Partisan Voting and Uncertainty

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Abstract

In this paper, we study a model in which partisan voting is rationalized by Knightian decision theory under uncertainty (Bewley, 1986). When uncertainty is large, some voters become hard-core supporters of their current party due to status quo bias. We characterize equilibria in a setting similar to Krishna and Morgan (2012). Under Knightian uncertainty, mixed-strategy equilibrium is more flexible, because indifference among strategies used in equilibrium is no longer required. With costly information, there always exists an equilibrium. In some scenarios, swing voters acquire positive amount of information in equilibrium.

1   Introduction

In everyday politics, partisans are considered as hard-core supporters who do not change their position no matter what happens. Partisan voting is an important phenomenon. According to Huffpost Politics, the latest pollster estimate of party identification[1] in the U.S. is: independent 34.3%, Democrat 33.5%, and Republican 26.4%. Around 60% of the population consider themselves partisan voters. Moreover, voters are more likely to consider themselves independent than they actually are. Burden and Klofstad (2005) identify more partisan voters by asking a set of questions related to party identification than asking about party identity directly.

Is partisan voting rational? Party supporters may appear to be stubborn and irresponsive to persuasion. In some voting models, partisans are assumed to stick to some parties (i.e. Feddersen and Pesendorfer, 1996; Palfrey and Rosenthal, 1983; Myatt, 2007). Under such assumption, partisan voters are not rational, since they do not take useful information into account. In other models, there are no fundamental differences between swing voters and party supporters in terms of rational calculation (i.e. Feddersen and Pesendorfer, 1999; Aragones and Palfrey, 2002; Gul and Pesendorfer, 2008; Krishna and Morgan, 2011). Some voters vote according to their information, and others don't. Swing voters and partisan

[1]On August 27, 2012.

voters are classified by their responsiveness to information. Moreover, a voter's responsiveness to information depends on intensity of his preference, in the sense that voters with less intensive preference are more responsive[2].

In this paper, we consider an alternative rationalization for partisan voting, and discuss its implications. Facing uncertainty, a Knightian decision maker's behavior is affected by her status quo. Voters who have one party as their status quo behave differently from those who do not have any particular party as their status quo. The status quo bias is more powerful with larger uncertainty. When the status quo bias is strong enough, partisan voters become hard-core supporters who are loyal to their own party regardless of any useful information. When the status quo bias is not strong enough, partisan voters may vote against their own party.

The model is built on Myerson's large Poisson game in a common value setting similar to Krishna and Morgan (2012). In such games, the size of the electorate is random. Myerson (1998 & 2000) shows the equivalence of qualitative predictions between Poisson voting model and standard voting model with a fixed electorate. The advantage of the Poisson game is that it simplifies the analysis. Moreover, considering a random electorate rather than a fixed one in large election is more natural. Voters are assumed to be Knightian decision makers with multiple priors. Party preference is defined by voters' status quo. Facing multiple priors, it happens that two alternatives are incomparable. If one voter consider some party as her status quo, she will stick to her own party when she is not able to compare two alternatives.

Four types of equilibria are characterized: 1) sincere voting equilibrium, 2) uninformative voting equilibrium, 3) partisan voting equilibrium, and 4) partial partisan voting equilibrium. In the standard expected utility model, mixed-strategy equilibrium requires indifference among strategies used with positive probability, which is not required for strategies that are incomparable to each other for a Knightian decision maker. Therefore, voting under uncertainty allows more flexible equilibrium behaviors. In a large election, standard expected utility model has to struggle with the equilibrium existence. With less stringent condition imposed by use of mixed strategies, we can support some the first three types of equilibrium in a generic set of parameters. Moreover, if costly information acquisition is introduced, there always exists an equilibrium, with positive information acquisition in some circumstances.

In Section 2, we briefly go through Knightian decision model and inertia assumption, and argue that party identity can be naturally considered as a status quo. In Section 3, the model is presented. The large population property is discussed in Section 4. Section 5 is an extension with costly information acquisition, and Section 6 consists of a comparison between this model and two expected utility models. Section 7 concludes and discusses some future work.

2   Incomplete Preferences and Status Quo

Under uncertainty, completeness is not necessarily a reasonable axiom for individual decision problems. Bewley (1986, 1987 & 1989) develops Knightian decision theory, which relaxes the axiom of completeness.

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[2]More detailed discussion of models with party identity as preference intensity is in a later section, where these models are compared with the model with party identity as status quo.

Under the completeness axiom, individual decision maker is able to rank any pair of alternatives. If the preference is not complete, some alternatives are incomparable. Bewley (1986) axiomatizes a model allowing for incompleteness with subjective probabilities.

Consider a finite state space N, the set of all probability distributions over N, $\triangle (N) := \{\pi \in R^N : \pi_i \geq 0 \ \forall i = 1,...N, \ \sum_{i=1}^{N} \pi_i = 1\}$, and two random monetary payoffs, $x, y \in X^N$, where $X \subset R$ is finite. Bewley characterizes incomplete preference relations represented by a unique nonempty, closed, convex set of probability distribution Π and a continuous, strictly increasing, concave function $u : X \to R$, unique up to positive affine transformation, such that

$$x \succ y \quad if \ and \ only \ if \quad \sum_{i=1}^{N} \pi_i u(x_i) > \sum_{i=1}^{N} \pi_i u(x_i) \ \ for \ all \ \pi \in \Pi.$$

If the set of probabilities Π is a singleton, this is equivalent to an expected utility representation, so ordering is complete.

If Π is not a singleton, comparison between two alternatives are done "one probability distribution at a time". A strict preference is obtained, only when one alternative is "strictly preferred" to the other unanimously under all $\pi \in \Pi$.

In some situations, a Knightian decision maker can not make up her mind. Bewley's inertia assumption helps to settle some choice problems among incomparable alternatives. If there is a *status quo*, a Knightian decision maker always choose the status quo as long as no other alternative is strictly preferred to it according to every probability distribution. For instance, consider two alternatives a and b, a is strictly preferred to b for some p, and b is strictly preferred to a for some other p. Knightian decision rule concludes that a and b are incomparable. A decision maker without any status quo will choose either a or b, or randomize. A decision maker with a as status quo will always choose a, and b is always chosen by a decision maker with b as status quo. When a and b are comparable to each other, these three types of decision maker will make the same choice.

2.1 Party Identity as Status Quo

Campbell, Converse, Miller and Stokes (1960) in their classic *The American Voter* say:

> *Only in the exceptional case does the sense of individual attachment to party reflects a formal membership or an active connection with a party apparatus. Nor does it simply denote a voting record, although the influence of party allegiance on electoral behavior is strong, generally this tie is a psychological identification, which can persist without a consistent record of party support. Most Americans have this sense of attachment with one party or the other. And for the individual who does, the strength and direction of party identification are facts of central importance in accounting for attitude and behavior.*
>
> *In characterizing the relation of individual to party as a psychological identification we invoke a concept that has played an important if somewhat varied role in psychological theories of the relation of individual or individual to group. We use the concept here to characterize the individual's affective orientation to an important group-object in his environment.*

The difficulty in Bewley's inertia assumption is "identifying a plausible candidate for the role of status quo" (Lopomo, Rigotti and Shannon, 2009). In the case of partisan voting, party identity, as an "affective orientation", is a natural candidate for a status quo. For all possible states, party supporters compare two parties. They stick to their own parties as long as it is preferred in some states. There must be strong enough incentive to motivate a party supporter to vote against his own party. Swing voters can be considered as voters without a party identity. As long as two parties are incomparable in a Knightian sense, a swing voter can cast a vote in any manner he likes. If a complete preference ordering is always taken for granted, such behaviors can never be justified.

3   The Model

There are two party candidates, A and B. There are two states of the world, α and β. The prior probabilities of state α and β are p and $1 - p$, respectively, and p is between $\underline{p}$ and $\overline{p}$. Assume $\overline{p} > \frac{1}{2}$. Candidate A is the better choice in state α, and candidate B is the better choice in state β. In state α, the payoff of any citizen is 1 if candidate A is elected and -1 if B is elected. In state β, things reverse.

The size of the electorate is a random variable that follows the Poisson distribution with mean n. The probability that there are m voters is $e^{-n}\frac{n^m}{m!}$. After the electorate is drawn, their party identities are determined randomly. There are three types of voters: one type of partisan voters , labeled A or partisan of A, treats party candidate A as their status quo choice; another type of partisan voters, labeled B or partisan of B, treats party candidate B as their status quo, swing voters, labeled S, have no party candidate as their status quo choice. A voter's type is A and B with probability λ_A and λ_B, respectively, independent of the state. Otherwise, he is a swing voter, with probability of λ_S. No abstention implies $\lambda_A + \lambda_B + \lambda_S = 1$. For each type i voter, $\lambda_i > 0$. Therefore, the expected numbers of partisan A voters, partisan B voters and swing voters are:

$$n_A = \lambda_A n, \ n_B = \lambda_B n, \ n_S = \lambda_S n.$$

Before casting a vote, every voter receives a private signal regarding the true state of world. Signals are independent. The signal takes one of two values, a or b. The probability of receiving each signal is

$$\frac{1}{2} < P[a \mid \alpha] = P[b \mid \beta] = q < 1.$$

The signal is always informative but noisy, hence q is greater than $\frac{1}{2}$ and less than 1. Signal a is associated with state α, while signal b is associated with state β. The posterior probabilities of the states after receiving the signals are

$$q(\alpha \mid a) = \frac{pq}{pq + (1 - p)(1 - q)}, \ q(\beta \mid b) = \frac{(1 - p)q}{(1 - p)q + p(1 - q)}.$$

3.1   Pivotal Voting

An elementary event is a singleton consisting of a pair of vote totals (k, l), where k is the number of votes for party candidate A and l the votes for party candidate B. An event is an union of elementary events. An event is pivotal if a single vote can affect the final

outcome of the election. There are two types of elementary events where one vote can have an effect on the final outcome: 1) there is a tie, and 2) party candidate A has one vote less or more than party candidate B. Let $T = \{(k, k) : k \geq 0\}$ denote the event that there is a tie, and let $T_{-1} = \{(k-1, k) : k \geq 1\}$ denote the event that A has one vote less than B, and let $T_{+1} = \{(k, k-1) : k \geq 1\}$ denote the event that A has one vote more than B. The event piv_A (pivotal if vote for A) is defined by $T \cup T_{-1}$. event piv_B is defined similarly.

Let σ_A and σ_B be the expected number of votes for A and B in state α, respectively. Abstention is not allowed, so $\sigma_A + \sigma_B = n$. τ_A and τ_B are defined similarly for the corresponding expected votes in state β.

Let $\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\}$ be the voting profile. $(\gamma_a, \gamma_b)_i$ is the probability of voting for party candidate A of type i with a signal a and b.

Consider an event where the size of the realized electorate is m and there are k votes in favor of party candidate A and l votes in favor of party candidate B. The probability of such event in state α is

$$\Pr[(k, l; m) \mid \alpha] = (e^{-\sigma_A} \frac{\sigma_A^k}{k!})(e^{-\sigma_B} \frac{\sigma_B^l}{l!}).$$

Since $k + l = m$, $\sigma_A + \sigma_B = n$, the expression becomes

$$
\begin{aligned}
\Pr[(k, l; m) \mid \alpha] &= \Pr[(k, l) \mid \alpha] \\
&= e^{-n} \frac{\sigma_A^k}{k!} \frac{\sigma_B^l}{l!} \\
&= \Pr[(k, l) \mid \alpha].
\end{aligned}
$$

The probability of the event (k, l) in state β is defined similarly.

The probability of a tie in state α is

$$
\begin{aligned}
\Pr[T \mid \alpha] &= \sum_{k=0}^{\infty} e^{-n} \frac{\sigma_A^k}{k!} \frac{\sigma_B^k}{k!} \\
&= e^{-n} \sum_{k=0}^{\infty} \frac{\sigma_A^k}{k!} \frac{\sigma_B^k}{k!},
\end{aligned}
$$

while the probability that A has one vote less than B in state α is

$$\Pr[T_{-1} \mid \alpha] = e^{-n} \sum_{k=1}^{\infty} \frac{\sigma_A^{k-1}}{(k-1)!} \frac{\sigma_B^k}{k!},$$

and the probability that B has one vote less than A in state α is

$$\Pr[T_{+1} \mid \alpha] = e^{-n} \sum_{k=1}^{\infty} \frac{\sigma_A^k}{k!} \frac{\sigma_B^{k-1}}{(k-1)!}.$$

The corresponding probabilities in state β are obtained by substituting σ for τ. In state α,

$$
\begin{aligned}
\Pr[piv_A \mid \alpha] &= \Pr[T \mid \alpha] + \Pr[T_{-1} \mid \alpha], \\
\Pr[piv_B \mid \alpha] &= \Pr[T \mid \alpha] + \Pr[T_{+1} \mid \alpha].
\end{aligned}
$$

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The corresponding probability of pivotal events are similarly defined for state β. The probability of pivotal voting could be approximated using modified Bessel functions[3]:

$$\Pr[T \mid \alpha] \approx e^{-n} I_0(2\sqrt{\sigma_A \sigma_B}) = e^{-n} \frac{e^{2\sqrt{\sigma_A \sigma_B}}}{\sqrt{2\pi \cdot 2\sqrt{\sigma_A \sigma_B}}}$$

$$\Pr[T_{\pm 1} \mid \alpha] \approx \left(\frac{\sigma_A}{\sigma_B}\right)^{\pm \frac{1}{2}} \Pr[T \mid \alpha]$$

The approximation is useful when we study the large population property. Since

$$\Pr[piv_A \mid \alpha] \approx \Pr[T \mid \alpha] \left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}\right],$$

so $\Pr[piv_A \mid \alpha]$ is the product of $\Pr[T \mid \alpha]$ and a function independent of population size.

