Optimal Delegation Under Uncertain Bias

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April 14, 2013

Abstract

We study a principal-agent relationship without monetary transfers. The principal is uncertain of the agent's preferences. We show that if the principal is restricted to offering convex menus (menus consisting of only convex sets), then a pooling menu, a menu consisting of a single delegation set, is optimal. We also show that the restriction to convex menus is without loss in various settings. Thus, in these settings, the optimal menu consists of a single interval. We also show that the optimal pooling menu is convex (in all settings studied). In addition, we provide some comparative statics. Finally, this paper generalizes a main result of Melumad and Shibano (1991) to a larger class of loss functions. The proof of this generalization provides additional intuition for the result: nonconvex menus are mean-preserving spreads of convex menus. Thus, nonconvex menus are suboptimal for the principal.

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†The author gratefully acknowledges the help of Johannes Hörner and Larry Samuelson for their valuable comments, discussions, and support throughout this entire project.

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1 Introduction

It's time to apply the same rules from top to bottom.

-President Barack Obama, 2012 State of the Union Address [1]

In order to succeed, companies must design prudent rules and guidelines for the actions of their employees. One reason for the significance of this design is that those managing an organization frequently have to rely on the information (and actions) of others. Yet, the people they rely on may not share the same goals as the managers. Thus, they may provide noisy information or may implement actions contrary to the desires of the managers. These concerns are not just relevant to companies, but are also relevant to legislatures forming committees or regulatory agencies (Melumad and Shibano, 1994, Gilligan and Krehbiel, 1987).

In certain cases, this difficulty can be alleviated by offering monetary transfers. These transfers may incentivize the agents to implement actions in the best interest of the managers or to reveal their hidden information (and preferences) to the manager (Laffont and Martimort, 2002). Yet in industries where the wages are flat, a manager will not be able to use monetary transfers to try and screen between agents. In fact, the lower-level managers may not have the authority to collect or provide the required monetary transfers to their workers to induce them to take the optimal actions. In addition, designing contracts specifying the optimal transfers for every task may be too complex or costly for the manager to design. In certain situations, such as committees of legislative bodies (Gilligan and Krehbiel, 1987, Melumad and Shibano, 1994), monetary transfers are not permitted for legal reasons.

This paper analyzes settings where a principal faces uncertainty in two respects. First, the principal faces uncertainty regarding the underlying state of nature (where the optimal decision depends on the state of nature). In addition, the principal does not know the preferences of an agent to which he may delegate a task. The goal of this paper is to understand how the principal designs delegation rules (the decision rights available to the agents) to optimally use the information of the agents. The principal could offer all agents the same set of responsibilities. This set could be smooth or disjointed (these will be defined below and the example in two paragraphs should make these descriptions clearer).

Alternatively, the principal may design the decision rights in order to learn about the preferences of the agents. The principal may design different sets of responsibilities in order to screen between agents with with different preferences. By allowing agents to choose between different sets of responsibilities a principal will be able to

[1]The 2012 State of the Union transcript is available at: http://www.whitehouse.gov/the-press-office/2012/01/24/remarks-president-state-union-address

infer agent preferences. This paper explores whether a principal can design different responsibilities to take advantage of the information revealed by this responsibility choice. This paper argues that the principal should require all agents to follow the same guidelines (and offer them all the same set of guidelines). In addition, the menu should not be disjointed: if the principal allows the agent to take two actions, a and b, he should allow the agent to take all actions "in between" a and b. The choices from the optimally designed screening menu of responsibilities provide the principal with little useful information. Only final actions provide useful information (about the state) to the principal, not the choice of responsibility.

In order to clarify this description, consider the following example (adapted from Frankel, 2011). Consider a school where teachers are paid flat wages and assign grades to their students. Both the school and teacher desire to assign higher grades to better performing students. The school may prefer that the teacher assign a lower amount of A-level grades (to maintain the reputation of a high grade point average from the school). In contrast, the teacher may prefer to give a higher amount of A-level grades to students (perhaps from a desire to make the students happier, or to encourage students to take the course by offering them the prospect of higher grades). In order, to control their agent (the teacher) the school may set a required grading curve that all teachers may obey.

Thus, the school may limit the grading practices of the teachers by requiring them to assign grades according to a particular curve. For example, one type of curve could be bimodal around A-level and C-level grades. Another type of curve could be more uniform. Alternatively, the school may offer each teacher a menu of sets of allowable curves. For example, the menu could consist of one set of permissible bimodal distributions and one set of allowable uniform distributions of grades. The teachers would pick set of curves from the menu at the beginning of the semester. By doing this, the school could allow the different types of teachers to reveal their preferences about grading, while still preserving a desired grade-point average. We show that the school should give each teacher one allowable set of curves. In addition, if both bimodal and uniform curves are allowed, the school should offer a smooth gradation of choices of curves between bimodal and uniform.

More generally, we would like to analyze the setting of communication without transfers. In particular, we are interested in studying how uncertainty in agent preferences influences communication (under commitment on the part of the principal). In this case, a principal offers an agent a menu of sets (of decisions). The agent picks a set, then observes the state of the world, and then chooses an action from the set of actions that he had chosen. The principal implements the action the agent selected. We interpret offering a rule with a larger range of actions as inducing more communication between principal and agent than a rule that offers a smaller range of actions. In general, we want to explore how the nature of communication changes when an additional channel of communication is added: a channel for the agent to reveal preferences (as opposed to just the state of the world). We show that in the case of no transfers, it is optimal for the principal to just offer one possible set of actions. In other words, attempting to screen between the different types of agents by offering a menu of actions does not yield the principal any extra expected utility. The optimal set is larger than that offered to the most biased agent, but smaller than that offered to the least biased agent when preferences are known.

These results depend on multiple assumptions. First, we assume that the state is distributed uniformly over the unit interval. Another crucial assumption is that the bias term is constant and does not depend on the state. At the beginning of the paper, we restrict attention to *convex menus* (menus that only contain convex sets). Under this restriction, the results depend on the agents having a symmetric, differentiable, and strictly concave loss functions. The loss also depends on the distance of the action, x, from the state, s, (or from the state plus a bias term). More formally, the loss function is $L(x, s) = U(|x - s - k_i|)$, where k_i is the bias term and U is a symmetric, differentiable, and strictly concave loss function. Later we show that the above restriction to convex menus is without loss for quadratic loss functions and for a two type case (where one type has no bias).

1.1 Outline

In Section 2, we describe the related literature. In Section 3, we present the formal model. In Section 4, we provide a characterization of the best response of the agent. In Section 5, we present examples to provide intuition for the proof of the main result. In Section 6, we summarize our main results. In Section 7, we show that the optimal delegation set is pooling of the principal is restricted to convex menus. In Section 8, we show that this restriction is without loss. In Section 9, we provide some comparative statics. Section 10 concludes the paper.

2 Related Literature

The literature most relevant to this project is that of cheap talk and mechanism design without transfers (delegation).The work most influential to this project from the cheap talk literature is that of Crawford and Sobel (1982). The case of cheap talk with unknown preferences has been studied by Sobel (1985), Morgan and Stocken (2003), Wolinsky (2003), Dimitrakas and Sarafidis (2005), and Li and Madarasz (2009). These papers do not discuss screening using different communication protocols.

Of the delegation literature the two most important papers for this project are those of Holmstrom (1984), and Melumad and Shibano (1991) (abbreviated as MS).

Martimort and Semenov (2006) provide conditions for when the optimal delegation set is an interval (but still consider the case when the preferences of the agent are known). Mylovanov (2008) studies veto-based delegation and Kovac and Mylovanov (2009) also study when stochastic mechanisms yield optimal payoffs to the principal. Alonso and Matouschek (2008) study the optimal delegation problem under more general preferences and distributions over the state space, but when the preference of the agent is known.

Alonso and Matouschek (2007) provide a synthesis of the two literatures by studying a setting of cheap talk with partial commitment. Dessein (2002) provides a different synthesis of these two literatures. Amador and Bagwell (2012a) and (2012b) provide applications of the theory of delegation to tariff caps. Amador, Werning, and Angeletos (2006) apply the theory of delegation to study commitment and flexibility in saving rules.

Armstrong (1995) and Frankel (2011) consider the case of preference uncertainty but do not study the optimal screening contracts. Work that does study the optimal screening menu, has been done by Kovac and Krahmer (2012). Yet the agents in their model have known preferences. What distinguishes the agents is the precision of their knowledge over the future state of nature. Carrasco and Fuchs (2009) consider a setting of implementing a decision with agents who have different preferences, but the preferences of the agents are known. Another related literature is that of sequential screening (2000). In that paper, the functional form of the agents' utility is monotonic. In this case, the utilities are not monotonic.

3 The Model

3.1 Preferences

The setting is similar to that of Melumad and Shibano (1991), except that we introduce uncertainty in the bias between the agent and principal Thus, the payoff functions of the agent depend on the state of nature ($s \in [0, 1]$), the action implemented by the principal ($x \in \mathbb{R}$), and the bias of the agent ($k \in \mathbb{R}_+$). Let $U : \mathbb{R} \to \mathbb{R}$, where U is a symmetric, differentiable, strictly concave function, that is maximized at zero (without loss of generality, we normalize U so that $U(0) = 0$)[2].The utility function of the principal is $U^P(x, s) = U(x - s)$. Let $0 \leq k_1 < k_2 < \cdots < k_N$ The utility function of agent i is $U^i(x, s) = U(x - s - k_i)$ where $i \in \{1, 2, \ldots, N\} = \mathcal{N}$. k_i and s are random and statistically independent, where s is distributed uniformly over $[0, 1]$. The probability that bias k_i is chosen is denoted by p_i. We call this environ-

[2]These conditions also imply that $U(\cdot)$ is strictly decreasing over $\mathbb{R}_+$ and strictly increasing over $\mathbb{R}_-$.

ment (of utility functions, arbitrary number of types, and nonnegative bias), setting D (for default setting). Later on we will restrict attention to two simpler settings.

3.2 Actions and Timing

Before describing the timing of the game, I first define a delegation set. A *delegation set* is a set of actions, D, that the agent will be restricted to take. D must be compact and I denote the set of compact subsets of the real line by $\mathcal{D}$. (We only need to restrict attention to closed sets since for every unbounded closed set, there is a bounded closed set that produces the same outcome and provides identical incentives). By the taxation principle, the principal will offer the agents a menu of set, $m = \{D_1, \ldots, D_N\}$ (I will describe the timing of the game below). Let $\mathcal{D}^N = \mathcal{M}$ be the set of all menus of delegation sets. We call a menu, $m \in \mathcal{M}$, a *convex menu* if every $D \in m$ is a convex set. In other words, all the delegation sets in a convex menu are convex. We denote the set of convex menus by $\mathcal{M}_{\mathcal{C}}$. We call a menu, $m \in \mathcal{M}$, a *nonconvex menu* if there exists a nonconvex set, $D' \in m$.

The timing of the game is as follows:

Time 0: Nature chooses k_i (bias) for the agent. The agent observes this value, but the principal does not.

Time 1: The principal offers the agent a menu of delegation sets, $m = \{D_i\}_{i \in \mathcal{N}} \in \mathcal{M}$.

Time 2: The agent selects one of the sets, D_i, and this selection is observed by the principal.

Time 3: The state of the world, s, is chosen by nature. It is observed by the the agent but not the principal.

Time 4: The agent picks a *final action* $d \in D_i$, which is observed by the principal.

Time 5: The agent's action choice is implemented and payoffs are determined.

The interpretation of this formulation is that the agent takes the final, payoff-relevant action. Yet, the action the agent takes is restricted by the principal. The action chosen by the agent must be an element of his chosen delegation set, which was designed by the principal. Hence, the principal's strategy is an element $\{D_i\}_{i \in \mathcal{N}} \in \mathcal{M} = \mathcal{D}^N$.

The agent's strategy is an action at each information set in the game tree. Thus, the agent's strategy is σ, where

$$\sigma : \mathcal{N} \times \mathcal{M} \times [0, 1] \to \mathcal{D} \times \mathbb{R}, \tag{3.1}$$

where

$$\sigma(i,m,s) = (\sigma_{\mathcal{M}}(i,m), \sigma_{\mathcal{D}}(i,m,s)), \tag{3.2}$$

where:

$$\sigma_{\mathcal{M}}(i,m) \in m, \tag{3.3}$$

$$\sigma_{\mathcal{D}}(i,m,s) \in \sigma_{\mathcal{M}}(i,m), \tag{3.4}$$

$$i \in \mathcal{N} = \{1,\ldots,N\}, s \in [0,1]. \tag{3.5}$$

$\sigma_{\mathcal{M}}(i,m)$ represents the delegation set agent i chooses from the menu offered by the principal (implied by condition 3.3). $\sigma_{\mathcal{D}}(i,m,s)$ represents the final action chosen by the agent after observing the state of nature. Notice that the final action must be an element of the delegation set chosen (implied by condition 3.4).

3.3 Solution Concept

The solution concept used throughout this paper is *Perfect Bayes-Nash Equilibrium.* The principal chooses $m = \{D_1,\ldots,D_N\}$ to maximize ex-ante expected utility:

$$\max_{m\in\mathcal{M}} \sum_{i=1}^{n} p_i \Bigg(\int_0^1 U^P\big(\sigma_{\mathcal{D}}(i,m,s), s\big) ds \Bigg). \tag{3.6}$$

Each type of agent ($i \in \{1,\ldots,N\}$), chooses the final action, $\sigma_{\mathcal{D}}(i,m,s)$ to maximize ex-post utility conditional on the original choice of delegation set from the menu, $\sigma_{\mathcal{M}}(i,m)$:

$$\sigma_{\mathcal{D}}(i,m,s) \in \operatorname*{argmax}_{d\in\sigma_{\mathcal{M}}(i,m)} U^i(d,s) = \operatorname*{argmax}_{d\in\sigma_{\mathcal{M}}(i,m)} U(d-s-k_i). \tag{3.7}$$

Their are two points to notice. First, notice that $\sigma_{\mathcal{D}}$ is determined by $\sigma_{\mathcal{M}}$. We call the value of $\sigma_{\mathcal{M}}(i,m)$, agent i's choice of delegation set. Second, we don't need to assume that the menu contains only bounded closed sets. In other words, $\sigma_{\mathcal{M}}(i,m)$ need not be compact. Since the loss function is symmetric and decreasing in the distance from $s+k_i$ action, we know that there exists an $Q(i,m)$ such that $|\sigma_{\mathcal{D}}(i,m,s)| \leq Q(i,m), \forall s \in [0,1]$. Thus, we know that even for menus containing closed (but not bounded sets), $\sigma_{\mathcal{D}}$ is well-defined. Each agent type, chooses the delegation set, D_j, from the menu, $m = \{D_1,\ldots,D_N\}$ in order to maximize interim expected utility given his future final actions, $\mathbb{E}^i D_j$, where:

$$\mathbb{E}^i D_j = \int_0^1 U^i\big(\sigma_{\mathcal{D}}(i,m,s), s\big) ds \tag{3.8}$$

where $\sigma_{\mathcal{D}}(i, m, s) \in D_j$.

Before deriving results about the optimal convex and pooling menus, we need to derive some properties about equilibrium strategies of the agents. We will state the useful properties of the equilibrium best responses in the next section (we will characterize $\sigma_{\mathcal{D}}(i, m, s)$ and prove some useful lemmas about this function). Melumad and Shibano (1991) prove the relevant properties of these best response functions for a general class of utility functions. We will show below that the utility functions assumed here satisfy the properties necessary for Melumad and Shibano's proof. Hence, their results apply in this setting.

4 Characterizing the Agent's Best Response

The agent type's ($i \in \mathcal{N}$) behavior is very simple conditional on the choice of a delegation set, $\sigma_{\mathcal{M}}(i, m) = D \in m$. Agent type i will choose, for each state s, the element in D closest to s. Let's call this point $x_i^D(s)$. More formally, for every compact set $D \subseteq \mathbb{R}$ (we just need the set to be closed, but we assume compactness for a smoother description) :

$$x_i^D(s) \in \operatorname*{argmax}_{d \in D} U^i(d, s) = \operatorname*{argmax}_{d \in D} U(d - s - k_i). \tag{4.1}$$

Comparing (3.7) and (4.1) we define

$$x_i^D(s) := \sigma_{\mathcal{D}}(i, m, s), \tag{4.2}$$

when $\sigma_{\mathcal{M}}(i, m) = D$.

