An Experimental Investigation on Belief and Higher-Order Belief in the Centipede Games*

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Abstract

This paper explores people's beliefs behind non-backward induction behavior in laboratory centipede games. We elicit players' beliefs about opponents' strategies and 1st-order beliefs. The measured beliefs help us to infer the conditional probability systems of both players. The inferred CPS's and players' actual strategy choices identify *why* they fail to reach the BI outcomes. First, we examine whether the player's strategies are best response to the measured beliefs, i.e. players are rational. In all the treatments, the frequency of players' being rational is significantly less than probability 1; but the frequency in the Constant-Sum treatment is significantly higher than that in other treatments. Second, as regarding players' beliefs and higher-order beliefs of rationality, we find that neither *common initial belief of rationality* nor *common strong belief of rationality* always holds. Nevertheless, in the Constant-Sum treatment both the frequency of players' initially believing in others' rationality and the frequency of players' higher-order initial belief of rationality are significantly than those in the other treatments.

Keywords: Centipede Game; Rationality; Belief and Higher Order Belief; Laboratory Experiments; Learning

JEL Classification: C72; C92; D83

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1 Introduction

This paper studies rationality, belief of rationality, and higher-order belief of rationality in the centipede game experiment. Actual play in centipede experiments seldom ends as backward induction predicts. Existing literature attributes the departure from backward induction (BI thereafter) prediction either to players' lack of rationality, or to players' inconsistent beliefs and higher-order beliefs of others' rationality. In this paper, we evaluate these arguments in a more direct fashion. We elicit the first mover's belief about the second mover's strategy as well as the second mover's initial and conditional beliefs about the first mover's strategy and 1st-order belief. The measured beliefs help us to infer the conditional probability systems (CPS thereafter) of both players. The inferred CPS's and players' actual strategy choices identify *why* they fail to reach the BI outcomes.

The first strand of the existing experimental literature focuses on players' lack of rationality. It presumes presence of behavioral types who fail to or do not maximize monetary payoffs[1]. For example, McKelvey and Palfrey [24] assume that ex-ante a player chooses to not play along the BI path with probability p. But assuming irrationality before a game starts is restrictive; people could be right but think others are wrong. In this paper, the inferred CPS and players' strategies allow us to directly examine players' rationality. We define rationality as a player's strategy best responding to the measured belief. We find, in all three treatments, the frequency of either player's being rational is significantly lower than 100 percent. But in the Constant-Sum treatment, which excludes the efficiency property as well as any possibility of mutual benefits, the frequency of the first-mover being rational is significantly higher than those in the other two treatments.

The other strand of literature attributes the experimental anomalies to lack of common knowledge of rationality. Two field centipede experiments (Palacios-Huerta and Volij [27] and Levitt et.al [22]) are in this fashion. Both use professional chess players as experimental subjects; the authors assume there is always rationality and common knowledge of rationality among chess players. The authors' approach is based on Aumann's [2] claim[2] "if common knowledge of rationality holds then the backward induction outcome results." Nevertheless, the notion of "common knowledge" is not empirically verifiable; one can never ensure the existence of "common knowledge" among chess-players or the non-existence of it among ordinary laboratory subjects. This suggests that the knowledge-based approach may have limited explanatory power for the anomalies in the centipede experiments. Thus

[1]See McKelvey and Palfrey [24], Fey et.al.[15], Zauner [37], Kawagoe and Takizawa [20].

[2]For more knowledge-based theoretical discussion on the "backward induction paradox," see Bicchieri [9][10], Pettit and Sugden [29], Reny [31][32], Bonanno [13], Aumann [2][4][3], Binmore [11][12].

in this paper we follow an alternative approach, the belief-based epistemic game theory[3] to address the notion of common *belief* of rationality. The measured beliefs, high-order beliefs, and players' actual strategy choices help us to identify whether *rationality and common initial belief of rationality* and/or *rationality and common strong belief of rationality* hold.

We find, in fact, that common *initial* belief of rationality does not always exist in the laboratory. In all three treatments, the frequency of players' believing opponents' rationality is significantly less than 100 percent. Nevertheless, in the Constant-Sum treatment this frequency is significantly higher than that in the other two treatments; whereas the frequency in the Baseline Centipede treatment does not differ significantly form that in the No-Mutual-Benefit treatment, a treatment that excludes the mutually beneficial outcome but not the efficiency property from the Baseline game. Moreover, in all treatments the average frequency of the second mover's initially believing the first mover's rationality and 2nd-order rationality is significantly less than 100 percent. This frequency in the Constant-Sum treatment is significantly higher than those in the other two treatments. Also it gradually increases towards 100 percent as subjects gain experience in later rounds of the experiment; whereas in the other two treatments there is no such increasing pattern as more rounds are played.

Furthermore, we find that common *strong* belief of rationality is seldom observed in the laboratory, especially for the second-mover. In all three treatments, the the average frequency of the second mover's strongly believing the first-mover's rationality and 2nd-order rationality is significantly less than 100 percent. And this frequency in the Constant-Sum treatment does not significantly differ from those in the other two treatments. Notice that the second-movers are informed that the first-mover has chosen a non-BI strategy for the first stage before being asked to state their conditional beliefs. Thus our result indicates that once the second-movers observe the first-movers' deviating from the BI path, the former can hardly believe that the latter's rationality AND higher-order belief of rationality.

Last but not least, let us close this section by emphasizing the difference between this belief-based approach and the *level-k* model. The level-k analysis assumes the presence of behavior types *before* the game starts: there always exists a level-zero who is the least sophisticated; each player believes her opponent to be less sophisticated than herself and respond to those types optimally. Nevertheless, our approach does not impose any presumptions on players' beliefs and behavior: we elicit the true patterns of them. We do not assume *ex-ante* that players best respond to others' types; nor do we restrict players' beliefs

[3]See Aumann and Brandenburger [1], Battigalli [5], Battigalli and Siniscalchi [6][7], Ben-Porath [8], Brandenburger [14].

about their opponents' degree of sophistication. Strategies and reported beliefs from our experiment can be used to examine the level-k model, but not vice versa.

The remainder of the paper is organized as follows. Section 2 formally defines players' beliefs, rationality, and beliefs of rationality in the centipede game. Section 3 presents the experimental design in detail, with Section 3.1 introducing experimental treatments and testing hypothesis and Section 3.2 introducing the procedure and belief elicitation method in the laboratory. Section 4 presents the experimental findings on players' strategies, players' beliefs about opponents' strategies, rationality, and higher-order beliefs of rationality. Section 5 reviews related theoretical literature on backward induction and epistemic game theory and previous experimental studies on the centipede games. Section 6 concludes.

2 Defining Belief, Rationality, and Belief of Rationality

We follow Brandenburger's [14] notation of players' belief types and epistemic states throughout this section. Denote the two-player (Ann and Bob) finite centipede game $\langle S^a, S^b, \Pi^a, \Pi^b \rangle$ where S^i and Π^i represent player i's set of pure strategies and set of payoffs, respectively.

Definition 1. *We call the structure* $\langle S^a, S^b; T^a, T^b; \lambda^a(\cdot), \lambda^b(\cdot) \rangle$ *a type structure for the players of a two-person finite game where* T^a *and* T^b *are compact metrizable space, and each* $\lambda^i : T^i \to \Delta(S^{-i} \times T^{-i}), i = a, b$ *is continuous. An element* $t^i \in T^i$ *is called a* ***type*** *for player* $i, (i = a, b)$*. An elements* $(s^a, s^b, t^a, t^b) \in S \times T$ *(where* $S = S^a \times S^b$ *and* $T = T^a \times T^b$*) is called a* ***state****.*

We first define *rationality* using the type-state language:

Definition 2. *A strategy-type pair of player* $i, (i = a, b)$*,* (s^i, t^i) *is* ***rational*** *if* s^i *maximizes player* i*'s expected payoff under the measure* $\lambda^i(t^i)$*'s marginal on* S^{-i}*.*

Next, we define a player's believing an event as:

Definition 3. *Player* i*'s type* t^i *believes an event* $E \subseteq S^{-i} \times T^{-i}$ *if* $\lambda^i(t^i)(E) = 1, i = a, b$*. Denote*

$$B^i(E) = \{t^i \in T^i : t^i \text{ believes } E\}, i = a, b$$

the set of player i*'s types that believe the event* E*.*

For each player i, denote R_1^i the set of all rational strategy-type pairs (s^i, t^i). Thus R_1^{-i} stands for the set of all rational strategy-type pairs of opponent $-i$, i.e.

$$R_1^{-i} = \{(s^{-i}, t^{-i}) \in S^{-i} \times T^{-i} : (s^{-i}, t^{-i}) \text{ is rational.}\}, i = a, b$$

Now we can define a player's believing in his or her opponent's rationality as player i believes an event $E = R_1^{-i}$:

Definition 4. *Player i's type t^i* ***believes his or her opponent's rationality*** *$R_1^{-i} \subseteq S^{-i} \times T^{-i}$ if $\lambda^i(t^i)(R_1^{-i}) = 1$. Denote*

$$B^i(R_1^{-i}) = \{t^i \in T^i : t^i \textit{ believes } R_1^{-i}\}, i = a, b$$

the set of player i's types that believe opponent $-i$'s rationality.

Then for all $m \in \mathbf{N}$ and $m > 1$, we can define R_m^i inductively by

$$R_m^i = R_{m-1}^i \cap (S^i \times B^i(R_{m-1}^{-i})), i = a, b$$

And write $R_m = R_m^a \times R_m^b$. Then players' higher order beliefs of rationality is defined in the following way:

Definition 5. *If a state $(s^a, s^b, t^a, t^b) \in R_{m+1}$, we say that there is* ***rationality and mth-order belief of rationality*** *(RmBR) at this state.*
If a state $(s^a, s^b, t^a, t^b) \in \cap_{m=1}^{\infty} R_m$, we say that there is ***rationality and common belief of rationality*** *(RCBR) at this state.*

For a perfect-information sequential move game such as the centipede game, in case the game situation involves the players not playing the backward-induction path (BI path thereafter), we also need to describe players' beliefs of probability-0 events. We use the tool of *conditional probability systems* (CPS thereafter) introduced by Renyi's [33]. It consists of a family of conditional events and one probability measure for each of these events. For the centipede game under analysis, we define player i ***initially believes event E*** if i's CPS assigns probability 1 to event E at the *root* of the perfect-information game tree. We denote the set of player i's types that initially believe event E as $\mathrm{IB}^i(E), i = a, b$. We also define player i ***strongly believes event E*** if for any information set H that is reached, i.e. $E \cap (H \times T^{-i}) \neq \emptyset$, i's CPS assigns probability 1 to event E. We denote the set of a player's types who strongly believe event E as $\mathrm{SB}^i(E), i = a, b$.

3 Experimental Design

We experimentally investigate players' rationality, beliefs and higher-order beliefs about opponents' rationality. Section 3.1 describes the treatments and testing hypothesis. Section 3.2 details the laboratory environments, belief elicitation, and other experimental procedures.

3.1 Treatments and Hypothesis

Our experiment consists of three treatments, each of which is a three-legged centipede game. The first treatment, "Baseline Centipede Game" is shown in Figure 1. Both the Nash equilibrium outcome and Subgame Perfect equilibrium outcome involve player A choosing OUT at the first stage and the two players ending up with a $20-10$ split of payoffs.

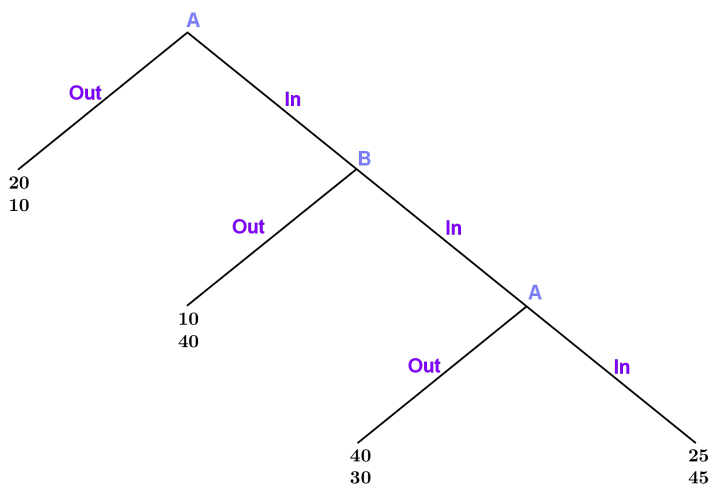


Figure 1: Baseline Centipede Game

Here we emphasize three feature of this baseline game: first, the same as the "backward induction paradox" discussed in the theoretical literature, the sum of the players' payoffs grows at each stage. Had the players not played the BI path, the outcome would yield the players a larger *sum* of payoffs. We call this an *efficient* outcome. Second, had player A played IN at both of his/her decision stages and had player B played IN at his/her decision stage, the game would end up with a mutually beneficial $25-45$ payoff split. This is because 25 is greater than 20, the payoff that player A gets if he/she plays OUT at the first stage; and 45 is greater than 40, the payoff that player B gets if he/she plays OUT at the second stage. Third, allowing for probabilistic belief, it is easy to calculate that if player

A expects player B to play IN with a probability greater than $\frac{1}{3}$, his/her best response is to play IN for the first stage and OUT for the third stage. As for player B, if he/she expects player A to play IN at the third stage with a probability greater than $\frac{2}{3}$, his/her best response is to play IN for the second stage.

