal and T distributions in statistical modeling of asset returns by both in-sample and out-of-sample tests in later sections.

2.2.1 Normal Tempered Stable Distribution:

The Normal Tempered Stable (NTS) distribution is built upon the Tempered Stable subordinator with parameters $\alpha \in (0, 2)$ and $\theta > 0$ with characteristic function:

$$\phi_T(u) = \exp \left(-\frac{2\theta^{1-\frac{\alpha}{2}}}{\alpha} ((\theta - iu)^{\frac{\alpha}{2}} - \theta^{\frac{\alpha}{2}}) \right)$$

The n -dimensional NTS distributed random vector $X = (X_1, \dots, X_n)^\top$ is defined to be:

$$X = \mu + \beta(T - 1) + \sqrt{T}(\gamma \circ \xi)$$

where $\mu, \beta \in \mathbb{R}^n$, $\gamma \in \mathbb{R}_+^n$, $\xi \sim \mathcal{N}(0, \Sigma)$, $(\gamma \circ \xi) = (\gamma_1 \xi_1, \dots, \gamma_n \xi_n)^\top$; and T is the Tempered Stable subordinator with parameters α and θ . The subordinator is independent of ξ . The n -dimensional NTS random vector so constructed, is denoted as $X \sim \text{NTS}_n(\alpha, \theta, \beta, \gamma, \mu, \Sigma)$.

Standardization of the NTS random vector involves setting $\mu = (0, \dots, 0)^\top$, $\gamma_i = \sqrt{1 - \beta_i^2 \left(\frac{2-\alpha}{2\theta} \right)}$ with $|\beta_i| < \sqrt{\frac{2\theta}{2-\alpha}}$, where β_i and γ_i are the i -th elements of β and γ , respectively, for $i = 1, 2, \dots, n$. This yields a standard NTS random vector with 0 mean and unit variance and is denoted as

⁹see [Barndorff-Nielsen and Shephard \(2001\)](#) and [Barndorff-Nielsen and Levendorskii \(2001\)](#).

¹⁰We note that while tempering the Stable distribution results in finite variance, thereby implying convergence to the Normal distribution via the Central Limit Theorem, the speed of convergence is slow; and the distribution of the summands remains close to that of the corresponding Stable distribution. This is an instance of a *pre-limit theorem* more details on which, and applications thereof, may be found in [Rachev and Mittnik \(2000\)](#).

$X \sim \text{std NTS}_n(\alpha, \theta, \beta, \Sigma)$.¹¹

We use the standard (multivariate) NTS random vector to model G-SIB returns.

2.2.2 Modeling Temporal Dependencies

Real life asset return data exhibit autocorrelation and volatility clustering. Postulating an underlying return distribution, no matter how sophisticated, cannot capture the dependence effect owing to the implicit assumption of observed returns being iid.

Hence the common assumption in the literature is the use of ARMA(1,1)-GARCH(1,1) stochastic process:

$$r_{t+1} = \mu_{t+1} + \sigma_{t+1}\epsilon_{t+1}$$

where r_{t+1} denotes asset return at time $t + 1$, μ_{t+1} is the conditional mean, while σ_{t+1} is the conditional standard deviation. ϵ_{t+1} is the “innovation” and is a standard random variable with 0 mean and unit standard deviation.

The conditional mean and the conditional standard deviation themselves are modeled as autoregressive processes, giving rise to the nomenclature of ARMA (Autoregressive Moving Average) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity).

The ARMA(1,1) process for the conditional mean is:

$$\mu_{t+1} = c + ar_t + b\sigma_t\epsilon_t$$

The GARCH(1,1) process for the conditional standard deviation is:

$$\sigma_{t+1}^2 = d + f(\sigma_t\epsilon_t)^2 + g\sigma_t^2$$

We assume the innovations to be standard NTS, i.e., $\{\epsilon_t\}_{t \in \{1,2,\dots\}}$ are iid and

$$\epsilon_t \sim \text{std NTS}(\alpha, \theta, \beta, 1)$$

3 Data

We construct an equally weighted portfolio of the “*Global Systemically Important Banks*” (G-SIBs) — a list of 29 banks compiled by the Financial Stability Board (FSB) in November 2013.¹² Our data set comprises daily log returns of 28 of these 29 banks from January 4th, 2000 to February 28th,

¹¹We note that the sum of standard NTS random variables with common α and θ parameters is again an NTS random variable (Kim *et al.*, 2012) — a fact that we use in portfolio analysis with NTS innovations in later sections.

¹² The list of Global Systemically Important Banks as it stood in November 2013, is presented in the Appendices.

2014. The only bank among the G-SIBs that we exclude is Groupe BPCE since it is unlisted.¹³ We use the S&P 1200 Global Financial Sector (SGFS) index to represent the global banking stock market.

We exclude American non-business days from the data set, which gives us 3694 observations for each bank’s stocks. The two Chinese banks: Bank of China and Industrial and Commercial Bank of China, the three Japanese banks: Mitsubishi UFJ FG, Mizuho FG and Sumitomo Mitsui FG; and the French bank Group Crédit Agricole do not have sufficient historical data to cover the entire sample period. For all of these G-SIBs, we perform imputation using the one-factor bootstrapped method with the SGFS index (FinAnalytica Inc., 2014). We note that our approach in dealing with missing data via imputation and substitution is consistent with recent literature (Kurosaki and Kim, 2013).

4 Parameter Estimation and Risk Computation

Our methodology may be considered to be built up of two steps — the first one being the estimation of the ARMA-GARCH and innovation process’s parameters — and the second one being the generation of scenarios from the fitted model and subsequent computation of risk by the Monte Carlo method.¹⁴

4.1 Estimation Technique

We model the log returns of banking stocks as an ARMA(1,1)-GARCH(1,1) stochastic process:

$$\begin{aligned} r_{t+1}^i &= \mu_{t+1}^i + \sigma_{t+1}^i \epsilon_{t+1}^i \\ \mu_{t+1}^i &= c_i + a_i r_t^i + b_i \sigma_t^i \epsilon_t^i \\ (\sigma_{t+1}^i)^2 &= d_i + f_i (\sigma_t^i)^2 + g_i (\sigma_t^i \epsilon_t^i)^2 \end{aligned}$$

The index $i \in \{1, 2, \dots, 28\}$ runs through the list of banks in the G-SIB list.