We call x_i^D the *delegation schedule generated by* D for type i. This function maps the current state, s, to an optimal action of the agent within his chosen delegation set, D. Let $x_i^O(s) = x_i^{\mathbb{R}}(s) = \operatorname{argmax}_{d \in \mathbb{R}} U(d - s - k_i) = s + k_i$. x_i^O is the optimal delegation schedule for type i: for each state, s, x_i^O yields the best possible final action for the agent type. Call the range of x_i^O the *set of ideal actions for type* i. Notice that the range of $x_i^O = [k_i, 1 + k_i]$. In addition, the ideal action set of the principal is $[0, 1]$. The properties of x_i^D are listed in Appendix A. The most important result is that the delegation schedule is an increasing function.

We call a delegation set D *nonredundant for agent type* i if $I(x_i^D) := Image(x_i^D) = D$. In words, a delegation set D is nonredundant for player i if every action in set D is taken by player i for a particular state. We now show that for every closed set D, there is a nonredundant set $D' = I(x_i^D)$ for player i (which we know is compact). This is Result 4.3. Before proving it we will need a lemma and a corollary.

Lemma 4.1. *For all $D \in \mathcal{D}$ and $i \in \mathcal{N}$, $I(x_i^D)$ is compact.*

Proof. See Appendix A. □

Corollary 4.2. *For all $D \in \mathcal{D}$ and $i \in \mathcal{N}$, $I(x_i^{I(x_i^D)}) = I(x_i^D)$.*

The following result puts this lemma and corollary together and allows us to reduce attention to nonredundant sets:

Result 4.3. *For every set $D \in \mathcal{D}$ and $i \in \mathcal{N}$, there is a nonredundant set $D' \in \mathcal{D}$ such that $I(x_i^{D'}) = I(x_i^D)$ and $x_i^{D'}(s) = x_i^D(s), \forall s \in [0,1]$.*

Result 4.3 (the nonredundancy result) will simplify analysis when studying screening menus (defined below).

The interim expected utility to the agent of type i after choosing the delegation set D is:

$$\mathbb{E}^i D = \int_0^1 U\big(x_i^D(t) - t - k_i\big)dt.$$

Thus, by the Taxation Principle, the principal's optimization program is :

$$\max_{\{D_1,\ldots,D_n\}\in\mathcal{M}} \sum_{i=1}^n p_i \left(\int_0^1 U\big(x_i^{D_i}(t) - t\big)dt\right) \tag{4.3}$$

subject to the type incentive constraint for delegation sets (IC_k^i):

$$\mathbb{E}^i D_i \geq \mathbb{E}^i D_j, \forall i, j.$$

A menu solving the principal's problem subject to the IC_k^i constraints[3] is called *$\mathcal{M}$-optimal.* If the set of menus in (4.3) is restricted to convex menus, then the expected utility maximizing, incentive compatible menu will be called *$\mathcal{M}_\mathcal{C}$-optimal.* We call

$$\mathbb{E}_i^P D_i = \int_0^1 U(x_i^{D_i}(t) - t)dt$$

the *expected payoff to the principal from type i.*

We say that a set D' *improves upon D for type i* if

$$\mathbb{E}_i^P D' = \int_0^1 U(x_i^{D'}(t) - t)dt > \int_0^1 U(x_i^D(t) - t)dt = \mathbb{E}_i^P D$$

and

$$\mathbb{E}^i D' \geq \mathbb{E}^i D.$$

[3]For the rest of this paper, we will just call these constraints the IC_k constraints.

Notice that the menu that improves upon another only satisfies the IC_k constraint for type i (and not necessarily type j).

A menu is *convex* if all sets in it are convex. A menu $m \in \mathcal{D}^N$ is *pooling* if it is singleton ($m = \{D\}$). We call D the *pooling set*. A menu $m \in \mathcal{D}^N$ is *screening* if it is non-singleton and all sets are nonredundant. This is not such a demanding requirement since $I(x_i^{D_i}) \subseteq D_i$, and, therefore,

$$\mathbb{E}^i\bigg(I(x_i^{D_i})\bigg) = \mathbb{E}^i(D_i) \geq \mathbb{E}^i(D_j) \geq \mathbb{E}^i\bigg(I(x_j^{D_j})\bigg),$$

$\forall i, j \in \mathcal{N}$. Hence, the expected payoff and incentive constraints are preserved.

The decision of the agent is determined by the menu that is offered. The two types of menus are convex menus (menus containing only convex sets) and nonconvex menus. In order to to better analyze nonconvex menus, we will use some results derived in the next section about nonconvex menus and the gaps in them.

5 An Example That Illustrates the Proofs of the Main Results

We provide an example to show that the problem is nontrivial and to give the reader a flavor for the proofs of the main results of this paper.

Example 1: Feasibility of Convex Menus and Optimality of Pooling

First we show that the problem is feasible (even under the restriction to convex menus) and then provide intuition for the optimality of pooling menus. We show that there exist menus that are convex, incentive compatible, and screening. Let $U^P(x,t) = -(x-t)^2$, $U^1(x,t) = -(x-t-.05)^2$, and $U^2(x,t) = -(x-t-.5)^2$. (Hence, $k_1 = .05$ and $k_2 = .5$.) Let $m = \{D_1, D_2\}$, where $D_1 = [0.05, .45]$ and $D_2 = \{\frac{1}{2}\}$. Hence, we have $\mathbb{E}^1 D_1 > \mathbb{E}^1 D_2$, $\mathbb{E}^2 D_2 > \mathbb{E}^2 D_1$.

Notice that menu D_1 is "lower" than menu D_2. We will prove below, via a single-crossing lemma, that incentive compatibility implies that the delegation sets in a screening menu must "increase" with bias (the delegation set chosen by type i, D_i, must be lower than the delegation set chosen by type j, D_j, with $k_j > k_i$). Notice that if D_1 and D_2 were reversed ($D_1 = \{\frac{1}{2}\}$ and $D_2 = [0.05, .45]$), the menu would not be incentive compatible.

On the other hand, if the principal knows the bias of the agent, he would want to choose the set $[k_i, 1-k_i]$ (this will be shown below- what is important to notice for intuition is that the optimal delegation set is lower for types with lower bias). Thus, in order to screen, one must choose sets according to an order contrary to that desired by expected payoff maximization. Hence, the optimal delegation set is

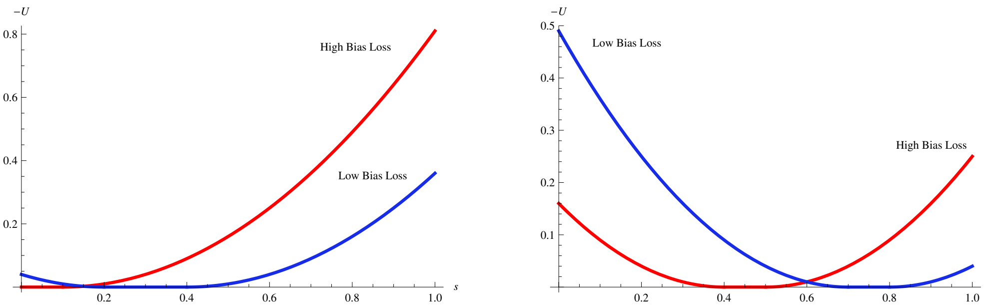

Figure 1: Figures 1a and 1b: 1a plots the loss for each type for set D_1.
1b plots the loss for each type for set D_2.

pooling. In this case, the pooling menu $m' = \{[.05, .5]\}$ yields the principal strictly higher expected payoff than the screening menu m.

Example 2: Improving On Nonconvex Menus

We now provide an example to show why convex menus may be assumed without loss of generality (under quadratic loss functions). Let $U^P(x,t) = -(x-t)^2$ be the utility of the principal, $U^1(x,t) = -(x-t)^2$ be the utility of the unbiased type, and $U^2(x,t) = -(x-t-.5)^2$ be the utility of the biased type. Let $m = \{D_1, D_2\}$, where $D_1 = [0, 0.15] \cup \{0.45\}$ and $D_2 = \{0.5\}$. This menu is incentive compatible ($\mathbb{E}^1 D_1 > \mathbb{E}^1 D_2$, $\mathbb{E}^2 D_2 > \mathbb{E}^2 D_1$).

Notice that D_1 has a gap, $G = (.15, .45)$. Yet, we can "fill in" the gap in set D_1 (replace D_1 with $\hat{D} = [0, .45]$), and then "thin" it (replace $\hat{D}$ with a new set $D' = [0, a]$ such that $a < .45$) in a way that:

- The unbiased agent is indifferent between the new (convex set), $D' = [0, a]$, and the original set, D_1 ($\mathbb{E}^1 D' = \mathbb{E}^1 D_1$). In this case, $a \approx 0.443$.
- The the expected utility to the principal is higher from the new set than the original set: $\mathbb{E}^P_1 D' > \mathbb{E}^P_1 D_1$
- The new set D' is even worse for the other agent than the original set D_1: $\mathbb{E}^2 D' < \mathbb{E}^2 D_1 < \mathbb{E}^2 D_2$. Hence the new menu is incentive compatible.

Under quadratic loss, the *gain* in expected payoff to the principal from *filling in* a gap, is equal to the gain in expected payoff of the agent (type 1) from filling in a gap. Yet, the *loss* in expected payoff from *thinning* the filled in set is lower for the principal than the agent (type 1). Thus, the "filled in and thinned" set keeps type 1 at the same expected payoff (and preserves incentive compatibility), but strictly benefits the principal. The "fill in" and "thin" variational argument will be used to prove that the restriction to convex menus is without loss. Finally, we point out that by offering the convex, pooling menu $m = [0, \frac{1}{2}]$, the principal can achieve a strictly higher expected payoff than from offering the menu $\{D', D_2\}$.

We are now ready to state the main results of this paper.

6 Main Results

If $U(\cdot) = -(\cdot)^2$ and there are an arbitrary (but finite) amount of types with nonnegative bias we say that we are in *setting* Q. If $U(\cdot)$ is defined as in section 3, but there are only two types, one unbiased ($k_1 = 0$) and the other biased ($k_2 > 0$), we say that we are in *setting* T. Notice that setting D is a generalization of both settings Q and T. The main result of this paper is:

Proposition 6.1 (No Need to Screen- NNS)**.** *In settings Q and T the optimal delegation menu is pooling and convex. An optimal menu is of the form: $m^* = \{D^*\}$, where*

$$D^* = \begin{cases} [k_1, \gamma^*] & \text{if } k_1 \leq \frac{1}{2}, \\ \{\frac{1}{2}\} & \text{if } k_1 > \frac{1}{2}. \end{cases} \tag{6.1}$$

where $\gamma^* \leq 1$.

In other words, under the appropriate restrictions, no screening menu can yield the principal strictly higher expected payoff than a pooling menu. The principal should not worry about screening and just find the optimal pooling menu. In addition, the principal doesn't even need to worry about discontinuities, and his program is reduced from an infinite dimensional optimization problem to a two dimensional optimization problem. Alternatively, the result shows that the additional information the principal extracts about bias from an optimal (incentive compatible) screening menu is worthless. The principal would perhaps be better suited trying to infer the bias of his agent from repeated interaction, or by investing in technology for acquiring information about bias (though these are subjects for further work).

This main result is proved in stages. We first prove (in section 7) that if attention is restricted to convex menus, the optimal delegation menu is pooling (under no restrictions on utility functions, setting D):

Proposition 6.2 (No Need to Screen: Convex Menu Version)**.** *Under setting D (defined at the beginning section 3), the optimal menu in $\mathcal{M}_{\mathcal{C}}$ (the set of convex menus) is pooling. An optimal menu is of the form: $m^*_{\mathcal{C}} = \{D^*\}$, where*

$$D^* = \begin{cases} [k_1, \gamma^*] & \text{if } k_1 \leq \frac{1}{2}, \\ \{\frac{1}{2}\} & \text{if } k_1 > \frac{1}{2}. \end{cases} \tag{6.2}$$

where $\gamma^* \leq 1$.

We then proceed to show that this restriction is without loss in settings Q and T (in section 8).

Proposition 6.3 (W.L.O.G. for Q and T)**.** *The restriction to convex sets is without loss in settings Q and T.*

In order to prove this result, we will need a result that filling in gaps of nonconvex sets is beneficial to the principal (Lemma 8.2). The proof of this result allows us to generalize a result of Melumad and Shibano (1991) that the optimal delegation set (under known preferences) is convex. In addition, the proof will provide intuition for their convexity result. The deviation from the principal's ideal action under a nonconvex delegation set is a mean-preserving spread of the deviation under a convex delegation set (which we also discuss in section 8). With this result, it is simple to show that the optimal pooling set, the set in an optimal pooling menu (which is the optimal menu in settings Q, T, and D restricted to convex menus), is convex and approximately unique: all other optimal menus differ on a set of actions that will be played with probability zero.

Proposition 6.4 (Convex Optimal Pooling)**.** *Given any distribution over N types of agents, their exists a convex set D_P^* that could serve as an optimal pooling set. All other optimal pooling menus differ from D_P^* on a set that will be played with probability zero in equilibrium.*

Section 9 contains some comparative static results. We show some comparative statics of the optimal pooling delegation set. Let $\mathbf{p}_N := (p_1, \ldots, p_N)$, where $\sum_{i=1}^N p_i \leq 1$ (the subscript N denotes the dimension of the vector ($\mathbf{p}_L$ would be an L-tuple). We also define $\mathbf{k}_N := (k_1, \ldots, k_N)$ (again, the subscript N denotes the dimension of the vector ($\mathbf{k}_L$ would be an L-tuple). Denote the optimal pooling delegation set by $D^*(\mathbf{p}_N, \mathbf{k}_N)$.

Result 6.5. *Fix $\boldsymbol{p}_N$. The optimal pooling delegation set, $D^*(\boldsymbol{p}_N, \boldsymbol{k}_N)$ is weakly decreasing in $\boldsymbol{k}_N$. Formally,*

$$\boldsymbol{k}'_N \geq \boldsymbol{k}_N \Rightarrow D^*(\boldsymbol{p}_N, \boldsymbol{k}'_N) \subseteq D^*(\boldsymbol{p}_N, \boldsymbol{k}_N), \tag{6.3}$$

where $\boldsymbol{k}'_N \geq \boldsymbol{k}_N$ iff $k'_i \geq k_i$ for all $i \in \{1, \ldots, N\}$.

In order to introduce the next comparative static result, let $F_{(\mathbf{p}_N, \mathbf{k}_N)}(z)$ denote the cumulative distribution function of the biases $\mathbf{k}_N$ under the probability distribution $\mathbf{p}_N$:

Result 6.6. *The optimal pooling delegation set, $D^*(\boldsymbol{p}_N, \boldsymbol{k}_N)$ is weakly decreasing in first-order stochastic dominance ($\succsim_{1st}$). Formally,*

$$(\boldsymbol{p}'_N, \boldsymbol{k}'_N) \succsim_{1st} (\boldsymbol{p}_L, \boldsymbol{k}_L) \Rightarrow D^*(\boldsymbol{p}'_N, \boldsymbol{k}'_N) \subseteq D^*(\boldsymbol{p}_L, \boldsymbol{k}_L), \tag{6.4}$$

where $(\boldsymbol{p}'_N, \boldsymbol{k}'_N) \succsim_{1st} (\boldsymbol{p}_L, \boldsymbol{k}_L)$ iff $F_{(\boldsymbol{p}'_N, \boldsymbol{k}'_N)}(z) \leq F_{(\boldsymbol{p}_L, \boldsymbol{k}_L)}(z)$ for all $z \in \mathbb{R}$.

7 Convex Menus

We first prove that if the principal is restricted to offer convex menus, then the optimal convex menu will be a pooling menu. We use a variational argument. We will prove this by finding a pooling menu that increases the expected payoff from each type. In order to find this pooling menu we need three lemmas about convex delegation sets. The first establishes that filling in the gap between zero and the lowest point of the set can only help the principal (no matter what type of agent). The second lemma will tell us how high to set the highest point of the pooling delegation set. The final lemma is a version of single-crossing and will allow us to order the sets in the proposed delegation menu. We use this order to construct a pooling menu that yields the principal higher expected payoff.

Recall that the principal's program is to maximize :

$$\mathbb{E}^P(\mathcal{M}) = \max_{m=\{D_1,\ldots,D_N\}} \mathbb{E}^P(m) = \max_{m=\{D_1,\ldots,D_N\}} \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D_i}(t) - t)dt \right)$$

subject to the type incentive constraint for delegation sets (IC_k^i):

$$\mathbb{E}^i D_i \geq \mathbb{E}^i D_j, \forall\ i, j.$$

Note that $\mathbb{E}^P(m = \{D_1, \ldots, D_N\}) = \sum_{i=1}^N p_i \left(\int_0^1 U(x_i^{D_i}(t) - t)dt \right)$.