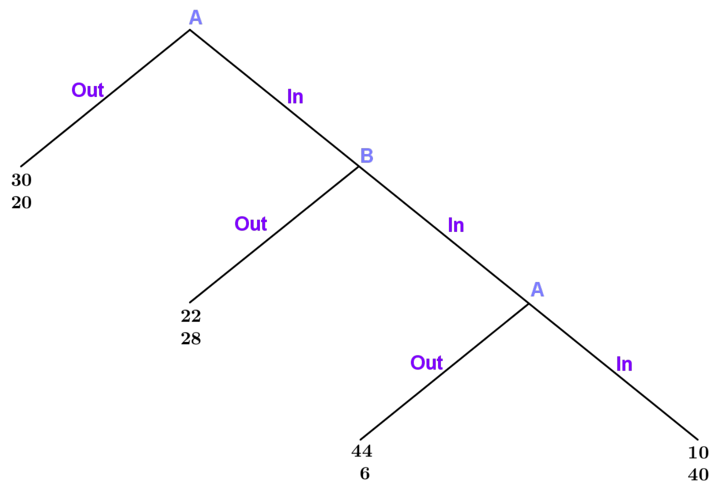


Figure 2: Constant-Sum Centipede Game

Our second treatment, the "Constant-Sum Centipede Game," is shown in Figure 2. The sum of the players' payoffs at all stages is a constant. This version of the centipede game eliminates the *efficient concern* presented in the Baseline Centipede treatment; and similar experimental treatments *without* examining players' beliefs has been conducted by Fey et.al. [15], Levitt et.al. [22]. We choose the constant-sum payoff to be 50 because this is the actual average sum of the payoffs subjects earned in the laboratory in the Baseline Centipede treatment. And we choose the split of the players' payoffs at each stage such that the cutoff probabilistic belief for each player is the same as that in the Baseline Centipede treatment. Namely, if player A expects player B to play IN with $p \geq \frac{1}{3}$, his/her best response is to play IN-OUT; if player B expects player A to play IN at the third stage with $q \geq \frac{2}{3}$, his/her best response is to play IN for the second stage.

Notice that the Constant-Sum Centipede excludes both the *efficiency* property and *mutual-beneficial payoff* property from the Baseline Centipede. To further investigate the key driving force underlying the observed differences between the first two treatments, we conduct a "No-Mutual-Benefit Centipede" treatment as shown in Figure 3. The sum of the players' payoffs at each stage remains the same as in the Baseline Centipede; the only change is the $15-55$ split of payoffs had both players played the IN-IN-IN path. In this case player A's cutoff probabilistic belief for playing IN-OUT remains as $\frac{1}{3}$, whereas player B's cutoff probabilistic belief for playing IN changes to $\frac{2}{5}$.

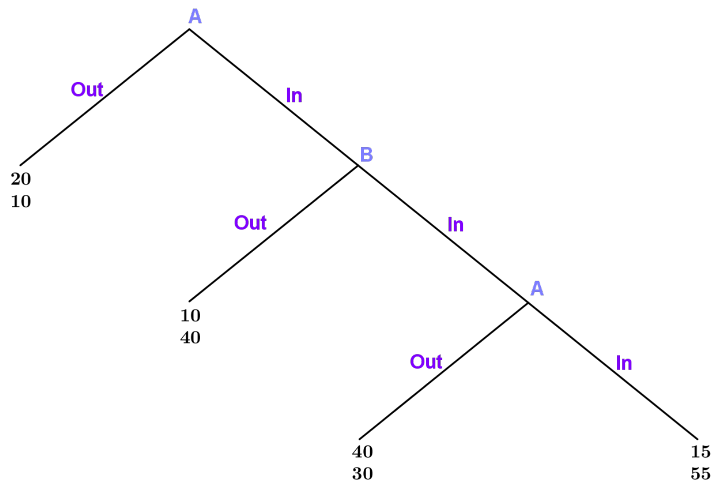


Figure 3: No-Mutual-Benefit Centipede Game

Table 1 below summarizes the treatments and number of sessions, subjects, and matches of games for each treatment.

Table 1: Experimental Treatments

Treatments	# of Sessions	# of Subjects	Total # of Games
Baseline Centipede	5	60	450
Constant-Sum	5	60	450
No-Mutual-Benefit	3	36	270

Next, we list the testing hypotheses as comparisons between the treatments, and as comparisons with theoretical predictions. Our first set of hypotheses are on the players' strategy choices. In all three treatments, the Nash equilibrium outcome involves the game ending as player A plays OUT at the first stage; and the Subgame Perfect equilibrium prescribes both player's choosing OUT at each's decision stage(s). Therefore we have the following hypotheses:

Hypothesis 1. *In all three treatments, the frequency of player A's choosing IN at the first stage does not differ significantly from* 0*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 2. *In all three treatments, the frequency of player B's choosing IN at the second stage does not differ significantly from* 0*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

The second set of hypotheses describes players' rationality. As defined in Section 2, a player is rational if his or her strategy choice maximizes the expected payoffs *given* his or her belief. A fully rational player best responds to both the initial belief and conditional belief with probability 1. In Appendix 7.1 we demonstrate the following hypotheses by proving five observations.

Hypothesis 3. *If player A is* ***rational****, then in all three treatments, the frequency of A's strategy best responding to A's belief does not significantly differ from* 1*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 4. *If player B is* ***rational****, then in all three treatments, the frequency of B's strategy best responding to B's belief does not significantly differ from* 1*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

We then move to players' belief about the opponents' rationality and common belief of rationality. As defined in Section 2, a player believes one's opponent being rational if he/she assigns probability 1 to all the states (s^{-i}, t^{-i}) in which opponent $-i$'s strategy best responds to the belief in that state. For player A, this probability is the one he/she states before the game starts. For player B, the probability he/she assigns to A's strategy-belief pair at the *root* of the game tree is the *initial* belief, while the probability he/she assigns once called upon to move at the second stage (if observed) is the *conditional* belief in the definition of "strong belief" in Section 2. Thus we have the following hypotheses. In Appendix 7.1 we prove them by demonstrating five observations.

Hypothesis 5. *If* ***rationality and common strong belief of rationality*** *holds, then in all three treatments, the frequency of A's believing B's choosing IN at the second stage does not significantly differ from* 0*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 5 comes from the fact that *rationality common strong belief of rationality (RCSBR)* implies that player A's believing in B's rationality, believing in B's (initially and conditionally) believing A's rationality, and so on. Thus as shown in Section 7.1, there is no state that involves player A's believing player B's choosing IN satisfying RCSBR.

Hypothesis 6. *If* ***common belief of rationality*** *holds, then in all three treatments, the frequency of B's believing A's rationality does not significantly differ from* 1*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 7. *If* ***rationality and common initial belief of rationality*** *holds, then in all three treatments, the frequency of B's* ***initially*** *believing in A's rationality* and 2*nd-Order rationality does not differ significantly from* 1*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 8. *If* ***rationality and common strong belief of rationality*** *holds, then in all three treatments, the frequency of B's* ***conditionally*** *believing in A's rationality* and 2*nd-Order rationality does not differ significantly from* 1*. Specifically, this frequency in the Baseline Centipede treatment does not differ significantly from that in the Constant-Sum treatment.*

Hypothesis 6 comes from the fact that *common belief of rationality* implies player B's (both initially and conditionally) assigning probability 1 to the event of A's rationality. Hypothesis 7 comes from the fact that *rationality and common* ***initial*** *belief of rationality* implies player B's assigning probability 1 to A's rationality AND A's believing B's rationality at the *root* of the game tree. Hypothesis 8 is from the fact that **rationality and common *strong* belief of rationality** implies player B's still assigning probability 1 to A's rationality and 2nd-order rationality even after observing A has chosen IN for the first stage.

3.2 Design and Procedure

All sessions were conducted at the Pittsburgh Experimental Economics Lab (PEEL) in Spring 2013. A total of 156 subjects are recruited from the undergraduate population of the University of Pittsburgh who have no prior experience in our experiment. The experiment adopts between-subject design, with 5 sessions for the Baseline Centipede treatment, 5 sessions for the Constant-Sum treatment, and 3 sessions for the No-Mutual-Benefit treatment. The experiment is programmed and conducted with z-Tree (Fischbacher [16]).

Upon arrival at the lab, we seat the subjects at separate computer terminals. After we have enough subjects to start the session[4], we hand out instructions and then read the instruction aloud. A quiz which tests the subjects' understanding of the instruction follows. We pass the quiz's answer key after the subjects finish it, explaining in private to whomever have questions.

[4]Each session has 12 subjects. We over-recruit as many as 16 subjects each time. By arrival time, from the 13th subject on, we pay them a $5.00 show-up fee and ask them to leave.

In each session, 12 subjects participate in 15 rounds of one variation of the centipede game. Half of the subjects are randomly assigned the role of Member A and the other half the role of Member B. The role remain fixed throughout the experiment. In each round, one Member A is paired with one Member B to form a group of two. The two members in a group would then play the centipede game in that treatment. Subjects are randomly rematched with another member of the opposite role after each round.

For the aim of collecting enough data, we first use strategy method to elicit the subjects' strategy choice[5]. We ask the subjects to specify their choice at *each* decision stage had it been reached. Then the subjects' choice(s) are carried out automatically by the programme and one would not have a chance to revise it if one's decision stage is reached.

After the subjects finish the choice task, they enter a "forecast task" phase which is to elicit their beliefs about opponent's choices. Member A is asked to choose from one of the two statements which he/she thinks more likely[6]: "Member B has chosen IN" or "Member B has chosen OUT." Member A's predictions are incentivized by a linear rule: 5 points if correct, 0 if incorrect. Member B is informed that his/her partner A has made a selection of choices for stage 1 and 3, AND have chosen a statement about Member B's choice. Then Member B's are asked to enter six numbers as the percent chance into a table, each cell of which represents a choice-forecast pair that Member A has chosen. For example, as shown in the table below, the upper-left cell represents the event that Member A has chosen OUT for the 1st stage and "Statement I."

Statement I	■		
Statement O			
	1st Stage *Out*, 3rd *In* or *Out*	1st Stage *In*, 3rd Stage *Out*	1st Stage *In*, 3rd Stage *In*

If B's decision stage is reached (which means his/her partner Member A has chosen IN), he/she will be asked to make a second forecast about the percent chance for each possible

[5]Another advantage of the strategy method is to exclude subjects' incentives to signal, hedge, or bluff their opponent. Had we not adopted this method, in the baseline treatment we would have observed an even higher frequency of player A's choosing IN for the first stage. Player A might find it optimal to "bluff opponent" if player B is tempted by the *efficient* and *mutually beneficial* payoff split in the Baseline Centipede treatment AND B would not strongly believe A's rationality after observing A's choosing IN for the first stage.

[6]Since in all treatments player A's cutoff probabilistic belief is $\frac{1}{3}$, which is smaller than 50 percent, the point prediction Member A is making here is without loss of generality.

outcome of A's choices. Member B's predictions are incentivized by the quadratic rule:

$$5 - 2.5 \times [(1 - \beta_{ij})^2 + \sum_{kl \neq ij} \beta_{kl}^2]$$

where β_{kl} stands for Member B's stated percent chance in row k column l of the table, and i, j represents that row i column j is the outcome from Member A's choices[7].

At the end of the experiment, one round is randomly selected to count for payment. A subject's earning in each round is the sum of the points he/she earn from the choice task and the forecast task(s). The exchange rate between points and US dollars is 2.5 : 1. A subject receivers his/her earning in that selected round plus the \$5.00 show-up fee.

4 Experimental Findings

4.1 Players' Strategy Choices

Our first set of results compares the frequency of players' strategy choices with that predicted by the Subgame Perfect equilibrium. We first state the result addressing Hypothesis 1, then move to the result addressing Hypothesis 2.