3.2 Voting Under Knightian Uncertainty

If the prior is unique, there is no difference among partisans of A, partisans of B, and swing voters S. Voters vote rationally. Therefore, this non-expected utility model incorporates the standard strategic voting model with utiliy-maximizing voters as a special case. Once the electorate has multiple priors, party identity has different impact for different voter.

Facing multiple priors, voters are Knightian. If one party is strictly preferred to another, all voters vote the dominant party. If two parties are incomparable, partisans stick to their own parties, and voting behavior of swing voters is not determined. The strict preference defined in this paper slightly differs from Bewley's definition:

$$\begin{aligned} A \succ B \iff & \forall p \in [\underline{p}, \overline{p}], \ E_p[u(A)] \geq E_p[u(B)], \\ & \exists p \in [\underline{p}, \overline{p}], \ E_p[u(A)] > E_p[u(B)], \end{aligned}$$

where $[\underline{p}, \overline{p}]$ denotes the set of priors. If A is strictly preferred to B, we say A strictly dominates B. As discussed in section 3.3, $E_p[u(A)] - E_p[u(B)]$ is strictly increasing in p. Therefore, when $\underline{p} \neq \overline{p}$, it is equivalent to define the strict preference as

$$A \succ B \iff \forall p \in [\underline{p}, \overline{p}], \ E_p[u(A)] \geq E_p[u(B)] .$$

Definition 1 *Party candidate i dominates party candidate j if*

$$E_p[u(i)] \geq E_p[u(j)], \ \forall p \in [\underline{p}, \overline{p}].$$

This definition incorporates both multiple priors and single prior. Strict dominance requires at least one of the equalities hold strict.

We define a voting equilibrium under uncertainty formally. First, we define a maximal voting choice and an optimal voting choice in an environment with uncertainty. Second, a voting equilibrium under uncertainty is defined in terms of maximal and optimal choices.

Definition 2 *Party candidate i is an optimal choice if i dominates all j. Party candidate i is a maximal choice if i is not strictly dominated by any j.*

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[3]For detail, see Krishna and Morgan (2012).

In other words, consider a prior belief $[\underline{p}, \overline{p}]$ which is not a singleton, two party candidates A and B. A is optimal if $E_p[u(A)]$ is larger than $E_p[u(B)]$ for all $p \in [\underline{p}, \overline{p}]$. A is maximal $E_p[u(B)]$ is not larger than $E_p[u(A)]$ for all $p \in [\underline{p}, \overline{p}]$. An alternative is optimal when it is preferred to the other alternative for all p, and it is maximal when it is strictly preferred to the other alternative for some p. Obviously, an optimal choice is maximal, but not the other way around. Again, in our setting, if a party dominates its opponent for all p, it strictly dominates it for some p. If the prior is a singleton, these two concepts are equivalent.

Definition 3 *A voting profile $\{(\gamma_a, \gamma_b)_i : i \in \{A, B, S\}\}$ is a voting equilibrium under uncertainty if*
    i) partisans vote for their own party exclusively if it is a maximal choice, and
    ii) if there is an optimal choice, swing voters vote for it exclusively.

In equilibrium, partisans switch to the opponent of their own party only when the opponent is preferred in every state. Otherwise, they always vote for their own party candidate. They do not mix. When there is a optimal choice, the model has a clear prediction on swing voter's voting behavior. However, when there is no optimal choice, swing voters can swing in any manner they like.

The following table summarizes equilibrium voting behaviors of all the partisans and swing voters.

Table 1: Voting behaviors of partisans of A, partisans of B and swing voters

	Partisan of A	*Swing voter*	*Partisan of B*
$\forall p \in [\underline{p}, \overline{p}],\ E_p[u(A)] \geq E_p[u(B)]$	A	A	A
$\exists p \in [\underline{p}, \overline{p}],\ E_p[u(A)] > E_p[u(B)]$ $\exists p \in [\underline{p}, \overline{p}],\ E_p[u(A)] < E_p[u(B)]$	A	*not determined*	B
$\forall p \in [\underline{p}, \overline{p}],\ E_p[u(A)] \leq E_p[u(B)]$	B	B	B

3.3 Equilibria Characterization

In equilibrium with uncertainty, a rational voter, no matter he is a partisan or a swing voter, compares the expected utility of voting for two candidates for every p:

Given a signal s, the expected utility of voting for party candidate A for a particular p is

$$E_p[u(A)] = q_p(\alpha \mid s) \Pr[piv_A \mid \alpha] - q_p(\beta \mid s) \Pr[piv_A \mid \beta].$$

$q_p(\alpha \mid a) \Pr[piv_A \mid \alpha]$ is the expected utility of making a right decision, choosing A in the state of α, while $q_p(\beta \mid a) \Pr[piv_A \mid \beta]$ is the expected utility of making a mistake, choosing A in the state of β. Similarly, given a signal s, the expected utility of voting for party candidate B for a particular p is

$$E_p[u(B)] = q_p(\beta \mid s) \Pr[piv_B \mid \beta] - q_p(\alpha \mid s) \Pr[piv_B \mid \alpha].$$

In equilibrium with uncertainty, party candidate A is a maximal voting choice given a signal s if

$$\exists p \in [\underline{p}, \overline{p}], \ s.t. \qquad E_p[u(A)] > E_p[u(B)],$$

and party candidate A is an optimal voting choice given a signal s if

$$\exists p \in [\underline{p}, \overline{p}], \ s.t. \qquad E_p\left[u\left(A\right)\right] \geq E_p\left[u\left(B\right)\right].$$

Denote $\frac{\Pr[piv_A \mid \alpha] + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta] + \Pr[piv_B \mid \beta]}$ as Ω, $\frac{q_p(\beta \mid a)}{q_p(\alpha \mid a)}$ as Q_p^a, and $\frac{q_p(\beta \mid b)}{q_p(\alpha \mid b)}$ as Q_p^b.

$$Q_p^a = \frac{(1-p)(1-q)}{pq}, Q_p^b = \frac{(1-p)q}{p(1-q)}.$$

Ω is the ratio of pivotal probability in two states, and Q_p^s is the ratio of posterior probability of two states given a signal s. By comparing these two ratios, Ω and Q_p^s, a voter can decide to vote based on the information derived from the scenario of being pivotal for the whole election or the information from his private signal. If it is more likely to be pivotal in one state, it is a safe choice to vote for the corresponding party candidate, since not much damage can be made even the decision is incorrect.

Obviously, Q_p^a and Q_p^b are decreasing in p. If party candidate A is maximal given a signal s, it must be the case that $\Omega > Q_{\overline{p}}^s$. Similarly, if party candidate A is optimal given a signal s, it must be the case that $\Omega \geq Q_{\underline{p}}^s$. This simplification is helpful for equilibrium characterization in the following sections. Instead of the whole set of priors. We only need to check whether these conditions hold for the boundary beliefs, $\underline{p}$ and $\overline{p}$.

According to Table 1, there are four possible types of voting equilibrium, illustrated in Table 2, and we will discuss each type of equilibrium in a separate subsection. The rows represent the signals received, and the columns represent voters' party identities. For instance, the first row and first column can be read as "in a sincere voting equilibrium, given a signal a, partisans of A vote for party candidate A."

Table 2: Four types of voting equilibrium and equilibrium voting profile

2.1: Sincere voting			
	A	S	B
a	A	A	A
b	B	B	B

2.2: Uninformative voting								
	A	S	B			A	S	B
a	A	A	A	or	a	A	A	A
b	A	A	A		b	A	A	A

2.3: Full partisan voting			
	A	S	B
a	A	γ_a	B
b	A	γ_b	B

2.4: Partial partisan voting								
	A	S	B			A	S	B
a	A	A	A	or	a	A	γ_a	B
b	A	γ_b	B		b	B	B	B

In a sincere voting equilibrium, actual votes represents the exact signals. In an uninformative voting equilibrium, there is no information, even preference, revealed. Besides these two extreme cases, in partisan voting equilibrium, both preference and information find their way to express themselves. If the prior is not biased towards one party, we end up with a full partisan voting equilibrium, where partisans vote along their loyalty, while swing voters contribute to information aggregation. If the prior is biased, the partisans of the advantaged party stick to their status quo, while swing voters and partisans of the less advantaged party respond to their signals.

3.3.1 Sincere Voting Equilibrium

In a sincere voting equilibrium, all voters vote according to their private signals. When signal a is received, all voters vote for party candidate A. When signal b is received, all voters vote for party candidate B. Sincere equilibrium is possible if party candidate A is

an optimal choice with signal a, while party candidate B is an optimal choice with signal b, as illustrated in Table 2.1.

In the equilibrium,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1,0), (1,0), (1,0)\}.$$

Since the voting behavior is deterministic with respect to the corresponding signal, the expected number of votes in each state only depends on the signal precision and voter population:

$$\sigma_A = qn = \tau_B, \sigma_B = (1-q)n = \tau_A$$

Given the expected number of votes in state α and β, several lines of calculation show the ratio of pivotal probability in two states always equals to one.

Lemma 1 *In any sincere voting equilibrium, $\Omega = 1$.*

Proof. All proofs are in the Appendix. ∎

Party candidate A is an optimal choice with signal a, therefore, it is strictly preferred to party candidate B for $\underline{p}$:

$$\Omega = 1 \geq Q_{\underline{p}}^a = \frac{(1-\underline{p})(1-q)}{\overline{p}q} \Leftrightarrow q \geq 1 - \underline{p}.$$

On the other hand, party candidate B is an optimal choice with signal b, therefore, it is strictly preferred to party candidate A for $\overline{p}$:

$$\Omega = 1 \leq Q_{\overline{p}}^b = \frac{(1-\overline{p})q}{\overline{p}(1-q)} \Leftrightarrow q \geq \overline{p}.$$

The following proposition summarizes the necessary and sufficient condition for the existence of a sincere voting equilibrium under uncertainty.

Proposition 1 *A sincere voting equilibrium exists if and only if*

$$q \geq \max(1 - \underline{p}, \overline{p}).$$

To support the existence of a sincere voting equilibrium, signal has to be precise enough overcome any uncertainty existed in the prior belief. If q is lower than $\overline{p}$, a signal b is not able to persuade partisans of A to vote against their own party. If q is higher than $1 - \underline{p}$, there is no signal can induce a vote for party candidate A from partisans of B. Given a signal a, voters are pretty sure the true state is α. *Vice versa.* When the prior is a singleton, $\underline{p} = \overline{p}$, the condition is simply $q \geq p$. It corresponds to the condition required to support a sincere voting in an environment without uncertainty.

3.3.2 Uninformative Voting Equilibrium

Uninformative equilibria, in which all voters vote for one party regardless of their own signal, are also possible. In voting games with a fixed electorate size, an uninformative equilibrium arises because the probability of being pivotal is zero. However, in voting games with unknown electorate size, the probability of being pivotal is always positive.

In the equilibrium where all voters always vote for party candidate A,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1,1), (1,1), (1,1)\},$$

and in the equilibrium where all voters always vote for party candidate B,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(0,0), (0,0), (0,0)\}.$$

With an unknown electorate size, there is always positive probability that there are less than three voters. When there are three or more voters, a single voter can never be pivotal in the equilibrium where all voters vote for one party. When there are only two voters, a voter can cast a vote to cancel the vote cast by the other voter. When there is only one voter, the election result is determined by this single voter alone[4].

Again, we have the ratio of pivotal probability in two states, Ω, be exactly one.

Lemma 2 *In any uninformative equilibrium, $\Omega = 1$.*

Since party candidate A is the optimal choice given both signals, party candidate A is preferred to party candidate B for $\underline{p}$ given both signal a and b:

$$\Omega \geq Q^a_{\underline{p}} \text{ and } \Omega \geq Q^b_{\underline{p}} \Leftrightarrow \underline{p} \geq \max(q, 1-q) = q$$

Similarly, the necessary and sufficient condition for the existence of an uninformative voting equilibrium, in which every voter votes for party candidate B, is $\bar{p} \leq 1 - q$. Since $q > \frac{1}{2}$, it contradicts the assumption that $\bar{p} > \frac{1}{2}$. Therefore, there does not exist an uninformative equilibrium with every voter voting for party candidate B.