Remember that

$$\mathbb{E}_i^P D_i = \int_0^1 U(x_i^{D_i}(t) - t)dt$$

the *expected payoff to the principal from type i.*

Since we first restrict attention to the case when all the D_i are convex we may let, $D_i = [a_i, b_i]$.

We have the following lemma:

Lemma 7.1. *(Down to k_i Lemma) Let $D = [a, b]$, where $a > k_i \geq 0$, then $\mathbb{E}_i^P D < \mathbb{E}_i^P D'$, when $D' = [k_i, b]$*

Proof. See Appendix. □

Thus, if the agent's type was known, Lemma 12 implies that an optimal convex delegation set is of the form $D = [k_i, b]$. Thus, the expected payoff to the principal from the set $[k_i, b]$ when $1 + k_i \geq b \geq k_i$ would be:

$$\Psi_i(b) = \mathbb{E}_i^P[k_i, b] = \int_0^{b-k_i} U(k_i)ds + \int_{b-k_i}^1 U(b - s)ds \tag{7.1}$$

$$= U(k_i)(b-k_i)+\int_{-k_i}^{1-b} U(s)ds = U(k)(b-k_i)+\int_{-k_i}^{\min\{1-b,0\}} U(s)ds+\int_{0}^{\max\{1-b,0\}} U(s)ds.$$

For $b < k_i$, $\Psi_i(b) = \int_0^1 U(b-s)ds$. Thus, we have the following lemma:

Lemma 7.2. *(Known Bias Optimum Lemma) Let the bias, k_i, of the agent be known by the principal. If $k_i \geq \frac{1}{2}$, then an optimal convex delegation set is $D_i^* = \{\frac{1}{2}\}$. If $k_i < \frac{1}{2}$, then an optimal convex delegation set is $D_i^* = [0, 1-b_i^*]$, where $b_i^* = 1-k_i$. For this case, $\Psi_i(\cdot)$ is strictly increasing from $[0, 1-k_i]$ and strictly decreasing from $[1-k_i, 1+k_i]$. Thus, $D_i^* = [0, q_i]$ where $q_i = \max\{\frac{1}{2}, 1-k_i\}$.*

Proof. See Appendix. □

7.1 Single-Crossing Lemma and Proof of Main Result for Convex Menus

We now want to prove a single-crossing result. First, if we have two sets $D_1 = [a_1, b_1] \neq D_2 = [a_2, b_2]$ such that $a_1 < a_2$ and $b_1 \geq b_2$, or $a_1 \leq a_2$ and $b_1 > b_2$ (or the case with both inequalities reversed), then D_1 would always be strictly preferred by both types and the incentive constraints would not hold.

Thus, the only remaining possibilities are:
(A) $a_1 < a_2$ and $b_1 < b_2$
(B) $a_1 > a_2$ and $b_1 > b_2$
We will use the single-crossing condition to rule out case (B).

Lemma 7.3. *(Single-Crossing Lemma) If $D_i, D_j \in m$ such that m is IC_k and $k_i < k_j$. We cannot have $a_i > a_j$ and $b_i > b_j$. If $a_i > a_j$ and $b_i > b_j$, then $\mathbb{E}^j D_j \geq \mathbb{E}^j D_i$ implies that $\mathbb{E}^i D_j > \mathbb{E}^i D_i$, which violates the IC_k condition.*

Proof. See Appendix. □

Armed with these three lemmas, we are ready to prove that the optimal convex menu is pooling. First, if the bias of all types is $\geq \frac{1}{2}$, then by the Known Bias Optimum Lemma (Lemma 7.2), the optimal convex menu is $[0, \frac{1}{2}]$. This yields the principal the optimal expected utility from each type. Thus, we assume that $k_1 < \frac{1}{2}$. We provide a complete proof in the Appendix. Here we provide the intuition:

The goal is to find a pooling menu that yields the principal higher expected payoff. We construct this pooling menu in three steps. First, we know that an IC_k menu must be of the form $m = \{D_1 = [a_1, b_1], \ldots, D_N = [a_N, b_N]\}$ where $b_i \leq b_j$ for all $j > i$. By

Lemma 7.1, we know that if we were to replace each D_i by $D_i^0 = [k_i, b_i]$, we would improve the principal's expected utility. There is one problem though, the menu of D_i^0's may not be IC_k. Yet, we can replace this menu with a pooling menu. From single-crossing, we know that the b_i are increasing in i. In addition, from Lemma 7.2 (Known Bias Optimum Lemma), we know that the $q_i = 1 - k_i$ (the optimal end points under known bias), are decreasing in i. Thus, if some $b_i > q_i$, then $b_j > q_j$ for all $j > i$. In words, if type i's delegation is too large, all higher type's delegation sets are too large. Thus, by shrinking the delegation sets of all such types (and expanding the delegation sets of the types whose sets are not too large) we can achieve a pooling delegation set that yields the principal higher utility than the original menu m. We know state the result:

Proposition 7.4 (No Need to Screen: Convex Menus)**.** *If menus in a delegation set are restricted to contain only convex sets, then there exists an optimal pooling menu,* $m = \{P^*\}$ *that is optimal.*

Proof. See Appendix. □

Hence, if an organization is restricted to offering a convex menu of guidelines, then the organization should set the same guidelines for each member. We also note that incentive compatibility was only used to show that the sets are ordered ($b_i < b_{i+1}$ for all i). Once the sets are ordered, the argument did not use incentive compatibility. Thus, let $d_i^* = \max_{d \in D_i} d$. Thus, if we are given a menu $m = \{D_1, \ldots, D_N\}$ where: (i) each D_i is convex, (ii) $d_i^* \leq 1 + k_i$ for all i, and (iii) $d_i^* \leq d_{i+1}^*$ for all i (and strict inequality holds for, at least, $i = 1$), then we can use the argument in the proof to find a (convex) pooling menu that yields the principal strictly higher expected utility. Call a menu satisfying (i)-(iii) a *nice menu* (notice that a nice menu may not be IC_k). Thus, we have the following lemma:

Lemma 7.5. *Given a nice menu,* m*, then there exists a pooling menu* (m') *with convex delegation set* $D' = [k_1, \gamma]$ ($\hat{m} = \{[k_1, \gamma]\}$) *such that:*

$$\mathbb{E}^P(\hat{m}) = \sum_{i=1}^{N} \mathbb{E}_i^P[k_1, \gamma] > \sum_{i=1}^{N} \mathbb{E}_i^P D_i = \mathbb{E}^P(m). \tag{7.2}$$

This lemma will prove useful in the next section. Suppose an incentive-compatible menu, $m = \{D_1, \ldots, D_N\}$, yields the principal less expected payoff than a nice menu, $m^n = \{D_1^n, \ldots, D_N^n\}$:

$$\mathbb{E}^P(m^n) = \sum_{i=1}^{N} \mathbb{E}_i^P D_i^n > \sum_{i=1}^{N} \mathbb{E}_i^P D_i = \mathbb{E}^P(m). \tag{7.3}$$

Thus, Lemma 7.5 shows that we can find a pooling menu $\hat{m} = \{\hat{D}\}$ such that :

$$\mathbb{E}^P(\hat{m}) = \sum_{i=1}^{N} \mathbb{E}_i^P \hat{D} > \sum_{i=1}^{N} \mathbb{E}_i^P D_i^n = \mathbb{E}^P(m^n) > \mathbb{E}^P(m). \tag{7.4}$$

We state this as the following corollary:

Corollary 7.6. *Let m be an incentive-compatible menu. If there is a nice menu (that is not necessarily incentive-compatible), m^n, that yields the principal higher expected payoff as in equation (7.3), then there is a convex, pooling menu (a singleton delegation set composed of a convex set) that yields the principal strictly higher expected payoff as in equation (7.4).*

In this section, we restricted the analysis to convex menus. In the next section we will show that this analysis is without loss in two important settings. We will do so by showing that for each incentive compatible menu, there is a nice menu that yields the principal strictly higher expected payoff. Hence, by Corollary 7.6, there is a convex pooling menu that yields the principal strictly higher expected payoff.

8 When the Restriction to Convex Menus is Without Loss: Settings Q and T

In this section, we prove that the restriction to convex menus is without loss for settings Q (quadratic loss) or T (two types, one unbiased). In order to analyze nonconvex menus, we will need to analyze nonconvex sets. Hence, we will need to study sets with gaps:

8.1 A Note on Gaps

In the class of games studied in this paper, the principal offers the agent a menu of sets. In order to discuss the types of sets the principal may find optimal to offer we introduce a useful definition. We define carefully the definition of a *gap* in a delegation set. Assume that a delegation set, D, is not convex. Thus, if there exists a point y such that $x, z \in D$ and $x < y < z$, let $G_D^+(y) = [y, u_y^D)$, where

$$u_y^D = \sup_t \{t \in \mathbb{R} | t > y, [y, t) \cap D = \emptyset\}.$$

In addition, let $G_D^-(y) = [d_y^D, y)$, where

$$d_y^D = \inf_t \{t \in \mathbb{R} | y > t, (t, y] \cap D = \emptyset\}.$$

Thus, define $G_D(y) := G_D^-(y) \cup G_D^+(y)$, where $G_D(y)$ is the largest gap containing y.

We state one more lemma which will be useful later.

Lemma 8.1. *$d_y^D, u_y^D \in D$.*

Proof. This follows from the openness of the complement of a compact set (D is compact). Since compact sets are closed, these points must be contained in a compact set, D. □

With this result, we can define the expected payoff increase from filling in a gap. This will be used in the proof that the restriction to convex sets is without loss (Proposition 8.5). This proof will require us to fill in gaps to convert (nonconvex) incentive-compatible menus into nice menus. In the next subsection, we show that filling in a gap that is not too high ,$G \subseteq (-\infty, 1 + k_i]$, raises the expected payoff of the principal (this will be used in the proof that the restriction to convex menus is without loss).

8.2 Gap Filling Lemma, Intuition for Convex Sets, and the Optimal Pooling Menu

We first show how to improve upon an arbitrary set with a gap. We shall show that the same modification strictly improves utility, *independent of the bias of the agent.* Thus, if there is a pooling menu with a gap, we can use this particular modification and raise the principal's expected payoff. The modification used in this section will be to completely fill in the gap (but only when the gap does not contain the point $1 + k_i$, the largest point the agent would choose if the delegation set were $\mathbb{R}$). In other words, if D contains a gap, (l, h) such that $h \leq 1 + k_1$, the set $D' = D \cup (l, h)$ will yield the principal strictly higher expected utility. We call this modification *gap filling.* We restrict attention to the case when $h \leq 1+k_i$. Thus, there are two possible cases for the gap $G = (l, h)$: (a) $l \geq k_i$ or (b) $l < k_i$. In either case, filling in the gap raises the expected payoff of the principal. We state this in the following lemma:

Lemma 8.2. *(Gap Filling Lemma) Let $k_i \geq 0$. Let D be a set with gap, $G = (l, h)$, such that $G \subseteq (-\infty, 1 + k_i]$. Let $D(\epsilon, l, h) := D \cup [l, l + \epsilon) \cup (h - \epsilon, h]$, where $\epsilon \leq \frac{h-l}{2}$. Then we know that for all $\epsilon \in [0, \frac{h-l}{2}]$ we have*

$$\mathbb{E}_i^P D = \int_0^1 U(x_i^D(s) - s)ds \leq \int_0^1 U(x_i^{D(\epsilon,l,h)}(s) - s)ds = \mathbb{E}_i^P D(\epsilon, l, h).$$

Hence, completely filling in the gap (replacing D with $D' = D \cup G$) would yield the principle higher utility: $\mathbb{E}_i^P D \leq \mathbb{E}_i^P D'$, where the inequalities in the expected payoffs are strict if and only if $h > k_i$.

Proof. See Appendix. □

The intuition for this result is simple. The deviation from a principal's ideal choice (absolute value of distance of action chosen by agent from the state, the ideal choice of the principal) can be viewed as a random variable. The proof shows that the deviation of a set with a gap is a mean-preserving spread of the deviation of a set with a (partially) filled in gap. Thus, we have further intuition for Melumad and Shibano's (1991) result about the optimality of intervals. Gaps in delegation sets generate "riskier" lotteries for the principal. Hence, convex sets are optimal when preferences are known.

Yet, the Gap Filling Lemma leaves us on the cusp of proving a new result about the optimal pooling menu. We know that if the highest point of a delegation set in an optimal pooling menu is $\leq 1 + k_1$ then we can just (completely) fill in all the gaps and raise the expected utility of the principal. Thus, it remains to show that the optimal pooling menu is contained in $(-\infty, 1 + k_1]$:

Lemma 8.3. *Let D_P^* denote the optimal pooling delegation set. It is without loss of generality to assume that $D_P^* \subseteq (-\infty, 1 + k_1]$.*

Proof. See Appendix. □

Thus, as argued in the previous paragraph we have the following proposition:

Proposition 8.4 (Convex Optimal Pooling)**.** *Given any distribution over N types of agents, their exists a convex set D_P^* that could serve as an optimal pooling set. All other optimal pooling menus differ from D_P^* on a set that will be played with probability zero in equilibrium.*

8.3 Proving the Main Result: Proposition 8.5

Recall that the proof that the restriction to convex sets is without loss (Proposition 8.5) shows that for every incentive compatible menu $m = \{D_1, \ldots, D_N\}$, there is a nice menu yielding the principal higher expected payoff. Thus, by Corollary 7.6, we know that there is a convex pooling menu that yields the principal strictly higher payoff than m. The construction of the nice menu proceeds according to the following steps:

Step 1: If $D_1 \subseteq (-\infty, 1 + k_1]$, let $\hat{D}_1 = D_1$ and skip to step 2. If $D_1 \subseteq (-\infty, k_i]$, let $d^* = \max D_1 = \max_{d \in D_1} d$. Let $I_1 = \{d^*\}$ and skip to step 3. If $D_1 \nsubseteq (-\infty, 1 + k_1]$, replace D_1 with $\hat{D}_1 \subseteq (-\infty, 1 + k_1]$ such that:

$$\mathbb{E}_1^1 \hat{D}_1 = \mathbb{E}_1^1 D_1, \tag{8.1}$$

$$\mathbb{E}_1^P \hat{D}_1 \geq \mathbb{E}_1^P D_1, \tag{8.2}$$

and

$$\mathbb{E}_1^j \hat{D}_1 \leq \mathbb{E}_1^j D_1, \tag{8.3}$$

for all $j > 1$.

Step 2: Find $I_1 = [k_1, a_1^*]$ such that:

$$\mathbb{E}^1 I_1 = \mathbb{E}^1 \hat{D}_1, \tag{8.4}$$

$$\mathbb{E}^P I_1 \geq \mathbb{E}^P \hat{D}_1, \tag{8.5}$$

and

$$\mathbb{E}^j I_1 \leq \mathbb{E}^j \hat{D}_1, \tag{8.6}$$

for all $j > 1$.

Notice that for all $j > 1$, $\mathbb{E}^j D_j \geq \mathbb{E}^j I_1$.

Step 3: Repeat steps 1 and 2 replacing 1 with i for all $i > 1$.

Notice again that for all $j > i$, $\mathbb{E}^j D_j \geq \mathbb{E}^j I_i$.

Note that by equations (8.1), (8.3), (8.4), and (8.6), and incentive compatibility of the original menu we have $a_i^* \geq a_j^*$, for all $j < i$. This holds since $\mathbb{E}^i I_i \geq \mathbb{E}^i I_j$ if and only if $a_i^* \geq a_j^*$.

Thus, the menu $\{I_1, \ldots, I_N\}$ is a nice menu and we have the following result.

Proposition 8.5. *In settings Q and T, the restriction to convex sets is without loss: the optimal delegation set is pooling and convex.*

Proof. See Appendix. □

While we leave the details of the construction to the Appendix. The construction of $\hat{D}_i$ is valid for setting D. The properties of settings Q and T are used for the construction of I_i. In all settings, the agent gains expected payoff from filling in a gap. In all settings, the principal gains expected payoff from filling in a gap (this is the gap filling lemma). In setting Q, the expected gain to the principal from filling in a gap is equal to the expected gain to an agent from filling in a gap (this is the meaning of equation (14.100) in the Appendix).

In setting T, the gain to the principal from filling in a gap is the same as the gain to the unbiased agent (since they have the same preferences). The only difference is in the final stage. As in setting Q, we replace D_1 with I_1 and D_2, with $\hat{D}_2$. Yet, the gain to the principal from filling in a gap may be less than that of the agent (since the preference function is more general than quadratic and agent 2 has positive bias). However, by the Gap Filling Lemma, we can simply fill in all the gaps in $\hat{D}_2$ (resulting in the new set $\tilde{I}_2 = [k_2, \tilde{a}_2])$ and be assured that $\tilde{a}_2 \geq a_1^*$, where $I_1 = [k_1, a_1^*]$. Thus, we found a nice menu that yields the principal higher expected payoff than the the original menu. Hence, by Corollary 7.6, we know that there is a convex, pooling menu that yields the principal higher expected payoff than the original menu. Thus, we know that convex, pooling menus are optimal. In the next section, we discuss the comparative statics of these menus.