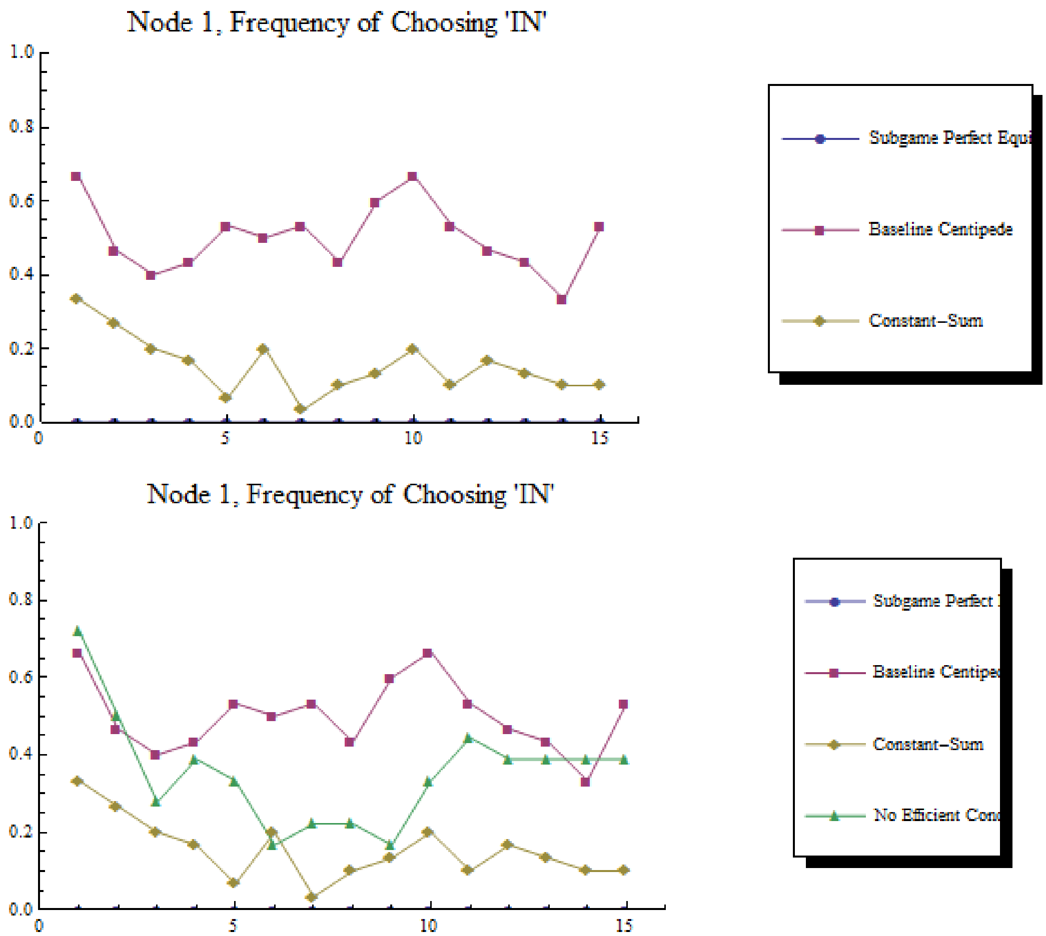
Result 1. *(1) In all three treatments, the average frequency of A's choosing IN at the first stage is significantly higher than* 0. *(2) The average frequency of A's choosing IN at the first stage in the Constant-Sum treatment is significantly lower than that in the Baseline Centipede treatment.*

Result 1 addresses Hypothesis 1. Figure 4 depicts the treatment-average frequency of player A's choosing IN at the first stage across all periods. This frequency in the Constant-Sum treatment is significantly lower than that in the Baseline treatment; but both of them are significantly higher than 0, the Subgame Perfect equilibrium prediction.

It is natural to ask what these player A's would play given that they had deviated from the equilibrium path. Namely, what is the frequency of choosing strategy IN-OUT

[7] Palfrey and Wang [28] and Wang [36] have discussed eliciting subjects' beliefs using *proper* scoring rules. This is the major reason we adopt a quadratic scoring rule. We are also aware of the risk-neutrality assumption behind the quadratic rule and the possibility to use an alternative belief elicitation method proposed by Karni [19]. But concerning the complexity of explaining Karni's method to the subjects, we adopt the quadratic rule which is simpler in explanation.

Figure 4: Average Frequency of A's Strategy Choice, Across Periods

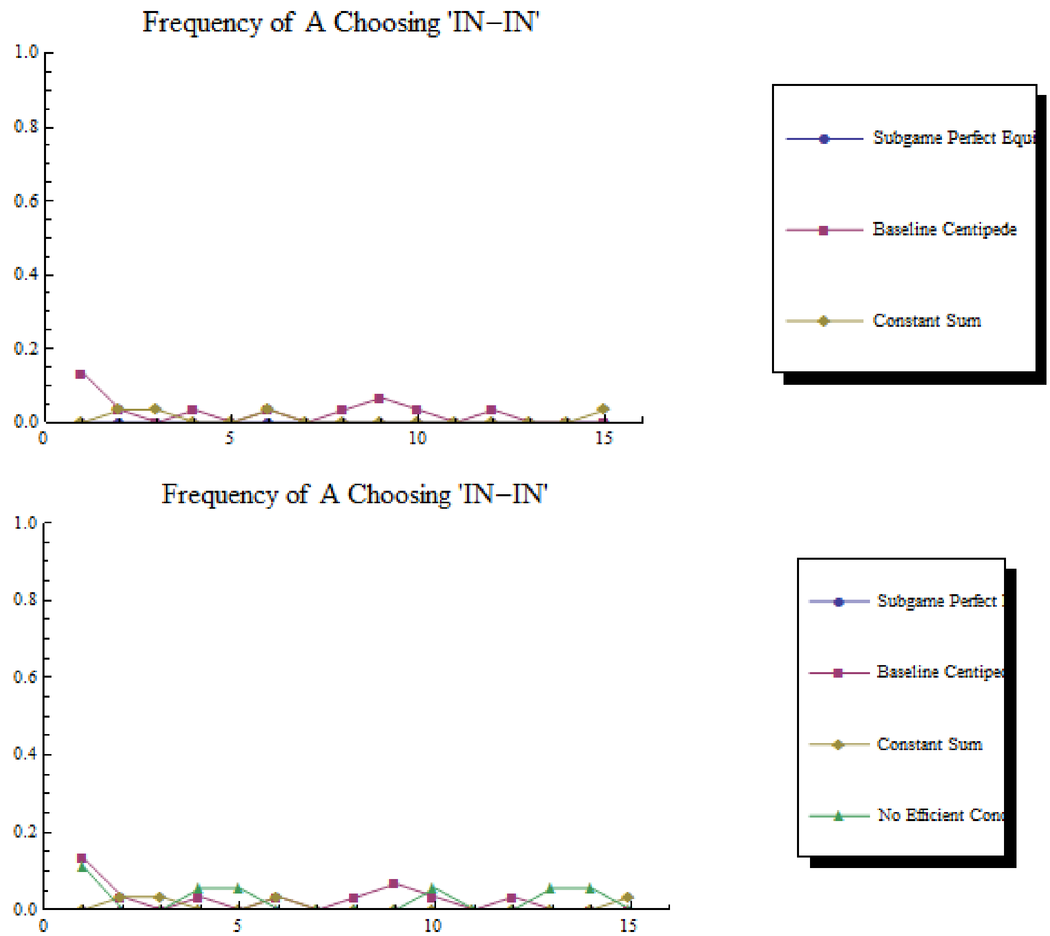


Note: Figure on top compares the average frequency of A's choosing IN predicted by the Subgame Perfect equilibrium (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

versus the frequency of choosing IN-IN? Figure 5 depicts the treatment-average frequency of player A's choosing strategy IN-IN. It is interesting to note that the frequency in all three treatment is not significantly different from 0; and there is no significant difference across treatments. Notably, this is true even for the Baseline Centipede treatment. In other words, despite the *efficiency* property and *mutual benefit* property of the Baseline Centipede, actual plays seldom end up with the "mutually beneficial" $25-45$ payoff split. Conditional on the third node being reached, almost all player A's optimally choose OUT for the third stage.

Result 2. *(1) In all three treatments, the average frequency of B's choosing IN at the second stage is significantly higher than* 0*. (2) The average frequency of B's choosing IN at the second stage in the Constant-Sum treatment is significantly lower than that in the*

Figure 5: Average Frequency of A's Choosing IN-IN at Both Decision Stages, Across Periods

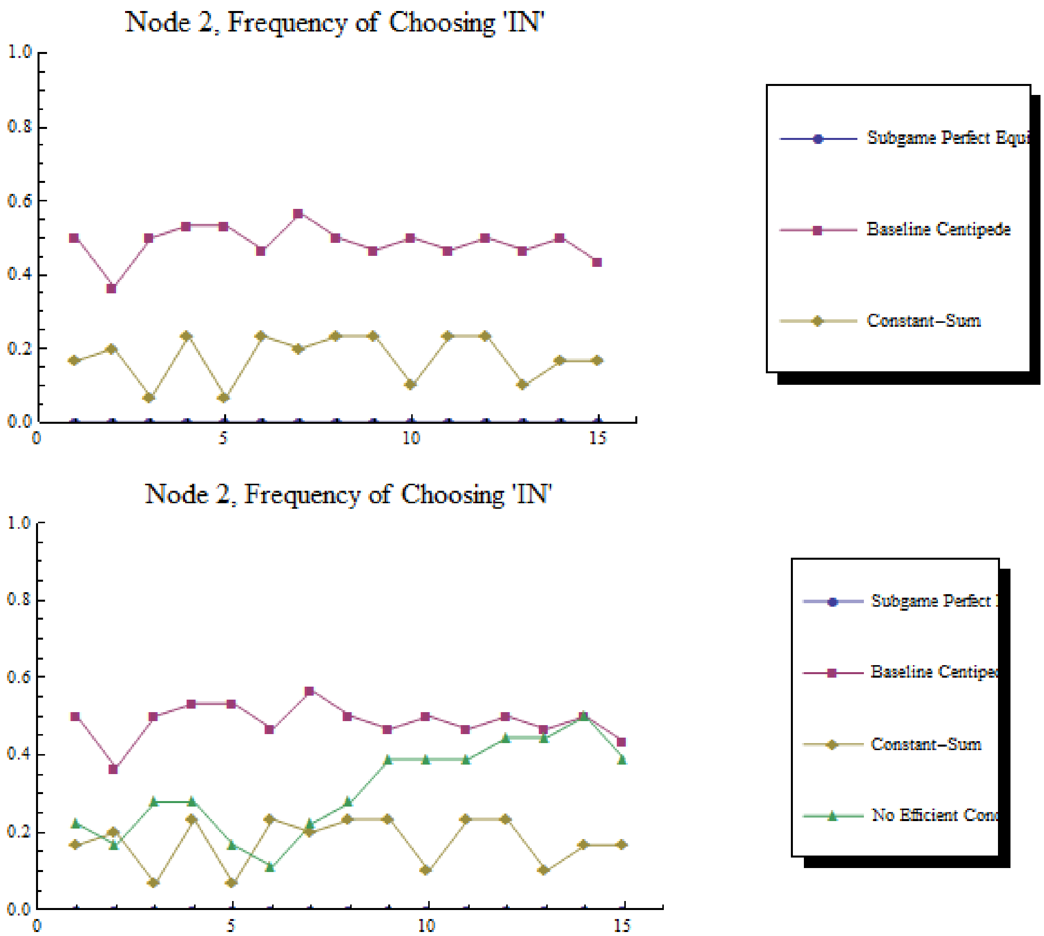


Note: Figure on top compares the average frequency of A's choosing IN at the first stage and IN at the third stage predicted by the Subgame Perfect equilibrium (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

Baseline Centipede treatment.

Result 2 addresses Hypothesis 2. Figure 6 depicts the treatment-average frequency of player B's choosing IN at the second stage across all periods. This frequency in the Constant-Sum treatment is significantly lower than that in the Baseline treatment; but both of them are significantly higher than 0, the Subgame Perfect equilibrium prediction.