Proposition 2 *An uninformative voting equilibrium with every voter voting for party candidate A exists if and only if*

$$q \leq \underline{p} \text{ and } \underline{p} > \frac{1}{2}.$$

To support an uninformative voting equilibrium, on the one hand, the quality of signal is pretty low relative to the prior probability. On the other hand, it also suggests $\underline{p} > \frac{1}{2}$, which means the prior belief is highly biased towards the state α.

3.3.3 Full Partisan Voting Equilibrium

In a full partisan voting equilibrium, partisans of A always vote for party candidate A, and partisans of B always vote for party candidate B. When partisans do not agree on their choices, no party candidate is an optimal choice. Swing voters are free to use any strategy as both party candidates are maximal choices. Swing voters might or might not vote sincerely. To support a full partisan voting equilibrium, it is necessary that upon receiving a signal, either a or b, neither party candidate is strictly preferred to the other.

In a full partisan voting equilibrium, the voting profile for partisans are fixed,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1,1), (\gamma_a, \gamma_b), (0,0)\}.$$

For any pair of (γ_a, γ_b), to support a full partisan voting equilibrium, we need to have party candidate A is strictly preferred to party candidate B for some p, and party candidate

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[4]If there is no voter, which is still possible, there is no voting problem.

B is strictly preferred to party candidate A in some other p. Therefore, given both signals, party candidate A is strictly preferred to party candidate B for $\bar{p}$, and party candidate B is strictly preferred to party candidate A for $\underline{p}$:

$$\Omega > Q_{\bar{p}}^s, \ \Omega < Q_{\underline{p}}^s \ \forall s \in \{a, b\}.$$

The above condition directly leads to the following proposition.

Lemma 3 *A full partisan voting equilibrium exists only if*

$$\frac{(1 - \bar{p})q}{\bar{p}(1 - q)} < \Omega < \frac{(1 - \underline{p})(1 - q)}{\underline{p}q}.$$

There is not much intuition provided at this stage. We don't even know when Ω falls into this interval. The condition merely says that the ratio of pivotal probability in two states are bounded above as well below by some positive constants, which are uniquely defined by $\left(\underline{p}, \bar{p}, q\right)$.

To ensure $\frac{(1-\bar{p})q}{\bar{p}(1-q)} < \frac{(1-\underline{p})(1-q)}{\underline{p}q}$, we can rewrite the equilibrium condition as

$$\underline{p} < \left(\frac{(1 - \bar{p})q^2}{\bar{p}(1 - q)^2} + 1\right)^{-1}.$$

Since we assume $\bar{p} > \frac{1}{2}$, given $\bar{p}$, the right-hand side of the inequality is strictly decreasing in q. As information precision grows, larger uncertainty is required to sustain full partisan voting in equilibrium.

3.3.4   Partial Partisan Voting Equilibrium

The last type of equilibrium is partial partisan voting equilibrium, where only one side of the partisans vote regardless of their signals, see Table 2.4. To support such equilibrium, it is necessary to have that one party candidate is optimal when its corresponding signal is received, and is maximal when the other signal is received. As a result, the swing voters vote according to the signal upon receiving one of the two signals, and are free to use any mixed strategy upon receiving the other one.

In a partial partisan voting equilibrium, in which partisans of A are not responsive to their signals, party candidate A is optimal with a signal a, while both party candidate A and B are maximal with a signal b,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1, 1), (1, \gamma_b), (1, 0)\}.$$

In equilibrium, party candidate A strictly dominates B given a signal a, and both party candidates are maximal choices given a signal b. Therefore, party candidate A is strictly preferred to party candidate B for $\underline{p}$ and $\bar{p}$ given signal a and b respectively, and party candidate B is strictly preferred to party candidate A for $\underline{p}$ given signal b.

$$\Omega \geq Q_{\underline{p}}^a \ and \ \Omega > Q_{\bar{p}}^b, \Omega < Q_{\underline{p}}^b.$$

Lemma 4 *A partial partisan voting equilibrium favoring party candidate A exists only if*

$$\frac{(1-\bar{p})q}{\bar{p}(1-q)} < \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} \quad if \; \frac{(1-\bar{p})q}{\bar{p}(1-q)} \geq \frac{(1-\underline{p})(1-q)}{\underline{p}q},$$
$$\frac{(1-\underline{p})(1-q)}{\underline{p}q} \leq \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} \quad if \; \frac{(1-\bar{p})q}{\bar{p}(1-q)} < \frac{(1-\underline{p})(1-q)}{\underline{p}q}.$$

And a partial partisan voting equilibrium favoring party candidate B exists only if

$$\frac{(1-\bar{p})(1-q)}{\bar{p}q} < \Omega < \frac{(1-\underline{p})(1-q)}{\underline{p}q} \quad if \; \frac{(1-\underline{p})(1-q)}{\underline{p}q} \leq \frac{(1-\bar{p})q}{\bar{p}(1-q)},$$
$$\frac{(1-\bar{p})(1-q)}{\bar{p}q} < \Omega \leq \frac{(1-\bar{p})q}{\bar{p}(1-q)} \quad if \; \frac{(1-\underline{p})(1-q)}{\underline{p}q} > \frac{(1-\bar{p})q}{\bar{p}(1-q)}.$$

The equilibrium conditions look similar to those for a full partisan equilibrium. To see it clear, consider a fixed Ω, the relative positions among $Q_{\bar{p}}^a, Q_{\underline{p}}^a$ and $Q_{\bar{p}}^b$, and among $Q_{\bar{p}}^a, Q_{\bar{p}}^b$ and $Q_{\underline{p}}^b$ are fixed by the assumption on $(\underline{p}, \bar{p}, q)$. Given any triple of $(\underline{p}, \bar{p}, q)$, there are only two possible cases: $Q_{\bar{p}}^a < Q_{\underline{p}}^a \leq Q_{\bar{p}}^b < Q_{\underline{p}}^b$ or $Q_{\bar{p}}^a < Q_{\bar{p}}^b \leq Q_{\underline{p}}^a < Q_{\underline{p}}^b$.

Figure 1: Supports of partisan voting equilibrium
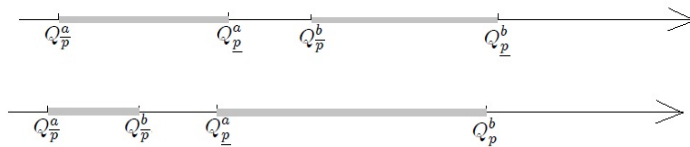
In Figure 1, the grey segment on the left represents the set of Ω that support a partial partisan equilibrium favoring party candidate B, while the grey segment on the right represents the set of Ω that support a partial partisan equilibrium favoring party candidate A. The set of Ω between $Q_{\underline{p}}^a$ and $Q_{\bar{p}}^b$ is the set supporting a full partisan equilibrium.

4   Large Population Properties

The previous section characterizes all the possible equilibria that arise in the election under uncertainty. The voting profiles in the sincere voting equilibrium and the uninformative voting equilibrium deliver clear analytical results on equilibrium conditions. We already know both sufficient and necessary conditions for the existence of a sincere voting equilibrium and a uninformative equilibrium. The specifications of sincere voting equilibrium and uninformative equilibrium also pin down the equilibrium voting profile. However, we do not have much idea of what conditions can support a partisan voting equilibrium, and how voters, especially swing voters, vote.

The difficulty to derive analytical results for partisan voting equilibrium is that the ratio of pivotal probability in two states, Ω, is determined by the corresponding voting profile. For any finite electorate size, Ω is endogenous in a partisan voting equilibrium. When electorate size goes to infinite, the sequence of corresponding equilibrium Ω converges to a unique constant invariant to equilibrium voting profiles.

Krishna and Morgan (2011 & 2012) develop the Bessel function approximation method to pin down the limit property of the equilibrium ratio of pivotal probability in two states as the electorate size goes to infinity. They study the large population property of sincere voting equilibrium with endogenous participation without uncertainty. Martinelli (2011)

does the same exercise in a nearly common interest setting. Here, we also use this method to find equilibrium conditions for partisan voting in a large election. On the one hand, we are also able to characterize the set of equilibrium voting profile for full partisan voting equilibrium, and it is always a set. On the other hand, there is no voting profile that supports a partial partisan voting profile. That is to say, there does not exist a partial partisan voting equilibrium in a large election.

4.1 Equilibrium Vote Share

We use the Bessel function approximation to get some analytical results on the equilibrium voting profile of a large election. The limit property of the ratio of pivotal probability in two states, Ω, helps to pin down the voting profile when the election is large, as $n \to \infty$. At the limit, the equilibrium voting profile has to equalize the probability of being pivotal in the two states, in order to provide sufficient incentive for each voter to vote informatively.

Lemma 5 *When $n \to \infty$, in any equilibrium,*

$$\Omega \to 1.$$

We have $\Omega = 1$ for every type of equilibrium. This also gives us linear equilibrium conditions for all partisan equilibria.

Lemma 6 *When $n \to \infty$, a full partisan voting equilibrium exists only if*

$$q < \min\left(\bar{p}, 1 - \underline{p}\right) \ \text{and} \ \underline{p} < \frac{1}{2},$$

and a partial partisan voting equilibrium exists only if

$$
\begin{aligned}
\underline{p} &< q < \bar{p} \ \text{if} \ \underline{p} \geq \frac{1}{2}, \\
1 - \underline{p} &< q < \bar{p} \ \text{if} \ \underline{p} < \frac{1}{2}.
\end{aligned}
$$

We illustrate these sufficient conditions graphically in Figures 2.1 to 2.3. $\underline{p}$ is on the $x - axis$, and $\bar{p}$ is on the $y - axis$. The area above the 45-degree line represents the set of $\left(\underline{p}, \bar{p}\right)$ such that $\bar{p} > \underline{p}$. The shaded area represents the the set of $\left(\underline{p}, \bar{p}\right)$ supporting the same types of equilibrium given some q.

Figure 2: $(\underline{p}, \overline{p}, q)$ and the equilibrium existence
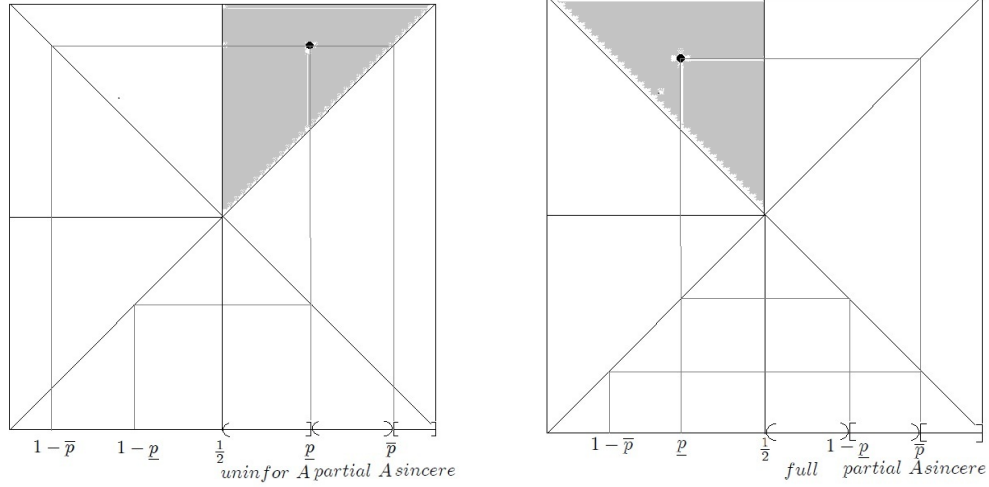
2.1
$\frac{1}{2} \leq \underline{p} < \overline{p}$

2.2
$\underline{p} < \frac{1}{2} < 1 - \underline{p} \leq \overline{p}$

2.3
$\underline{p} \leq \frac{1}{2} < \overline{p} \leq 1 - \underline{p}$

1. Figure 2.1 shows the potential equilibria occurs when $\frac{1}{2} \leq \underline{p} < \overline{p}$. Voters hold a biased prior belief, which strongly favors party candidate A. When si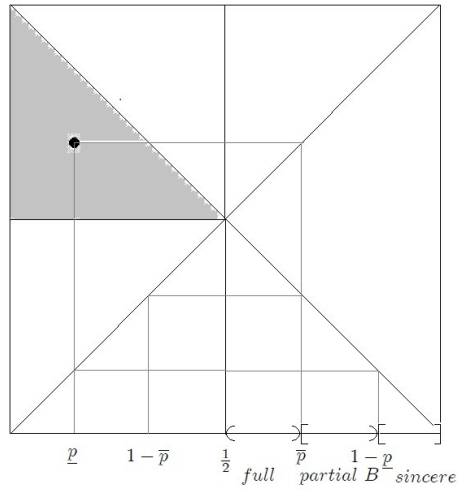gnal is highly precise, $q \geq \overline{p}$, there exists a sincere voting equilibrium. When signal is highly imprecise, $q \leq \underline{p}$, there exists an uninformative voting equilibrium where every voter vote for party candidate A. Otherwise, there may exist a partial partisan voting equilibrium favoring party candidate A.