$\epsilon_t = [\epsilon_t^1, \epsilon_t^2, \dots, \epsilon_t^{28}]^\top$ is the multivariate innovation. We choose the 28-dimensional NTS distribution to model the joint distribution of innovations since it captures the skewed, interdependent and leptokurtotic nature of real life stock returns.

¹³On November 6th 2014, the Financial Stability Board updated its list of G-SIBs. Apart from all the previous 29 banks, there is an additional member: the *Agricultural Bank of China*. Since it was listed on the Shanghai and Hong Kong Stock Exchange in August 2010 and does not contain sufficient historical stock price data, we drop it from the list of the G-SIBs and focus on the list as it stood last year.

¹⁴ We broadly follow the approach of Kim *et al.* (2011), which studied ARMA-GARCH processes with Tempered Stable innovations.

The NTS distribution has two tail parameters α and θ and one skewness parameter β . We represent G-SIB stocks by the S&P 1200 financial index (SGFS) and assume common tail parameters α and θ for G-SIBs' NTS marginals and estimate them from the SGFS index. This leaves the skewness parameters $\{\beta_i\}_{i=1}^{28}$ to be calibrated for each bank in the portfolio. We follow [Kim *et al.* \(2012\)](#) in joining NTS marginals by means of their covariance matrix into multivariate NTS and note that such a method is computationally feasible even in very high dimensional settings. For comparative purposes, we also estimate an ARMA(1,1)-GARCH(1,1) model with standard (multivariate) T innovations. In order to compute the degrees of freedom of the T distributed innovations, we first fit our representative SGFS index to an ARMA(1,1)-GARCH(1,1)-T process (abbreviated as an AGT process from hereon) and use the degrees of freedom that give the best fit to the SGFS time series. For the sake of completion, we also compare these results with that of the standard specification of stock returns following ARMA(1,1)-GARCH(1,1)-(multivariate) Normal process.

Computation of the covariance matrix Σ for the NTS, T and Normal innovations is done on the basis of the most recent 250 days of daily returns.¹⁵ The reason for explicitly modeling the dependence between various banking stocks' innovations by means of their covariance matrix is because at each time period it is unreasonable to assume that movements in the stock price of one banking stock are unrelated to those of another G-SIB. Hence while there is no temporal correlation between innovations (they remain an iid process with 0 mean and unit variance), they do depend on each other by means of a covariance matrix.

4.2 Scenario Generation and Computation of Risk

Using the above methodology, we can estimate the parameters of statistical models — ARMA(1,1)-GARCH(1,1) with NTS (AGNTS), T (AGT) and Normal (AGNormal) innovations respectively. We then employ the estimated models to compute the time series of risk.

We employ the rolling window estimation technique with a 1250 days' forward moving time window to generate such a series.¹⁶ Our first time window starts at January 4th, 2000 and ends at October 18th, 2004. For this time window, we estimate all parameters of the AGNTS, AGNormal

¹⁵There are about 250 trading days in a typical year. Hence our choice of a 250-day window corresponds to the most recent one year of return data for computing the covariance matrix.

¹⁶1250 daily returns correspond to about five years of financial data, since each year has about 250 daily returns. The reason for choosing a five year time window is prompted by pragmatics. Very long time windows (comprising say, around eleven years' daily data) cause stack overflow issues in MATLAB, while very short time windows are not able to capture the stylized facts of skewness and fat-tailedness. We note that the choice of optimal time window length is very much an open problem.

and AGT model, thereby automatically estimating all parameters of the G-SIB portfolio.¹⁷ Once all parameters are estimated, risk is computed corresponding to that particular time window and the time window is then shifted one period ahead. Using such a moving time window entails estimation of model parameters for 2445 different time windows — each window shifted one trading day to the right of its preceding one.

We then generate a large number, N ($= 10^6$, in our paper) of scenarios for the one dimensional portfolio log return one period ahead: $\{r_{t+1}^{\text{port},1}, \dots, r_{t+1}^{\text{port},N}\}$. These N simulations are the realizations of the one dimensional return distribution of the G-SIB portfolio.

On the basis of the above generated scenarios, we compute three different measures of risk for each specification of a statistical model (AGNTS, AGT and AGNormal): Value at Risk (VaR), Average Value at Risk (AVaR) and Foster-Hart Risk (FH). The time horizon for both VaR and AVaR is taken to be one day.

Finally, we update the conditional means and conditional standard deviations of the three statistical models for the next period and move the time window one period forward after which we re-estimate all parameters of AGNTS, AGT and AGNormal models, generate scenarios, compute risks and again move the time window one period ahead.

5 Empirical Study

Our empirical study is divided in three broad categories:

1. Tests of the validity of the statistical model (AGNTS versus AGT and AGNormal).
2. Backtesting of the computed VaR and AVaR estimates.
3. Analysis of the time series of portfolio risk (FH risk versus VaR and AVaR).

5.1 Tests of Statistical Model Validity

In order to test which statistical model provides a better fit to the observed data, we rely on the Kolmogorov-Smirnov (KS) test, which is a goodness of fit based test that compares empirical distributions to a specific, hypothesized distribution. We test the standardized innovations of each banking stock against distributional hypotheses of standard Normal Tempered Stable (NTS), standard T and standard Normal random variables respectively.

Based on 2445 daily estimations of the AGNTS, AGT and the AGNormal models, we apply the Kolmogorov-Smirnov test 2445 times for each

¹⁷For the case with NTS distributed assets, [Kim et al. \(2012\)](#) show that the portfolio remains NTS distributed.

banking stock. Table 1 reports the number of days on which NTS, T and Normal assumptions for each banking stock are rejected at 0.5, 1, 5 and 10% significance levels respectively.

As can be seen from Table 1, the distributional hypothesis of portfolio returns being NTS distributed comprehensively outperforms the T and Normal hypotheses. This superior performance is observed not just for the levels of 10% and 5% but also for the relatively conservative significance levels of 1% and 0.5% for *25 of the 28 banks* — strong evidence of fatter tails than even that of the T distribution.