Figure 6: Average Frequency of B's Strategy Choice, Across Periods



Note: Figure on top compares the average frequency of B's choosing IN predicted by the Subgame Perfect equilibrium (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

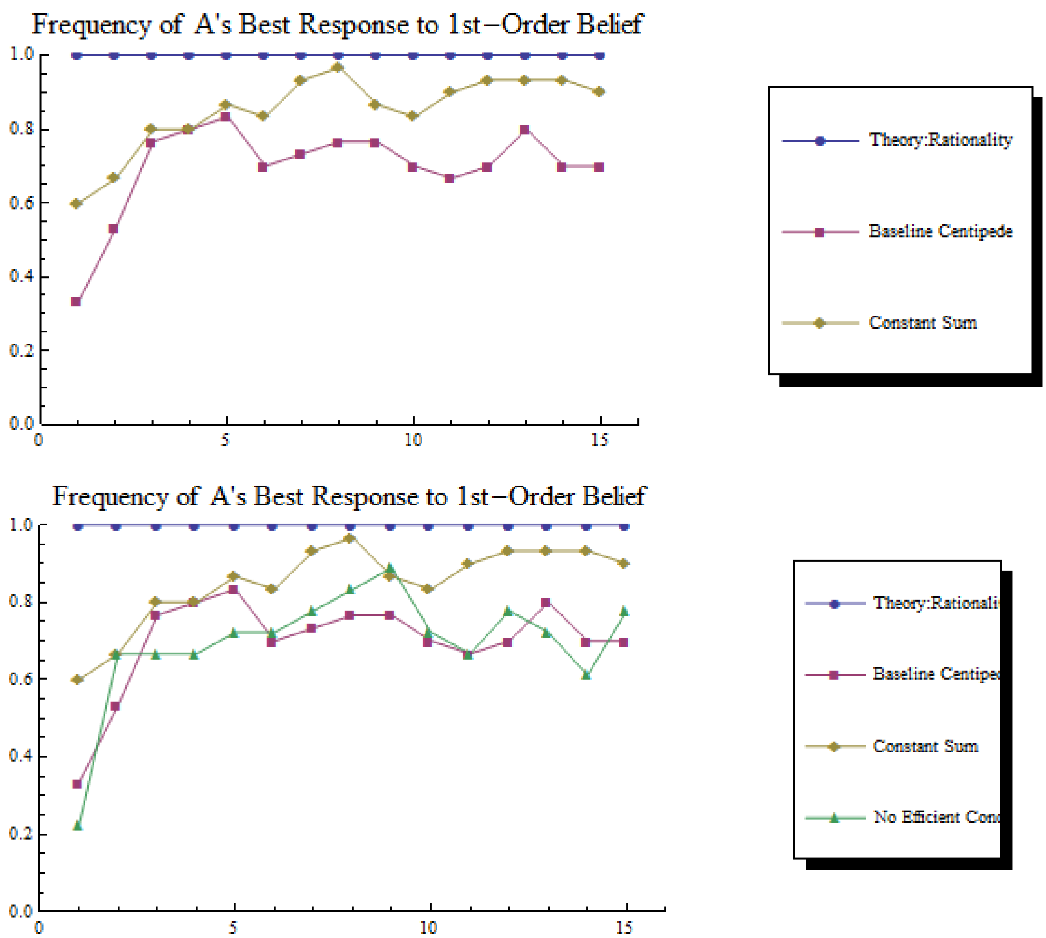
4.2 Rationality

In this section we present comparison results on players' rationality across treatments. We first examine the frequency of A's best responding to his/her stated belief. Notice that there are two data points from A's strategy-belief choices that can be identified as "rational." Either player A chooses strategy IN-OUT and believes that B has chosen IN, or chooses OUT for the first stage and believes that B has chosen OUT. We sum up the frequencies from the two cases as we calculate the overall frequency of A's being rational.

Result 3. *(1) In all three treatments, the average frequency of player A's being rational is significantly lower than* 1. *(2) The average frequency of player A's being rational in the Constant-Sum treatment is significantly higher than that in the Baseline Centipede* *treatment.*

Result 3 addresses Hypothesis 3. Figure 7 depicts the treatment-average frequency of player A's being rational across all periods. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatment; but both of them are significantly lower than 1 as required by the notion of *rationality*.

Figure 7: Average Frequency of A's Best Responding to Own Belief, Across Periods



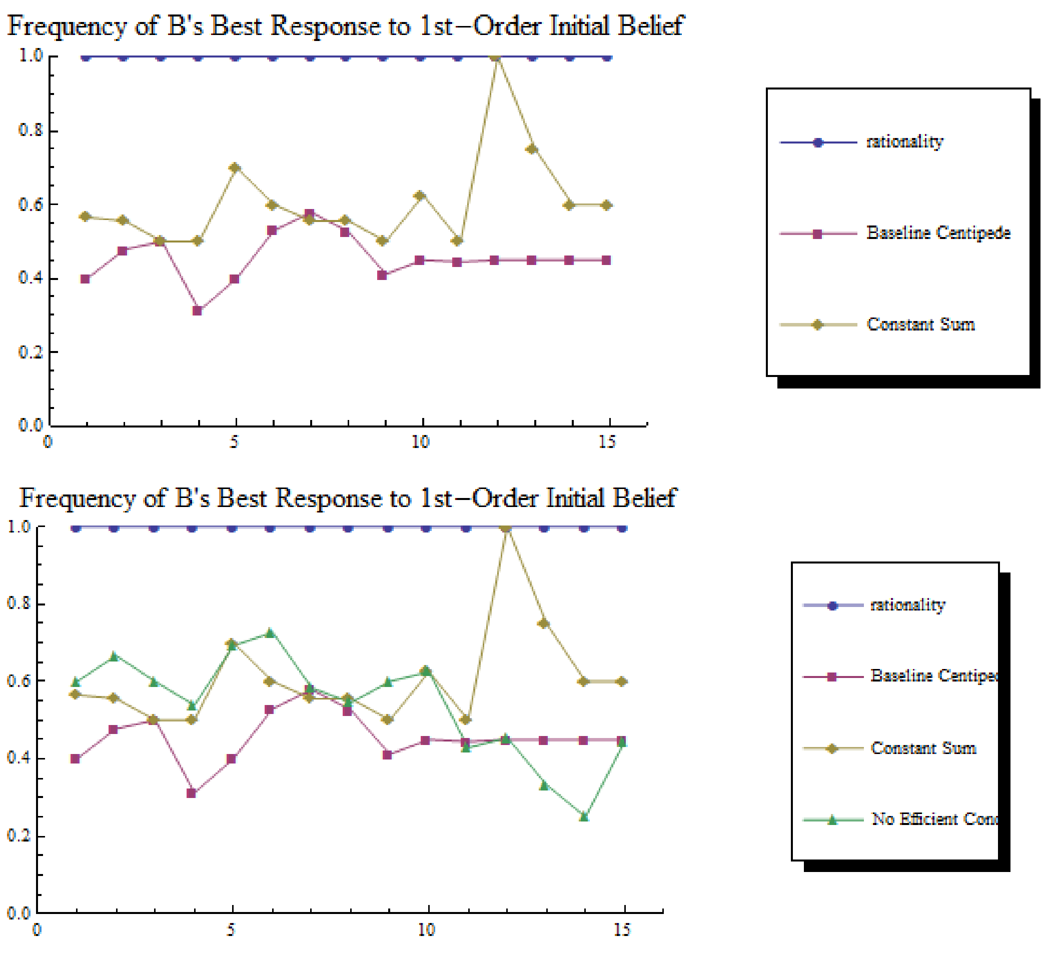
Note: Figure on top compares the average frequency of A's best responding to his/her stated belief if A is rational (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

We then investigate the frequency of B's best responding to his/her stated belief. From B's stated belief, if the probability he/she assigns to A's choosing strategy IN-IN is greater than his/her cutoff probabilistic belief, it is rational for B to choose IN for the second stage; otherwise, it is rational to choose OUT for the second stage. We sum up the frequencies from the two cases as we calculate the overall frequency of B's being rational.

Result 4. *(1) In all three treatments, the average frequency of player B's being rational is significantly lower than* 1*. (2) The average frequency of player B's being rational in the Constant-Sum treatment is not significantly different from that in the Baseline Centipede treatment.*

Result 4 addresses Hypothesis 4. Figure 8 depicts the treatment-average frequency of player B's being rational across all periods. This frequency in the Constant-Sum treatment is not significantly higher than that in the Baseline treatment; and both of them are significantly lower than 1 as required by the notion of *rationality*.

Figure 8: Average Frequency of B's Best Responding to Own Belief, Across Periods



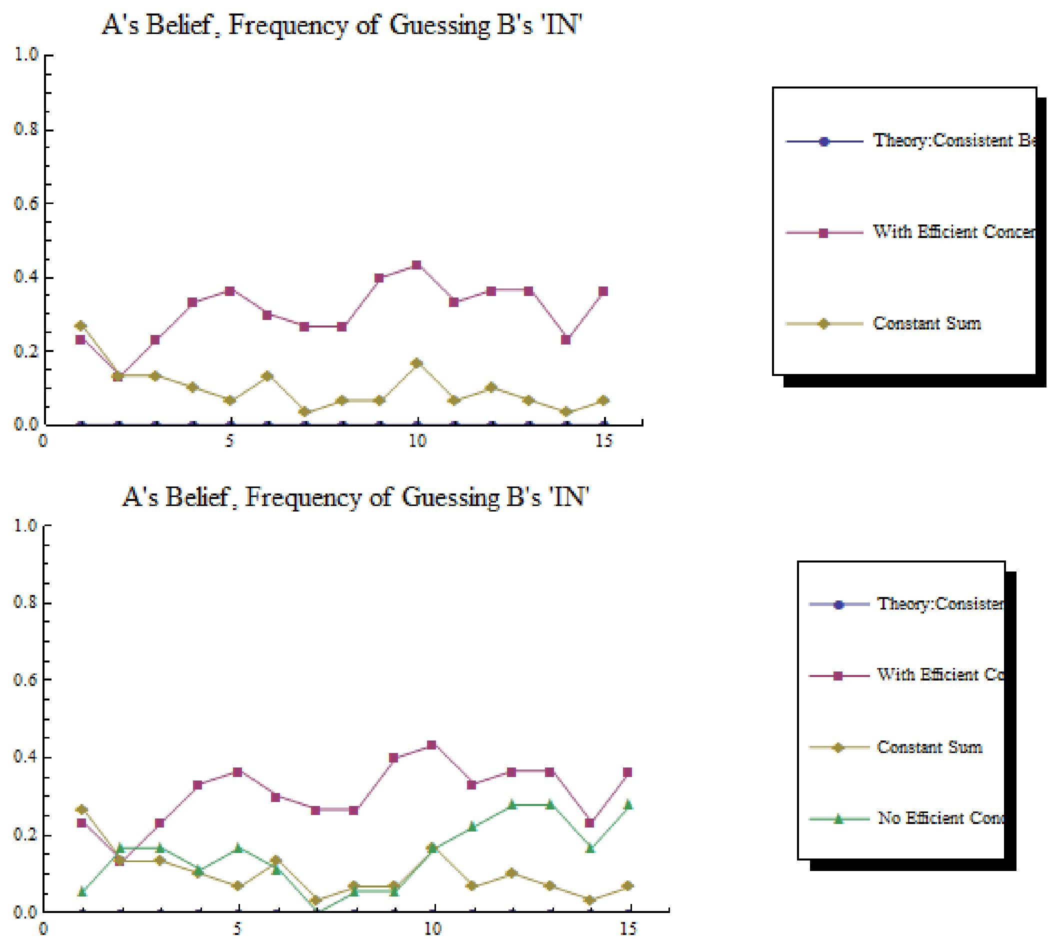
Note: Figure on top compares the average frequency of B's best responding to his/her own belief if B is rational (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

4.3 Belief of Rationality and Higher-Order Belief of Rationality

In this section we present comparison results on players' belief of rationality and higher-order belief of rationality across treatments. We first examine the frequency of A's believing B's choosing IN for the second stage.

Result 5. *(1) In all three treatments, the average frequency of player A's believing B's choosing IN is significantly higher than* 0*. (2) The average frequency of player A's believing B's choosing IN in the Constant-Sum treatment is significantly lower than that in the Baseline Centipede treatment.*

Figure 9: Average Frequency of A's Believing B's Choosing IN, Across Periods



Note: Figure on top compares the average frequency of A's believing B's rationality and 2nd-Order rationality if RCSBR holds (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

Result 5 addresses Hypothesis 5. As shown in Section 3.1, if *rationality and common strong belief of rationality* holds, in all states player A should not believe that B would ever chosen IN for the second stage. Figure 9 depicts the treatment-average frequency of player A's believing B's choosing IN across all periods. This frequency in the Constant-Sum treatment is significantly lower than that in the Baseline treatment; but both of them are significantly higher than 0 as required by the notion of RCSBR. The comparison of player A's belief accuracy across treatments is included in Appendix 7.2.

We then examine player B's believing A's rationality. If player B's stated belief assigns a sum of probability 1 to the two cases in which player A is rational (either A chooses strategy IN-OUT and believes B has chosen IN, or A chooses OUT for the first stage and believes B has chosen OUT), we say that player B believes A's rationality.

