STRATEGY-PROOFNESS AND ASYMPTOTIC EFFICIENCY IN EXCHANGE ECONOMIES

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1. Extended Abstract

We study classical economies: goods are divisible, preferences monotone and convex, and budget balance is required for feasibility. It is well established now that if a social choice rule in this setting is *strategy-proof* and *efficient,* it will fail even rudimentary requirements of equity. One such requirement is that there exist a reference bundle such that each agent finds their assignment under the rule at least as good as the reference bundle. Serizawa and Weymark [1] showed that no strictly positive reference bundle can be found for a rule that satisfies *strategy-proofness* and *efficiency*. Our response in this paper is to insist upon *strategy-proofness* and *individual rationality,* and to ask if we can achieve *asymptotic efficiency* in this setting, as the population size increases. A rule with these properties would be attractive for many reasons. Firstly, *strategy-proofness* is a robust form of implementation, so we can be reasonably sure of the properties of such a rule, even in small populations. *Strategy-proofness*, moreover, posesses some welfare consequences of its own: (i) no time need be spent calculating equilibrium strategies; (ii) strategically sophisticated agents cannot gain an advantage over less sophisticated agents. Furthermore, the *efficiency* properties of our most central social choice rule, the Walrasian correspondance, only hold in large economies; in finite economies, price-taking is rarely an optimal strategy. Given this manipulability of the Walrasian rule, it is concievable that there exists a *strategy-proof* rule that is preferable at every population size. A *strategy-proof* rule that has good *efficiency* properties in large economies would be a prime candidate.

Unfortunately, our findings are not encouraging. If a rule is *strategy-proof* and satisfies the *equal division lower bound,* there are many sequences of economies that, under the operation of the rule, will remain bounded away from any *efficient* allocation. We find these sequences even when restricting our study to quasilinear preferences. Moreover, even in the quasilinear domain, these sequences are not rare, as the following three conditions are sufficient: (i) the aggregate offer curve of $n-1$ agents has non-zero curvature for each n; (ii) the Walrasian price ratio is bounded away from zero; (iii) the second derivative of the utility function bounded above.

Thankfully this is not the end of the story. For the case of 2 goods, we find a *strategy-proof* rule that, properly calibrated, is *asymptotically efficient* on a small domain. The rule calculates for each agent i her marginal rate of substitution s_i through the point of equal division. The rule chooses a value s_{i^*} from this list based upon its parameters. Agents are then uniformly rationed along the line of slope s_{i^*} through equal division. The slope s_{i^*} is chosen in a manner that is *strategy-proof* for any domain of preferences satisfying the gross-substitutes condition. It is clear that the pivotal agent i^* has no incentive to manipulate. Other agents can become pivotal only at the cost of not trading, or, if they

move to the other side of the market, by paying a sort of "premium" for their increased demand of the other good. We further show that in fact, for a one-dimensional domain $\mathcal{D}$ satisfying the *single-crossing* condition, the parameters can be chosen such that the rule is *asymptotically efficient.* A domain $\mathcal{D}$ satisfies the *single-crossing* condition if, at each point in the consumption space, the marginal rates of substitution of the relations in $\mathcal{D}$ are ordered in the same way. That is, if at point x, R has a steeper MRS than R', then the same is true at any other point x'.

References

[1] Shigehiro Serizawa and John A. Weymark. Efficient strategy-proof exchange and minimum consumption guarantees. *Journal of Economic Theory*, 109(2):246–263, April 2003.

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