There are three exceptions to this observation — the Spanish bank Santander, the Dutch ING Group and the Swiss bank UBS. Among the three of them, however, only for the case of Santander do we see T outperforming NTS comprehensively — for the other two banks, the relative victory margins of T are slender.

The Normal hypothesis provides terrible fits in general. In fact, the Normal hypothesis is uniformly rejected much more numerously than the T and the NTS hypotheses for *all* significance levels. Hence we conclude that the Normal distribution is a poor contender for fitting standardized innovations of G-SIB stocks.

For all the other 25 banks, the NTS substantially outperforms the T hypothesis at *all* confidence levels. We believe that this constitutes comprehensive evidence of NTS’s superiority over the T.

To conclude, the NTS hypothesis is rejected far fewer times than the T and Normal hypotheses at all significance levels. The overall result is that the NTS distributional hypothesis provides much better fits than those of the T and Normal distributions. Such overwhelming evidence, in our opinion, provides strong support in favor of the AGNTS model over the AGT and AGNormal for stock returns for G-SIBs.

5.2 Backtesting

In order to evaluate the accuracy of forecasted VaR according to the AGNTS, AGT and AGNormal statistical models, we perform backtesting using the Christofferson Likelihood Ratio (CLR) test (Christofferson, 1998) and the Berkowitz Likelihood Ratio (BLR) test (Berkowitz, 2001). We check the VaR of the equally weighted portfolio by both the CLR and the BLR tests; and further analyze our backtests using a multi-period framework.

The CLR test has three parts: the CLR unconditional coverage test (‘uc’),¹⁸ the CLR independence test (‘ind’); and the CLR joint test of coverage and independence (‘cc’). We report the p -values of the equally weighted portfolio’s CLR and BLR tests later on in this section.

We conduct the CLR test on the equally weighted G-SIB portfolio to

¹⁸This test is also called the “proportion of failures” test in Kupiec (1995).

Table 1: Number of rejections of distributional assumptions for each banking stock on the basis of KS test (out of a total of 2445 estimations)

	Significance Level 10%			Significance Level 5%			Significance Level 1%			Significance Level 0.5%		
	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
BAC	2443	1976	1522	2445	1569	1220	2406	716	400	2362	630	96
BK	2445	2361	1889	2443	2283	1583	2436	1892	931	2431	1773	679
C	2391	2007	1787	2278	1552	1419	1742	958	523	1493	666	432
GS	2331	1350	899	2030	792	580	1203	32	1	835	8	0
JPM	2443	1855	1487	2400	1399	846	1787	673	92	1375	426	28
MS	2249	1621	1725	1938	1377	1129	1457	570	370	1259	352	154
STT	2444	2443	1403	2444	2390	1268	2443	1616	946	2440	1453	708
WFC	1878	1337	742	1718	1198	606	1540	655	418	1370	558	280
BARC	1967	154	65	827	14	7	9	6	5	8	6	3
BNP	1922	71	13	1692	25	4	1024	7	1	598	6	0
CSGN	2135	307	329	1705	157	53	1042	11	2	834	4	0
DBK	2087	430	332	1637	229	74	869	23	8	790	20	8
ACA	1624	8	3	1132	4	1	362	2	1	126	2	1
HSBA	2421	189	11	2193	40	0	609	2	0	303	0	0
INGA	1998	869	1048	1522	360	772	655	13	15	245	10	6
NDA	2442	1481	668	2348	1235	574	1778	364	346	1395	175	109
RBS	2351	1263	959	2184	1126	796	1613	819	348	1448	765	227
SAN	2354	1148	1726	2164	758	1439	1293	209	768	1159	78	455
GLE	2419	765	663	2212	347	290	871	5	7	493	4	2
UBSN	1664	441	531	1136	337	312	528	21	75	354	1	39
UCG	1616	387	90	1307	161	15	709	18	7	308	16	6
BBVA	2405	1078	941	2294	583	428	1832	160	27	1593	55	3
STAN	1635	376	12	1383	20	2	677	3	2	477	3	1
BOC	1991	1431	1455	1737	1407	1420	1445	1337	1279	1383	1295	1172
MUFG	1976	87	5	1341	4	2	319	1	1	118	0	1
MHFG	2445	1911	1461	2442	1633	1403	2041	1461	900	1586	1383	613
SMFG	2435	1725	979	2433	1560	180	2359	775	5	2069	469	3
ICBC	1936	1344	1190	1475	1209	1065	1210	1037	946	1103	996	871

Table 2: p -values of CLR test for portfolio VaR based on 2444 daily VaR estimations from Oct 18th, 2004 to February 27th, 2014

99.5% VaR								
uc			ind			cc		
AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
0.0000	0.4415	0.6179	0.4039	0.6668	0.6889	0.0002	0.6778	0.8146
99% VaR								
uc			ind			cc		
AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
0.0004	0.3678	0.2751	0.8194	0.4039	0.3878	0.0016	0.4705	0.3796
95% VaR								
uc			ind			cc		
AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
0.0126	0.0034	0.0729	0.3466	0.1788	0.3314	0.0285	0.0056	0.1249

Table 3: BLR tail test for the equally weighted G-SIB portfolio, based on 2444 daily VaR estimations from Oct 18th, 2004 to February 27th, 2014

Tail probability	0.5%			1%			5%		
Model	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
p -value	0.1347	0.3754	0.6250	0.1139	0.4086	0.6292	0.0870	0.4864	0.6468

check if our estimations of portfolio VaR are reasonable. Since the portfolio risk estimation is based on the 28 dimensional joint distribution model, this test actually examines the reliability of risk estimates computed via the AGNormal, AGT and AGNTS models.

Table 2 reports the p -values of CLR tests, based on 2444 daily VaR estimations starting from Oct 18th, 2004 and ending on February 27th, 2014 (the last daily VaR recorded on February 28th cannot be backtested). On the basis of such tests, we conclude that both the AGT and the AGNTS models comprehensively beat the AGNormal model's risk backtests for all confidence levels and all three test criteria. Among the AGT and the AGNTS models, we observe the AGNTS to be superior, although the relationship is reversed for the 99% VaR risk backtests.