9 Comparative Statics

Let $I(u) = [0, u]$

$$V_i(u) := \mathbb{E}_i^P I(u) = \begin{cases} \int_0^1 U(u-s)ds & \text{when } u \in [0, k_i] \ , \\ (u-k_i)U(k_i) + \int_{u-k_i}^1 U(u-s) & \text{when } u \in [k_i, 1+k_i] \ , \\ U(k_i) & \text{when } u \geq 1+k_i. \end{cases} \tag{9.1}$$

Notice that $V_i(k_i, u)$ is twice differentiable and strictly concave in u.

Differentiating we get:

$$\frac{\partial V}{\partial u}(k_i, u) := \frac{d}{du}\big(\mathbb{E}_i^P I(u)\big) = \begin{cases} \int_0^1 U'(u-s)ds = U(u) - U(1-u) & \text{when } u \in [0, k_i] \ , \\ U(k_i) + \int_{u-k_i}^1 U'(u-s) = U(k_i) - U(1-u) & \text{when } u \in [k_i, 1+k_i] \ , \\ 0 & \text{when } u \geq 1+k_i. \end{cases} \tag{9.2}$$

Notice that $\frac{\partial V}{\partial u}(k_i, u)$ is nonincreasing in k_i: if $k_i' \geq k_i$, then $\frac{\partial V}{\partial u}(k_i', u) \leq \frac{\partial V}{\partial u}(k_i, u)$.

Let $m_u = \{I(u)\}$. Differentiability and strict concavity imply that $\sum_{i=1}^N p_i V(k_i, u) = \mathbb{E}^P(m_u)$ is strictly concave and twice differentiable and is maximized at u such that

$$\sum_{i=1}^N p_i \frac{\partial V}{\partial u}(k_i, u) = 0 \tag{9.3}$$

In addition, if the p_i are held fixed, but and if $k_i' \geq k_i$, then the fact that $\frac{\partial V}{\partial u}(k_i, u)$ is nonincreasing in k_i' gives us:

$$\sum_{i=1}^{N} p_i \frac{\partial V}{\partial u}(k_i, u) \leq 0. \tag{9.4}$$

Hence, let $\mathbf{p}_N := (p_1, \ldots, p_N)$, where $\sum_{i=1}^{N} p_i \leq 1$ (the subscript N denotes the dimension of the vector- $\mathbf{p}_L$ would be an L-tuple). and $\mathbf{k}_N := (k_1, \ldots, k_N)$ (again, the subscript N denotes the dimension of the vector- $\mathbf{k}_L$ would be an L-tuple). Denote the optimal pooling delegation set by $D^*(\mathbf{p}_N, \mathbf{k}_N)$. Remembering that that if $k_1 > \frac{1}{2}$ the optimal delegation menu is $\{\frac{1}{2}\}$, we have the following result:

Result 9.1. *Fix $\boldsymbol{p}_N$. The optimal pooling delegation set, $D^*(\boldsymbol{p}_N, \boldsymbol{k}_N)$ is weakly decreasing in $\boldsymbol{k}_N$. Formally,*

$$\boldsymbol{k}'_N \geq \boldsymbol{k}_N \Rightarrow D^*(\boldsymbol{p}_N, \boldsymbol{k}'_N) \subseteq D^*(\boldsymbol{p}_N, \boldsymbol{k}_N), \tag{9.5}$$

where $\boldsymbol{k}'_N \geq \boldsymbol{k}_N$ iff $k'_i \geq k_i$ for all $i \in \{1, \ldots, N\}$.

Now, let $F_{(\mathbf{p}_N, \mathbf{k}_N)}(z)$ denote the cumulative density function of the distribution of the biases $\mathbf{k}_N$ under the probability distribution $\mathbf{p}_N$. We can extend the previous result:

Result 9.2. *The optimal pooling delegation set, $D^*(\boldsymbol{p}_N, \boldsymbol{k}_N)$ is weakly decreasing in first-order stochastic dominance ($\succsim_{1st}$). Formally,*

$$(\boldsymbol{p}'_N, \boldsymbol{k}'_N) \succsim_{1st} (\boldsymbol{p}_L, \boldsymbol{k}_L) \Rightarrow D^*(\boldsymbol{p}'_N, \boldsymbol{k}'_N) \subseteq D^*(\boldsymbol{p}_L, \boldsymbol{k}_L), \tag{9.6}$$

where $(\boldsymbol{p}'_N, \boldsymbol{k}'_N) \succsim_{1st} (\boldsymbol{p}_L, \boldsymbol{k}_L)$ iff $F_{(\boldsymbol{p}'_N, \boldsymbol{k}'_N)}(z) \leq F_{(\boldsymbol{p}_L, \boldsymbol{k}_L)}(z)$ for all $z \in \mathbb{R}$.

Proof. We provide a complete proof in the Appendix. The intuition of the proof is to show that if one lottery, a, over types (first-order) stochastically dominates another, b, then we can convert a into b through a sequences of monotonic adjustments to bias and monotonic adjustments to the probabilities. □

Thus, if one draw of types is "more biased" (according to first-order stochastic dominance), then the principal will offer the riskier draw a smaller delegation set (of the optimal menu).

10 Conclusion

In this paper we have shown that under uncertainty over the preferences of agents (and the technical conditions over distributions and preferences), pooling delegation menus perform as well (or better) than screening menus if menus are restricted to those that only contain convex sets. We also showed that this restriction was without loss in multiple settings. In these settings, we showed that there is no benefit to screening. Thus, the informational role of the delegation set should be to reveal the state of nature and not the preferences of the agent. Another implication of this paper is that the principal cannot screen with partial commitment (committing to take certain actions but not others). This holds since the case of cheap talk is equivalent to committing to take the equilibrium decisions of the model. Thus, screening between agents of different bias will require different channels.

The results in this paper suggest several directions for future work. First, it remains to generalize the results of this paper to settings where the states are not distributed uniformly, the bias is not constant, and the utility is not necessarily quadratic. One can also explore whether stochastic mechanisms yield the principal higher expected payoff than deterministic mechanisms. In addition, one may generalize the results of this paper to the case of more than 1 agent (potentially to analyze hierarchies). In addition, as in Amador and Bagwell (2012) one may consider alternate quasilinear utilities to incorporate the possibility of money burning in delegation. One may consider (finite) repeated interaction to see if there is a screening menu that yields the principal strictly higher expected payoff than the optimal (repeated pooling) delegation menu.

11 Appendix A: Proofs for Results in Section 4

Lemmas 11.1-11.4 are the delegation schedule analogs of *delegation rule* lemmas in Proposition 1 of Melumad and Shibano (1991). Notice that $U^i = U^P$, for $k_i = 0$. In addition, U^i is single-peaked (for each s, there is an x such that $\frac{\partial U^i}{\partial x}\big(x - s - k_i\big) = 0$), $\frac{\partial^2 U^i}{\partial x^2}\big(x - s - k_i\big) < 0$, and $\frac{\partial^2 U^i}{\partial x \partial s}\big(x - s - k_i\big) > 0$. These are the conditions on the the utility function for Proposition 1 of Melumad and Shibano (1991). Hence, we can cite a few results of Proposition 1 from their paper:

Lemma 11.1 (Delegation schedules are weakly increasing)**.** *For all $i \in \mathcal{N}$, $x_i^D(s)$ is weakly increasing in s and the only discontinuities of it are jump discontinuities.*

Thus, we know that for all D and i, $x_i^D(s)$ is weakly increasing and, hence, has only jump discontinuities. Let $x_i^{D+}(s) = \lim_{r \to s^+} x_i^D(s)$ and $x_i^{D-}(s) = \lim_{r \to s^-} x_i^D(s)$. By part (iii) of Proposition 1 of Melumad and Shibano (1991) we have:

Lemma 11.2. *At a point of discontinuity, $\tau \in [0, 1]$, of x_i^D, we have that: (a) $|x_i^{D+}(\tau) - \tau - k_i| = |x_i^{D-}(\tau) - \tau - k_i|$. (b) $x_i^D(\tau) \in \{x_i^{D-}(\tau), x_i^{D+}(\tau)\}$.*

In addition, we have the following corollary:

Corollary 11.3. *If τ is a point of discontinuity of the delegation schedule,*

$$x_i^{D-}(\tau) < \tau + k_i < x_i^{D+}(\tau). \tag{11.1}$$

Finally, we know that the function x_i^D achieves both the right and left hand limits at a point of discontinuity (though not at the same point):

Lemma 11.4. *If τ is a point of discontinuity, there exist $s_1, s_2 \in [0,1]$ such that $x_i^D(s_1) = x_i^{D+}(\tau)$ and $x_i^D(s_2) = x_i^{D-}(\tau)$.*

Proof. By Lemma 11.2, we know that $x_i^D(\tau) \in \{x_i^{D-}(\tau), x_i^{D+}(\tau)\}$. W.L.O.G. assume $x_i^D(\tau) = x_i^{D-}(\tau)$.

By Corollary 11.3, we know that there is s_1 close enough to τ that $\tau + k_i < s_1 + k_i < x_i^{D+}(\tau)$. In addition, we know that $x_i^{D+}(\tau) \in D$, by compactness (closure) of D (since compactness implies closure). Also, we know

$$\left(x_i^{D-}(\tau), x_i^{D+}(\tau)\right) \cap D = \emptyset.$$

Otherwise, $x_i^D(\tau)$ is not optimal.

By Lemma 11.2,

$$|x_i^{D-}(\tau) - \tau - k_i| = |x_i^{D+}(\tau) - \tau - k_i|.$$

Thus,

$$|x_i^{D-}(\tau) - s_1 - k_i| > |x_i^{D+}(\tau) - s_1 - k_i|.$$

Thus, $x_i^{D+}(\tau) = x_i^D(s_1)$. □

Comment: In addition, this proof shows that if the point of discontinuity, τ, is contained in the interior of the unit interval, then $\exists s_1$, such that $x_i^D(s) = x_i^{D+}(\tau), \forall s \in (\tau, s_1]$. By a similar argument, $\exists s_2$, such that $x_i^D(s) = x_i^{D-}(\tau), \forall s \in [s_2, \tau)$.

Proof of Lemma 4.1:

Proof. We prove this by showing that the set $I(x_i^D)$ is bounded and closed. Hence, by the Heine-Borel theorem it is compact.

Step 1: $I(x_i^D)$ is bounded.

Proof. First, since x_i^D weakly-increasing by Lemma 11.1. Thus, it's range is bounded: $x_i^D(0) \le x_i^D(s) \le x_i^D(1)$ for all $s \in [0,1]$. □

Step 2: Admissible sets are closed.

Proof. We prove this lemma by showing that the complement of $I(x_i^D)$ is open.

Let $q \in \mathbb{R}$ where q is in the complement of $I(x_i^D)$, $I(x_i^D)^C$. Then, $\exists \epsilon > 0$ such that $(q-\epsilon, q+\epsilon) \subseteq I(x_i^D)^C$.

Otherwise, q would be a right or left hand limit of x_i^D by Lemma 11.1. But then, by Lemma 11.4, $q \in I(x_i^D)$. But this is a contradiction. Thus, $I(x_i^D)^C$ is open and $I(x_i^D)$ is closed. □

Since $I(x_i^D)$ is closed and bounded, by the Heine-Borel Theorem it is compact. □

12 Appendix B: Proofs for Results in Section 7

Proof of Lemma (Down to k_i Lemma):

Proof. If $a \geq 1 + k_i$, then $U(x_i^D(s) - s) < U(k_i)$ for all $s \in [0,1]$. Thus,

$$\mathbb{E}_i^P D = \int_0^1 U(x_i^D(s) - s)ds < \int_0^1 U(k_i)ds = \mathbb{E}_i^P D'. \tag{12.1}$$

If $a < 1 + k_i$, then

$$E_i^P D = \int_0^1 U(x_i^D(s) - s)ds = \int_0^{a-k_i} U(a_i - s)ds + \int_{a-k_i}^1 U(x_i^D(s) - s)ds \tag{12.2}$$

$$= \int_{k_i}^a U(s)ds + \int_{a-k_i}^1 U(x_i^D(s) - s)ds < \int_0^{a-k_i} U(k_i)ds + \int_{a-k_i}^1 U(x_i^D(s) - s)ds = \mathbb{E}_i^P D' \tag{12.3}$$

since $a > k_i$. Thus, D is not optimal. □

Proof of Lemma 7.2:

Proof. Notice that if $k_i \geq \frac{1}{2}$, then $\Psi_i(k_i) > \Psi_i(b), \forall\, b > k_i \geq \frac{1}{2}$. In addition, for strictly concave $U(\cdot)$

$$\operatorname*{argmax}_{b \in [0,1]} \int_0^1 U(b - s)ds = \frac{1}{2}. \tag{12.4}$$

Thus, for $k_i \geq \frac{1}{2}$, an optimal convex delegation set is $[0, \frac{1}{2}]$.

If $k_i < \frac{1}{2}$, then

$$\Psi_i(k_i) = \int_0^1 U(k_i - s)ds > \int_0^1 U(b - s)ds = \Psi_i(b), \forall\, b \in [0, k_i).$$

But over $[k_i, 1]$ Ψ is differentiable and strictly concave. Hence, $\Psi_i(b)$ is maximized by b^* such that

$$\Psi_i'(b^*) = U(k) - U(1-b^*) = 0$$

$$\iff b^* = 1 - k_i,$$

since $\Psi_i''(b) = U'(1-b) < 0$, for $b \in [k_i, 1)$. Thus, we know that $\Psi_i(\cdot)$ is strictly increasing until b^* and decreasing after. □

Proof of Lemma 7.3 (Single-Crossing Lemma):

Proof. Assume that $a_i > a_j$ and $b_i > b_j$. W.L.O.G. We can assume (by the nonredundancy result) that $b_i \leq 1 + k_i$ and $a_j \geq k_j$.

$$\mathbb{E}^j D_j = \int_0^{a_j - k_j} U(a_j - s - k_j) ds + \int_{b_j - k_j}^1 U(b_j - s - k_j) ds = \int_0^{a_j - k_j} U(s) ds + \int_0^{1 + k_j - b_j} U(s) ds, \tag{12.5}$$

In addition, we have:

$$\mathbb{E}^j D_i = \int_0^{a_i - k_j} U(a_i - s - k_j) ds + \int_{b_i - k_j}^1 U(b_i - s - k_j) ds = \int_0^{a_i - k_j} U(s) ds + \int_0^{1 + k_j - b_i} U(s) ds, \tag{12.6}$$

The IC_k condition implies that $\mathbb{E}^j D_j \geq \mathbb{E}^j D_i$.

$$\iff \int_{1+k_j-b_i}^{1+k_j-b_j} U(s) ds \geq \int_{a_j-k_j}^{a_i-k_j} U(s) ds \tag{12.7}$$

$$\iff \int_{1+k_i-b_i}^{1+k_i-b_j} U(s) ds > \int_{a_j-k_i}^{a_i-k_i} U(s) ds, \tag{12.8}$$

since $k_i < k_j$ and $U(\cdot)$ is strictly concave

$$\iff \mathbb{E}^i D_j > \mathbb{E}_i^D \tag{12.9}$$

and the IC_k condition is violated. □

Proof of the No Need to Screen Result for Convex Menus (Proposition 7.4):

Proof. If there is a type with bias strictly less than $\frac{1}{2}$, then by the Single-Crossing Lemma (Lemma 7.3), we know that if a menu $m = \{D_1 = [a_1, b_1], \ldots, D_N = [a_N, b_N]\}$ satisfies the IC_k constraints we need

$$b_1 < b_2 < \cdots < b_N \tag{12.10}$$

By Lemma 12, we know that if we were to replace each D_i by $D_i^0 = [k_i, b_i]$ (forming the menu m^0), then

$$\int_0^1 U(x_i^{D_i}(t) - t)dt \leq \int_0^1 U(x_i^{D_i^0}(t) - t)dt, \tag{12.11}$$

and, therefore,

$$\mathbb{E}^P(m) = \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D_i}(t) - t)dt \right) \leq \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D_i^0}(t) - t)dt \right) = \mathbb{E}^P(m^0). \tag{12.12}$$

Thus, while m^0 may yield the principal a higher expected payoff, it may not be incentive compatible. Thus, we will modify this menu further (making it both a incentive compatible and a pooling menu).