Result 6. *(1) In all three treatments, the average frequency of player B's believing A's rationality is significantly lower than* 1*. (2) The average frequency of player B's believing A's rationality in the Constant-Sum treatment is significantly higher than that in the Baseline Centipede treatment.*

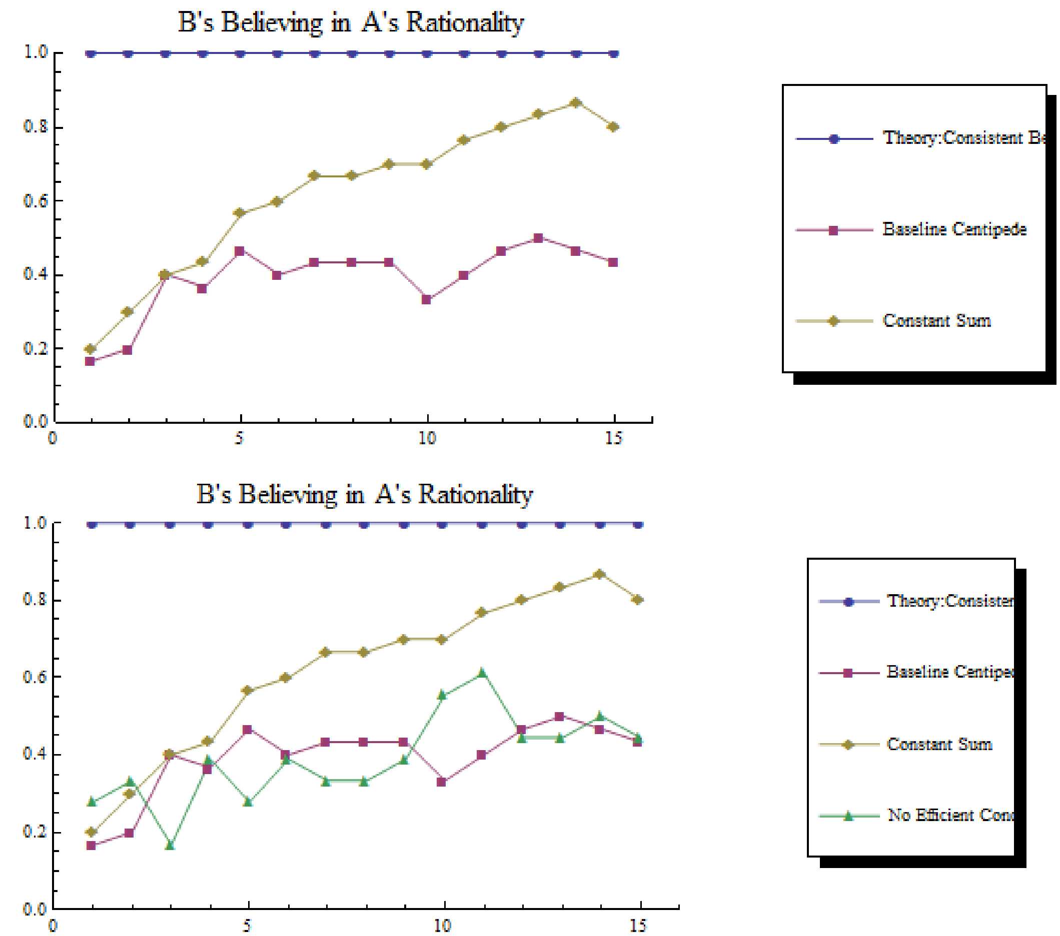
Result 6 addresses Hypothesis 6. Figure 10 depicts the treatment-average frequency of player B's believing A's rationality across all periods. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatment; but both of them are significantly lower than 1 as required by the notion of *common belief of rationality*. It is also worth noting that in the Constant-Sum treatment this frequency increases towards 1 gradually as more rounds are played.

Next we examine player B's believing A's rationality AND believing A's believing B's rationality (2nd-Order rationality). If player B's initial belief assigns probability 1 to the event that player A chooses OUT for the first stage and believes B has chosen OUT, we say that player B initially believes A's rationality and 2nd-Order rationality.

Result 7. *(1) In all three treatments, the average frequency of player B's initially believing A's rationality and 2nd-order rationality is significantly lower than* 1*. (2) The average frequency of player B's initially believing A's rationality and 2nd-order rationality in the Constant-Sum treatment is significantly higher than that in the Baseline Centipede treatment.*

Result 7 addresses Hypothesis 7. Figure 11 depicts the treatment-average frequency of player B's believing A's rationality and 2nd-order rationality across all periods. This frequency in the Constant-Sum treatment is significantly higher than that in the Baseline treatment; but both of them are significantly lower than 1 as required by the notion of

Figure 10: Average Frequency of B's Believing A's Rationality, Across Periods



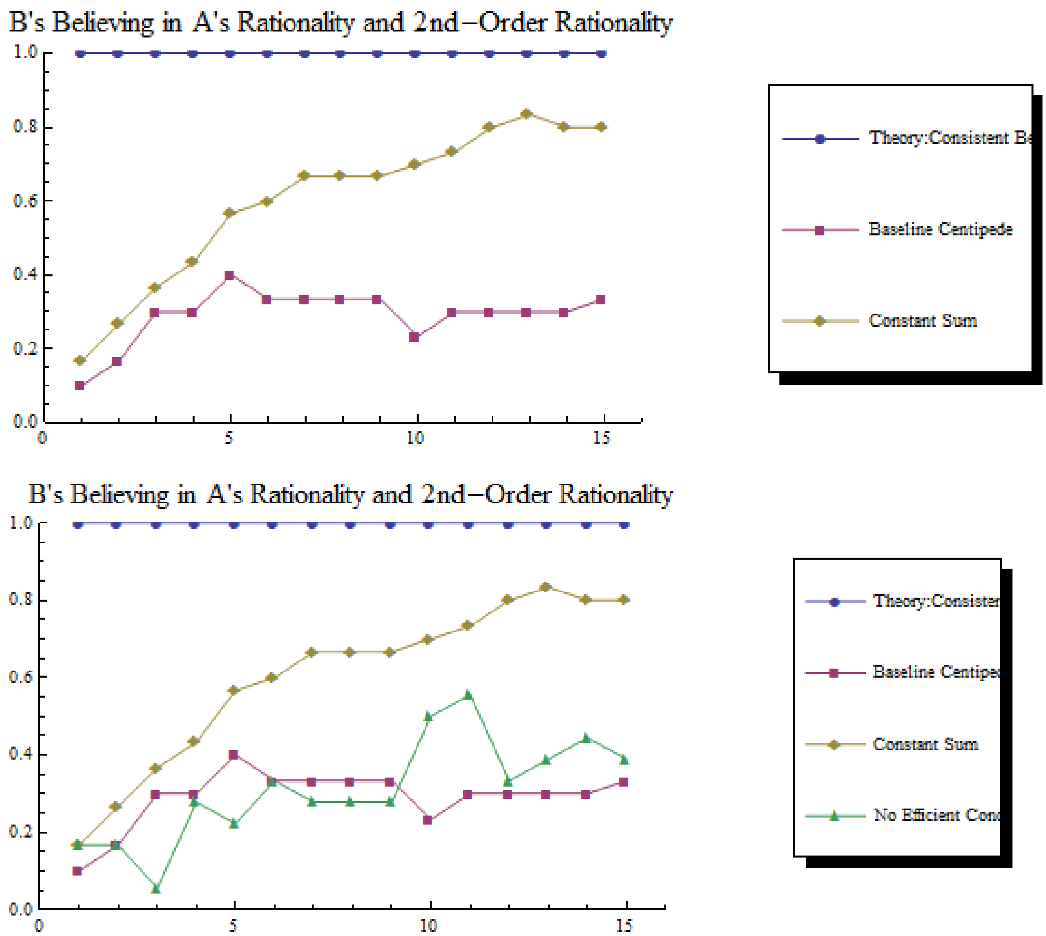
Note: Figure on top compares the average frequency of B's believing in A's rationality if *common belief in rationality* holds (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

rationality and common initial belief of rationality. It is also worth noting that in the Constant-Sum treatment this frequency increases towards 1 gradually as more rounds are played.

Last we look into player B's strongly believing A's rationality AND 2nd-Order rationality conditional on A has chosen IN for the first stage. If player B's conditional belief assigns probability 1 to the event that player A chooses strategy IN-OUT and believes B has chosen IN, we say that player B strongly believes A's rationality and 2nd-Order rationality.

Result 8. *(1) In all three treatments, the average frequency of player B's strongly believing A's rationality and 2nd-order rationality is significantly lower than* 1. *(2) The average frequency of player B's strongly believing A's rationality and 2nd-order rationality in the Constant-Sum treatment is not significantly different from that in the Baseline Centipede*

Figure 11: Average Frequency of B's Believing A's Rationality and 2nd-Order Rationality, Across Periods

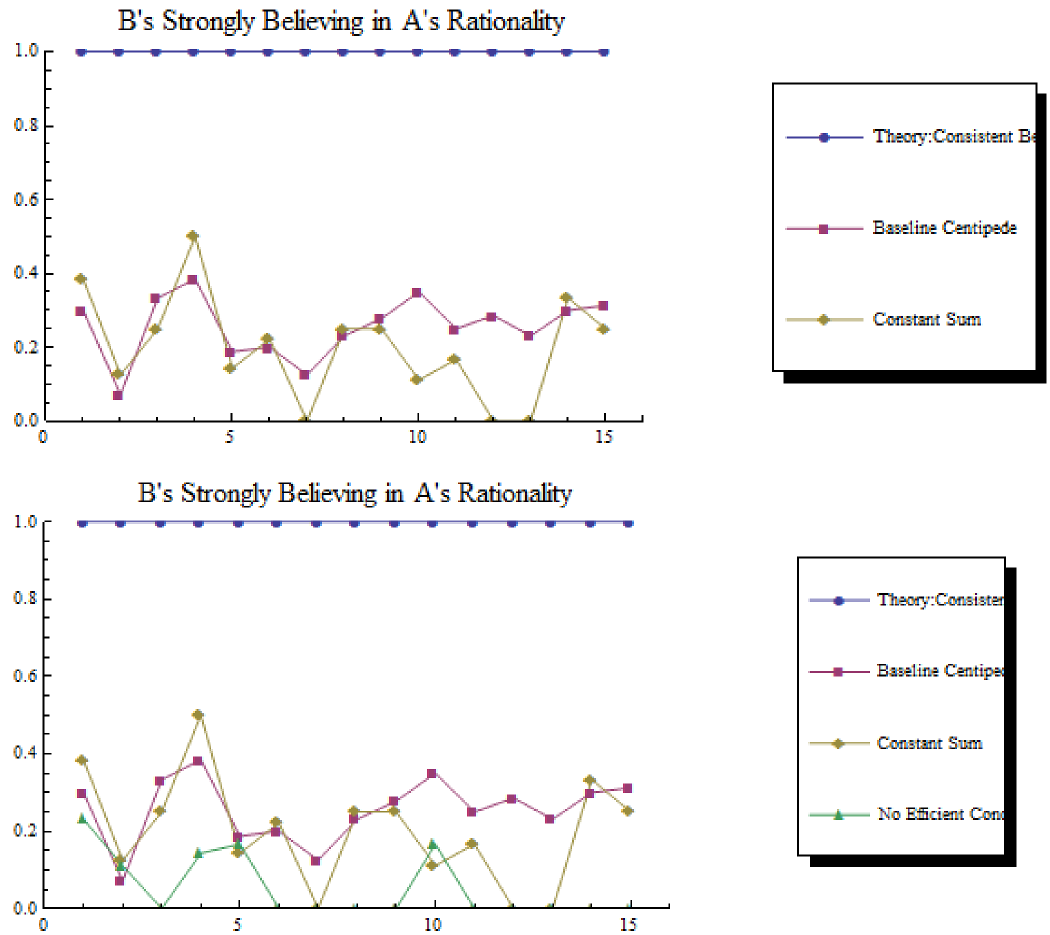


Note: Figure on top compares the average frequency of B's believing A's rationality and 2nd-Order rationality if RCIBR holds (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

treatment.

Result 8 addresses Hypothesis 8. Figure 12 depicts the across-period treatment-average frequency of player B's strongly believing A's rationality and 2nd-order rationality conditional on B is informed that A has chosen IN for the first stage. This frequency in the Constant-Sum treatment is not significantly different from that in the Baseline treatment; and both of them are significantly lower than 1 as required by the notion of *rationality and common strong belief of rationality.* In other words, once player B observes player A's deviating from the equilibrium path, B hardly believes A's being rational AND A's believing B's rationality.