In addition to the CLR tests, we also perform the BLR tail test to examine the reliability of the estimated AVaR. In Table 3, we backtest for three levels of tail probability: $\{0.5\%, 1\%, 5\%\}$. As expected, the AGNormal model is beaten uniformly by both the AGT and the AGNTS models. Between the AGT and AGNTS models, the latter uniformly outperforms the former for all three chosen levels of tail probabilities. Hence we can conclude that the AVaR risk backtests are better according to the AGNTS model than the AGT or the AGNormal models.

As a final exercise in backtesting, we divide the entire time series into five equally spaced periods as follows:

1. 2004–2006: from 11/02/2004 to 09/12/2006
2. 2006–2008: from 09/13/2006 to 07/23/2008

Table 4: Multi-period p -values of CLR test for portfolio 99.5% VaR

	uc			ind			cc		
	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
2004–2006	0.3596	0.7287	0.7706	0.7969	0.8471	0.8978	0.6358	0.9243	0.9505
2006–2008	0.0012	0.0543	0.1503	0.5604	0.6988	0.7471	0.0045	0.1457	0.3379
2008–2010	0.0172	0.3596	0.1504	0.6514	0.7969	0.7474	0.0528	0.6358	0.3376
2010–2012	0.1501	0.7707	0.7706	0.7479	0.8978	0.8989	0.3376	0.9505	0.9506
2012–2014	0.3670	0.0264	0.0274	0.7977	0.7965	0.9473	0.6442	0.7977	0.8913

3. 2008–2010: from 07/24/2008 to 06/03/2010

4. 2010–2012: from 06/04/2010 to 04/13/2012

5. 2012–2014: from 04/16/2012 to 02/24/2014

We then repeat the CLR test on the 99.5% VaR period by period; and illustrate the results in Table 4. The AGNormal model reports uniformly lower p -values than its competitors. Among the AGT and AGNTS models, however, we see roughly the same level of robustness — while the AGT model beats the AGNTS during 2008–2010, it is outperformed for 2004–2006, 2006–2008 and 2012–2014. Both of them show equally good performances during 2010–2012. Hence we conclude that both AGT and AGNTS models report robust VaR values but overall the level of robustness for AGNTS is even more than that of AGT.

We backtest for AVaR values for the aforementioned periods by using multi period BLR tail tests (see Table 5). With the exception of 2004–2006, the AGNormal model loses again to the AGT and the AGNTS models. The AGT model proves marginally superior to AGNTS during the more tranquil 2004–2006 and 2012–2014 while the AGNTS model yields uniformly better p -values for the periods 2006–2008, 2008–2010 and 2010–2012.

To conclude, a battery of backtesting procedures are employed to check if the VaR and AVaR estimates computed under the AGNormal, AGT and AGNTS models are indeed reliable. The performance of the AGNTS model is generally the best while the AGT model finishes a close second. This prompts us to conclude that on the basis of both in-sample tests (KS) and out-of-sample tests (CLR and BLR) AGNTS is a better statistical model for the skewed, fat-tailed returns of G-SIBs than the conventional AGT and AGNormal models. Hence, in the following subsection, we carry out portfolio risk analysis assuming an underlying AGNTS statistical model with three different measures of risk — VaR, AVaR and FH.

5.3 Portfolio Risk Analysis

Before analyzing the time series of the portfolio equity risk for the G-SIBs, we briefly discuss the behavior of the (log) return time series of the SGFS index.

Table 5: Multi-period p -values of BLR tail test

Tail probability	0.5%			1%			5%		
Model	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS	AGNormal	AGT	AGNTS
2004–2006	0.1741	0.2555	0.0217	0.1955	0.2706	0.0246	0.2550	0.3316	0.0284
2006–2008	0.0002	0.0019	0.0056	0.0002	0.0020	0.0057	0.0002	0.0017	0.0055
2008–2010	0.3489	0.5093	0.5316	0.4018	0.5046	0.5244	0.4627	0.5305	0.5536
2010–2012	0.3495	0.4547	0.5562	0.3416	0.4484	0.5515	0.3061	0.4016	0.5181
2012–2014	0.0002	0.0036	0.0020	0.0001	0.0039	0.0021	0.0000	0.0048	0.0024

We plot the (log) return time series of the index and superimpose on it, a Hodrick-Prescott (abbreviated as ‘HP’ from hereon) filtered time series of the index returns with $\lambda = 20000$ to obtain a smoothed representation of the index by removing its cyclical component (Hodrick and Prescott, 1997). As is clear from the SGFS time series graph presented in Figure 1, there are two sharp downturns in the index returns — one during the first quarter of 2008 — corresponding to the collapse and eventual sale of Bear Stearns to JP Morgan Chase; and a second, even more pronounced fall in the final quarter of 2008 and the first quarter of 2009 — corresponding to the bankruptcy of Lehman Brothers in September 2008 and the ensuing panic. A second, though less severe phase of large negative returns occurred in the second quarter of 2010, with the onset of the Greek crisis, Standard and Poor’s downgrading of Greek debt ratings to junk bond status (April 27th) and the subsequent nationwide protests against austerity policies in Greece. Finally there is another heavy downslide in the third and fourth quarters of 2011, coinciding with Standard and Poor’s downgrading of US federal government debt in August 2011; the bankruptcy of Dexia, an ex-G-SIB, in October 2011; and the controversial Greek announcement of a referendum on the Eurozone debt deal in November 2011.

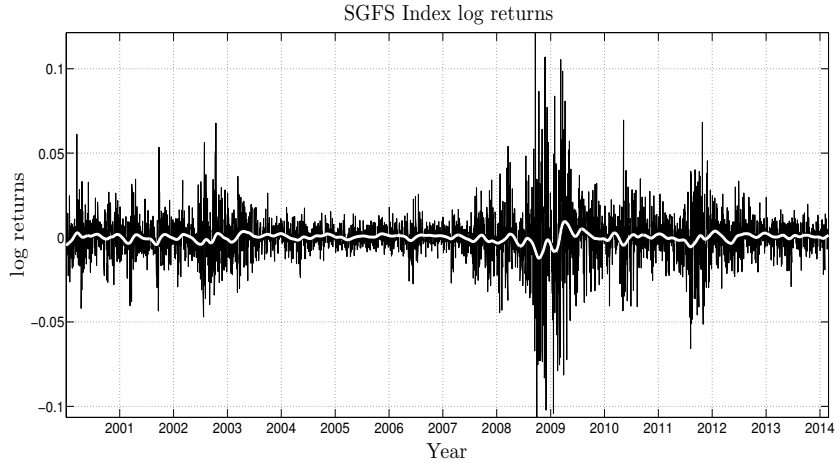


Figure 1: Log-returns of the S&P Global 1200 Financial (SGFS) index. The white graph corresponds to an HP filtered log returns of the SGFS index ($\lambda = 20000$ for the HP filter).