2. Figure 2.2 shows the potential equilibria occurs when $\underline{p} < \frac{1}{2} < 1 - \underline{p} < \overline{p}$. Voters hold a balanced prior belief, in the sense that $\underline{p}$ is not too low, as $1 - \underline{p} \leq \overline{p}$. Similarly, when signal quality is high enough, $q \geq \overline{p}$, there exists a sincere voting equilibrium. When information is quite noisy, $q < 1 - \underline{p}$, there may exist a full partisan voting

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equilibrium. Otherwise, there may exist a partial partisan voting equilibrium favoring party candidate A.

3. Figure 2.3 shows the potential equilibria occurs when $\underline{p} \leq \frac{1}{2} < \overline{p} \leq 1 - \underline{p}$. Such belief is less balanced than that in Figure 2.2, as $\overline{p} \leq 1 - \underline{p}$. It differs from the situation in Figure 2.2 only when signal precision is moderate, as $\overline{p} \leq q < 1 - \underline{p}$, the partial partisan voting equilibrium arising favors party candidate B, rather than party candidate A.

Therefore, in a large election, given a triple $(\underline{p}, \overline{p}, q)$, one and at most one equilibrium may exist in equilibrium. To ensure the existence of a equilibrium, it is also necessary to check whether the equilibrium voting profile satisfying $\nu_A (1 - \nu_A) = \omega_A (1 - \omega_A)$ exists or not.

4.2   Equilibrium Strategy

As *Propositions* 1 and 2 suggested, when q is high enough or low enough with biased prior, there always exists a sincere voting equilibrium or an uninformative voting equilibrium. Moreover, the voting profile is given by the equilibrium configuration. We study the partisan voting profile needed to support $\Omega = 1$ in this section. It turns out that there is a set of voting profiles that support a full partisan voting equilibrium, while no voting profile could support a partial partisan voting equilibrium in a large election.

4.2.1   Full partisan voting equilibrium

In a full partisan voting equilibrium,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1,1), (\gamma_a, \gamma_b), (0,0)\}.$$

In state α,

$$\sigma_A = n_A + n_S [q\gamma_a + (1-q)\gamma_b], \ \ \sigma_B = n_B + n_S [q(1-\gamma_a) + (1-q)(1-\gamma_b)],$$

while in state β,

$$\tau_A = n_A + n_S [(1-q)\gamma_a + q\gamma_b], \ \ \tau_B = n_B + n_S [(1-q)(1-\gamma_a) + q(1-\gamma_b)].$$

After spelling out the vote share in state α and β, we can find there is a large set of voting profiles can make $\Omega = 1$. Some of the voting profiles are uninformative and have no restriction on parameters. Thus such voting profiles always exist. Others are informative and require a relatively balanced partisan voter population. Then we can conclude there always exists a full partisan voting equilibrium when signal quality is low and the prior belief does not favor one party too much.

Proposition 3 *When $n \to \infty$, a full partisan voting equilibrium exists if and only if*

$$q < \min\left(\overline{p}, 1 - \underline{p}\right) \ and \ \underline{p} < \frac{1}{2}.$$

Moreover, in the full partisan voting equilibrium,

$$\gamma_a = \gamma_b \ or \ \gamma_a + \gamma_b = 1 + \frac{n_B - n_A}{n_S}.$$

In a full partisan equilibrium, two possible scenarios could occur. In one scenario, $\gamma_a = \gamma_b$, swing voters may mix or not, but they are not responding to their private signals as if they do not have any valuable information. There is no information revealed in the equilibrium. It is an uninformative full partisan voting equilibrium, and it differs from an uninformative voting equilibrium by having partisans stick to their own parties. In such equilibrium, partisans express their preferences, while swing voters express their indecisiveness. It happens when there is no useful information revealed by other's votes, and private signal is not precise enough to help a voter reach a decision out of a balanced prior belief.

In the other scenario, $\gamma_a + \gamma_b = \frac{n_B - n_A}{n_S} + 1$, swing voters vote informatively, except $\gamma_a = \gamma_b = \frac{1}{2}\left(1 + \frac{n_B - n_A}{n_S}\right)$. The necessary condition for the existence of a informative full partisan voting equilibrium is $\frac{n_B - n_A}{n_S} \in [0, 1]$. Swing voters mix their votes to the extent that enables others to vote informatively after counter-balancing impact of partisan voters. $\frac{n_B - n_A}{n_S}$ and 1 on the right-hand side represent the balancing consideration and pivotal consideration, respectively. When $n_B = n_A$, this balancing component disappears. It also suggests two sides of partisans can not be too unbalanced relative to the population size of swing voters, otherwise swing voters are not able to counter-balance impact of the party with over-populated party supporters. Then a swing voter adjusts her own voting profile in order to make others vote informatively by provide them positive probability being pivotal.

In contrast, we have no clear knowledge about how a Knightian voter should vote in equilibrium. The equilibrium condition only imposes conditions on the relation between γ_a and γ_b. This might be the weakness of Knightian decision rule, in the sense that we are not able to pin down a unique equilibrium strategy. If we consider the symmetric equilibrium in a weaker sense, it is also the strength of Knightian decision making. The equilibrium is symmetric with respect to a mixing rule, but it does not require everyone do exactly the same thing. If everyone follows the same mixing rule, symmetric equilibrium sustains without strictly symmetric equilibrium behavior.

4.2.2 Partial partisan voting equilibrium

In a partial partisan voting equilibrium favoring party candidate A,

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1, 1), (1, \gamma_b), (1, 0)\}.$$

In state α,

$$\sigma_A = n_A + n_S\left[q + (1 - q)\gamma_b\right] + n_B q, \ \ \sigma_B = n_B(1 - q) + n_S(1 - q)(1 - \gamma_b),$$

while in state β,

$$\tau_A = n_A + n_S\left[(1 - q) + q\gamma_b\right] + n_B(1 - q), \ \ \tau_B = n_B q + n_S q(1 - \gamma_b).$$

Expressions for vote shares in a partial partisan voting equilibrium favoring party candidate B are similar. The following proposition shows that no feasible voting profile can support a partial partisan voting equilibrium.

Proposition 4 *When $n \to \infty$, there does not exist a partial partisan voting equilibrium.*

When n_A and n_B are strictly positive, there is no partial partisan voting equilibrium. Without abstention, partial partisan voting equilibrium is not possible, because having one side of the partisans stick to their own party has already nullified any benefit of strategic voting for other voters.

5   Costly Information

As discussed in last section, uncertainty sometimes hampers information revealing. We showed that whenever there exists an informative full partisan voting equilibrium, there exist an uninformative full partisan voting equilibrium. When information is costless and voting is compulsory, it is still possible not to have informative voting. A natural question is: is there any information aggregation when information is costly?

In this section, we consider an extension of the original model, where voters need to acquire costly information before the election. If no information is acquired, then no information can be aggregated by voting.

The cost of information is a function of information precision, $c(q)$:

$$c\left(\frac{1}{2}\right) = 0, c(1) = \infty, c'(q) > 0, c''(q) > 0.$$

$c\left(\frac{1}{2}\right) = 0$ suggests when the signal is uninformative, it is free. When it is extremely precise, it costs a lot. The cost of information is strictly increasing in information quality, and the marginal cost is also strictly increasing.

The expected payoff of acquiring information q, with an expected voting profile (γ_a, γ_b) for $p \in \left[\underline{p}, \overline{p}\right]$ is

$$
\begin{aligned}
V_p(q) &= p(a,q)\left\{ \begin{array}{c} \gamma_a\left(q(\alpha \mid a)\Pr[piv_A \mid \alpha] - q(\beta \mid a)\Pr[piv_A \mid \beta]\right) \\ +(1-\gamma_a)\left(q(\beta \mid a)\Pr[piv_B \mid \beta] - q(\alpha \mid a)\Pr[piv_B \mid \alpha]\right) \end{array} \right\} \\
&\quad + p(b,q)\left\{ \begin{array}{c} \gamma_b\left(q(\alpha \mid b)\Pr[piv_A \mid \alpha] - q(\beta \mid b)\Pr[piv_A \mid \beta]\right) \\ +(1-\gamma_b)\left(q(\beta \mid b)\Pr[piv_B \mid \beta] - q(\alpha \mid b)\Pr[piv_B \mid \alpha]\right) \end{array} \right\} - c(q),
\end{aligned}
$$

where $p(a,q)$ and $p(b,q)$ are the probabilities of acquiring signal a and b, given signal precision q, respectively. We also denote $V_p(q) + c(q)$ as $v_p(q)$.

Then we define a maximal choice of information level before defining an equilibrium in this voting game with endogenous information acquisition.

Definition 4 *If q and q' are choices of information level $(q \neq q')$, q dominates q' if $V_p(q) \geq V_p(q') \; \forall p \in \left[\underline{p}, \overline{p}\right]$.*

If q dominates q', it is never a worse choice in any circumstance.

Definition 5 *q^* is a maximal choice of information level if no other q dominates it.*

Facing information acquisition decision, voters are also under uncertainty. In this environment, uncertainty never goes away, but it may shrink or exaggerate. We define the following equilibrium concept with costly information under uncertainty.

Definition 6 $\{(q_A, q_B, q_S), \{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_B, (\gamma_a, \gamma_b)_S\}\}$ *forms an equilibrium with costly information under uncertainty if*

i) $\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_B, (\gamma_a, \gamma_b)_S\}$ *is an equilibrium voting profile, given* (q_A, q_B, q_S) *and* $[\underline{p}, \overline{p}]$,

ii) q_i *is a maximal choice of information level given* $(\gamma_a, \gamma_b)_i$, $[\underline{p}, \overline{p}]$ *and* $c(q)$.

As last section, we only study the large population property in this game. Without the help of Bessel function approximation for a large population, we hardly know what happens in a full partisan voting equilibrium.

Lemma 7 *In an sequence of equilibrium with costly information,* $q_i = \frac{1}{2}$ *if* $\{(\gamma_a, \gamma_b)_i : \gamma_a = \gamma_b\}$.

If voters do not expect to vote informatively, they are not going to acquire any information before the election. Therefore, uninformative voting equilibrium and uninformative full partisan voting equilibrium are still an equilibrium with costly information under uncertainty with zero equilibrium information acquisition: $(q_A, q_B, q_S) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$.

Lemma 8 *For any sequence of equilibrium,* $\lim_{n \to \infty} v_p(q) = 0$.

It is also obvious that the benefit of getting information shrinks as electorate size grows, as the impact of a single vote diminishes quickly. Immediately, we have the following proposition.

Proposition 5 *In any sequence of equilibrium with costly information,*

$$\lim_{n \to \infty} q_i = \frac{1}{2} \ \forall i \in \{A, B, S\}.$$

In any large election, there will not be any high level of information acquisition. However, recall the equilibrium condition for a sincere voting equilibrium, $q \geq \max(1 - \underline{p}, \overline{p})$. Hence, the following proposition says that in any sequence of equilibrium, in the limit we have either uninformative voting or full partisan voting. Sincere voting for everyone is impossible. We conclude this line of argument in the following proposition.

Proposition 6 *If* $\frac{1}{2} \leq \underline{p} < \overline{p}$, *in any sequence of equilibrium with costly information, voting is uninformative,*

$$q_i = \frac{1}{2} \ \forall i \in \{A, B, S\} \ \text{and} \ \lim_{n \to \infty} (\gamma_a, \gamma_b)_i = (1, 1) \ \forall i \in \{A, B, S\}.$$

If $\underline{p} < \frac{1}{2} \leq \overline{p}$, *in any sequence of equilibrium with costly information, voting is full partisan,*

(uniformative full partisan voting)
$q_i = \frac{1}{2} \ \forall i \in \{A, B, S\}$ *and*
$\lim_{n \to \infty} \{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{\{(1, 1), (\gamma, \gamma), (0, 0)\} : \gamma \in [0, 1]\}$
OR
(informative full partisan voting)
$q_A = q_B = \frac{1}{2}$, $q_S > \frac{1}{2}$ *and*
$\lim_{n \to \infty} \{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\}$
$= \left\{\{(1, 1), (\gamma_a, \gamma_b), (0, 0)\} : \gamma_a + \gamma_b = 1 + \frac{n_B - n_A}{n_S}, \gamma_a > \gamma_b, \gamma_s \in [0, 1]\right\}.$

Proposition 7 is the main result of this section. Three important messages:

1. There is no equilibrium with everyone voting sincerely in a large election with costly information and large population. To support a sincere voting equilibrium, we need very high level of signal precision. As the electorate size goes to infinity, *ex ante* benefit from acquiring information and voting accordingly also diminishes. Nobody will spend much on information, so the signals acquired in equilibrium are always quite noisy, too noisy for a sincere voting equilibrium to exist.

2. Partisans do not acquire information, and if there is positive amount of information acquisition, it must be the swing voter who is acquiring it. It is too costly for partisans to get signal informative enough to vote against their own party, so they choose not to acquire any. Voting along their loyalty is costless and fully rational.

3. If swing voters acquire positive amount of information, they are more likely to vote for party candidate A with a signal a, and party candidate B with a signal b. They do not expect themselves using information unreasonably when they decide to pay for it.

6   Partisan Voting without Uncertainty

In this section, We provide a short discussion on how our non-expected utility model compares with a standard expected utility model.