Figure 12: Average Frequency of B's Strongly Believing A's Rationality and 2nd-Order Rationality, Across Periods



Note: Figure on top compares the average frequency of B's believing in A's rationality and 2nd-Order rationality if RCSBR holds (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

5 Related Literature

McKelvey and Palfrey's [24] seminal centipede game experiment shows individuals' behavior inconsistent with standard game theory prediction. Neither do they find convergence to subgame perfect equilibrium prediction as subjects gain experience in later rounds of the experiment. The authors attribute such inconsistent behavior to uncertainties over players' payoff functions; specifically, the subjects might believe a certain fraction of the population is altruist. They establish a structural econometric model to incorporate players' selfish/altruistic types, error probability in actions, and error probability in beliefs. If most of the players are altruistic, the altruistic type always chooses PASS except on the last node while the selfish type might mimic the altruist for the first several moves as in standard reputation models. As pointed out, the equilibrium prediction of this incomplete information game is sensitive to the beliefs about the proportion of the altruistic type. In our design we try to avoid this complication by allowing sorting.

Subsequent experimental studies on centipede games tend to view this failure of backward induction as individuals' irrationality. Fey et.al. [15] examine a constant-sum centipede game which excludes the possibility of Pareto improvement by not backward inducting. Among the non-equilibrium models, they find that the Quantal Response Equilibrium, in which players err when playing their best responses, fit the data best. Zauner [37] estimates the variance of uncertainties about players' preferences and payoff types and makes comparison between the altruism models and the quantal response models. Kawagoe and Takizawa [20] offer an alternative explanation for the deviations in centipede games adopting level-k analysis. They claim that the level-k model provide good predictions for the major features in the centipede game experiment without the complication to incorporate incomplete information on "types." Nagel and Tang [26] investigate centipede games in a variation of the strategy method: the game is played in the reduced normal form, which is considered as "strategically equivalent" to the extensive form counterpart, but precisely to identify "learning." They examine behavior across periods according to learning direction theory. They show significant differences in patterns of choices between the cases when a player has to split the cake before her opponent and when she moves after her opponent.

Consequently, other research tries to restore the subgame perfect equilibrium outcome by providing the subjects with aids in their decision-making processes. Bornstein et. al. [17] show that groups tend to terminate the game earlier than individual players, once free communication is allowed within each group. Maniadis [23] examines a set of centipede games with different stakes and finds that providing aggregate information causes strong convergence to the subgame perfect equilibrium outcome. However, after uncertainties are incorporated into the payoff structure, the effect of information provision shifts in the opposite direction. Rapoport et. al. [30] study a three-person centipede game. They show that when the number of players increases and the stakes are sufficiently high, results converge to theoretical predictions more quickly. But when the game is played with low stakes, both convergence to equilibrium and learning across iterations of the stage game are weakened. Palacios-Huerta and Volij [27] cast their doubt on average people's full rationality by recruiting expert chess players to play a field centipede. Strong convergence to subgame perfect prediction is observed.

It is helpful to link the attempted arguments for the experimental centipede game anomaly with literature on equilibrium refinement. These concepts – trembling hand perfection, stability, sequential equilibrium, and forward induction – attempt to distinguish whether a deviation to the off-equilibrium path is by mistake or by intention.

"Mistakes in actions" corresponds to Selten's [34] *trembling hand perfectness.* This refinement does not "completely" exclude a small probability of "irrational" errors, while it requires the limit of such irrationality coinciding with fully rational choices. Practically, if in sequential games such as the centipede game, a player fails to do the backward calculations and chooses "Pass," the next mover should believe that such an event results from the first mover's mistake.

Uncertainty about payoff types, namely, players being altruistic, follows the *stability* argument by Kohlberg and Mertens [21], who consider the correlation among un-scheduled deviations. The probability of trembling is excluded in their refinement concept, as the authors state, "the probabilities of error must not be interpreted as probabilities that the players will ACTUALLY err." Therefore, players' apparently irrational choices are made deliberately and strategically, e.g. the player is altruistic.

Errors-in-belief roughly echoes the concept of *sequential equilibrium* proposed by Kreps and Wilson [25]. This concept requires players' beliefs being consistent with completely mixed strategies at each information set, where "completely mixed" excludes information sets that will not be reached with positive probability. Therefore, once a player sees "Pass" from the previous mover, he knows that he is on an "off-equilibrium" path. As noted by Govindan and Wilson [18], a player can hold any beliefs off the equilibrium path.

Efficiency concerns come from van Damme's [35] *forward induction* argument. He points out that the weaker equilibrium refinement concepts do not require back-to-rational behavior in later stages of the game after somebody's off-equilibrium action, either by mistakenly or deliberately. For example, in the centipede game, the second mover believes that the first mover will continue to be insane all through the rest of the game. On the contrary, van Damme [35] captures the off-schedule intention which looks irrational but is actually not. This forward induction argument indicates that any unexpected deviation from certain equilibrium should be viewed as rational and more ambitious strategies are expected in succeeding stages. His argument suggests that a player will never make mistake and behaves rationally all the way to the end, if there are potentially better payoffs.

6 Conclusion and Discussion

This paper explores people's beliefs behind non-backward induction behavior in laboratory centipede games. We elicit the first mover's belief about the second mover's strategy as well as the second mover's initial and conditional beliefs about the first mover's strategy and 1st-order belief. The measured beliefs help me infer the conditional probability systems of both players. The inferred CPS's and players' actual strategy choices identify why they fail to reach the BI outcomes. First, we examine whether the player's strategies are best response to the stated beliefs. In both the Baseline Centipede treatment and the Constant-Sum treatment, the frequency of players' best responding to own beliefs is significantly lower than 1. Specifically, the frequency in the Constant-Sum treatment is higher than that in the Baseline treatment; and the frequency in the No-Mutual-Benefit treatment is not significantly different from that in the Baseline treatment. Second, we investigate players' belief of opponents' rationality and higher-order belief of rationality. In all treatments, both the frequency of players' believing in others' rationality and the frequency of higher-order belief of rationality are significantly smaller than 1. Nevertheless, the frequency in the Constant-Sum treatment dominates that in the Baseline and the No-Mutual-Benefit treatment. Third, when it comes to the second mover's conditional beliefs once the first-mover has chosen a non-BI strategy, the frequency of the second movers' strongly believing the first movers' rationality is very low; and there is no significant different across treatments.

7 Appendix

7.1 Proofs and Calculations

This section demonstrating the hypotheses specified in the main text by proving five observations. The first observation is about B's belief of A's rationality. The rest four observations identify the states that satisfy RCIBR and RCSBE. In summary, when the strategy choices and inferred CPS's constitute a state that satisfies *rationality and common strong belief of rationality* (RCSBR henceforth), players *do not* fail to reach the backward induction (BI henceforth) outcome in this state. But the reverse is not true. It is possible that Role A's strategy choice leads to the BI outcome, but Role B's strategy and belief are not consistent with RCSBR. Moreover, there exists a state in which Role B's strategy and belief are consistent with the BI outcome but Role A's are not. There also exists a state in which neither player's strategy and belief are consistent with the BI outcome, but a weaker notion of common belief in rationality, *rationality and common initial belief of rationality*, still holds.

For the east of demonstration, we alter the notations of the players' moves slightly, as shown in Figure 13. Since we shall prove the following observations for all three treatments, we use (x_j, y_j) to represent the players' payoffs associated with each terminal node. And u^A represents Statement OUT, t^A represents Statement IN in the instruction.

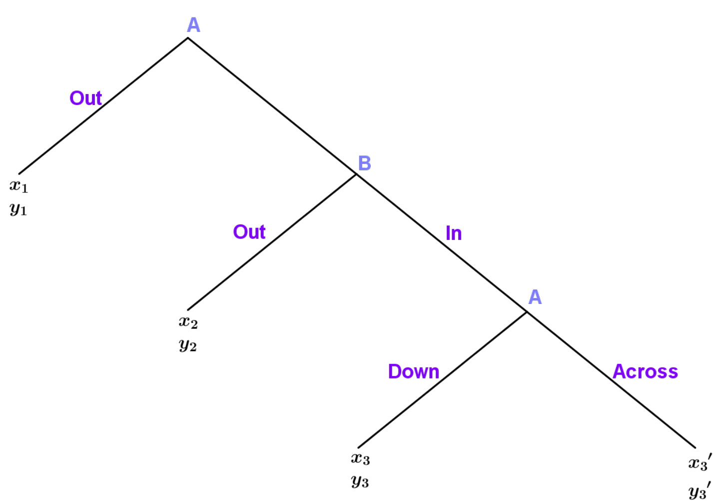


Figure 13: The Centipede Game

Observation 1. *From the measured initial belief of player B,*

u^A	β_{11}	β_{12}	β_{13}
t^A	β_{21}	β_{22}	β_{23}
	Out	*Down*	*Across*

if $\beta_{11} + \beta_{22} = 1$, *player B initially believes player A's rationality.*

From the measured conditional belief of player B,

u^A	γ_{12}	γ_{13}
t^A	γ_{22}	γ_{23}
	Down	*Across*

if $\gamma_{22} = 1$, *player B strongly believes player A's rationality and 2nd-Order rationality.*

Observation 2. If the following data point is observed, the players' strategies and beliefs constitute a state that satisfies RCSBR:

- Role A chooses *Out* and statement u^A
- Role B chooses *Out* and the measured beliefs take the form:

u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[1]	0[0]
	Out	Down	Across

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Observation 3. If the following data point is observed, Role B's strategy and belief are not consistent with RCSBR. Nevertheless, the BI outcome still obtains.

- Role A chooses *Out* and statement u^A
- Role B chooses *In* and the measured beliefs take the form:

u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[0]	0[1]
	Out	Down	Across

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Remark: *1st-order strong belief of rationality* of both players because Role B's measured belief indicates that he does not *strongly* believe Role A's rationality. However, since Role A chooses *Out* at the first node, the BI outcome still obtains. Although RCSBR does not hold in this state, a weaker notion, *rationality and common initial belief of rationality* (RCIBR), still holds. RCIBR only requires the belief consistency given the root of the game tree.

Observation 4. If the following data point is observed, Role B's strategy and belief are consistent with the BI outcome. But the BI outcome does not obtain.

- Role A chooses *Down* and statement t^A

- Role B chooses *Out* and the measured beliefs take the form:

u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[1]	0[0]
	Out	Down	Across

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Remark: In this state there is no *1st-order* ***strong*** *belief of rationality*, nor *1st-order* ***initial*** *belief of rationality* because Role A's measured belief indicates that she does not strongly, nor initially believe Role B's rationality. Since Role A chooses *Down* and Role B chooses *Out*, the BI outcome does not obtain. The game ends at the second node by Role B's playing *Out*.

Observation 5. If the following data point is observed, neither player's strategy and belief is consistent with the BI outcome. The BI outcome does not obtain. Nevertheless, the strategies and beliefs constitute a state that satisfies *rationality and common* ***initial*** *belief of rationality.*

- Role A chooses *Down* and statement t^A

- Role B chooses *In* and the measured beliefs take the form:

u^A	1[0]	0[0]	0[0]
t^A	0[0]	0[0]	0[1]
	Out	Down	Across

Note: The first number in each cell represents Role B's belief in task (2). The second number in [] represents Role B's revised belief in task (3).

Remark: In this state there is no *1st-order strong belief of rationality* of both players because (1) Role B's measured belief indicates that he does not *strongly* believe Role A's rationality, and (2) Role A's measured beliefs indicates that she believes Role B's rationality in response to his belief, but she does not believe that Role B believes her rationality. Since Role A chooses *Down* and Role B chooses *In*, the BI outcome does not obtain. The game ends at the last node by Role A's choosing *Down*.

Compare observation 3 and 5. Role B's inferred CPS is the same, which assigns probability 0 to Role A's rationality if Role B's decision node is reached. Therefore, whenever Role B does not believe Role A's rationality conditional on a zero-probability event, Role A can attain a higher payoff by playing *Down* instead of *Out*. Both states satisfy *rationality and common* ***initial*** *belief of rationality*, but not *rationality and common* ***strong*** *belief of rationality*.