By the Known Bias Optimum Lemma (Lemma 7.2) we know that the optimal complete information (over types) delegation set for type i is equal to $D^{i*} = [0, q_i]$, where $q_i = \max\{\frac{1}{2}, 1 - k_i\}$. Thus, we have:

$$q_1 \geq q_2 \geq \cdots \geq q_N. \tag{12.13}$$

Recalling Equation (12.10):

$$b_N > \cdots > b_1. \tag{12.14}$$

Roughly, these equations state that the optimal delegation sets (under complete information) are decreasing. In contrast, the sets in a non-pooling, but IC_k menu must be increasing. We will use this contrast to achieve a contradiction.

If $b_1 \geq q_1$, then the pooling menu with pooling set $[0, q_1]$ is IC_k (trivially) and yields the principal strictly higher expected payoff than m^0 (and m) from the Known Bias Optimum Lemma (since the expected payoff to the principal is decreasing when the delegation set is too large- thus, there is a gain from shrinking each D_i^0).

If $b_N \leq q_N$, then the pooling menu with pooling set $[0, q_N]$ is IC_k (trivially) and yields the principal strictly higher expected payoff than m^0 (and m) by the Known Bias Optimum Lemma. If $b_1 < q_1$ and $b_N > q_N$ we define the *Turning Point Type*

$$i^* = \max\{i \in \mathcal{N} | b_i \leq q_i\}. \tag{12.15}$$

By equations (12.14) and (12.13) we know that i^* is well-defined.

W.L.O.G assume $b_{i^*} < q_{i^*+1} \leq q_{i^*} < b_{i^*+1}$. Let the pooling delegation set be $D^* = [0, q_{i^*+1}]$. From the Known Bias Optimum Lemma, we know the the pooling menu yields the principal strictly higher expected payoff than menu m^0 (and m). In addition, it satisfies IC_k (trivially). Hence, we have shown by contradiction, that the optimal convex menu must be pooling. $\square$

13 Appendix C: Proofs for Section 8.2

Proof of the Gap Filling Lemma, Lemma 13.2:

There are two possible cases for the gap $G = (l, h)$: (a) $l \geq k_1$ or (b) $l < k_1$. We prove the Gap Filling Lemma for each case separately.

In each case we show that the deviation generated by the unfilled set is a mean-preserving spread of the (partially) filled in set. Let's begin with case (a): $l \geq k_1$.

13.1 Proof of Case (a)

Let's track the deviation of the delegation schedule from the ideal point of the principal.

$$\mathbb{X}_i^D(s) = x_i^D(s) - s = \begin{cases} l - s & \text{when } s \in [l - k_i, \frac{h+l}{2} - k_i) \text{ ,} \\ h - s & \text{when } s \in (\frac{h+l}{2} - k_i, h - k_i] \text{ ,} \end{cases} \tag{13.1}$$

Let $\epsilon \leq \frac{h-l}{2}$, $D(\epsilon, l, h) = D \cup [l, l + \epsilon] \cup [h - \epsilon, h]$. First, notice that

$$x_i^D(s) - s = x_i^{D(\epsilon,l,h)}(s) - s, \tag{13.2}$$

when $s \in [0, l - k_i] \cup [h - k_i, 1]$. When $s \in [l - k_i, h - k_i]$,

$$\mathbb{X}_i^{D(\epsilon,l,h)}(s) = x_i^{D(\epsilon,l,h)}(s) - s, \tag{13.3}$$

where

$$\mathbb{X}_i^{D(\epsilon,l,h)}(s) = \begin{cases} k_i & \text{when } s \in [l - k_i, l + \epsilon - k_i] \text{ ,} \\ l + \epsilon - s & \text{when } s \in [l + \epsilon - k_i, \frac{h+l}{2} - k_i) \text{ ,} \\ h - \epsilon - s & \text{when } s \in (\frac{h+l}{2} - k_i, h - \epsilon - k_i] \text{ ,} \\ k_i & \text{when } s \in [h - \epsilon - k_i, h - k_i] \text{ ,} \end{cases} \tag{13.4}$$

We argue that $\mathbb{X}_i^D$ is a mean-preserving spread of $\mathbb{X}_{D(\epsilon,l,h)}(s)$ when s is uniformly distributed between the interval $[l - k_i, h - k_i]$.

Lemma 13.1. *$\mathbb{X}_i^D(s)$ is a mean-preserving spread of $\mathbb{X}_i^{D(\epsilon,l,h)}(s)$ when s is uniformly distributed between the interval $[l - k_i, h - k_i]$.*

Proof. For this proof, we fix l and h. Thus, we denote $D(\epsilon, l, h)$ by $D(\epsilon)$.

Part A: We first show that $\mathbb{X}_i^D(s)$ and $\mathbb{X}_i^{D(\epsilon)}(s)$ have the same mean.

Let $\eta = h - l$.

$$\mathbb{E}\mathbb{X}_i^D = \frac{1}{\eta}\left(\int_{l-k_i}^{\frac{h+l}{2}-k_i}(l-s)ds+\int_{\frac{h+l}{2}-k_i}^{h-k_i}(h-s)ds\right) = \frac{1}{\eta}\left(\int_0^{\frac{h-l}{2}}(k_i-t)dt+\int_{-\frac{h-l}{2}}^{0}(k_i-t)dt\right), \tag{13.5}$$

where equality was obtained by a change of variables $s = t + l - k_i$ in the first integral and $s = t + h - k_i$ in the second integral.

But by (13.5) we have that

$$\mathbb{E}\mathbb{X}_i^D = k_i \tag{13.6}$$

but then

$$\mathbb{E}\mathbb{X}_i^D = \mathbb{E}\mathbb{X}_i^{D(\epsilon)} = k_i \tag{13.7}$$

since

$$\mathbb{E}\mathbb{X}_i^{D(\epsilon)} = \frac{1}{\eta}\left(\int_{l-k_i}^{l+\epsilon-k_i}k_i ds+\int_{l+\epsilon-k_i}^{\frac{h+l}{2}-k_i}(l+\epsilon-s)ds+\int_{\frac{h+l}{2}-k_i}^{h-\epsilon-k_i}(h-\epsilon-s)ds+\int_{h-\epsilon-k_i}^{h-k_i}k_i ds\right) \tag{13.8}$$

$$= \frac{1}{\eta}\left(2\epsilon k_i + \int_0^{\frac{h-l}{2}-\epsilon}(k_i - s)ds + \int_{-\frac{h-l}{2}+\epsilon}^{0}(k_i - s)ds\right) = k_i, \tag{13.9}$$

where the equality between equations (13.8) and (13.9) follows a change of variables similar to that in (13.5).

Part B: We now show that for all $t \in \mathbb{R}$, we have:

$$\int_{-\infty}^{t}\big(F_D(s) - F_{D(\epsilon)}(s)\big)ds \geq 0.$$

(and the inequality holds strictly for $s \in (k_i - \frac{h-l}{2}, k_i + \frac{h-l}{2})$.

Let $F_D(x)$ denote the cdf of $\mathbb{X}_i^D$ and let $F_{D(\epsilon)}(x)$ denote the cdf of $\mathbb{X}_{D(\epsilon,l,h)}$.

$$F_D(x) = \begin{cases} 0 & \text{when } s \in (\infty, k_i - \frac{h-l}{2}] \ , \\ \frac{1}{\eta}(s - (k_i - \frac{h-l}{2})) & \text{when } s \in [k_i, \frac{h-l}{2} + k_i] \ , \\ 1 & \text{when } s \in [\frac{h-l}{2} + k_i, \infty) \ , \end{cases} \tag{13.10}$$

$$F_{D(\epsilon)}(x) = \begin{cases} 0 & \text{when } s \in (-\infty, k_i - \frac{h-l}{2} + \epsilon] \ , \\ \frac{1}{\eta}(s - (k_i - \frac{h-l}{2} + \epsilon)) & \text{when } s \in [k_i - \frac{h-l}{2} + \epsilon, k_i) \ , \\ \frac{2\epsilon}{\eta} + \frac{1}{\eta}(s - (k_i - \frac{h-l}{2} + \epsilon)) & \text{when } s \in [k_i, k_i + \frac{h-l}{2} - \epsilon) \ , \\ 1 & \text{when } s \in [k_i + \frac{h-l}{2} - \epsilon, \infty) \ , \end{cases} \tag{13.11}$$

Notice that $\int_{-\infty}^{t} \big(F_D(s) - F_{D(\epsilon)}(s)\big) ds \geq 0$,for all $t \in (-\infty, k_i)$ since $F_D(s) \geq F_{D(\epsilon)}(s)$ for all such s (and the inequality is strict from $[k_i - \frac{h-l}{2}, k_i)$).

Define $\psi(\cdot)$ such that:

$$\psi(s) := F_D(k_i + s) - F_{D(\epsilon)}(k_i + s).$$

Notice that for $s \in [-\frac{h-l}{2}, \frac{h-l}{2}]$ we have:

$$\psi(s) = -\psi(s). \tag{13.12}$$

Thus, since $F_D(s) = F_{D(\epsilon)}(s)$ for $s \in (-\infty, k_i - \frac{h-l}{2}] \cup [k_i + \frac{h-l}{2}, \infty)$ and since $F_D(s) > F_{D(\epsilon)}(s)$ for all such $s \in [k_i - \frac{h-l}{2}, k_i)$, equation (13.12) implies that

$\int_{-\infty}^{t} \big(F_D(s) - F_{D(\epsilon)}(s)\big) ds \geq 0$ for all $s \in \mathbb{R}$ (and the inequality is strict for $s \in [k_i - \frac{h-l}{2}, k_i + \frac{h-l}{2})$. □

Lemma 13.1 that filling in a gap (by any amount) strictly increases the utility of the principal (and, of course, the agent), *independent of the agent's bias!* This is summarized in the following proposition:

Lemma 13.2. *(Gap Filling Lemma- Case (a), $G \subseteq [k_i, 1 + k_i]$) Let $k_i \geq 0$. Let D be a set with gap, $G = (l, h)$, such that $G \subseteq [k_i, 1 + k_i]$. Then we know that $\mathbb{E}_i^P D = \int_0^1 U(x_i^D(s) - s) ds < \int_0^1 U(x_i^{D(\epsilon)}(s) - s) ds = \mathbb{E}_i^P D(\epsilon)$. Hence, completely filling in the gap (replacing D with $D' = D \cup G$) would yield the principle strictly higher utility:*

$$\mathbb{E}_i^P D < \mathbb{E}_i^P D' = \mathbb{E}_i^P (D \cup G). \tag{13.13}$$

Proof. We know that $x_i^D(s) = x_i^{D(\epsilon)}(s)$ for all $s \in [0, l] \cup [h, 1]$. Thus, we just need to show that

$$\int_l^h U(x_i^D(s) - s) ds < \int_l^h U(x_i^{D(\epsilon)}(s) - s) ds. \tag{13.14}$$

But from Lemma 13.1 that $\mathbb{X}_i^D = x_i^D(s) - s$ is a mean-preserving spread of $\mathbb{X}_i^{D(\epsilon)} = x_i^{D(\epsilon)}(s) - s$. So since $U(\cdot)$ is strictly concave we have that:

$$\frac{1}{\eta}\int_l^h U(x_i^D(s)-s)ds < \frac{1}{\eta}\int_l^h U(x_i^{D(\epsilon)}(s)-s)ds, \tag{13.15}$$

which is equivalent to equation (13.14).

Equation (13.13) is obtained by replacing D with $D(\epsilon)$, where $\epsilon = \frac{h-l}{2}$ (since $D(\frac{h-l}{2}) = D \cup G$).

This concludes the proof of case (a). □

For case (b), the argument is similar:

13.2 Proof for Case (b) $l < k_1$

In order to to prove the result, we need to prove an analogous mean-preserving spread lemma to Lemma 13.1. In this case, the gap, $G = (l, h)$ will not be contained in $[k_i, 1 + k_i]$. Yet, $h \in (k_i, 1 + k_i]$ (if $h = k_i$, the filling in the gap has no effect on the agent's actions). Another condition needed to make the case nontrivial is that $\frac{h+l}{2} - k_i > 0$. Otherwise, l would be played with zero probability and filling in the gap is equivalent to "lower" the delegation set as in the down to k Lemma (Lemma 12). As in the down to k Lemma, filling in the gap (or, equivalently in this special case, adding to the delegation set from below) strictly increases the expected payoff of the principal.

Once again, we define the deviation function, $\mathbb{X}_i^D(s)$ as before:

$$\mathbb{X}_i^D(s) = x_i^D(s) - s = \begin{cases} l - s & \text{when } s \in [0, \frac{h+l}{2} - k_i) \\ h - s & \text{when } s \in (\frac{h+l}{2} - k_i, h - k_i] \end{cases}. \tag{13.16}$$

Notice that $k - l < \frac{h-l}{2}$. Thus, $k - l + \frac{h+l}{2} \leq h - \epsilon$. In add

$$\mathbb{X}_i^D(s) = x_i^D(s) - s, \tag{13.17}$$

when $s \in [l - k_i, h - k_i]$.

Let $\epsilon \leq k_i - l$, $D(\epsilon, l, h) = D \cup [l, l + \epsilon] \cup [h - \epsilon, h]$. The inequalities $l < k_i$ and $\epsilon \leq k_i - l$ imply that the type i will never play an action lower than $l + \epsilon$. In fact, type i will play l until the state he is indifferent between l and h.

Hence, if we define the deviation function as before, we have:

$$\mathbb{X}_i^{D(\epsilon,l,h)}(s) = x_i^{D(\epsilon,l,h)}(s) - s, \tag{13.18}$$

where

$$\mathbb{X}_i^{D(\epsilon,l,h)}(s) = \begin{cases} l + \epsilon - s & \text{when } s \in [0, \frac{h+l}{2} - k_i) \\ h - \epsilon - s & \text{when } s \in (\frac{h+l}{2} - k_i, h - \epsilon - k_i] \\ k_i & \text{when } s \in [h - \epsilon - k_i, h - k_i] \end{cases}. \tag{13.19}$$

In addition, notice that $\mathbb{X}_i^{D(\epsilon,l,h)}(s) = \mathbb{X}_i^D(s)$ for all other s in $[0,1]$.

We argue that $\mathbb{X}_i^D$ is a mean-preserving spread of $\mathbb{X}_i^{D(\epsilon,l,h)}(s)$ when s is uniformly distributed between the interval $[0, h+l-2k_i]$.

Lemma 13.3. *$\mathbb{X}_i^D(s)$ is a mean-preserving spread of $\mathbb{X}_i^{D(\epsilon,l,h)}(s)$ when s is uniformly distributed between the interval $[0, 2\big(\frac{h+l}{2} - k_i\big)]$.*

Proof. Part A: We first show that $\mathbb{X}_i^D(s)$ and $\mathbb{X}_i^{D(\epsilon,l,h)}(s)$ have the same mean.

Let $\eta = h + l - 2k_i$.

$$\begin{aligned}\mathbb{E}\mathbb{X}_i^D &= \frac{1}{\eta}\bigg(\int_0^{\frac{h+l}{2}-k_i}(l-s)ds + \int_{\frac{h+l}{2}-k_i}^{h-k_i-(k_i-l)}(h-s)ds\bigg) \qquad (13.20)\\ &= \frac{1}{\eta}\bigg(\int_{k_i-l}^{\frac{h-l}{2}}(k_i-t)dt + \int_{-\frac{h-l}{2}}^{-(k_i-l)}(k_i-t)dt\bigg),\end{aligned}$$

where equality was obtained by a change of variables $s = t + l - k_i$ in the first integral and $s = t + h - k_i$ in the second integral.

But by (13.20) we have that

$$\mathbb{E}\mathbb{X}_i^D = k_i \qquad (13.21)$$

but then

$$\mathbb{E}\mathbb{X}_i^D = \mathbb{E}\mathbb{X}_i^{D(\epsilon,l,h)} = k_i \qquad (13.22)$$

since

$$\begin{aligned}\mathbb{E}\mathbb{X}_i^{D(\epsilon,l,h)} &= \frac{1}{\eta}\bigg(\int_0^{\frac{h+l}{2}-k_i}(l+\epsilon-s)ds + \int_{\frac{h+l}{2}-k_i}^{h-\epsilon-k_i-(k_i-l-\epsilon)}(h-\epsilon-s)ds\bigg) \qquad (13.23)\\ &= \frac{1}{\eta}\bigg(\int_{k_i-l-\epsilon}^{\frac{h-l}{2}-\epsilon}(k_i-s)ds + \int_{-\left(\frac{h-l}{2}-\epsilon\right)}^{-\left(k_i-l-\epsilon\right)}(k_i-s)ds\bigg) = k_i, \qquad (13.24)\end{aligned}$$

where the equality between equations (13.23) and (13.24) follows a change of variables similar to that in (13.5).