As discussed in the introduction, party identity can be treated as difference in prior belief, or difference in preference intensity. In this section, we consider two types of expected utility partisan voting model.

6.1   Partisan as Difference in Prior Belief

Consider the following setting:

Partisans of A have prior belief of state α equal to $\bar{p}$. Partisans B have prior belief of state α equal to $\underline{p}$. Swing voters have prior belief of state α equal to p, and $p \in \left(\underline{p}, \bar{p} \right) \frac{\underline{p} + \bar{p}}{2}$. So, $\bar{p} > \frac{\underline{p} + \bar{p}}{2} > \underline{p}$. Also, $\bar{p} > \frac{1}{2}$. They gain the same utility when the elected candidate is the right candidate and when the elected candidate is not. In this setting, voters differ from each other in terms of their prior belief of the true state. Partisans A think that state α is more likely than partisans B do. Swing voters have a moderate belief. Signal is noisy and informative, $q \in \left(\frac{1}{2}, 1 \right)$.

This characterization does not affect the sincere voting and uninformative voting equilibrium at all[5]. However, it imposes a strict requirement on mixed strategy equilibrium. In an expected utility model, use of mixed strategy requires indifference among all strategies used with positive probability. Also, the large population property requires that we have $\Omega = 1$, which implies that $\sigma_A = \tau_A$ or $\sigma_A + \tau_A = n$. Moreover, in equilibrium, monotonicity in strategy poses extra restrictions. This line of argument is summarized in the following lemma.

Lemma 9 *When $n \to \infty$, all mixed-strategy equilibria are non-generic.*

---

[5]It is because that the voting profile in these two types of equilibrium ensures $\Omega = 1$.

When equilibrium conditions require parameters satisfy some equalities, it is not generic in the parameter space. Nongeneric equilibria are way less important than generic ones. All mixed-strategy equilibria are nongeneric. Therefore, we can focus on pure-strategy equilibria. We conclude the conditions for all generic pure-strategy equilibria when $n \to \infty$. Additionally, we also list all the nongeneric equilibria in the appendix at the end of proof for *Proposition* 8.

Proposition 7 *When $n \to \infty$,*

1. *a sincere voting equilibrium exists if and only if*

$$q \geq \max(1 - \underline{p}, \overline{p}),$$

2. *an uninformative voting equilibrium exists if and only if*

$$q \leq \underline{p} \text{ and } \underline{p} > \frac{1}{2},$$

3. *an uninformative full partisan voting equilibrium exists if and only if*

$$\max\left(1 - \overline{p}, p\right) \leq q \leq \min\left(1 - p, \overline{p}\right)$$
the corresponding voting profile is $\{(1,1), (0,0), (0,0)\}$,
$$\max\left(1 - p, \underline{p}\right) \leq q \leq \min\left(1 - \underline{p}, p\right)$$
the corresponding voting profile is $\{(1,1), (1,1), (0,0)\}$,

4. *no generic equilibrium otherwise.*

The proof is obvious, and the message is clear. Except the sincere voting equilibrium when signal is precise, there is no informative voting in almost all other circumstances.

6.2 Partisan as Difference in Preference Intensity

In Feddersen and Pesendorfer (1997 & 1999), Aragones and Palfrey (2002), Gul and Pesendorfer (2008), Krishna and Morgan (2011) and Martinelli(2011), party preference is modeled as preference intensity. Feddersen and Pesendorfer (1997,1999) assume a continuum of voter preference types. In Aragones and Palfrey (2002), one candidate enjoys an advantage over the other candidate by some fixed distance δ. Gul and Pesendorfer (2008) adopt a weaker assumption that voters have personality preference when both candidates offer the same policy. In a very similar setting as this paper, both Krishna and Morgan (2011) and Martinelli(2011)

Consider the following setup:

Partisans A gain an extra θ_A if candidate A is elected, regardless of the true state. Similarly, partisans B gain an extra θ_B if candidate B is elected. Swing voters do not gain extra utility from either candidate. This setting can be axiomatized as a model of separable status quo bias (Masatlioglu & Ok, 2005)[6]. By Theorem 2 in Masatlioglu and

---

[6]The crucial difference between Masatlioglu and Ok's model and a standard Knightian model is **Axiom SQI** (Status-quo independence). This axiom seperates the status quo bias from a choice problem with a status quo choice. Adding the status quo choice does not have any effect on the relative ordering of

Ok (2005), there exists a continuous mapping U, associating with ordering without status quo choice, and a function φ, as "status quo bonus". Therefore, we can write

$$U(a|\alpha) = U(b|\beta) = 1, U(b|\alpha) = U(a|\beta) = 0,$$

$$\varphi_A(a) = \varphi_B(b) = \theta \in [0,1)^7,$$

$$\varphi_A(b) = \varphi_B(a) = 0, \varphi_S(a) = \varphi_S(b) = 0.$$

A voter will make her decision according to the sum of U and φ. Voters have the same prior belief of state α, $p > \frac{1}{2}$. And signal is informative but noisy, $q \in \left(\frac{1}{2}, 1\right)$.

After some algebra, we have

$$
\begin{aligned}
\Omega_A &= \frac{\Pr[piv_A \mid \alpha](1+\theta) + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta](1-\theta) + \Pr[piv_B \mid \beta]}, \\
\Omega_S &= \frac{\Pr[piv_A \mid \alpha] + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta] + \Pr[piv_B \mid \beta]}, \\
\Omega_B &= \frac{\Pr[piv_A \mid \alpha] + \Pr[piv_B \mid \alpha](1-\theta)}{\Pr[piv_A \mid \beta] + \Pr[piv_B \mid \beta](1+\theta)}.
\end{aligned}
$$

For partisans of A, the additional utility gain from getting their favored candidate elected is attached to party candidate A's pivotal probability. It is clear the pivotal ratio Ω_A is always higher than that for a unbiased swing voter. Partisans set a higher bar to ask for compensation for voting against their favored party candidate. The large population property still applies. Hence,

$$\lim_{n\to\infty} \Omega_A > \lim_{n\to\infty} \Omega_S = 1 > \lim_{n\to\infty} \Omega_B.$$

We also focus on generic equilibria, because *Lemma* 8 still applies in this environment. Sincere voting and uninformative voting equilibria are generic. Similarly, informative full partisan voting equilibria are non-generic. We skip the full list of nongeneric equilibria, as it will be a repetition of the list in *Proposition* 8.

Proposition 8 *A sincere voting equilibrium exists if and only if*

$$q \geq \max\left(q^*, q^{**}\right),$$

where q^* is uniquely defined by

$$\frac{1-p}{p} = \left[\frac{2-\theta+\left(\frac{q}{1-q}\right)^{-\frac{1}{2}}+\left(\frac{q}{1-q}\right)^{\frac{1}{2}}(1-\theta)}{2+\theta+\left(\frac{q}{1-q}\right)^{\frac{1}{2}}+\left(\frac{q}{1-q}\right)^{-\frac{1}{2}}(1+\theta)}\frac{q}{1-q}\right]^{-1}.$$

---

alternatives that are not status quo, which is not ensured in a Knightian model where we have a incomplete ordering.

Consider two states s_1 and s_2, a eletment (x_1, x_2) in X represents getting x_1 in state s_1 and x_2 in state s_2. Consider $x = (1,2)$, $y = (3,1)$ and $z = (2,3)$. y and z belong to the choice set of $\{x, y, z\}$ with no status quo, and z belongs to the choice set of $\{x, y, z\}$ with x as status quo. **Axiom SQI** suggests that y also belongs to the choice set of $\{x, y, z\}$ with x as status quo. However, in a Knightian model, y is not preferred to x if x is status quo.

[7] For analysis for $\theta \geq 1$, see Krishna and Morgan (2011).

This setting gives a higher and a lower bound on q for the existence of a sincere voting equilibrium. The higher θ is, the higher the lower bound $\max(q^*, q^{**})$ is.

Proposition 9 *When $n \to \infty$, an uninformative voting equilibrium with every voter voting for party candidate A exists if and only if*

$$q \leq p.$$

We obtain similar results on information precision and partisanship as in the non-expected utility model. If the magnitude of θ represents the strength of party bias, to have a party supporter to vote against his own party as well as his own signal, it is necessary that he is pretty sure that his own party is not a good choice before any signal. Correspondingly, a Knightian decision maker needs to have a prior that strongly favors his own party's rival candidate.

After excluding all mixed equilibria, the following proposition says there only exists one type of full partisan voting equilibrium, and we get the similar equilibrium condition.

Proposition 10 *When $n \to \infty$, an uninformative full partisan voting equilibrium favoring party candidate A exists if and only if*

$$q \leq \min\left(p, q^*\right),$$

where

$$q^* = \left[\frac{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right](1 - \theta)}{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right](1 + \theta)} \frac{p}{1 - p} + 1 \right]^{-1}.$$

And the corresponding voting profile is $\{(1, 1), (1, 1), (0, 0)\}$.

It is difficult to compare θ with $\left[\underline{p}, \overline{p}\right]$. One may get close resemblance if introducing continuous type of voters. However, two types of expected utility model suggests the same thing: only a Knightian model has enough flexibility to sustain informative voting when signal is noisy. This conclusion can be extended to all models with a utility function defined, including types of model concerning uncertainty. The logic behind is straightforward: any mixed strategies used in the equilibrium imposes stringent conditions on the parameters, which makes any informative partisan voting equilibria nongeneric. As long as we have a utility function defined, in this setting, we are going back to the simple world where partisans are stubborn, and swing voters are rationally ignorant without a notice of their valuable information.

7   Conclusion

In this paper, equilibrium existence is established, and equilibrium strategies are identified for large election. Even with costly information acquisition, swing voters still acquire information in some circumstances, but partisans do not.

Naturally, we would like to investigate the welfare implications of this model. However, to compare welfare among all possible equilibria, we need to do comparison according to

a set of priors. If there are some better-informed outsiders (say, government or election experts), it might be possible to conclude that some equilibria that is "better" than the others by using a "correct" prior. Otherwise, to be a "good" equilibrium in a Knightian world, it has to be really "good".

The main messages of this paper are: 1) partisan voting can be rationalized by Knightian uncertainty with party identity as a status quo, and 2) in a world of uncertainty, swing voters may vote informatively, even information is costly.

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8 Appendix

Lemma 1 *In any sincere voting equilibrium, $\Omega = 1$.*
Proof.

$$
\begin{aligned}
\Omega &= \frac{\Pr[piv_A \mid \alpha] + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta] + \Pr[piv_B \mid \beta]} \\[2mm]
&= \frac{2\Pr[T \mid \alpha] + \Pr[T_{-1} \mid \alpha] + \Pr[T_{+1} \mid \alpha]}{2\Pr[T \mid \beta] + \Pr[T_{-1} \mid \beta] + \Pr[T_{+1} \mid \beta]} \\[2mm]
&= \frac{2e^{-n}\sum\limits_{k=0}^{\infty}\frac{\sigma_A^k}{k!}\frac{\sigma_B^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\sigma_A^{k-1}}{(k-1)!}\frac{\sigma_B^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\sigma_A^k}{k!}\frac{\sigma_B^{k-1}}{(k-1)!}}{2e^{-n}\sum\limits_{k=0}^{\infty}\frac{\tau_A^k}{k!}\frac{\tau_B^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\tau_A^{k-1}}{(k-1)!}\frac{\tau_B^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\tau_A^k}{k!}\frac{\tau_B^{k-1}}{(k-1)!}} \\[2mm]
&= \frac{2e^{-n}\sum\limits_{k=0}^{\infty}\frac{\sigma_A^k}{k!}\frac{\sigma_B^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\sigma_A^{k-1}}{(k-1)!}\frac{\sigma_B^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\sigma_A^k}{k!}\frac{\sigma_B^{k-1}}{(k-1)!}}{2e^{-n}\sum\limits_{k=0}^{\infty}\frac{\sigma_B^k}{k!}\frac{\sigma_A^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\sigma_B^{k-1}}{(k-1)!}\frac{\sigma_A^k}{k!} + e^{-n}\sum\limits_{k=1}^{\infty}\frac{\sigma_B^k}{k!}\frac{\sigma_A^{k-1}}{(k-1)!}} \\[2mm]
&= \frac{2\Pr[T \mid \alpha] + \Pr[T_{-1} \mid \alpha] + \Pr[T_{+1} \mid \alpha]}{2\Pr[T \mid \alpha] + \Pr[T_{+1} \mid \alpha] + \Pr[T_{-1} \mid \alpha]} = 1.
\end{aligned}
$$

∎

Proposition 1 *A sincere voting equilibrium exists if and only if*

$$q \geq \max(1 - \underline{p}, \overline{p}).$$

Proof. If there is a sincere voting equilibrium, $\Omega = 1$ in a sincere equilibrium.

$$\Omega = 1 \geq Q_{\underline{p}}^a = \frac{(1 - \underline{p})(1 - q)}{\overline{p}q} \Leftrightarrow q \geq 1 - \underline{p},$$

$$and \ \Omega = 1 \leq Q_{\overline{p}}^b = \frac{(1 - \overline{p})q}{\overline{p}(1 - q)} \Leftrightarrow q \geq \overline{p}.$$

Therefore, $q \geq \max\left(1 - \underline{p}, \overline{p}\right)$.