Proof for Observation 2 and 4

The inferred CPS's of both players are as follows:

$\lambda^a(t^a)$

		S^b: Out	S^b: In
T^b	t^b	0	1

$\lambda^a(u^a)$

		S^b: Out	S^b: In
T^b	t^b	1[0]	0[1]

$\lambda^b(t^b)$

		S^a: Out	S^a: Down	S^a: Across
T^a	u^a	1[0]	0[0]	0[0]
T^a	t^a	0[0]	0[1]	0[0]

We are going to show:

1. The state $(\text{Out}, u^a, \text{Out}, t^b)$ satisfies both RCIBR and RCSBR

2. The state $(\text{Down}, t^a, \text{Out}, t^b)$ satisfies neither RCIBR nor RCSBR

First notice that the strategy-type pair (Out, u^a) and (Down, t^a) are rational for player Ann. The strategy-type pair (Out, t^b) is rational for player Bob. For the **initial** belief we have:

$$\text{IB}^a(R_1^b) = \{u^a\}, \text{IB}^b(R_1^a) = \{t^b\},$$

then we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \text{IB}^a(R_1^b)) = \{(\text{Out}, u^a)\} \\ R_2^b &= R_1^b \cap (S^b \times \text{IB}^b(R_1^a)) = \{(\text{Out}, t^b)\} \end{aligned}$$

Inductively, we have $R_m^a\{(\text{Out}, u^a)\}$ and $R_m^b = (\text{Out}, t^b)$, $\forall m \in \mathbf{N}$. Therefore we have:

$$\cap_{m=1}^{\infty} R_m = (\text{Out}, u^a, \text{Out}, t^b)$$
$$\text{and } (\text{Down}, t^a, \text{Out}, t^b) \notin \cap_{m=1}^{\infty} R_m$$

As for **strong** beliefs, at the second node of the game, Bob's information set $H = \{\text{Ann would play "Down" or "Across"}\}$. Thus

$$H \times T^a = \{(\text{Down}, t^a), (\text{Down}, u^a), (\text{Across}, t^a), (\text{Across}, u^a)\}$$

Bob's type t^b is the only type who assigns probability 1 to any event E s.t. $E \cap (H \times T^a) \neq \emptyset$. So we have $\text{SB}^b(R_1^a) = \{t^b\}$.

At the first node of the game, $H = \emptyset$ for Ann. So Ann's strong beliefs at this node are degenerate. At the third node of the game, Ann's information set $H = \{\text{Bob played "In"}\}$. Both Ann's type assigns probability 1 to any event E s.t. $E \cap (H \times T^a) \neq \emptyset$. So we have $\text{SB}^a(R_1^b) = \{t^a, u^a\}$.

Inductively we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \text{SB}^a(R_1^b)) = R_1^a \\ R_2^b &= R_1^b \cap (S^b \times \text{SB}^b(R_1^a)) = \{(\text{Out}, t^b)\} \end{aligned}$$

Iterate one more level, we have:

$$\begin{aligned}
\mathrm{SB}^b(R_2^a) &= \mathrm{SB}^b(R_1^a) = \{t^b\} \\
\mathrm{SB}^a(R_2^b) &= \{t^a \in T^a : \forall H \text{ s.t. } R_2^b \cap (H \times T^b) \neq \emptyset, \lambda^a(t^a)(R_2^b) = 1\} \\
&= \{u^a\}
\end{aligned}$$

and

$$R_3^a = R_2^a \cap (S^a \times \mathrm{SB}^a(R_2^b)) = \{(\mathrm{Out}, u^a)\}$$
$$R_3^b = R_2^b \cap (S^b \times \mathrm{SB}^b(R_2^a)) = \{(\mathrm{Out}, t^b)\}$$

Then we have $\mathrm{SB}^b(R_3^a) = \{t^b\}$ and $\mathrm{SB}^a(R_3^b) = \{u^a\}$. For any $m \geq 3$, we have $R_m^a = \{(\mathrm{Out}, u^a)\}$ and $R_m^b = \{(\mathrm{Out}, t^b)\}$. Thus:

$$\cap_{m=1}^{\infty} R_m = \{(\mathrm{Out}, u^a, \mathrm{Out}, t^b)\}$$

Therefore, the only state that satisfies *rationality and common strong belief of rationality* is $(\mathrm{Out}, u^a, \mathrm{Out}, t^b)$.

The results are summarized in the following table:

State	RCIBR	RCSBR
$(\mathrm{Down}, t^a, \mathrm{Out}, t^b)$	$\times$	$\times$
$(\mathrm{Out}, u^a, \mathrm{Out}, t^b)$	$\surd$	$\surd$

Proof for Observation 3 and 5

The inferred CPS's of both players are:

$\lambda^a(t^a)$

T^b		Out	In
	u^b	0	0
	t^b	0	1
		S^b	

$\lambda^a(u^a)$

T^b		Out	In
	u^b	1[0]	0[0]
	t^b	0[0]	0[1]
		S^b	

$\lambda^b(t^b)$

T^a	u^a	1[0]	0[0]	0[0]
	t^a	0[0]	0[0]	0[1]
		Out	Down	Across
			S^a	

We are going to show:

- Both states $(\text{Down}, t^a, \text{In}, t^b)$ and $(\text{Out}, u^a, \text{In}, t^b)$ satisfy RCIBR but not RCSBR.

We first identify strategy-type pairs that are rational. For player Ann, it is easy to show that strategy $s^a = \text{Down}$ maximizes type t^a's expected payoff and strategy $s^a = \text{Out}$ maximizes type u^a's expected payoff. For player Bob, strategy $s^b = \text{In}$ maximizes type t^b's expected payoff. Thus we have:

$$\begin{aligned} R_1^a &= \{(\text{Down}, t^a), (\text{Out}, u^a)\} \\ R_1^b &= \{(\text{In}, t^b)\} \end{aligned}$$

Initial beliefs:

Both Ann's type t^a and u^aassign probability 1 to (In, t^b), so we have $\text{IB}^a(R_1^b) = \{t^a, u^a\}$. Bob's type t^b assigns probability 1 to $(\text{Out}, u^a) \in R_1^a$, so we have $\text{IB}^b(R_1^a) = \{t^b\}$.

Then we have:

$$\begin{aligned} R_2^a &= R_1^a \cap (S^a \times \text{IB}^a(R_1^b)) \\ &= \{(\text{Down}, t^a), (\text{Out}, u^a)\} \cap (\{\text{Out}, \text{Down}, \text{Across}\} \times \{t^a, u^a\}) \\ &= R_1^a \end{aligned}$$

Similarly, $R_2^b = R_1^b$, and $R_3^a = R_2^a \cap (S^a \times \text{IB}^a(R_2^b)) = R_2^a = R_1^a$. Mathematical induction gives:

$$R_m^a = R_{m-1}^a = R_1^a \Rightarrow R_{m+1}^a = R_m^a = R_1^a$$

Similar result for Bob. Therefore we have

$$\begin{aligned} R_m &= R_m^a \times R_m^b = R_{m-1} \\ &\Rightarrow \cap_{m=1}^{\infty} R_m = R_1^a \times R_1^b \\ &= \{(\text{Down}, t^a, \text{In}, t^b), (\text{Out}, u^a, \text{In}, t^b)\} \end{aligned}$$

Thus both states satisfy *rationality and common initial belief of rationality.*

Strong beliefs:

At the second node of the game, Bob's information set $H = \{$Ann would play "Down" or "Across"$\}$. Thus

$$H \times T^a = \{(\text{Down}, t^a), (\text{Down}, u^a), (\text{Across}, t^a), (\text{Across}, u^a)\}$$

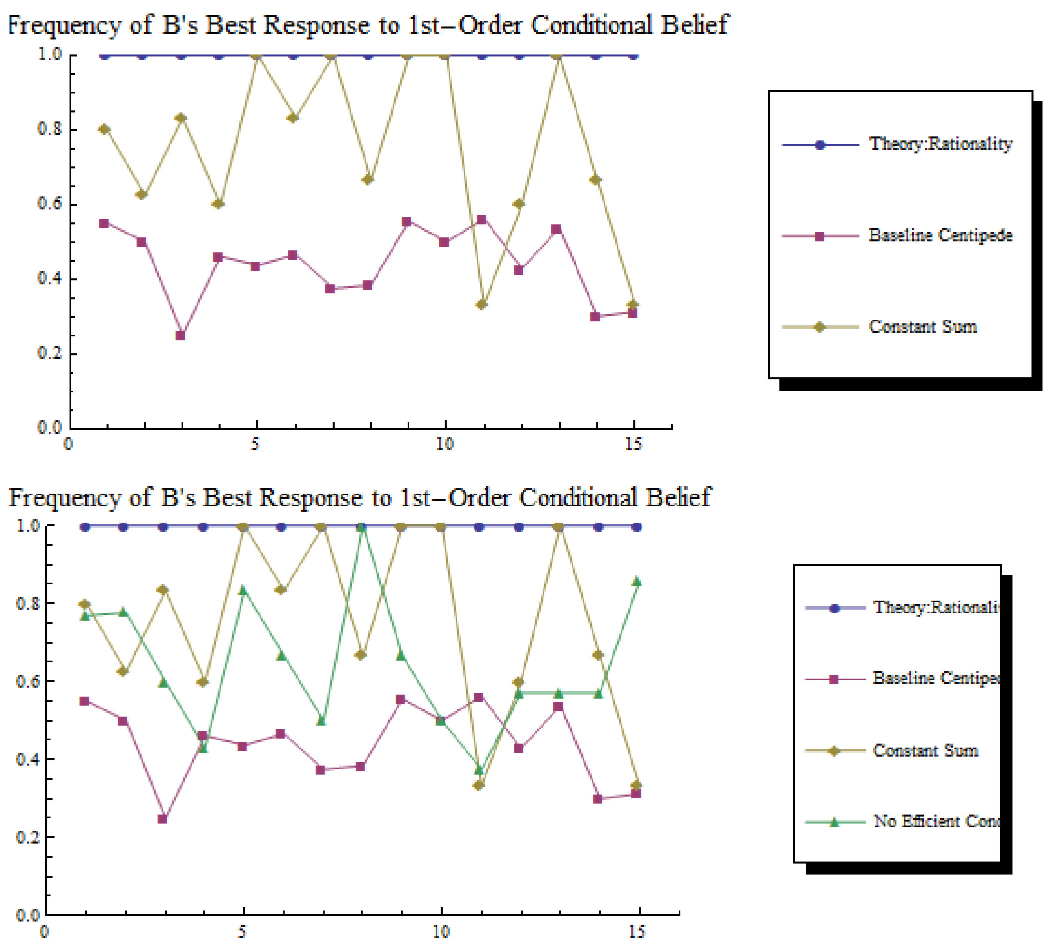
Bob's type t^b assigns probability 0 to $(\text{Down}, t^a) \in R_1^a$, but assigns probability 1 to $(\text{Across}, t^a) \notin R_1^a$. So we have $\text{SB}^b(R_1^a) = \emptyset$. Thus $\cap_{m=1}^{\infty} R_m = \emptyset$. No state belongs to $\emptyset$. Hence neither state satisfies RCSBR.

The results are summarized in the following table:

State	RCIBR	RCSBR
$(\text{Down}, t^a, \text{In}, t^b)$	$\surd$	$\times$
$(\text{Out}, u^a, \text{In}, t^b)$	$\surd$	$\times$

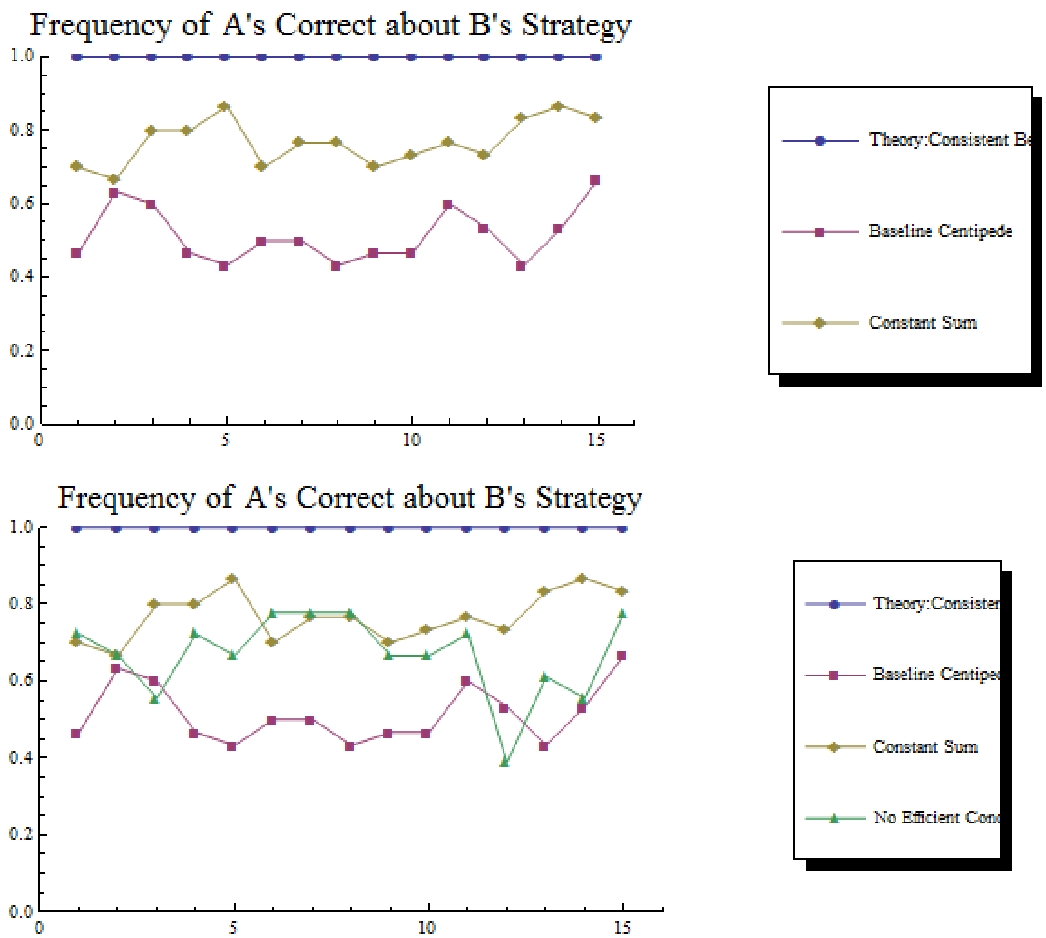
7.2 Other Figures and Tables

Figure 14: Average Frequency of B's Best Responding to Own Conditional Belief, Across Periods