Part B: We now show that for all $t \in \mathbb{R}$, we have:

$$\int_{-\infty}^{t}\big(F_D(s) - F_{D(\epsilon)}(s)\big)ds \geq 0$$

(and the inequality holds strictly for $s \in (k_i - \frac{h-l}{2}, k_i + \frac{h-l}{2})$.

As before we denote $D(\epsilon, l, h)$ by $D(\epsilon)$.

Let $F_D(x)$ denote the cdf of $\mathbb{X}_i^D$ and let $F_{D(\epsilon)}(x)$ denote the cdf of $\mathbb{X}_i^{D(\epsilon,l,h)}(\cdot)$. Recall that $\eta = h + l - 2k_i$.

$$F_D(x) = \begin{cases} 0 & \text{when } s \in (\infty, k_i - \frac{h-l}{2}] \\ \frac{1}{\eta}(s - (k_i - \frac{h-l}{2})) & \text{when } s \in [k_i - \frac{h-l}{2}, l] \\ \frac{1}{2} & \text{when } s \in [l, 2k_i - l] \\ \frac{1}{\eta}(s - (2(k_i - l))) + \frac{1}{2} & \text{when } s \in [2k_i - l, k_i + \frac{h-l}{2}] \ , \\ 1 & \text{when } s \in [k_i + \frac{h-l}{2}, \infty) \ . \end{cases} \tag{13.25}$$

$$F_{D(\epsilon)}(x) = \begin{cases} 0 & \text{when } s \in (-\infty, k_i - \frac{h-l}{2} + \epsilon] \\ \frac{1}{\eta}(s - (k_i - \frac{h-l}{2} + \epsilon)) & \text{when } s \in [k_i - \frac{h-l}{2} + \epsilon, l + \epsilon) \\ \frac{1}{2} & \text{when } s \in [l + \epsilon, 2k_i - l - \epsilon] \\ \frac{1}{\eta}(s - (2(k_i - l - \epsilon)) + \frac{1}{2} & \text{when } s \in [2k_i - l - \epsilon, k_i + \frac{h-l}{2} - \epsilon] \\ 1 & \text{when } s \in [k_i + \frac{h-l}{2} - \epsilon, \infty) \ . \end{cases} \tag{13.26}$$

Notice that $\int_{-\infty}^{t} \big(F_D(s) - F_{D(\epsilon)}(s)\big) ds \geq 0$ for all $t \in (-\infty, k_i)$ since $F_D(s) \geq F_{D(\epsilon)}(s)$ for all such s (and the inequality is strict from $[k_i - \frac{h-l}{2}, l + \epsilon)$).

Define $\psi(\cdot)$ such that:

$$\psi(s) := F_D(k_i + s) - F_{D(\epsilon)}(k_i + s).$$

Notice that for $s \in [l + \epsilon, 2k_i - l - \epsilon]$ we have:

$$\psi(s) = 0. \tag{13.27}$$

In addition, notice that for $s \in [k_i - \frac{h-l}{2}, l + \epsilon) \cup (2k_i - l - \epsilon, k_i + \frac{h-l}{2}]$ we have (similar to equation (13.12)):

$$\psi(s) = -\psi(s). \tag{13.28}$$

Thus, since $F_D(s) = F_{D(\epsilon)}(s)$ for $s \in (-\infty, k_i - \frac{h-l}{2}] \cup [k_i + \frac{h-l}{2}, \infty)$ and since $F_D(s) > F_{D(\epsilon)}(s)$ for all such $s \in [k_i - \frac{h-l}{2}, k_i)$, equations (13.27) and (13.28) imply that

$\int_{-\infty}^{t} \big(F_D(s) - F_{D(\epsilon)}(s)\big) ds \geq 0$ for all $s \in \mathbb{R}$ (and the inequality is strict for $s \in [k_i - \frac{h-l}{2}, k_i + \frac{h-l}{2})$. This concludes the proof of part (B).

Parts (A) and (B) imply that $\mathbb{X}_i^D$ is a mean-preserving spread of $\mathbb{X}_i^{D(\epsilon)}$

□

As before, this argument holds for all $k_i \geq 0$. In addition, we can now use Lemma 13.3 to extend the Gap Filling Lemma (Lemma **??**). We show that filling in a gap yields the principal strictly higher utility, even if the gap is not contained in the ideal action set of the agent. In other words, if $G \subseteq (-\infty, 1 + k_i]$ and $G \cap [k_i, 1 +_i] \neq \emptyset$, then $\mathbb{E}_i^P D < \mathbb{E}_i^P D(\epsilon, l, h)$. We state this more precisely in the following lemma:

Lemma 13.4. *(Gap Filling Lemma - Case (b) $G \not\subseteq [k_i, 1 + k_i]$). Let $G = (l, h)$ be a gap in the set D such that $G \subseteq (-\infty, 1 + k_i]$ and $G \cap [k_i, 1+_i] \neq \emptyset$. Let $\epsilon \leq \frac{h-l}{2}$. Hence,*

$$\mathbb{E}_i^P D \leq \mathbb{E}_i^P D(\epsilon, l, h). \tag{13.29}$$

Thus, completely filling in the gap (replacing D with $D\left(\frac{h-l}{2}, l, h\right) = D \cup (l, h)$ increases the expected payoff of the principal, where the inequalities are strict if $h > k_i$.

Proof. First, notice that $\mathbb{X}_i^D(s) = \mathbb{X}_i^{D(\epsilon,l,h)}(s)$ for all $s \in [h - k_i, 1]$. Thus, we compare the loss for the integral of the loss (utility) over the interval $[0, h - k_i]$. We break this comparison into two cases.

Case 1: $\epsilon \leq k_i - l$:

Recall that η (defined at the beginning of Lemma 13.3) is equal to $h + l - 2k_i$.

From Lemma 13.3 we know that $\mathbb{X}_i^D(s) = x_i^D(s) - s$ is a mean-preserving spread of $\mathbb{X}_i^{D(\epsilon,l,h)}(s) = x_i^{D(\epsilon,l,h)}(s) - s$ over the interval $[0, h + l - 2k_i]$. Since $U(\cdot)$ is strictly concave, we have that:

$$\frac{1}{\eta} \int_0^{h+l-2k_i} U(\mathbb{X}_i^D(s))ds < \frac{1}{\eta} \int_0^{h+l-2k_i} U(\mathbb{X}_i^{D(\epsilon,l,h)}(s))ds \tag{13.30}$$

$$\iff \int_0^{h+l-2k_i} U(\mathbb{X}_i^D(s))ds < \int_0^{h+l-2k_i} U(\mathbb{X}_i^{D(\epsilon,l,h)}(s))ds. \tag{13.31}$$

In addition, since $x_i^{D(\epsilon,l,h)}(s), x_i^D(s) > s + k_i$ for $s \in [h + l - 2k_i, h - k_i)$ and $|x_i^D(s) - s| > |x_i^{D(\epsilon,l,h)}(s) - s|$

$$\int_{h+l-2k_i}^{h-k_i} U(\mathbb{X}_i^D(s))ds < \int_{h+l-2k_i}^{h-k_i} U(\mathbb{X}_i^{D(\epsilon,l,h)}(s))ds. \tag{13.32}$$

Equations 13.31 and 13.32 imply

$$\int_0^{h-k_i} U(\mathbb{X}_i^D(s))ds < \int_0^{h-k_i} U(\mathbb{X}_i^{D(\epsilon,l,h)}(s))ds. \tag{13.33}$$

and since (as mentioned above) $\mathbb{X}_i^D(s) = \mathbb{X}_i^{D(\epsilon,l,h)}(s)$ for all $s \in [h - k_i, 1]$, equation 13.33 implies the conluding inequality for Case 1:

$$\mathbb{E}_i^P D < \mathbb{E}_i^P D(\epsilon, l, h). \tag{13.34}$$

Case 2: $k_i - l < \epsilon \leq \frac{h-l}{2}$ (Notice that $\epsilon = \frac{h-l}{2}$ fills in the gap.) Let ϵ be defined such that $k_i - l = \epsilon' < \epsilon \leq \frac{h-l}{2}$.

From Case 1, we know that

$$\mathbb{E}_i^P D < \mathbb{E}_i^P D(\epsilon', l, h). \tag{13.35}$$

Denote $D(\epsilon', l, h)$ by D'. But notice that D' is a set such that it's gap ($G' = (l' := l + \epsilon', h - \epsilon' =: h')$ is contained in $[k_i, 1 + k_i]$. Thus, by the Gap Filling Lemma for case (a) (Lemma 13.13), we know that

$$\mathbb{E}_i^P D' < \mathbb{E}_i^P D'(\epsilon - \epsilon', l', h') = \mathbb{E}_i^P D(\epsilon, l, h). \tag{13.36}$$

Hence, equations 13.35 and 13.36 imply the concluding inequality:

$$\mathbb{E}_i^P D < \mathbb{E}_i^P D(\epsilon, l, h). \tag{13.37}$$

If $\epsilon = \frac{h-l}{2}$, then equation (13.37) still holds and implies that a completely filled in gap strictly raises the expected utility of the principal (if $h > k_i$). □

13.3 Proof of Lemma 8.3, $D_P^* \subseteq (-\infty, 1 + k_1]$ WLOG

In order to prove this lemma, we show that we can ignore certain pathological cases. First, we can ignore the case when $l < 0$ and $h > 1$. For ϵ small enough, the set $\{l+\epsilon\} \cup \{h-\epsilon\}$ will yield the principal strictly higher expected payoff (this argument was made by Mylovanov, 2008). Next, we point out that in the optimal pooling menu, there cannot be more than one point higher than 1 in the optimal delegation set (played with strictly positive probability). The proof of this follows a similar argument to that of Mylovanov (Lemma 1, 2008).

Lemma 13.5. *(At Least One Point In* $[0, 1]$*) Let* D_P^* *be an optimal pooling menu, then* $D_P^* \cap [0, 1] \neq \emptyset$.

Proof. Assume not. So $D_P^* \cap [0, 1] = \emptyset$. Let l denote the largest point less than 0. Let h be the smallest point greater than 1 (both points are defined since D_P^* is closed.

Hence, there exists an $\epsilon > 0$ such that $D' \cap [0, 1] \neq \emptyset$, where $D' = \{l + \epsilon, h - \epsilon\}$. Then, $|x_i^{D'}(s) - s| < |x_i^{D_P^*}(s) - s|$, for all $s \in [0, 1]$ and for all $i \in \{1, \ldots, N\}$.

Thus, since $U(\cdot)$ is strictly increasing in the absolute value of loss

$$\mathbb{E}^P(D') = \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D'}(t) - t)dt \right) > \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D_P^*}(t) - t)dt \right) = \mathbb{E}^P(D_P^*), \tag{13.38}$$

which contradicts optimality of D_P^*. □

Next we show that there is, at most, 1 point $d \in D_P^*$ such that $d > 1$:

Lemma 13.6. *(At Most 1 Above 1 Lemma: 1A1 Lemma) Let D_P^* be an optimal pooling menu, then at most one element, d, such that $1 \geq d \in D_P^*$ can be played with positive probability.*

Proof. Assume not. Thus, there are d_1 and d_2 (assume without loss that $d_2 > d_1$) that are each played with positive probability. Let $d^* = \min\{d | d \in D_P^* \cap [1, \infty)\}$. This is well-defined since D_P^* is closed. Let $D' = D_P^* \cap (-\infty, d^*]$.

We now show that

$$\mathbb{E}^P(D') = \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D'}(t) - t)dt \right) > \sum_{i=1}^{N} p_i \left(\int_0^1 U(x_i^{D_P^*}(t) - t)dt \right) = \mathbb{E}^P(D_P^*). \tag{13.39}$$

Consider type i. We define the probability that type i selects an action greater than a point d, after choosing the delegation set, D.

$$T_i^D(d) := \{s | x_i^D(s) > d\}, \tag{13.40}$$

where we have that, if $T_i^D(d)$ has positive measure, then

$$\exists\ d_i' < 1, T_i^D(d) = \{s | x_i^D(s) > d\} \in \{(d_i', 1],\ [d_i', 1]\}, \tag{13.41}$$

since $x_i^D(\cdot)$ is increasing by Lemma 11.1.

If the probability that type i selects any action, d', such that $d' > d^*$ is zero, $\mathbb{P}\left(T_i^{D_P^*}(d^*) = \{s | x_i^{D_P^*}(s) > d^*\} \right) = 0$, then the expected payoff to the principal $(\int_0^1 U(x_i^{D_P^*}(t) - t)dt)$ is unchanged.

If $\mathbb{P}\big(T_i^{D_P^*}(d^*)\big) > 0$, recall from (13.40) that

$$T_i^{D_P^*}(d^*) = \{s | x_i^{D_P^*}(s) > d^*\} \in \{(d_i', 1],\ [d_i', 1]\}, \tag{13.42}$$

Thus, by (13.40),

$$x_i^{D'}(s) = d^* < x_i^{D_P^*}(s), \tag{13.43}$$

for all $s \in T_i^{D_P^*}(d^*)$. But since $d^* > 1$, $|x_i^{D'}(s) - s| < |x_i^{D_P^*}(s) - s|$ for all $s \in T_i^{d^*}$ (and equality holds for all other s).

Thus,

$$\int_0^1 U(x_i^{D'}(t) - t)dt\Big) > \int_0^1 U(x_i^{D_P^*}(t) - t)dt. \tag{13.44}$$

The result follows from plugging in (13.44) into (13.39).

□

Lemma (13.6) shows that we may restrict attention to sets with only one point strictly greater than 1.

Let d^* be the largest point in D_P^* that is contained in the unit interval. Notice that if h is played by type 1 (the type with the lowest bias) with zero probability, then we know that $1 + k_1 - d^* \leq h - 1 - k_1$, so $\frac{h+d^*}{2} \geq 1 + k_1$. Hence, $h - 1 > 1 - d^*$, and we are back to case 1 (where $\frac{h+d^*}{2} \geq 1$), which we ruled out already. Thus, we can assume that type 1 plays h with positive probability. Thus, we can assume that type 1 will play h in states $s \in (\frac{h+l}{2} - k_1, 1]$ Denote $\lambda_1 := \min\{1 - \big(\frac{h+l}{2} - k_1\big), 1\} > 0$. Similarly, let λ_i denote $\lambda_i := \min\{1 - \big(\frac{h+l}{2} - k_i\big), 1\} > 0$. Thus, $\lambda_i \leq \lambda_{i+1}$, for all i. In addition, $\lambda_i = \lambda_j$, if and only if $1 = \lambda_i = \lambda_j$. We can assume without loss of generality that $\lambda_1 < 1$. Otherwise, the set D_P^* contain a redundant point l, since if type 1 doesn't play l, then no type will ever play l.

Now remove h from D_P^* and add the interval $[l, l + \lambda_i]$ to it, resulting in $D_i' = (D_P^* \cap (-\infty, l]) \cup [l, l + \lambda_i]$. Notice that by the assumptions of the subcase and Lemma 13.5, l is in the unit interval. We now show why this adjustment strictly increases the utility of the principal. We prove this using the following lemmas.

Lemma 13.7. *(λ_i-Trick Lemma)*

Let i be such that $h > 1 + k_i$, $\lambda_i := \min\{1 - \big(\frac{h+l}{2} - k_i\big), 1\}$ (recall $\lambda_i > 0$ for all i), and $D_i' = (D_P^* \cap (-\infty, l]) \cup [l, l + \lambda_i]$.