If $q \geq \max\left(1 - \underline{p}, \overline{p}\right)$, consider a sincere voting voting profile

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1, 0), (1, 0), (1, 0)\}.$$

$\Omega = 1$.

For all voters, given a signal a, $\Omega = 1 \geq Q_{\underline{p}}^a = \frac{(1-\underline{p})(1-q)}{\overline{p}q}$, voting for party candidate A is optimal. Similarly, given a signal b, voting for party candidate B is optimal. Therefore, sincere voting is an equilibrium. ∎

Lemma 2 *In any uninformative equilibrium, $\Omega = 1$.*
Proof. Consider an equilibrium where all voters always vote for party candidate A,

$$
\begin{aligned}
\Omega &= \frac{\Pr[piv_A \mid \alpha] + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta] + \Pr[piv_B \mid \beta]} \\[2mm]
&= \frac{\Pr[m = 0] + (\Pr[m = 0] + \Pr[m = 1])}{\Pr[m = 0] + (\Pr[m = 0] + \Pr[m = 1])} \\[2mm]
&= 1.
\end{aligned}
$$

Similarly, in an equilibrium where all voters always vote for party candidate B, $\Omega = 1$.

∎

Proposition 2 *An uninformative voting equilibrium with every voter voting for party candidate A exists if and only if*

$$q \leq \underline{p} \text{ and } \underline{p} > \frac{1}{2}.$$

Proof. If there is an uninformative voting equilibrium, $\Omega = 1$ in an uninformative equilibrium with every voter voting for party candidate A.

$$\Omega = 1 \geq Q_{\underline{p}}^a \text{ and } \Omega \geq Q_{\underline{p}}^b \Rightarrow \underline{p} \geq \max(q, 1 - q) = q,$$

Therefore, $q \leq \underline{p}$. Since $q > \frac{1}{2}$, $\underline{p} > \frac{1}{2}$.

If $q \leq \underline{p}$ and $\underline{p} > \frac{1}{2}$, consider an uninformative voting voting profile with every voter voting for party candidate A

$$\{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{(1,1),(1,1),(1,1)\}.$$

$\Omega = 1$.

For all voters, given both signal a and b, $\Omega = 1 \geq Q_{\underline{p}}^a$ and $\Omega \geq Q_{\underline{p}}^b$, voting for party candidate A is optimal. Therefore, uninformative voting for party candidate A is an equilibrium. ∎

Lemma 3 *A full partisan voting equilibrium exists only if*

$$\frac{(1-\overline{p})q}{\overline{p}(1-q)} < \Omega < \frac{(1-\underline{p})(1-q)}{\underline{p}q}.$$

Proof. In a full partisan equilibrium,

$$\Omega > Q_{\overline{p}}^a \text{ and } \Omega < Q_{\underline{p}}^b \Rightarrow \frac{(1-\overline{p})q}{\overline{p}(1-q)} < \Omega < \frac{(1-\underline{p})(1-q)}{\underline{p}q}.$$

∎

Lemma 4 *A partial partisan voting equilibrium favoring party candidate A exists only if*

$$\begin{array}{ll} \frac{(1-\overline{p})q}{\overline{p}(1-q)} < \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} & \text{if } \frac{(1-\overline{p})q}{\overline{p}(1-q)} \geq \frac{(1-\underline{p})(1-q)}{\underline{p}q}, \\ \frac{(1-\underline{p})(1-q)}{\underline{p}q} \leq \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} & \text{if } \frac{(1-\overline{p})q}{\overline{p}(1-q)} < \frac{(1-\underline{p})(1-q)}{\underline{p}q}. \end{array}$$

And a partial partisan voting equilibrium favoring party candidate B exists only if

$$\begin{array}{ll} \frac{(1-\overline{p})(1-q)}{\overline{p}q} < \Omega < \frac{(1-\underline{p})(1-q)}{\underline{p}q} & \text{if } \frac{(1-\underline{p})(1-q)}{\underline{p}q} \leq \frac{(1-\overline{p})q}{\overline{p}(1-q)}, \\ \frac{(1-\overline{p})(1-q)}{\overline{p}q} < \Omega \leq \frac{(1-\overline{p})q}{\overline{p}(1-q)} & \text{if } \frac{(1-\underline{p})(1-q)}{\underline{p}q} > \frac{(1-\overline{p})q}{\overline{p}(1-q)}. \end{array}$$

Proof. In a partial partisan equilibrium favoring party candidate A,

$$\begin{array}{ccc} \Omega \geq Q_{\underline{p}}^a & \Rightarrow & \Omega \geq \frac{(1-\underline{p})(1-q)}{\underline{p}q}, \\ \text{and} \quad \Omega > Q_{\overline{p}}^b, \Omega < Q_{\underline{p}}^b & \Rightarrow & \frac{(1-\overline{p})q}{\overline{p}(1-q)} < \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)}. \end{array}$$

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Hence,

$$
\begin{array}{ll}
\frac{(1-\bar{p})q}{\bar{p}(1-q)} < \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} & if \ \frac{(1-\bar{p})q}{\bar{p}(1-q)} \geq \frac{(1-\underline{p})(1-q)}{pq}, \\
\frac{(1-\underline{p})(1-q)}{pq} \leq \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} & if \ \frac{(1-\bar{p})q}{\bar{p}(1-q)} < \frac{(1-\underline{p})(1-q)}{pq}.
\end{array}
$$

Similarly, in a partial partisan equilibrium favoring party candidate B,

$$
\begin{array}{ll}
\frac{(1-\bar{p})q}{\bar{p}(1-q)} < \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} & if \ \frac{(1-\bar{p})q}{\bar{p}(1-q)} \geq \frac{(1-\underline{p})(1-q)}{pq}, \\
\frac{(1-\underline{p})(1-q)}{pq} \leq \Omega < \frac{(1-\underline{p})q}{\underline{p}(1-q)} & if \ \frac{(1-\bar{p})q}{\bar{p}(1-q)} < \frac{(1-\underline{p})(1-q)}{pq}.
\end{array}
$$

∎

Lemma 5 *When $n \to \infty$, in any equilibrium,*

$$
\Omega \to 1.
$$

Proof. By *Lemma* 1 and 2, in any sincere or uninformative voting equilibrium, $\Omega = 1$.

Consider any partisan voting equilibrium, denote $\frac{\sigma_A}{n} = \nu_A$, $\frac{\tau_A}{n} = \omega_A$[8]. ν_A and ω_A are the expected vote share of candidate A in state α and β, which does not depend on population size.

$$
\begin{aligned}
\Omega &\approx \frac{\Pr[T \mid \alpha]}{\Pr[T \mid \beta]} \frac{[2 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}} + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}]}{[2 + \left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}} + \left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}]} \\
&= \frac{e^{-n} \frac{e^{2\sqrt{\sigma_A \sigma_B}}}{\sqrt{2\pi \cdot 2\sqrt{\sigma_A \sigma_B}}} [2 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}} + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}]}{e^{-n} \frac{e^{2\sqrt{\tau_A \tau_B}}}{\sqrt{2\pi \cdot 2\sqrt{\tau_A \tau_B}}} [2 + \left(\frac{\tau_A}{\tau_B}\right)^{\frac{1}{2}} + \left(\frac{\tau_A}{\tau_B}\right)^{-\frac{1}{2}}]} \\
&= \left(e^{2n}\right)^{\sqrt{\nu_A(1-\nu_A)} - \sqrt{\omega_A(1-\omega_A)}} \left(\frac{\nu_A(1-\nu_A)}{\omega_A(1-\omega_A)}\right)^{-\frac{1}{4}} \frac{[2 + \left(\frac{\nu_A}{1-\nu_A}\right)^{\frac{1}{2}} + \left(\frac{\nu_A}{1-\nu_A}\right)^{-\frac{1}{2}}]}{[2 + \left(\frac{\omega_A}{1-\omega_A}\right)^{\frac{1}{2}} + \left(\frac{\omega_A}{1-\omega_A}\right)^{-\frac{1}{2}}]} \\
&= g(n, \nu_A, \omega_A) f(\nu_A, \omega_A).
\end{aligned}
$$

where

$$
g(n, \nu_A, \omega_A) = \left(e^{2n}\right)^{\sqrt{\nu_A(1-\nu_A)} - \sqrt{\omega_A(1-\omega_A)}},
$$

and

$$
f(\nu_A, \omega_A) = \left(\frac{\nu_A(1-\nu_A)}{\omega_A(1-\omega_A)}\right)^{-\frac{1}{4}} \frac{[2 + \left(\frac{\nu_A}{1-\nu_A}\right)^{\frac{1}{2}} + \left(\frac{\nu_A}{1-\nu_A}\right)^{-\frac{1}{2}}]}{[2 + \left(\frac{\omega_A}{1-\omega_A}\right)^{\frac{1}{2}} + \left(\frac{\omega_A}{1-\omega_A}\right)^{-\frac{1}{2}}]}.
$$

$g(n, \nu_A, \omega_A)$ is a function of electorate size n, and vote share of party candidate A in two states, ν_A and ω_A. ν_A and ω_A merely depend on the voting profile $(\gamma_a, \gamma_b)_S$ and signal precision q. $f(\nu_A, \omega_A)$ is a function of ν_A and ω_A. Given a particular voting profile, $f(\nu_A, \omega_A)$ is a positive constant uniquely defined.

---

[8] $\nu_A, \omega_A \in [0, 1]$.

Obviously, $g(n, \nu_A, \omega_A)$ increases exponentially in n, if $\sqrt{\nu_A (1 - \nu_A)} \geq \sqrt{\omega_A (1 - \omega_A)}$. Otherwise, it decreases exponentially in n. Therefore,

$$\lim_{n \to \infty} g(\sigma_A, \tau_A) = \begin{cases} \infty & if \ \nu_A (1 - \nu_A) > \omega_A (1 - \omega_A) \\ 1 & if \ \nu_A (1 - \nu_A) = \omega_A (1 - \omega_A) \\ 0 & if \ \nu_A (1 - \nu_A) < \omega_A (1 - \omega_A) \end{cases}.$$

According to *Proposition* 3 and 4, we have derived a positive and finite upper and lower bound for Ω. Therefore, in any large population equilibrium, $\nu_A (1 - \nu_A) = \omega_A (1 - \omega_A)$. Then $\lim_{n \to \infty} g(\sigma_A, \tau_A) = 1$. This also implies

$$\nu_A = \omega_A \ or \ \nu_A = 1 - \omega_A.$$

In both cases, $f(\nu_A, \omega_A) = 1$. Therefore, $\lim_{n \to \infty} \Omega = 1$.

Hence, in any equilibrium, $\lim_{n \to \infty} \Omega = 1$. ∎

Lemma 6 *When $n \to \infty$, a full partisan voting equilibrium exists only if*

$$q < \min\left(\bar{p}, 1 - \underline{p}\right) \ and \ \underline{p} < \frac{1}{2},$$

and a partial partisan voting equilibrium exists only if

$$\underline{p} \ < \ q < \bar{p} \ if \ \underline{p} \geq \frac{1}{2},$$

$$1 - \underline{p} \ < \ q < \bar{p} \ if \ \underline{p} < \frac{1}{2}.$$

Proof. In any equilibrium, $\lim_{n \to \infty} \Omega = 1$.

By *Lemma 3*, $\frac{(1 - \bar{p})q}{\bar{p}(1 - q)} < 1 < \frac{(1 - \underline{p})(1 - q)}{\underline{p}q} \Rightarrow q < \min\left(\bar{p}, 1 - \underline{p}\right) \ and \ \underline{p} < \frac{1}{2}$.