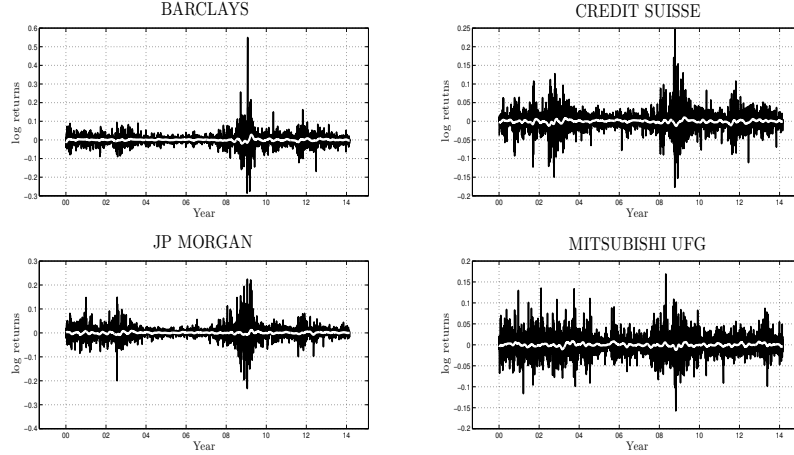


Figure 2: Log-returns of some G-SIBs ($\lambda = 20000$ for the HP filter)

A qualitatively similar, though not identical behavior is exhibited by the constituent banks of the index. For ease of comparison, we have included the log-return time series of four of the 29 such banks. These are: Barclays (UK) and Credit Suisse (Switzerland) from Europe; JP Morgan from the US; and Mitsubishi UFG from Asia. While all of these banks show sharp downturns in the last quarter of 2008 with the collapse of Lehman Brothers, the two European banks show much heavier losses from the Eurozone crisis than their American and Asian counterparts, as should be expected (see Figure 2).

Clearly, such sharp downturns amplified the equity risk posed by banks on their shareholders dramatically. Our portfolio of G-SIB stocks also exhibits these observations. In light of this information, we discuss the time series of our G-SIB portfolio equity risk below.

In order to track the risk profile of our equally weighted portfolio of G-SIB stocks, we plot the time series of portfolio risk according to three different risk measures — VaR, AVaR and FH — assuming an underlying AGNTS model. For VaR and AVaR, we compute risk for 90%, 95% 99% and 99.5% confidence levels.

Theory suggests that the Foster-Hart risk should be more than both VaR and AVaR at any confidence level. This is so because the risk as measured by Foster-Hart captures fat-tailed events — i.e., even if the maximum loss possible were to occur (with howsoever small a probability) wealth levels computed by means of the Foster-Hart riskiness will always prove adequate since it is always at least as large as the maximum loss. Such a property is not possessed by either VaR or AVaR. Indeed even for very high confidence levels, the probability that a large loss outstrips wealth levels computed via VaR or AVaR — an event that results in bankruptcy — is positive.

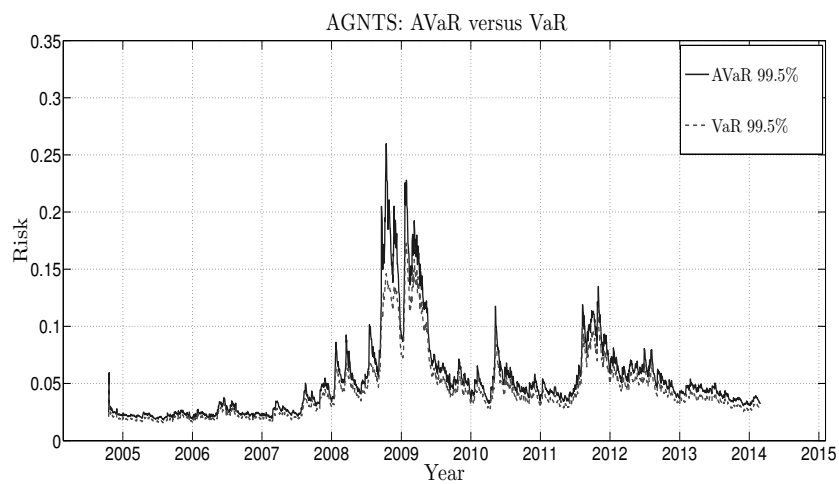


Figure 3: AVaR versus VaR at 99.5% confidence level under the AGNTS model

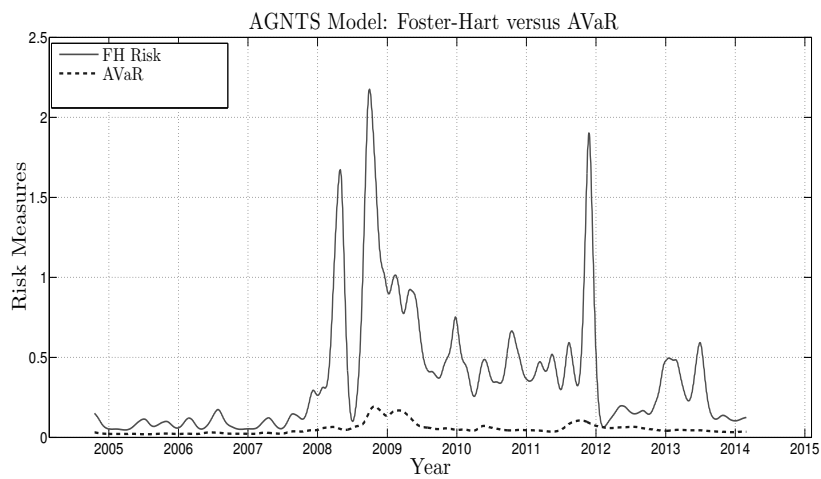


Figure 4: FH versus the 99.5% AVaR. All series have been smoothed by using the Hodrick-Prescott filter with $\lambda = 20,000$.