For all such i ,

$$\mathbb{E}_i^P D_P^* < \mathbb{E}_i^P D_i' \tag{13.45}$$

Proof. The principal's expected utility from type i is

$$\mathbb{E}_i^P D_P^* = \int_0^{\max\{0,l-k_i\}} U(x_i^{D_P^*}(s) - s)ds + \int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(x_i^{D_P^*}(s) - s)ds + \int_{\frac{h+l}{2}-k_i}^1 U(x_i^{D_P^*}(s) - s)ds \tag{13.46}$$

$$= C_i^l + \int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(l-s)ds + \int_{\frac{h+l}{2}-k_i}^{1} U(h-s)ds \tag{13.47}$$

(where we denoted $\int_0^{\max\{0,l-k_i\}} U(x_i^{D_P^*}(s)-s)ds$ by C_i^l)

$$= C_i^l + \int_{\max\{-l,-k_i\}}^{\frac{h-l}{2}-k_i} U(s)ds + \int_{-\frac{h-l}{2}-k_i}^{1-h} U(s)ds \tag{13.48}$$

$$= C_i^l + \int_{k_i-\frac{h-l}{2}}^{\min\{l,k_i\}} U(s)ds + \int_{h-1}^{k_i+\frac{h-l}{2}} U(s)ds. \tag{13.49}$$

Since $h > 1 + k_i$, notice that

$$\int_{h-1}^{k_i+\frac{h-l}{2}} U(s)ds < \lambda_i U(k_i). \tag{13.50}$$

Hence, by equations (13.47), (13.48), (13.49), and (13.50), we have:

$$\mathbb{E}_i^P D_P^* < C_i^l + \lambda_i U(k_i) + \int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(l-s)ds. \tag{13.51}$$

Now for set D_i' we have:

$$\mathbb{E}_1^P D_i' = \int_0^{\max\{0,l-k_i\}} U(x_i^{D_i'}(s)-s)ds + \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(x_i^{D_i'}(s)-s)ds + \int_{\max\{0,l+\lambda_i-k_i\}}^{1} U(x_i^{D_i'}(s)-s)ds \tag{13.52}$$

$$= C_i^l + \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(x_i^{D_i'}(s)-s)ds + \int_{\max\{0,l+\lambda_i-k_i\}}^{1} U(x_i^{D_i'}(s)-s)ds \tag{13.53}$$

(where we denoted $\int_0^{\max\{0,l-k_i\}} U(x_i^{D_i'}(s)-s)ds = \int_0^{\max\{0,l-k_i\}} U(x_i^{D_P^*}(s)-s)ds$ by C_i^l)

$$= C_i^l + \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(\min\{l+\lambda_i, s+k_i\}-s)ds + \int_{\max\{0,l+\lambda_i-k_i\}}^{1} U(l+\lambda_i-s)ds \tag{13.54}$$

$$\geq C_i^l + U(k_i)\lambda_i + \int_{\max\{0,l-k_i\}}^{1-\lambda_i} U(l-s)ds \tag{13.55}$$

$$\geq C_i^l + U(k_i)\lambda_i + \int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(l-s)ds. \tag{13.56}$$

Inequality (13.55) is explained carefully in the next subsection.

So by equations (13.52)-(13.56):

$$\mathbb{E}_i^P D_i' \geq C_i^l + U(k_i)\lambda_i + \int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(l-s)ds. \tag{13.57}$$

But by equations (13.51) and (13.57), we have:

$$\mathbb{E}_i^P D_i' \geq C_i^l + U(k_i)\lambda_i + \int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(l-s)ds > \mathbb{E}_i^P D_P^*. \tag{13.58}$$

This inequality concludes the proof of this lemma.

□

13.4 Proof of Inequality (13.55)

Recall that Inequality (13.55) states that

$$\mathbb{E}_i^P D_i' = C_i^l + \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(\min\{l+\lambda_i, s+k_i\}-s)ds + \int_{\max\{0,l+\lambda_i-k_i\}}^{1} U(l+\lambda_i-s)ds \tag{13.59}$$

$$\geq C_i^l + U(k_i)\lambda_i + \int_{\max\{0,l-k_i\}}^{1-\lambda_i} U(l-s)ds.$$

Proof. *Case A:* $l + \lambda_i - k_i \leq 0$ In this case:

$$\mathbb{E}_i^P D' = C_i^l + \int_0^1 U(l+\lambda_i-s)ds = \int_0^{\lambda_i}(l+\lambda_i-s)ds + \int_{\lambda_i}^1 (l+\lambda_i-s)ds \tag{13.60}$$

$$> C_i^l + \lambda_i U(k_i) + \int_0^{1-\lambda_i}(l-s)ds, \tag{13.61}$$

which is the desired inequality.

Case B: $l + \lambda_i - k_i > 0 \geq l - k_i$

In this case:

$$\mathbb{E}_i^P D_i' = C_i^l + \int_0^{l+\lambda_i-k_i} U(k_i)ds + \int_{l+\lambda_i-k_i}^1 U(l+\lambda_i-s)ds \tag{13.62}$$

$$= C_i^l + (\lambda_i - (k_i - l))U(k_i) + \int_{\lambda_i-(k_i-l)}^{\lambda_i} U(l + \lambda_i - s)ds + \int_{\lambda_i}^{1} U(l + \lambda_i - s)ds \quad (13.63)$$

$$> C_i^l + \lambda_i U(k_i) + \int_0^{1-\lambda_i} U(l - s)ds, \quad (13.64)$$

which is the desired inequality

Case C: $l - k_i > 0$. (Note This also implies that $\lambda_i < 1$, since $l < 1$.

In this case,

$$\mathbb{E}_i^P D_i' = C_i^l + \int_{l-k_i}^{l+\lambda_i-k_i} U(k_i)ds + \int_{l+\lambda_i-k_i}^{1} U(l + \lambda_i - s)ds \quad (13.65)$$

$$= C_i^l + \lambda_i U(k_i) + \int_{l-k_i}^{1-\lambda_i} U(l - s)ds. \quad (13.66)$$

Hence, the inequalities from Cases A-C, show that Inequality (13.54) holds.

□

We now use the λ_i Trick (Lemma 13.7) to find a convex delegation set, $\hat{D}$, to improve upon the original nonconvex delegation set, D_P^*, for all i such that $1 + k_i$. This will be defined formally in the following lemma:

Lemma 13.8. *(Finding the Right $\hat{D}, \hat{\lambda}$ Lemma)*

Let $\mathcal{I} := \{i | h > 1 + k_i\} = \{1, \ldots, I_h\}$. Then there exists a $\hat{\lambda}, \hat{D}$ such that $\hat{D}$ improves upon D_P^* for all $i \in \mathcal{I}$:

$$\sum_{i=1}^{I_h} p_i \mathbb{E}_i^P \hat{D} > \sum_{i=1}^{I_h} p_i \mathbb{E}_i^P D_P^*, \quad (13.67)$$

where $\hat{D} = (D_P^* \cap (-\infty, l]) \cup [l, l + \hat{\lambda}]$

In addition, we will show that this new, convex set $\hat{D}$ will strictly improve the principal's expected utility from all types:

$$\sum_{i=1}^{N} p_i \mathbb{E}_i^P \hat{D} > \sum_{i=1}^{N} p_i \mathbb{E}_i^P D_P^*, \quad (13.68)$$

Proof. Recall $\lambda_i := \min\{1 - \left(\frac{h+l}{2} - k_i\right), 1\} > 0$. Thus, $\lambda_i \leq \lambda_{i+1}$, for all i. In addition, $\lambda_i = \lambda_j$, if and only if $1 = \lambda_i = \lambda_j$. In addition, since (WLOG) $\lambda_1 < 1$, we know that:

$$\lambda_1 < \lambda_2 \leq \cdots \leq \lambda_{I_h}. \tag{13.69}$$

We denote $(D_P^* \cap (-\infty, l]) \cup [l, l+\lambda]$ by $D(\lambda)$ and define $\Psi_i^*(\lambda)$ as

$$\Psi_i^*(\lambda) := \mathbb{E}_i^P D(\lambda) \tag{13.70}$$

$$= \int_0^{\max\{0,l-k_i\}} U\big(x_i^{D(\lambda)}(s)-s\big)ds + \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda-k_i\}} U(k_i)ds + \int_{\max\{0,l+\lambda-k_i\}}^{1} U(l+\lambda-s)ds$$

$$= C_i^l + \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda-k_i\}} U(k_i)ds + \int_{\max\{0,l+\lambda-k_i\}}^{1} U(l+\lambda-s)ds, \tag{13.71}$$

where $C_i^l = \int_0^{\max\{0,l-k_i\}} U\big(x_i^{D(\lambda)}(s)-s\big)ds$.

In addition, we know from an argument similar to that in the proof of Lemma 7.2 (defined on page 18), that $\Psi_i^*(\lambda)$ is strictly increasing in λ for $l+\lambda \in [0, q_i]$ and is strictly decreasing for $l+\lambda \in [q_i, 1]$, where $q_i = \max\{\frac{1}{2}, 1-k_i\}$.

Thus, since we can assume that $\lambda_1 < 1$ and $k_1 < \frac{1}{2}$, we have the following two relations:

$$q_1 > q_2 \geq \cdots \geq q_{I_h}. \tag{13.72}$$

$$l+\lambda_1 < l+\lambda_2 \leq \cdots \leq l+\lambda_{I_h}. \tag{13.73}$$

Two equivalent inequalities (13.72) (13.73) are similar to inequalities (12.13) and (12.14). We employ a similar argument.

We know that if $l+\lambda_i > q_i$, then

$$\mathbb{E}_i^P D_P^* \leq \mathbb{E}_i^P D(\lambda_i) = \Psi_i^*(\lambda_i) < \Psi_i^*(\gamma) = \mathbb{E}_i^P D(\gamma), \tag{13.74}$$

for all $\gamma \in [0, \lambda_i]$ since $\Psi_i^*(\cdot)$ is strictly decreasing in γ over this interval.

Similarly, we know that if $l+\lambda_i < q_i$, then

$$\mathbb{E}_i^P D_P^* \leq \mathbb{E}_i^P D(\lambda_i) = \Psi_i^*(\lambda_i) < \Psi_i^*(\gamma) = \mathbb{E}_i^P D(\gamma), \tag{13.75}$$

for all $\gamma \in [\lambda_i, q_i - l]$ since $\Psi_i^*(\cdot)$ is strictly increasing in γ over this interval.

In addition, from inequalities (13.72) and (13.73) we know that

$$l+\lambda_i < q_i \Longrightarrow l+\lambda_j < q_j, \forall\ j < i \tag{13.76}$$

and

$$l+\lambda_i > q_i \Longrightarrow l+\lambda_j > q_j, \forall\, j > i. \tag{13.77}$$

. We break the remaining argument into cases:

Case A: $l+\lambda_1 > q_1$

If $l+\lambda_1 > q_1$, by equation (13.77) we know $l+\lambda_i > q_i$ for all$i$ $(I_h \geq i > 1)$. Thus, let $\hat{\lambda} = \max\{0, q_1 - l\}$, which implies that $\hat{D} = D(\hat{\lambda})$. But then by equation (13.74) we have that

$$\mathbb{E}_i^P D_P^* < \mathbb{E}_i^P \hat{D}, \tag{13.78}$$

for all $i \leq I_h$, which implies the desired equation, equation (13.67).

Case B: $l+\lambda_{I_h} \leq q_{I_h}$

If $l+\lambda_{I_h} \leq q_{I_h}$, by equation (13.76) we know $l+\lambda_i \leq q_i$ for all $i < I_h$ (and the inequality is strict for at least one i). Thus, let $\hat{\lambda} = q_{I_h}$, which implies that $\hat{D} = D(\hat{\lambda})$. But then by equation (13.75) we have that

$$\mathbb{E}_i^P D_P^* \leq \mathbb{E}_i^P \hat{D}, \tag{13.79}$$

for all $i \leq I_h$ (and the inequality is strict fo at least one i), which implies the desired equation, equation (13.67).

Case C: There exists type i such that $l+\lambda_i \leq q_i$ and $l+\lambda_{i+1} > q_{i+1}$

First, note that if $l+\lambda_i \leq q_i$, then $\lambda_i < 1$, since either $i = 1$ (and $\lambda_1 < 1$, as we can assume above) or $i > 1$ and $q_i < 1$ (and $k_i > 0$). Thus, $\lambda_j > \lambda_i$ for all $j > i$ and $\lambda_m < \lambda_i$ for all $m < i$.

If there exists type i such that $l+\lambda_i \leq q_i$ and $l+\lambda_{i+1} > q_{i+1}$ then we know that for all types $j < i$, $l+\lambda_j \leq q_j$ and for all types $m > i$, $l+\lambda_m > q_m$. In addition, for all types, $j < i$, $l+\lambda_j < l+\lambda_i \leq q_i \leq q_j$. Thus, for all such $j \leq i$:

$$\mathbb{E}_j^P D(\max\{\lambda_{i+1}, q_i - l\}) > \mathbb{E}_j^P D(\lambda_j) > \mathbb{E}_j^P D_P^*, \tag{13.80}$$

by the monotonicity properties of $\Psi_i^*(\cdot) = \mathbb{E}_i^P D(\cdot)$. In addition, by these monotonicity properties we have for all $m > i$

$$\mathbb{E}_m^P D(\max\{\lambda_{i+1}, q_i - l\}) \geq \mathbb{E}_m^P D(\lambda_m) > \mathbb{E}_j^P D_P^*. \tag{13.81}$$

Thus, set $\hat{\lambda} = \max\{\lambda_{i+1}, q_i - l\}$, $\hat{D} = D(\hat{\lambda})$, and by equations (13.80) and (13.81) we have the desired inequality:

$$\sum_{i=1}^{I_h} p_i \mathbb{E}_i^P \hat{D} > \sum_{i=1}^{I_h} p_i \mathbb{E}_i^P D_P^*. \tag{13.82}$$

This concludes the proof of Case C. We know show $\hat{D}$ yields the principal strictly higher ex-ante expected payoff (over all types) than D_P^*.

Notice that

$$q_{I_h} \leq l + \hat{\lambda} \leq 1 < 1 + k_{I_h} < h. \tag{13.83}$$

Thus, without loss of generality, we can assume $\hat{D}$ is convex. Otherwise, we can use the general Gap Filling Lemma (Lemma 13.4), to fill in all of the gaps and strictly improve expected utility at least one type.

To conclude the proof of the lemma, notice that if $m > I_h$, we know that $h \leq 1 + k_m$. Thus, if one fills in all of the gaps of D_P^*, replacing it with the $[0, h]$, we would have (for all $m > I_h$):

$$\mathbb{E}_i^P[0, h] \geq \mathbb{E}_i^P D_P^*. \tag{13.84}$$

But since $q_m \leq q_{I_h} \leq l + \hat{\lambda} < h$, then we know from the monotonicity properties of $\Psi_i^*(\cdot) = \mathbb{E}_i^P D(\cdot)$ we have for all $i > I_h$ that

$$\mathbb{E}_i^P[0, l + \hat{\lambda}] > \mathbb{E}_i^P[0, h] \geq \mathbb{E}_i^P D_P^*. \tag{13.85}$$

Thus, letting $\hat{D} = [0, l + \hat{\lambda}]$, by equations (13.82) and (13.85):

$$\sum_{i=1}^{N} p_i \mathbb{E}_i^P \hat{D} > \sum_{i=1}^{N} p_i \mathbb{E}_i^P D_P^*. \tag{13.86}$$

This concludes the proof of case (2ii), where $h > 1 + k_1$.

□

By the previous lemmas 14.5 through (13.8), we can restrict attention to delegation sets contained in $[0, 1]$. These previous lemmas show that if the set is not contained in the unit interval (which is contained in $(-\infty, 1 + k_1])$, we can find a another set that yields the principal strictly higher expected utility.

14 Appendix D: Proofs of Section 8 (Proposition 8.5)

Throughout this Appendix, we will modify a set D_i. This first modification will increase the expected payoff of the agent, effect the incentive compatibility conditions, and increase the expected utility of the principal. Then we will "thin" this set (defined below) so as to preserve the indifference of agent i. The lemma in the next section shows that thinning the set will maintain the added benefit to the principal and will preserve the original incentive compatibility conditions.

14.1 Aligned Thinning Lemma

Before we construct $\hat{D}_i$ and I_i that satisfy equations (8.1)-(8.6), we first prove a useful lemma, but before we that, we introduce some notation which will be used throughout this Appendix. Denote

$$\Delta_j^P(D', D) := \mathbb{E}_j^P D' - \mathbb{E}_j^P D',$$

where $j \in \{i, , i+1, \ldots, N\}$. Denote

$$\Delta^j(D', D) := \mathbb{E}^j D' - \mathbb{E}^j D',$$

where $i \in \{i, i+1, \ldots, N\}$. In addition, let A be a closed (and bounded) set. Denote $\max A := \max_{x \in A} x$. We call a closed (and bounded) delegation set D *thick at the top* if there exists an $\epsilon > 0$ such that $[\max D - \epsilon, \max D] \subseteq D$. We call the set $D^-(\delta)$ a *δ-thinning of D* if $D^-(\delta) = D \cap (-\infty, \max D - \delta]$ and $\delta \geq 0$ is chosen so that $[\max D - \delta, \max D] \subseteq D$.

Lemma 14.1. *(Aligned Thinning Lemma) Let $D \subseteq (-\infty, 1 + k_i]$ be thick at the top. Let $\delta > 0$ and $D^-(\delta)$ be a δ-thinning of D. We have the following inequality*

$$\Delta^j(D^-(\delta), D) < \Delta^i(D^-(\delta), D) < \Delta_i^P(D^-(\delta), D), \tag{14.1}$$

for all $j > i$.

In words, equation (14.1) lemma states that thinning certain thick at the top delegation sets causes the least expected utility loss to the principal (it may even be a gain) and causes more expected utility losses for higher-bias types. Thus, thinning sets will prove to be a powerful variation that preserves incentive compatibility while maintaining expected utility gains to the principal. This will become clearer in further proofs.