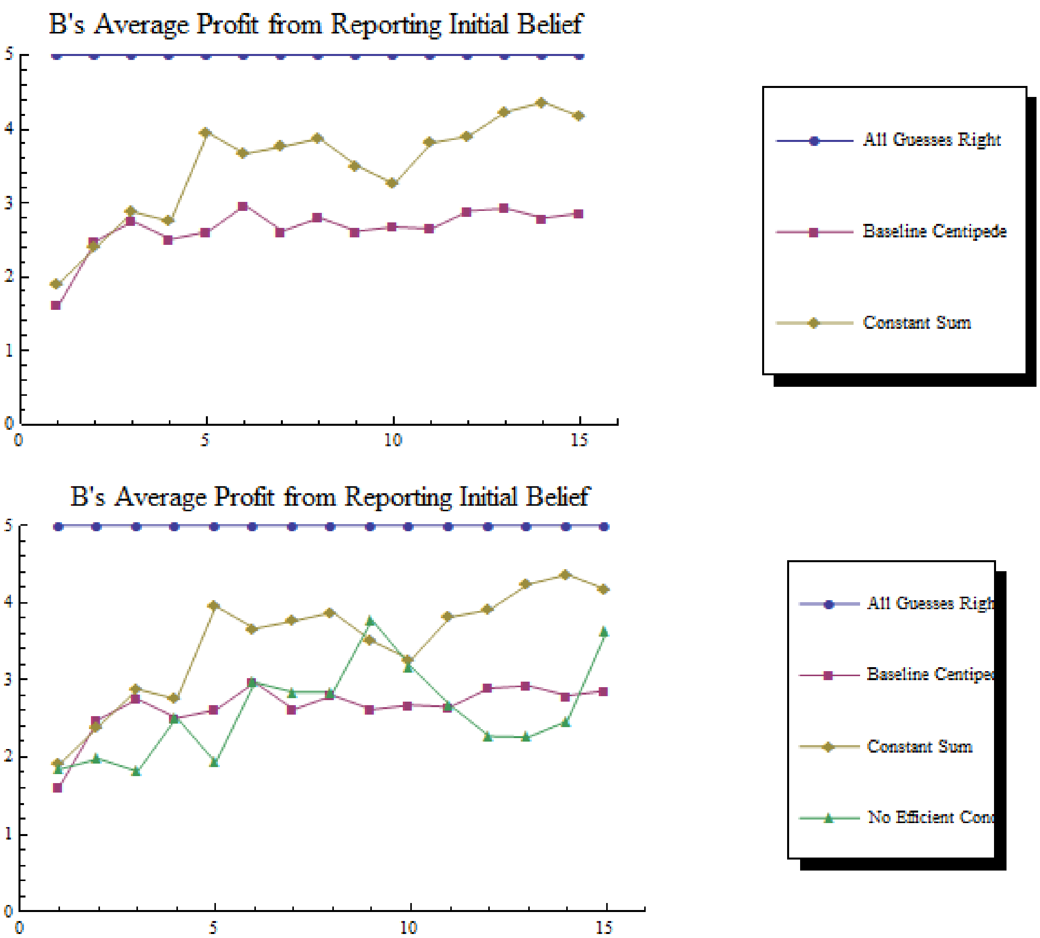
Note: Figure on top compares the average frequency of B's best responding to his/her stated conditional belief if B is conditionally consistent (blue curve), the frequency from the Baseline Centipede treatment (purple curve), and the frequency from the Constant-Sum treatment (yellow curve). Figure at bottom adds the average frequency from the No-Mutual-Benefit treatment (green curve), to the comparison.

Figure 15: Accuracy of A's Belief, Across Periods



Note: Figure on top compares the average accuracy of A's belief if RCBR holds (blue curve), the actual accuracy from the Baseline Centipede treatment (purple curve), and the actual accuracy from the Constant-Sum treatment (yellow curve). Figure at bottom adds the actual accuracy from the No-Mutual-Benefit treatment (green curve), to the comparison.

Figure 16: Accuracy of B's Belief, Across Periods



Note: Figure on top compares the average accuracy of B's initial belief if RCIBR holds(blue curve), the actual accuracy from the Baseline Centipede treatment (purple curve), and the actual accuracy from the Constant-Sum treatment (yellow curve). Figure at bottom adds the actual accuracy from the No-Mutual-Benefit treatment (green curve), to the comparison.

7.3 Laboratory Instructions

INSTRUCTIONS

Welcome! Thank you for participating in this experiment. This experiment studies decision-making between two individuals. In the following one hour or less, you will participate in 15 rounds of decision making. Please read the instructions carefully; the cash payment you earn at the end of the experiment may depend on how well you understand the instructions and make your decisions accordingly.

Your Role and Decision Group

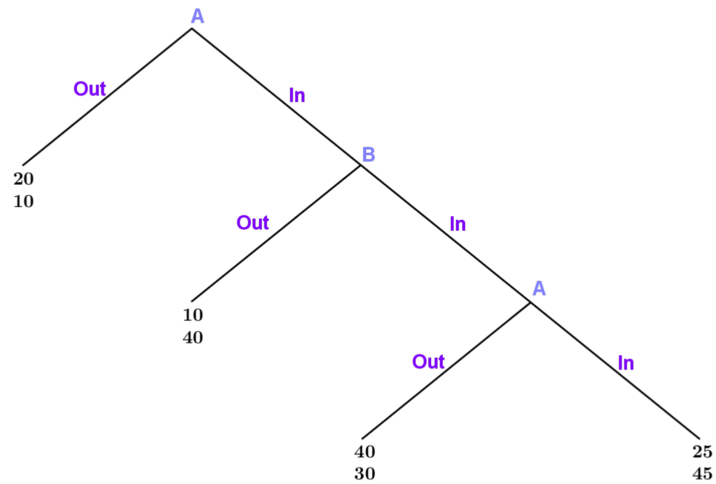
Half of the participants will be randomly assigned the role of Member A and half will be assigned the role of Member B. Your role will remain fixed throughout the experiment. In each round, one Member A will be paired with one Member B to form a group of two. The two members in a group make decisions that will affect their earnings in the round. Participants will be randomly rematched with another member of the opposite role after each round.

Your Choice Task(s) in Each Round

In each round, each group will face the three-stage decision task shown in Figure 17. The nodes represent choice stages, the letters above the nodes represent the member who is going to make a choice, and the numbers represent the points one will earn, with A's points on top and B's points at bottom.

- In the 1st stage A must decide between two options: *Out* or *In*. If A chooses *Out*, the task ends with A receiving 20 and B 10 points. If A chooses *In*, the task proceeds to the 2nd stage.
- In the 2nd stage B must decide between two options: *Out* or *In*. If B chooses *Out*, the task ends with A receiving 10 and B 40 points. If B chooses *In*, the task proceeds to the 3rd stage.
- In the 3rd stage A must choose again between two options: *Out* or *In*. If A chooses *Out*, A will receive 40 and B 30 points. If A chooses *In*, A will receive 25 and B 45 points.

Figure 17



Member A's Choice Task

You will be asked to specify your choices for **both** stage 1 and 3 through a computer interface. For each stage, you can choose one and only one option. Note that you will be making your choices **at the same time** your partner B is making his or her choice. So you don't know what B chooses. The choices you make here will be carried out automatically by the computer later on. You will not have an opportunity to revise them.

Member B's Choice Task

You will be asked to specify your choice for stage 2 through a computer interface. You can choose one and only one option. Note that you will be making your choice **at the same time** your partner A is making his or her choices. So you don't know what A chooses. The choice you make here will be carried out automatically by the computer later on. You will not have an opportunity to revise it.

Forecast Tasks in Each Round

Besides having the opportunity to earn points in the choice task, you will also be given the opportunity to earn extra points by making forecast(s).

Member A's Forecast Task

Your partner, Member B, has made a choice for stage 2. Please select the statement that you believe is more likely:

- **Statement I**: Member B has chosen *In*.

- **Statement O**: Member B has chosen *Out*.

You will earn 5 points if your forecast is correct (i.e. if Member B chooses *In* and you select Statement I, or B chooses *Out* and you select Statement O). You will earn nothing otherwise.

Member B's Forecast Task(s)

Your partner, Member A, has made choices for both stage 1 and 3; also, he or she is selecting between **Statement I** and **Statement O**, each of which is a statement about the choice you just made for stage 2. Which choices do you think your partner A has made for his or her stages, **and** which statement do you think your partner A is selecting?

Notice that A's selections can be expressed in the table below. The column represents A's selection of statement, the row represents A's choices for 1st and 3rd stages. So each cell represents an outcome of A's choices **and** statement. For example, the upper-left cell represents the outcome that A has chosen *Out* for 1st stage, *In* or *Out* for 3rd stage, **and** *Statement I*.

Statement I	■		
Statement O			
	1st Stage *Out*, 3rd *In* or *Out*	1st Stage *In*, 3rd Stage *Out*	1st Stage *In*, 3rd Stage *In*

Your first forecast task

Your first task is to forecast the percent chance that each of the six outcomes happens. A percent chance is a number between 0 and 100, where 100 means that you are certain that such outcome is the correct one, and 0 means that you are certain that such outcome is *not* the correct one. Enter the percent chance of each outcome into the corresponding cell. If you leave any cell as blank it will be viewed as 0. Make sure the six numbers sum up to 100.

You will earn 5 points if your forecast exactly coincide with your partner A's statement **and** choices. If your forecast does not exactly coincide with your partner A's choice and statement, you will receive 5 points minus 2.5 times a penalty amount. The penalty amount is the sum of squared distances between each of the six numbers you entered and the correct answer, i.e. the outcome from A's selection.

Example: Suppose you believe that with 80 percent chance A has chosen to play *In* for 1st and *Out* for 3rd stage, **and** has selected *Statement I*; with 15 percent chance A has chosen to play *In* for 1st and *Out* for 3rd stage, **and** has selected *Statement O*; with 5 percent chance A has chosen to play *In* for 1st and *In* for 3rd stage, **and** has selected *Statement O*, you should enter the numbers as below:

Statement I	0	80	0
Statement O	0	15	5 ■
	1st Stage *Out*, 3rd *In* or *Out*	1st Stage *In*, 3rd Stage *Out*	1st Stage *In*, 3rd Stage *In*

Now suppose your partner A has chosen *In* for 1st and *In* for 3rd stage, and has selected *statement O*. The penalty amount is $(100/100-5/100)^2+(0-80/100)^2+(0-15/100)^2=1.54$. So you earn $5-2.5*1.54=1.15$ from this forecast.

Your second forecast task

After the computer carries out your partner's and your choices, you will be informed if your partner A has chosen *In* for stage 1. Now you have a chance to make a second forecast. A four-cell table will be presented to you. (The first column of the table in your first forecast task is removed because A has chosen *In* for stage 1.) Please make a percent chance forecast again. Your penalty amount and earning point are calculated in the same way as in your first forecast task.

Final Comments

At the end of this experiment one round will be randomly selected to count for payment. Your earning in each round is the sum of the points you earn from the choice task and the forecast task(s). The exchange rate between points and US dollars is 2.5 : 1. Your cash payment will be your earning in US dollars plus the $5 show-up fee.

Your decisions and your payment will be kept confidential. You have to make decisions entirely on your own. Please do not talk to others. If you have any question at any time, raise your hand and the experimenter will come and assist you individually. Please turn off your cell phone and other electronic devices.

If you have any question, please raise your hand now. Otherwise we will proceed to the quiz.

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