By *Lemma 4*,

$$\begin{array}{ll} \frac{(1 - \bar{p})q}{\bar{p}(1 - q)} < 1 < \frac{(1 - \underline{p})q}{\underline{p}(1 - q)} & if \ \frac{(1 - \bar{p})q}{\bar{p}(1 - q)} \geq \frac{(1 - \underline{p})(1 - q)}{\underline{p}q} \\ \frac{(1 - \underline{p})(1 - q)}{\underline{p}q} \leq 1 < \frac{(1 - \underline{p})q}{\underline{p}(1 - q)} & if \ \frac{(1 - \bar{p})q}{\bar{p}(1 - q)} < \frac{(1 - \bar{p})(1 - q)}{\underline{p}q} \end{array} \Rightarrow \underline{p} < q < \bar{p} \ if \ \underline{p} \geq \frac{1}{2},$$

or

$$\begin{array}{ll} \frac{(1 - \bar{p})q}{\bar{p}(1 - q)} < \Omega < \frac{(1 - \underline{p})q}{\underline{p}(1 - q)} & if \ \frac{(1 - \bar{p})q}{\bar{p}(1 - q)} \geq \frac{(1 - \underline{p})(1 - q)}{\underline{p}q}, \\ \frac{(1 - \underline{p})(1 - q)}{\underline{p}q} \leq \Omega < \frac{(1 - \underline{p})q}{\underline{p}(1 - q)} & if \ \frac{(1 - \bar{p})q}{\bar{p}(1 - q)} < \frac{(1 - \bar{p})(1 - q)}{\underline{p}q}. \end{array} \Rightarrow 1 - \underline{p} < q < \bar{p} \ if \ \underline{p} < \frac{1}{2}.$$

∎

Proposition 3 *When $n \to \infty$, a full partisan voting equilibrium exists if and only if*

$$q < \min\left(\bar{p}, 1 - \underline{p}\right) \ and \ \underline{p} < \frac{1}{2}.$$

Moreover, in the full partisan voting equilibrium,

$$\gamma_a = \gamma_b \ or \ \gamma_a + \gamma_b = 1 + \frac{n_B - n_A}{n_S}.$$

Proof. In a full partisan voting equilibrium,

i) if $\nu_A = \omega_A$, $\sigma_A = \tau_A$. Then

$$n_A + n_S\left[q\gamma_a + (1-q)\gamma_b\right] = n_A + n_S\left[(1-q)\gamma_a + q\gamma_b\right]$$

$$\gamma_a = \gamma_b,$$

ii) if $\nu_A = 1 - \omega_A$, $\sigma_A = n - \tau_A$. Then

$$n_A + n_S\left[q\gamma_a + (1-q)\gamma_b\right] = n_B + n_S\left[(1-q)(1-\gamma_a) + q(1-\gamma_b)\right]$$

$$\gamma_a + \gamma_b = 1 + \frac{n_B - n_A}{n_S}.$$

As long as there exists a full partisan voting profile supporting $\lim_{n\to\infty}\Omega = 1$, a full partisan voting equilibrium exists if $q < \min\left(\overline{p}, 1 - \underline{p}\right)$ *and* $\underline{p} < \frac{1}{2}$.

In addition to *Lemma* 6, the sufficient and necessary condition for the existence of a full partisan voting equilibrium in large population is

$$q < \min\left(\overline{p}, 1 - \underline{p}\right) \ \ and \ \ \underline{p} < \frac{1}{2}.$$

∎

Proposition 4 *When $n \to \infty$, there does not exist a partial partisan voting equilibrium.*
Proof. If $\sigma_A = \tau_A$, then

$$n_A + n_S\left[q + (1-q)\gamma_b\right] + n_B q = n_A + n_S\left[(1-q) + q\gamma_b\right] + n_B\left(1-q\right)$$

$$n_S\left[1 - \gamma_b\right] + n_B = 0$$

Contradiction.
If $\sigma_A = n - \tau_A$, then

$$n_A + n_S\gamma_b = 0$$

Contradiction. Therefore, there never exist a partial partisan voting profile to support $\lim_{n\to\infty}\Omega = 1$. ∎

Lemma 7 *In an sequence of equilibrium with costly information, $q_i = \frac{1}{2}$ if $\{(\gamma_a, \gamma_b)_i : \gamma_a = \gamma_b\}$.*
Proof. If $\{(\gamma_a, \gamma_b)_i : \gamma_a = \gamma_b\}$, then

$$V_p(q) = \left\{ \begin{array}{c} \gamma_a\left(q(\alpha \mid a)\Pr[piv_A \mid \alpha] - q(\beta \mid a)\Pr[piv_A \mid \beta]\right) \\ +(1 - \gamma_a)\left(q(\beta \mid a)\Pr[piv_B \mid\beta] - q(\alpha \mid a)\Pr[piv_B \mid \alpha]\right) \end{array} \right\} - c(q).$$

$$\frac{\partial V_p}{\partial q} = -c'(q) < 0$$

Therefore, $q_i^* = \frac{1}{2}$. ∎
Lemma 8 *For any sequence of equilibrium, $\lim_{n\to\infty} v_p(q) = 0$.*
Proof.

$$\begin{aligned}
v_p(q) &= p(a,q)\left\{ \begin{array}{c} \gamma_a\left(q(\alpha \mid a)\Pr[piv_A \mid \alpha] - q(\beta \mid a)\Pr[piv_A \mid \beta]\right) \\ +(1 - \gamma_a)\left(q(\beta \mid a)\Pr[piv_B \mid\beta] - q(\alpha \mid a)\Pr[piv_B \mid \alpha]\right) \end{array} \right\} \\
&\quad + p(b,q)\left\{ \begin{array}{c} \gamma_b\left(q(\alpha \mid b)\Pr[piv_A \mid \alpha] - q(\beta \mid b)\Pr[piv_A \mid \beta]\right) \\ +(1 - \gamma_b)\left(q(\beta \mid b)\Pr[piv_B \mid\beta] - q(\alpha \mid b)\Pr[piv_B \mid \alpha]\right) \end{array} \right\} \\
&= \gamma_a\left\{pq\Pr[piv_A \mid \alpha] - (1-p)(1-q)\Pr[piv_A \mid \beta]\right\} \\
&\quad + (1 - \gamma_a)\left\{(1-p)(1-q)\Pr[piv_B \mid\beta] - pq\Pr[piv_B \mid \alpha]\right\} \\
&\quad + \gamma_b\left\{p(1-q)\Pr[piv_A \mid \alpha] - (1-p)q\Pr[piv_A \mid \beta]\right\} \\
&\quad + (1 - \gamma_b)\left\{(1-p)q\Pr[piv_B \mid\beta] - p(1-q)\Pr[piv_B \mid \alpha]\right\}
\end{aligned}$$

By *Lemma* 5, in any equilibrium, as $n \to \infty$, $\Pr[T \mid \alpha] = \Pr[T \mid \beta] = \Pr(T)$. In equilibrium,

i) $\nu_A = \omega_A$.

Let $\Pr[piv_A \mid \alpha] = \Pr[piv_A \mid \beta] = A$, and $\Pr[piv_B \mid \alpha] = \Pr[piv_B \mid \beta] = B$.

Denote $\left(\frac{\nu_A}{1-\nu_A} \right) = R$.

$$
\begin{aligned}
\lim_{n\to\infty} v_p(q) &= \lim_{n\to\infty} \left\{ \begin{array}{l} \gamma_a \left[pqA - (1-p)(1-q)A \right] + (1-\gamma_a)\left[(1-p)(1-q)B - pqB \right] \\ + \gamma_b \left[p(1-q)A - (1-p)qA \right] + (1-\gamma_b)\left[(1-p)qB - p(1-q)B \right] \end{array} \right\} \\
&= \lim_{n\to\infty} \left\{ \begin{array}{l} (p+q-1)\left[\gamma_a A - (1-\gamma_a)B \right] \\ + (p-q)\left[\gamma_b A - (1-\gamma_b)B \right] \end{array} \right\} \\
&= \lim_{n\to\infty} \Pr(T) \left\{ \begin{array}{l} (p+q-1)\left[\gamma_a \left(1 + R^{-\frac{1}{2}}\right) - (1-\gamma_a)\left(1 + R^{\frac{1}{2}}\right) \right] \\ + (p-q)\left[\gamma_b \left(1 + R^{-\frac{1}{2}}\right) - (1-\gamma_b)\left(1 + R^{\frac{1}{2}}\right) \right] \end{array} \right\} \\
&= c \lim_{n\to\infty} \Pr(T) \\
&= 0
\end{aligned}
$$

i) $\nu_A = 1 - \omega_A$.

Let $\Pr[piv_A \mid \alpha] = \Pr[piv_B \mid \beta] = A$, and $\Pr[piv_B \mid \alpha] = \Pr[piv_A \mid \beta] = B$.

Denote $\left(\frac{\nu_A}{1-\nu_A} \right) = R$.

$$
\begin{aligned}
\lim_{n\to\infty} v_p(q) &= \lim_{n\to\infty} \left\{ \begin{array}{l} \gamma_a \left[pqA - (1-p)(1-q)B \right] + (1-\gamma_a)\left[(1-p)(1-q)A - pqB \right] \\ + \gamma_b \left[p(1-q)A - (1-p)qB \right] + (1-\gamma_b)\left\{ (1-p)qA - p(1-q)B \right\} \end{array} \right\} \\
&= \lim_{n\to\infty} \Pr(T) \left\{ \begin{array}{l} \gamma_a \left[pq\left(1 + R^{-\frac{1}{2}}\right) - (1-p)(1-q)\left(1 + R^{\frac{1}{2}}\right) \right] \\ + (1-\gamma_a)\left[(1-p)(1-q)\left(1 + R^{-\frac{1}{2}}\right) - pq\left(1 + R^{\frac{1}{2}}\right) \right] \\ + \gamma_b \left[p(1-q)\left(1 + R^{-\frac{1}{2}}\right) - (1-p)q\left(1 + R^{\frac{1}{2}}\right) \right] \\ + (1-\gamma_b)\left\{ (1-p)q\left(1 + R^{-\frac{1}{2}}\right) - p(1-q)\left(1 + R^{\frac{1}{2}}\right) \right\} \end{array} \right\} \\
&= c \lim_{n\to\infty} \Pr(T) \\
&= 0
\end{aligned}
$$

∎

Proposition 5 *In any sequence of equilibrium with costly information,*

$$
\lim_{n\to\infty} q_i = \frac{1}{2} \; \forall i \in \{A, B, S\}.
$$

Proof. Suppose not. Therefore, in some sequence of equilibrium with costly information, for some voter of type, $\exists q^* > \frac{1}{2}$, $s.t. \forall N > 0, \exists n \geq N \; s.t. q^e \geq q^*$.

Pick some N s.t. $v_p(q^e) < c(q^*)$. Then,

$$
V_p(q^e) = v_p(q^e) - c(q^e) \leq v_p(q^e) - c(q^*) < 0
$$

Therefore, q^e is not an equilibrium information level. Contradiction. ∎

Proposition 6 *If $\frac{1}{2} \leq \underline{p} < \overline{p}$, in any sequence of equilibrium with costly information,*

$$
q_i = \frac{1}{2} \; and \; (\gamma_a, \gamma_b)_i = (1,1) \; \forall i \in \{A, B, S\}.
$$

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If $\underline{p} < \frac{1}{2} \le \overline{p}$, in any sequence of equilibrium with costly information,

(uniformative full partisan voting)
$q_i = \frac{1}{2} \ \forall i \in \{A, B, S\}$ *and*
$\lim_{n \to \infty} \{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\} = \{\{(1, 1), (\gamma, \gamma), (0, 0)\} : \gamma \in [0, 1]\}$
OR
(informative full partisan voting)
$q_A = q_B = \frac{1}{2}, \ q_S > \frac{1}{2}$ *and*
$\lim_{n \to \infty} \{(\gamma_a, \gamma_b)_A, (\gamma_a, \gamma_b)_S, (\gamma_a, \gamma_b)_B\}$
$= \left\{ \{(1, 1), (\gamma_a, \gamma_b), (0, 0)\} : \gamma_a + \gamma_b = 1 + \frac{n_B - n_A}{n_S}, \gamma_a > \gamma_b, \gamma_s \in [0, 1] \right\}.$

Proof. *i.* $\frac{1}{2} \le \underline{p} < \overline{p}$: by *Lemma* 7, an uninformative voting equilibrium is always an equilibrium with costly information. By *Proposition* 5, if $\lim_{n \to \infty} q_i = \frac{1}{2} \forall i \in \{A, B, S\}$, there does not exist a sincere voting equilibrium, since in a sincere voting equilibrium, $q \ge \max(1 - \underline{p}, \overline{p})$. For the same reason, we can also exclude the existence of a partial partisan voting equilibrium. Moreover, it does not exist even when $c(q) = 0 \ \forall q \in [\frac{1}{2}, 1]$. Therefore, uninformative voting equilibrium is the only type of equilibrium exists when $n \to \infty$.

ii. $\underline{p} < \frac{1}{2} \le \overline{p}$: $\lim_{n \to \infty} q_i = \frac{1}{2} \forall i \in \{A, B, S\}$, $\lim_{n \to \infty} q_i < \min(\overline{p}, 1 - \underline{p})$. There exists a full partisan voting equilibrium. Similarly, it is the only type of equilibrium exists when $n \to \infty$.

In a full partisan voting equilibrium, if $(\gamma_a, \gamma_b)_S = (\gamma, \gamma)$, by *Lemma* 6, there is no information acquisition. If $(\gamma_a, \gamma_b)_S \neq (\gamma, \gamma)$ and $(\gamma_a, \gamma_b)_S = \left(\gamma, 1 + \frac{n_B - n_A}{n_S} - \gamma\right)$, $\nu_A = 1 - \omega_A$.