This is exactly what we observe: a comparative study of risk profiles as measured by VaR, AVaR and FH risk shows that not only is the FH risk always more than that according to VaR and AVaR but in fact, the degree of divergence between the two is much higher during times of crises (see Figure 4). While it is true that during the worst of the financial crisis (March 2008 – Bear Stearns’s sale to JP Morgan and September 2008 – Lehman Brothers’ bankruptcy) VaR and AVaR are able to pinpoint the equity risk posed by, and reflected in, abysmal stock market performance; and exhibit local maxima at the aforementioned events, the corresponding FH risk puts the equity risk value to be much higher.

Such a result goes on to show that even though computed by an advanced, fat-tailed statistical model, consistent with skewness and leptokurtosis (ARMA(1,1)-GARCH(1,1)-NTS process) and the most widely used risk measures in the industry today (VaR and AVaR), the equity risk posed was severely underestimated during the financial crisis. In fact, it also implies that if the banks’ in-house risk management groups, based on the knowledge of their own internal portfolios, were to use VaR and AVaR as risk measures to compute critical reserves, their computations would have proved far from adequate.

However, without much difficulty, the internal risk management teams of banks can adopt our methodology and on the basis of their bank’s portfolio, come up with capital reserve computations that are much more accurate, robust, reliable and sensitive to information embedded in the stock market.

6 Mean Risk Analysis

The Foster-Hart risk is beneficial from the point of view of portfolio management as well. In the following discussion, we set up the portfolio optimization problem with different notions of portfolio risk: standard deviation, VaR, AVaR and FH. We assume that the portfolio is distributed according to the one dimensional NTS distribution, whose superiority has been demonstrated above.

We also note that under elliptically distributed portfolios, mean risk analysis with standard deviation, VaR and AVaR measures yields the same optimal portfolio weights; but when the portfolio is distributed according to a skewed, non Gaussian distribution like NTS, in general, we get different optimal weights for different minimization programs.

6.1 The Classical Optimization Programs

The classic framework devised by [Markowitz \(1952\)](#) uses standard deviation as a measure of portfolio risk. Hence the optimization program takes the

form:

$$\max \left(\sum_{i=1}^n w_i \mu_i \right)$$

subject to

$$w^\top \Sigma w \leq \sigma_*$$

$$\sum_{i=1}^n w_i = 1$$

$$0 \leq w_i \leq 1$$

The last two constraints rule out short selling and in essence, state that the feasible set of weights is the n dimensional simplex $w_i \in \Delta[0, 1]^n$. Moreover, we can rewrite the above program as:

$$\max_{w_i \in \Delta[0, 1]^n} \left(\sum_{i=1}^n w_i \mu_i - C w^\top \Sigma w \right)$$

or

$$\min_{w_i \in \Delta[0, 1]^n} \left(C w^\top \Sigma w - \sum_{i=1}^n w_i \mu_i \right)$$

here C may be interpreted to be a risk aversion parameter.

However, more sophisticated notions of risk may be employed to make the minimization program more useful. In general, if the measure of risk employed is $\rho(\cdot)$, the minimization program becomes:

$$\min_{w_i \in \Delta[0, 1]^n} \left(C \rho(w) - \sum_{i=1}^n w_i \mu_i \right)$$

Using VaR, AVaR and FH risk measures, we can analyse the optimal weights according to three different programs.

6.2 Transaction Costs

Rebalancing the portfolio is not without its downsides. Frequent rebalancing introduces transaction costs which reflect liquidity concerns in the markets, details of market microstructure and so on. We do not wish to ignore this very important issue since it eats up substantially into portfolio returns.

We propose a general quadratic transaction cost function and note that they have been commonly considered in literature ([Fabozzi et al., 2006](#)):

$$TC(w) = \lambda \left(\alpha + \beta \sum_{i=1}^n |w_{i,t} - w_{i,t-1}| + \gamma \sum_{i=1}^n (w_{i,t} - w_{i,t-1})^2 \right)$$

6.3 Portfolio Optimization

Hence the minimization program now becomes:

$$\min_{w_i \in \Delta[0,1]^n} \left(C\rho(w) + TC(w) - \sum_{i=1}^n w_i \mu_i \right)$$

or, in expanded form:

$$\min_{w_i \in \Delta[0,1]^n} \left(C\rho(w) + \lambda \left(\alpha + \beta \sum_{i=1}^n |w_{i,t} - w_{i,t-1}| + \gamma \sum_{i=1}^n (w_{i,t} - w_{i,t-1})^2 \right) - \sum_{i=1}^n w_i \mu_i \right)$$

In our set of minimization problems, the risk function $\rho(\cdot)$ takes four different forms — standard deviation, VaR, AVaR and FH. Except for standard deviation, there are no closed form solutions for any other risk function under the NTS distributional hypothesis. Hence we rely on the Monte Carlo technique to generate simulations for the four different programs and then find weights corresponding to the minimum.

We assume that the risk aversion parameter $C = \lambda = 1$ and that $\alpha = \beta = \gamma = 0.0050$ (50 basis points).

6.4 Results

We compare the cumulative returns of the portfolio under the four different strategies indicated above as shown in the figure below.