Let $\Pr[piv_A \mid \alpha] = \Pr[piv_B \mid \beta] = A$, and $\Pr[piv_B \mid \alpha] = \Pr[piv_A \mid \beta] = B$.

$$V_p(q) = \left\{ \begin{array}{l} \gamma_a \left[pqA - (1 - p)(1 - q)B \right] + (1 - \gamma_a)\left[(1 - p)(1 - q)A - pqB \right] \\ + \gamma_b \left[p(1 - q)A - (1 - p)qB \right] + (1 - \gamma_b)\left\{ (1 - p)qA - p(1 - q)B \right\} \end{array} \right\} - c(q)$$

$$\frac{\partial V_p}{\partial q} = (A + B)(\gamma_a - \gamma_b) - c'(q) = 0$$

Therefore,

$$(A + B)(\gamma_a - \gamma_b) = c'(q^e)$$

$$\gamma_a > \gamma_b$$

∎

Lemma 9 *When $n \to \infty$, all mixed-strategy equilibria are non-generic.*
Proof. When $n \to \infty$, given signal a, for partisans of A to use mixed strategy

$$1 = Q_{\overline{p}}^a \Rightarrow q = 1 - \overline{p}.$$

Therefore, any equilibrium with partisans of A using mixed strategy given signal a is nongeneric.

Similarly, any mixed-strategy equilibrium is nongeneric. ∎
Proposition 7 *When $n \to \infty$,*

1. a sincere voting equilibrium exists if and only if

$$q \geq \max(1 - \underline{p}, \overline{p}),$$

2. an uninformative voting equilibrium exists if and only if

$$q \leq \underline{p} \ and \ \underline{p} > \frac{1}{2},$$

3. an uninformative full partisan voting equilibrium exists if and only if

$$\max\left(1 - \overline{p}, p\right) \leq q \leq \min\left(1 - p, \overline{p}\right)$$
the corresponding voting profile is $\{(1,1), (0,0), (0,0)\}$,
$$\max\left(1 - p, \underline{p}\right) \leq q \leq \min\left(1 - \underline{p}, p\right)$$
the corresponding voting profile is $\{(1,1), (1,1), (0,0)\}$,

4. and no generic equilibrium otherwise.

Proof. In any equilibrium, $\lim_{n \to \infty} \Omega = 1$.

Equilibrium conditions for a sincere voting equilibrium and an uninformative equilibrium is the same as in *Proposition* 1 and 2, then we get $i)$ a sincere voting equilibrium exists if and only if $q \geq \max(1 - \underline{p}, \overline{p})$, and $ii)$ an uninformative voting equilibrium exists if and only if $q \leq \underline{p} \ and \ \underline{p} > \frac{1}{2}$.

By *Propostion* 5, there does not exist a partial partisan voting equilibrium. Therefore, we only need focus on full partisan voting equilibria.

In a full partisan voting equilibrium, by *Proposition* 3, the voting profile of a swing voter is

$$\gamma_a = \gamma_b \ or \ \gamma_a + \gamma_b = 1 + \frac{n_B - n_A}{n_S}.$$

Once mixed strategy is used, the equilibrium condition is tighten too much and the equilibrium existence is nongeneric. We only consider pure strategy equilibria.

The equilibrium voting profile of swing voters is either $(1,1)$ or $(0,0)$.

Given the full voting profile $\{(1,1), (0,0), (1,1)\}$,

$$\begin{aligned} Q_{\underline{p}}^a \geq 1 \geq Q_p^a \\ Q_{\underline{p}}^b \geq 1 \geq Q_p^b \end{aligned} \Leftrightarrow \max\left(1 - p, \underline{p}\right) \leq q \leq \min\left(1 - \underline{p}, p\right).$$

Therefore, a full partisan voting equilibrium with swing voters always voting for party candidate A exists if and only if $\max\left(1 - p, \underline{p}\right) \leq q \leq \min\left(1 - \underline{p}, p\right).$

Given the full voting profile $\{(1,1), (0,0), (0,0)\}$

$$\begin{aligned} Q_p^a \geq 1 \geq Q_{\overline{p}}^a \\ Q_p^b \geq 1 \geq Q_{\overline{p}}^b \end{aligned} \Leftrightarrow \max\left(1 - \overline{p}, p\right) \leq q \leq \min\left(1 - p, \overline{p}\right).$$

Similar, a full partisan voting equilibrium with swing voters always voting for party candidate B exists if and only if $\max\left(1 - \overline{p}, p\right) \leq q \leq \min\left(1 - p, \overline{p}\right).$

The full list of nongeneric equilibria:

Equilibrium conditions	Voting profile
	(full partisan voting)
$n_A = n_B$	$\{(1,1),(1,0),(0,0)\}$
$\frac{n_B - n_A}{n_S} \in (0,1), q = p$	$\left\{(1,1),\left(1,\frac{n_B - n_A}{n_S}\right),(0,0)\right\}$
$\frac{n_B - n_A}{n_S} \in (-1,0), q = 1 - p$	$\left\{(1,1),\left(\frac{n_S + n_B - n_A}{n_S},0\right),(0,0)\right\}$
	(partial partisan voting)
$\frac{n_B - n_A - n_S}{n_B} \in (0,1), q = 1 - \underline{p}$	$\{(1,1),(1,1),(\gamma,0)\}, \gamma = \frac{n_B - n_A - n_S}{n_B}$
$n_A < n_B, q = 1 - \underline{p} = p$	$\{(1,1),(1,\gamma_b),(\gamma_a,0)\}, \gamma_a = \frac{n_B - n_A}{n_B}$
$n_A < n_B, q = 1 - \underline{p}$	$\{(1,1),(1,0),(\gamma,0)\}, \gamma = \frac{n_B - n_A}{n_B}$
$\frac{n_B + n_S}{n_A} \in (0,1), q = 1 - p = \overline{p}$	$\{(1,\gamma),(0,0),(0,0)\}, \gamma = \frac{n_B + n_S}{n_A}$

The list above simply means that we can get some interesting equilibria if we are lucky enough, but most of time only equilibria mentioned in bullet points 1 to 3 exist. ■

Proposition 8 *A sincere voting equilibrium exists if and only if*

$$q \geq \max(q^*, q^{**}),$$

where q^* is uniquely defined by

$$\frac{1-p}{p} = \left[\frac{2 - \theta + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}} + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}(1-\theta)}{2 + \theta + \left(\frac{q}{1-q}\right)^{\frac{1}{2}} + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}(1+\theta)} \frac{q}{1-q}\right]^{-1},$$

and q^{**} is uniquely defined by

$$\frac{1-p}{p} = \frac{2 - \theta + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}} + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}(1-\theta)}{2 + \theta + \left(\frac{q}{1-q}\right)^{\frac{1}{2}} + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}(1+\theta)} \frac{q}{1-q}.$$

Proof. In a sincere voting equilibrium,

$$Q_p^b \geq \Omega_A \geq \Omega_S \geq \Omega_B \geq Q_p^a$$

If there is a sincere voting equilibrium,

$$
\begin{aligned}
\Omega_A &= \frac{\Pr[piv_A \mid \alpha](1+\theta) + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta](1-\theta) + \Pr[piv_B \mid \beta]} \\
&\approx \frac{\Pr[T \mid \alpha]}{\Pr[T \mid \beta]} \frac{\left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}\right](1+\theta) + \left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}\right]}{\left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}\right](1-\theta) + \left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}\right]} \\
&= \frac{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right](1+\theta) + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right]}{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right](1-\theta) + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right]}.
\end{aligned}
$$

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Similarly,
$$\Omega_S = 1,$$

$$\Omega_B = \frac{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right](1-\theta)}{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right](1+\theta)}.$$

The equilibrium conditions are:

$$Q_p^b \geq \Omega_A \Rightarrow$$

$$p \leq \left[\frac{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right](1+\theta) + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right]}{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right](1-\theta) + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right]}\frac{1-q}{q} + 1\right]^{-1} = G(\theta, q),$$

the right-hand side term, $G(\theta, q)$ is strictly increasing in q on $[0, 1) \times \left[\frac{1}{2}, 1\right)$.
Therefore, $p \leq p^*(\theta, q) \Rightarrow q \geq q^*(\theta, p)$.
Similarly,

$$\Omega_B \geq Q_p^a \Rightarrow$$

$$p \geq \left[\frac{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right](1-\theta)}{\left[1 + \left(\frac{q}{1-q}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{q}{1-q}\right)^{-\frac{1}{2}}\right](1+\theta)}\frac{q}{1-q} + 1\right]^{-1},$$

the right-hand side term $F(\theta, q)$ is strictly decreasing in q on $[0, 1) \times \left[\frac{1}{2}, 1\right)$.
Therefore, $p \geq p^{**}(\theta, q) \Rightarrow q \geq q^{**}(\theta, p)$.
If $q \geq \max(q^*, q^{**})$, then

$$p \leq G(\theta, q) \Rightarrow Q_p^b \geq \Omega_A,$$
$$\text{and } p \geq F(\theta, q) \Rightarrow \Omega_B \geq Q_p^a.$$

Therefore, sincere voting is an equilibrium voting profile. ■

Proposition 9 *When $n \to \infty$, an uninformative voting equilibrium with every voter voting for party candidate A exists if and only if*

$$q \leq p.$$

Proof. In an uninformative voting equilibrium where every voter votes for party candidate A,

$$\lim_{n \to \infty} \Omega_A \geq \lim_{n \to \infty} \Omega_S \geq \lim_{n \to \infty} \Omega_B \geq Q_p^b \geq Q_p^a.$$

$$
\begin{aligned}
\lim_{n \to \infty} \Omega_A &= \lim_{n \to \infty} \frac{\Pr[piv_A \mid \alpha](1+\theta) + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta](1-\theta) + \Pr[piv_B \mid \beta]} \\
&= \lim_{n \to \infty} \frac{\Pr[m = 0](1+\theta) + (\Pr[m=0] + \Pr[m=1])}{\Pr[m = 0](1-\theta) + (\Pr[m=0] + \Pr[m=1])} \\
&= \lim_{n \to \infty} \frac{(1+\theta) + (1+n)}{(1-\theta) + (1+n)} \\
&= 1,
\end{aligned}
$$

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Similarly, $\lim_{n\to\infty} \Omega_S = \lim_{n\to\infty} \Omega_B = 1.$

$$\lim_{n\to\infty} \Omega_B \geq Q_p^b \Rightarrow q \leq p.$$

If $q \leq p$, then $\lim_{n\to\infty} \Omega_A \geq \lim_{n\to\infty} \Omega_S \geq \lim_{n\to\infty} \Omega_B \geq Q_p^b \geq Q_p^a$.

In an uninformative voting equilibrium where every voter votes for party candidate B, $q \leq 1 - p$. Since we assume $p > \frac{1}{2}$, there does not exist such equilibrium.

Therefore, uninformative voting for party candidate A is an equilibrium voting profile.

■

Proposition 10 *When* $n \to \infty$, *an uninformative full partisan voting equilibrium favoring party candidate* A *exists if and only if*

$$q \leq \min\left(p, q^*\right),$$

where

$$q^* = \left[\frac{2 - \theta + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}} + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}(1-\theta)}{2 + \theta + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}} + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}(1+\theta)} \frac{p}{1-p} + 1 \right]^{-1}.$$

And the corresponding voting profile is $\{(1,1),(1,1),(0,0)\}.$

Proof. In an uninformative partisan voting equilibrium favoring party candidate A,

$$\lim_{n\to\infty} \Omega_A \geq \lim_{n\to\infty} \Omega_S \geq Q_p^b \geq Q_p^a \geq \lim_{n\to\infty} \Omega_B.$$

$$
\begin{aligned}
\lim_{n\to\infty} \Omega_A &= \lim_{n\to\infty} \frac{\Pr[piv_A \mid \alpha](1+\theta) + \Pr[piv_B \mid \alpha]}{\Pr[piv_A \mid \beta](1-\theta) + \Pr[piv_B \mid \beta]} \\
&\approx \frac{\Pr[T \mid \alpha]}{\Pr[T \mid \beta]} \frac{\left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}\right](1+\theta) + \left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}\right]}{\left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{\frac{1}{2}}\right](1-\theta) + \left[1 + \left(\frac{\sigma_A}{\sigma_B}\right)^{-\frac{1}{2}}\right]} \\
&= \frac{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right](1+\theta) + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right]}{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right](1-\theta) + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right]}.
\end{aligned}
$$

Similarly,

$$\lim_{n\to\infty} \Omega_S = 1,$$

$$\lim_{n\to\infty} \Omega_B \approx \frac{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right](1-\theta)}{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right](1+\theta)}.$$

$$\lim_{n\to\infty} \Omega_S \geq Q_p^b \Rightarrow q \leq p$$

$$Q_p^a \geq \lim_{n \to \infty} \Omega_B \Rightarrow$$

$$q \leq \left[\frac{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right](1-\theta)}{\left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{\frac{1}{2}}\right] + \left[1 + \left(\frac{n_A + n_S}{n_B}\right)^{-\frac{1}{2}}\right](1+\theta)} \frac{p}{1-p} + 1 \right]^{-1} = q^*,$$

Therefore, $q \leq \min(p, q^*)$.

Similarly, in an uninformative partisan voting equilibrium favoring party candidate B,

$$\lim_{n \to \infty} \Omega_A \geq Q_p^b \geq Q_p^a \geq \lim_{n \to \infty} \Omega_S \geq \lim_{n \to \infty} \Omega_B.$$

$$Q_p^a \geq \lim_{n \to \infty} \Omega_S \Rightarrow q \leq 1 - p.$$

Therefore, there does not exist such equilibrium. ∎