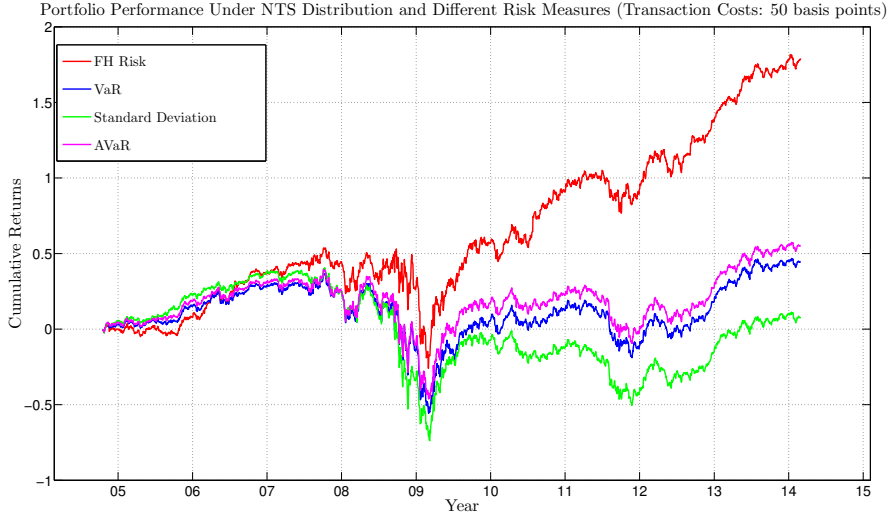


Figure 5: Portfolio Performance under AGNTS and quadratic transaction costs

As may be seen, since the FH risk is much more sensitive to tail events, it is able to perform the best — not only under market distress (as can be

observed for the period under the Great Recession) but also during periods of market tranquility. The only exceptions to this are the periods from 2005 to the first quarter of 2006. FH risk based portfolio performance strongly dominates all others for all periods other than the one indicated above.

Among the other risk measures, AVaR posts the strongest performance, followed by VaR; and then lastly by standard deviation. This is in keeping with the order of sophistication of the risk measures and the order in which each of them is able to respond to tail events.

In the appendices, we include portfolio performance results for portfolio distributional hypotheses under Normal and T assumptions. As may be observed, FH outperforms all rivals again although the margin of victory is lesser than in NTS.

7 Conclusions

In this paper, by a variety of methods, we compute the equity risks posed by the too-big-to-fail banks which constitute a very important part of the global financial system and are its most critical component from the point of view of maintenance of overall stability of the global financial markets.

In order to facilitate this exercise, we assume that stock returns of G-SIBs are distributed according to the ARMA(1,1)-GARCH(1,1) stochastic process whose innovations are distributed according to the NTS distribution — from a family of tempered α -Stable distributions with finite moments. Use of such a fat-tailed distribution for the innovations' process leads to much better statistical fits as compared to those that rely on standard T or standard Normal based techniques. Specifically, the superiority of the NTS to the T and Normal statistical model has been demonstrated for our dataset by both in-sample (the KS test) and out-of-sample tests (the CLR and BLR test).

In order to illustrate that current methods of risk assessment underestimate the real equity risk posed by the G-SIBs, we employ the Foster-Hart risk measure. The Foster-Hart risk measure is a recent breakthrough in the field of risk measurement which is based on the observation that irrespective of utility functions, the risk posed to each agent depends critically on the amount of wealth she needs to possess in order to stave off bankruptcy.

We show that during the Great Recession, equity risks posed by the G-SIBs were much higher than that suggested by conventional measures of risk such as VaR and AVaR. The difference between the FH risk and that computed by VaR and AVaR is not static — it evolves with time and is the most pronounced during times of high volatility and during times of crises. Indeed, even during periods of market tranquility, relying exclusively on VaR or AVaR may lull one into believing that equity risks are minimal when in fact they are high — an early indication of a potential crisis.

Our findings have important policy implications. Regulators looking to measure equity risk posed by banks on the financial system as a whole might end up underestimating it if they rely exclusively on VaR and AVaR. To accurately assess the risks posed, our approach, based on the Foster-Hart measure, is an improvement over current methodologies. Another application of our paper is in the area of critical reserve computations for banks. Current methods endorsed by regulators for computation of reserve requirements are not necessarily adequate and may be improved upon by employing the Foster-Hart measure of risk.

This paper should prove useful to the banking industry as well, since the banks' in-house risk management teams can adapt our methodology relatively easily and based on the knowledge of their internal portfolio composition, may track their own equity risks much more accurately.

Lastly we show that under the assumption of portfolios being distributed according to NTS, and under very general quadratic transaction costs, the FH risk measure strongly outperforms mean risk analysis based portfolio performance tests. In this regard, the Foster-Hart risk should prove very useful to portfolio managers who can benefit from its acute sensitivity to tail events, no matter how rare.

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References

- Admati, A. and Hellwig, M. (2013). *The Bankers' New Clothes: What's Wrong With Banking and What to Do About It*. Princeton, New Jersey: Princeton University Press. [2](#)
- Alexander, S. S. (1961). Price movements in speculative markets: Trends or random walks. *Industrial Management Review*, *2*, 7–25. [6](#)
- Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1998). Coherent measures of risk. *Mathematical Finance*, *6*(3), 203–228. [3](#), [5](#)
- Barndorff-Nielsen, O. E. and Levendorskii, S. Z. (2001). Feller processes of normal inverse gaussian type. *Quantitative Finance*, *1*(3), 318–331. [7](#)

- Barndorff-Nielsen, O. E. and Shephard, N. (2001). Normal modified stable processes. Economics Series Working Papers from University of Oxford, Department of Economics 72. 7
- Berkowitz, J. (2001). Testing density forecasts, with applications to risk management. *Journal of Business and Economic Statistics*, 19, 465–474. 12
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31, 307–327. 3
- Christofferson, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, 39(4), 841–862. 12
- Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica*, 4, 987–1008. 3
- Fabozzi, F. J., Focardi, S. M., and Kolm, P. N. (2006). *Financial Modeling of the Equity Market: From CAPM to Cointegration*. Hoboken, New Jersey: Wiley. 20
- Fama, E. F. (1963). Mandelbrot and the stable Paretian hypothesis. *Journal of Business*, 36, 420–429. 6
- Fama, E. F. (1965). The behavior of stock market prices. *Journal of Business*, 38, 34–105. 6
- FinAnalytica Inc. (2014). Cognition 4.0. <http://optimizer.cognition.net/>. 9
- Foster, D. P. and Hart, S. (2009). An operational measure of riskiness. *Journal of Political Economy*, 117(5), 785–814. 3, 5
- Haas, M. and Pigorsch, C. (2009). *Complex Systems in Finance and Econometrics*, New York: Springer, chap. Financial Economics, Fat-Tailed Distributions. 308–339. 6
- Hodrick, R. and Prescott, E. C. (1997). Postwar U.S. business cycles: An empirical investigation. *Journal of Money, Credit, and Banking*, 29(1), 1–16. 16
- Kim, Y. S., Giacometti, R., Rachev, S. T., Fabozzi, F. J., and Mignacca, D. (2012). Measuring financial risk and portfolio optimization with a non-Gaussian multivariate model. *Annals of Operations Research*, 325–343. 7, 8, 10, 11

- Kim, Y. S., Rachev, S. T., Bianchi, M. L., and Fabozzi, F. J. (2008). Financial market models with Levy processes and time-varying volatility. *Journal of Banking & Finance*, 32(7), 1363–1378. 7
- Kim, Y. S., Rachev, S. T., Bianchi, M. L., and Fabozzi, F. J. (2010). Tempered stable and tempered infinitely divisible GARCH models. *Journal of Banking & Finance*, 34, 2096–2109. 7
- Kim, Y. S., Rachev, S. T., Bianchi, M. L., Mitov, I., and Fabozzi, F. J. (2011). Time series analysis for financial market meltdowns. *Journal of Banking & Finance*, 35, 1879–1891. 9
- Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives*, 6, 6–24. 12
- Kurosaki, T. and Kim, Y. S. (2013). Sytematic risk measurement in the global banking stock market with time series analysis and CoVaR. *Investment Management and Financial Innovations*, 10(1), 184–196. 9
- Mandelbrot, B. (1963a). New methods in statistical economics. *Journal of Political Economy*, 71, 421–440. 6
- Mandelbrot, B. (1963b). The variation of certain speculative prices. *Journal of Business*, 36, 394–419. 6
- Mandelbrot, B. (1967). The variation of some other speculative prices. *Journal of Business*, 40, 393–413. 6
- Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance*, 7(1), 77–91. 19
- Rachev, S. T. and Mittnik, S. (2000). *Stable Paretian Models in Finance*. Hoboken, New Jersey: John Wiley and Sons. 6, 7
- Rachev, S. T., Schwartz, E., and Khindanova, I. (2003). *Handbook of Heavy Tailed Distributions in Finance*, Amsterdam: North Holland Handbooks of Finance, chap. Stable Modeling of Credit Risk. 6
- Rachev, S. T., Stoyanov, S., Biglova, A., and Fabozzi, F. (2005). *Data Analysis and Decision Support*, Berlin: Springer, chap. An Empirical Examination of Daily Stock Return Distributions for US Stocks. 6
- Riedel, F. and Hellmann, T. (2014). The Foster-Hart measure of riskiness for general gambles. *Theoretical Economics*, Forthcoming. 6

Table 6: List of the 29 G-SIBs (as of November 2013)

United States	Europe	Asia
Bank of America (BAC)	Groupe BPCE	Bank of China (BOC)
Bank of New York Mellon (BK)	Barclays (BARC)	Mitsubishi UFJ FG (MUFG)
Citigroup (C)	BNP Paribas (BNP)	Mizuho FG (MHFG)
Goldman Sachs (GS)	Standard Chartered (STAN)	Sumitomo Mitsui FG (SMFG)
JP Morgan Chase (JPM)	Credit Suisse (CSGN)	Industrial and Commercial Bank of China (ICBC)
Morgan Stanley (MS)	Deutsche Bank (DBK)	
State Street (STT)	Banco Bilbao Vizcaya Argentaria (BBVA)	
Wells Fargo (WFC)	Group Crédit Agricole (ACA)	
	HSBC (HSBA)	
	ING Bank (INGA)	
	Unicredit Group (UCG)	
	Nordea (NDA)	
	Royal Bank of Scotland (RBS)	
	Santander (SAN)	
	Société Générale (GLE)	
	UBS (UBSN)	

Appendices

The appendices include tables indicating the membership of G-SIBs, as assessed by the Financial Stability Board. Also contained are graphical results that chart variation in computed risk with changes in the underlying statistical model.

A The Global Systemically Important Banks

The list of Systemically Important Banks is included in table 6. This table was prepared in November 2013 by the Financial Stability Board.¹⁹ It features 8 banks from North America, 16 banks from Europe and 5 banks from Asia.

B Variation of FH Risk with the Underlying Statistical Model

As should be expected, due to the fat-tailed distributional hypothesis of the AGNTS model, the FH equity risk under AGNTS is higher than that computed under the AGT and the AGNormal model. While there are qualitative similarities, for almost all time periods, the FH risk under AGNTS dominates that under the others, as can be seen in Figure 6. (We note however, that this behavior is reversed for the third and fourth quarter of 2012 during which FH risk under AGNormal dominates the others.)

We use the unique, continuity property of the Foster-Hart risk to rule out the existence of outliers. We note that this technique is justified since the

¹⁹See http://www.financialstabilityboard.org/publications/r_131111.pdf for more details.

time series of maximum losses of the portfolio is (approximately) continuous.

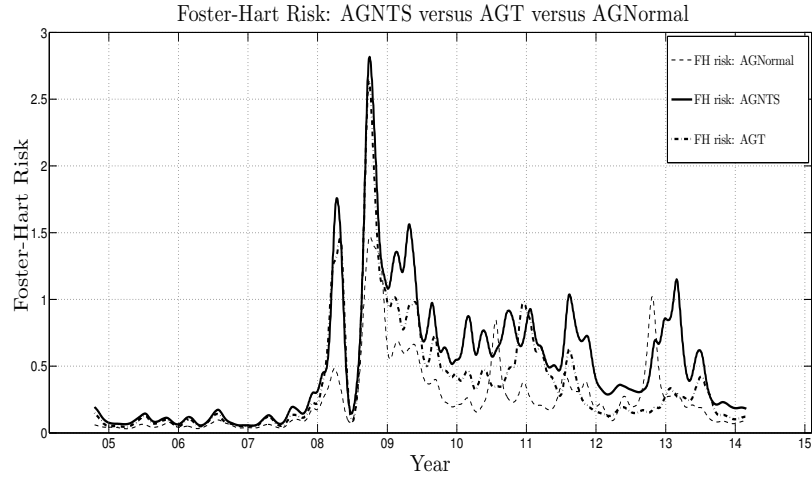


Figure 6: Variation of FH Risk with AGNTS, AGT and AGNormal models. All series have been smoothed by using the Hodrick-Prescott filter with $\lambda = 20000$.