Proof. We break this lemma into two results: (i) $\Delta^j(D^-(\delta), D) < \Delta^i(D^-(\delta), D)$ for all $j > i$ and (ii) $\Delta^i(D^-(\delta), D) < \Delta_i^P(D^-(\delta), D)$.

Proof of (i):

Proof. We first show that $\Delta^j(D^-(\delta), D) < \Delta^i(D^-(\delta), D)$ for all $j > i$. We will prove this relation by writing $\Delta^j(D^-(\delta), D)$ as a function of k_j and show that it is decreasing in k_j:

For all $j \in \{i, i+1, \ldots, N\}$

$$\Delta^j(D^-(\delta), D) = E^j D^-(\delta) - E^j D \tag{14.2}$$

$$= \int_0^{\max\{0, d^* - \delta - k_j\}} U(x_j^{D^-(\delta)}(s) - s - k_j) ds + \int_{\max\{0, d^* - \delta - k_j\}}^1 U(d^* - \delta - s - k_j) ds$$

$$-\int_0^{\max\{0,d^*-\delta-k_j\}} U(x_j^D(s)-s-k_j)ds-\int_{\max\{0,d^*-\delta-k_j\}}^{\max\{d^*-\delta-k_j,d^*-k_j\}} U(0)ds-\int_{\max\{0,d^*-k_j\}}^1 U(d^*-s-k_j)ds$$

$$= \int_{\max\{0,d^*-\delta-k_j\}}^1 U(d^*-\delta-s-k_j)ds - \int_{\max\{0,d^*-k_j\}}^1 U(d^*-\delta-s-k_j)ds \quad (14.3)$$

$\big($since $x_j^{D^-(\delta)}(s) = x_j^D(s)$ for all $s \in [0, \max\{0, d^* - \delta - k_j\}]$ and $U(0) = 0$ $\big)$.

$$= \int_{\max\{k_j+\delta-d^*,0\}}^{1+k_j-d^*+\delta} U(s)ds-\int_{\max\{k_j-d^*,0\}}^{1+k_j-d^*} U(s)ds = \int_{1+k_j-d^*}^{1+k_j-d^*+\delta} U(s)ds-\int_{\max\{k_j-d^*,0\}}^{\max\{k_j+\delta-d^*,0\}} U(s)ds \quad (14.4)$$

$$> \int_{1+k_l-d^*}^{1+k_l-d^*+\delta} U(s)ds - \int_{\max\{k_l-d^*,0\}}^{\max\{k_l+\delta-d^*,0\}} U(s)ds = E^l D^-(\delta) - E^l D, \quad (14.5)$$

for all $k_l > k_j$ since one of three cases holds: (A) $k_j + \delta - d^* < 0$ (B) $k_j - d^* < 0 \leq k_j + \delta - d^*$ (C) $k_j - d^* \geq 0$.

If (A) holds, then

$$E^j D^-(\delta) - E^j D = \int_{1+k_j-d^*}^{1+k_j-d^*+\delta} U(s)ds, \quad (14.6)$$

which is strictly decreasing in k_j.

If (B) holds then

$$E^j D^-(\delta) - E^j D = \int_{1+k_j-d^*}^{1+k_j-d^*+\delta} U(s)ds - \int_0^{k_j+\delta-d^*} U(s)ds. \quad (14.7)$$

Differentiating with respect to k_j we get

$$U(1 + k_j - d^* + \delta) - U(1 + k_j - d^*) - \big(U(k_j + \delta - d^*) - U(0)\big) \quad (14.8)$$

$$< U(1 + k_j - d^* + \delta) - U(1 + k_j - d^*) - \big(U(\delta) - U(0)\big) < 0 \quad (14.9)$$

since $k_j - d^* \leq 0 < k_j - d^* + \delta$ and since $U(\cdot)$ is strictly decreasing over $\mathbb{R}_+$, and since $U(\cdot)$ is strictly concave

$$U(a + \delta) - U(a) < U(b + \delta) - U(b) \quad (14.10)$$

for all $a > b \geq 0$. This concludes the argument for Case (B).

If (C) holds then

$$E^j D^-(\delta) - E^j D = \int_{1+k_j-d^*}^{1+k_j-d^*+\delta} U(s)ds - \int_{k_j-d^*}^{k_j+\delta-d^*} U(s)ds. \tag{14.11}$$

Differentiating with respect to k_j we get

$$U(1+k_j-d^*+\delta) - U(1+k_j-d^*) - \big(U(k_j+\delta-d^*) - U(k_j-d^*)\big) < 0 \tag{14.12}$$

by the same reasoning as in equation (14.10). □

Proof of (ii): $\Delta^i(D^-(\delta), D) < \Delta_i^P(D^-(\delta), D)$

Proof. We first reduce analysis to specific D and $\delta(D)$ (where the thinning will be function of the set D). The reason we can do this is because if D is thick at the top and $D^-(\delta)$ is a δ-thinning, then $D^-(\delta)$ is also a closed and bounded set. In addition, Recall that $\Delta_i^P(D^-(\delta), D) = E_i^P D^-(\delta) - E_j^P D$ Hence, letting $\delta_0 = 0$, $\sum_{r=1}^{L} \delta_r = \delta$, and $S_r = \sum_{h=0}^{r} \delta_h$ then:

$$\Delta_i^P(D^-(\delta), D) = \Delta_i^P(D^-(\sum_{r=1}^{L} \delta_r), D) \tag{14.13}$$

$$= \sum_{r=1}^{L} \Delta_i^P\bigg(D^-(\delta_r + S_{r-1}), D^-(S_{r-1}) = \sum_{r=1}^{L} \Delta_i^P\bigg(D^-(\sum_{h=0}^{r} \delta_h), D^-(\sum_{h=0}^{r-1} \delta_h)\bigg).$$

Hence, if $\Delta_i^P\bigg(D^-(\delta_r + S_{r-1}), D^-(S_{r-1}) > \Delta^i\bigg(D^-(\delta_r + S_{r-1}), D^-(S_{r-1})$, for all $r \in \{1, \ldots, L\}$ then:

$$\Delta_j^P(D^-(\delta), D) = \sum_{r=1}^{L} \Delta_j^P\bigg(D^-(\delta_r + S_{r-1}), D^-(S_{r-1})\bigg) > \sum_{r=1}^{L} \Delta^j\bigg(D^-(\delta_r + S_{r-1}), D^-(S_{r-1})\bigg) \tag{14.14}$$

$$= \Delta^j(D^-(\delta), D).$$

Cases: (I) $\max D > 1, \max D - 1 \geq \delta > 0$. (II) $1 \geq \max D > k_i, \max D - k_i \geq \delta > 0$ (III) $k_i \geq \max D > 0, \max D \geq \delta > 0$, and (IV) $\max D \leq 0, \delta > 0$

Proof for Case (I), $\max D > 1, \max D - 1 \geq \delta > 0$:

Proof.

$$\Delta_i^P(D^-(\delta), D) = E_i^P D^-(\delta) - E_i^P D = \int_0^{d^*-\delta-k_i} U(x_i^{D^-(\delta)}(s) - s)ds + \int_{d^*-\delta-k_i}^1 U(d^* - \delta - s)ds \tag{14.15}$$

$$- \int_0^{d^*-\delta-k_i} U(x_i^{D^-(\delta)}(s) - s)ds - \delta U(k_i) - \int_{d^*-k_i}^1 U(d^* - s)ds$$

(since $x_i^{D^-(\delta)}(s) = x_i^D(s)$ for all $s \in [0, d^* - \delta - k_i]$, and D is thick at the top so $x_i^D(s) = s + k_i$ for all $s \in [d^* - \delta - k_i, d^* - k_i]$)

$$= \int_{d^*-\delta-k_i}^1 U(d^* - \delta - s)ds - \delta U(k_i) - \int_{d^*-k_i}^1 U(d^* - s)ds \tag{14.16}$$

$$= \int_{-k_i}^{1+\delta-d^*} U(s)ds - \delta U(k_i) - \int_{-k_i}^{1-d^*} U(s)ds = \int_{1-d^*}^{1+\delta-d^*} U(s)ds - \delta U(k_i) \tag{14.17}$$

$$= \int_{d^*-\delta-1}^{d^*-1} U(s)ds - \delta U(k_i) > \int_{d^*-\delta-1}^{d^*-1} U(k_i)ds - \delta U(k_i) = 0 \tag{14.18}$$

(since nonredundancy of D and thickness at the top imply that $d^* \leq 1 + k_i$)

$$> \int_{1+k_i-d^*}^{1+k_i-d^*+\delta} U(s)ds = \int_{d^*-\delta-k_i}^1 U(d^* - \delta - s - k_i)ds - \int_{d^*-k_j}^1 U(d^* - s - k_i)ds \tag{14.19}$$

$$\int_0^{d^*-\delta-k_i} U(x_i^{D^-(\delta)}(s) - s)ds + \int_{d^*-\delta-k_i}^1 U(d^* - \delta - s)ds - \int_0^{d^*-k_i-\delta} U(x_i^{D^-(\delta)}(s) - s - k_i)ds \tag{14.20}$$

$$- \int_{d^*-k_i-\delta}^{d^*-k_i} U(s + k_i - s - k_i)ds - \int_{d^*-k_i}^1 U(d^* - s - k_i)d$$

$$= E^i D^-(\delta) - E^i D = \Delta^i(D^-(\delta), D). \tag{14.21}$$

Hence, for case (I), $\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$. □

Proof for Case (II), $1 \geq \max D > k_i, \max D - k_i \geq \delta > 0$

Proof.

$$\Delta_i^P(D^-(\delta), D) = E_i^P D^-(\delta) - E_i^P D = \int_0^{d^*-\delta-k_i} U(x_i^{D^-(\delta)}(s) - s)ds + \int_{d^*-\delta-k_i}^1 U(d^* - \delta - s)ds \tag{14.22}$$

$$-\int_0^{d^*-\delta-k_i} U(x_i^{D^-(\delta)}(s)-s)ds - \delta U(k_i) - \int_{d^*-k_i}^1 U(d^*-s)ds$$

(since $x_i^{D^-(\delta)}(s) = x_i^D(s)$ for all $s \in [0, d^*-\delta-k_i]$, and D is thick at the top so $x_i^D(s) = s + k_i$ for all $s \in [d^*-\delta-k_i, d^*-k_i]$)

$$= \int_{d^*-\delta-k_i}^1 U(d^*-\delta-s)ds - \delta U(k_i) - \int_{d^*-k_i}^1 U(d^*-s)ds \tag{14.23}$$

$$= \int_{-k_i}^{1+\delta-d^*} U(s)ds - \delta U(k_i) - \int_{-k_i}^{1-d^*} U(s)ds = \int_{1-d^*}^{1+\delta-d^*} U(s)ds - \delta U(k_i) \tag{14.24}$$

$$> \int_{1+k_i-d^*}^{1+k_i+\delta-d^*} U(s)ds = \int_{d^*-\delta-k_i}^1 U(d^*-\delta-s-k_i)ds - \int_{d^*-k_j}^1 U(d^*-s-k_i)ds \tag{14.25}$$

$$\int_0^{d^*-\delta-k_i} U(x_i^{D^-(\delta)}(s)-s)ds + \int_{d^*-\delta-k_i}^1 U(d^*-\delta-s)ds - \int_0^{d^*-k_i-\delta} U(x_i^{D^-(\delta)}(s)-s-k_i)ds \tag{14.26}$$

$$-\int_{d^*-k_i-\delta}^{d^*-k_i} U(s+k_i-s-k_i)ds - \int_{d^*-k_i}^1 U(d^*-s-k_i)d$$

$$= E^i D^-(\delta) - E^i D = \Delta^i(D^-(\delta), D). \tag{14.27}$$

Hence, for case (II), $\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$. □

Proof for Case (III), $k_i \geq \max D > 0, \max D \geq \delta > 0$

Proof. Since $\max D = d^* \leq k_i$, $x_i^D(s) = d^*$ for all $s \in [0,1]$. Similarly, $x_i^{D^-(\delta)}(s) = d^* - \delta$ for all $s \in [0,1]$. Hence,

$$\Delta_i^P(D^-(\delta), D) = E_i^P D^-(\delta) - E_i^P D = \int_0^1 U(d^*-\delta-s)ds - \int_0^1 U(d^*-s)ds \tag{14.28}$$

$$= \int_0^{d^*-\delta} U(d^*-\delta-s)ds + \int_{d^*-\delta}^1 U(d^*-\delta-s)ds - \int_0^{d^*} U(d^*-s)ds - \int_{d^*}^1 U(d^*-s)ds \tag{14.29}$$

$$= \int_{-d^*+\delta}^0 U(s)ds + \int_0^{1+\delta-d^*} U(s)ds - \int_{-d^*}^0 U(s)ds - \int_0^{1-d^*} U(s)ds \tag{14.30}$$

$$= -\int_{-d^*}^{-d^*+\delta} U(s)ds + \int_{1-d^*}^{1+\delta-d^*} U(s)ds \tag{14.31}$$

$$= -\int_{d^*-\delta}^{d^*} U(s)ds + \int_{1-d^*}^{1+\delta-d^*} U(s)ds \tag{14.32}$$

$$> \int_{1-d^*}^{1+\delta-d^*} U(s)ds > \int_{1-d^*}^{1+\delta-d^*} U(s)ds + \delta U(k_i) - \int_{k_i-d^*}^{k_i+\delta-d^*} U(s)ds \tag{14.33}$$

$$> \int_{1-d^*}^{1+\delta-d^*} U(s+k_i)ds - \int_{k_i-d^*}^{k_i+\delta-d^*} U(s)ds \tag{14.34}$$

(since $U(s) + U(k_i) > U(s + k_i)$ for $s \geq 0$)

$$= \int_{1+k_i-d^*}^{1+k_i+\delta-d^*} U(s)ds - \int_{k_i-d^*}^{k_i+\delta-d^*} U(s)ds = \int_{k_i+\delta-d^*}^{1+k_i+\delta-d^*} U(s)ds - \int_{k_i-d^*}^{1+k_i-d^*} U(s)ds \tag{14.35}$$

$$= \int_0^1 U(d^* - \delta - s - k_i)ds - \int_0^1 U(d^* - s - k_i)ds \tag{14.36}$$

$$= E^i D^-(\delta) - E^i D = \Delta^i(D^-(\delta), D). \tag{14.37}$$

Hence, for Case (III):$\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$. □

Proof of Case (IV), $\max D \leq 0, \delta > 0$

Proof. For $d, \delta \geq 0$ let

$$H(d, \delta) = \int_{d+\delta}^{1+d+\delta} U(s)ds - \int_d^{1+d} U(s)ds \tag{14.38}$$

$$\frac{\partial H}{\partial d}(d, \delta) = U(1 + d + \delta) - U(d + \delta) - [U(1 + d) - U(d)] < 0 \tag{14.39}$$

since $U(s)$ is strictly concave and maximized at $s = 0$. Hence, equation (14.39) implies that $H(d, \delta)$ is strictly decreasing in d.

We now use equation (14.39) to show the desired inequality $\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$.

As in Case (III), since $\max D = d^* \leq k_i$, $x_i^D(s) = d^*$ for all $s \in [0, 1]$. Similarly, $x_i^{D^-(\delta)}(s) = d^* - \delta$ for all $s \in [0, 1]$.

$$\Delta_i^P(D^-(\delta), D) = E_i^P D^-(\delta) - E_i^P D = \int_0^1 U(d^* - \delta - s)ds - \int_0^1 U(d^* - s)ds \tag{14.40}$$

$$= \int_{\delta-d^*}^{1+\delta-d^*} U(s)ds - \int_{-d^*}^{1-d^*} U(s)ds = H(-d^*, \delta) \tag{14.41}$$

$$> H(k_i - d^*, \delta) = \int_{k_i+\delta-d^*}^{1+k_i+\delta-d^*} U(s)ds - \int_{k_i-d^*}^{1+k_i-d^*} U(s)ds \tag{14.42}$$

(since $H(d,\delta)$ is strictly decreasing in d)

$$= \int_0^1 U(d^* - \delta - s - k_i)ds - \int_0^1 U(d^* - s - k_i)ds \tag{14.43}$$

$$= E^i D^-(\delta) - E^i D = \Delta^i(D^-(\delta), D). \tag{14.44}$$

Hence, for case (IV):$\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$. □

Thus, from cases (I)-(IV) and the argument at the beginning of case (ii) we know that for all D that are thick at the top, for any δ-thinning we have: $\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$. □

From case (i) we know that $\Delta^i(D^-(\delta), D) > \Delta^j(D^-(\delta), D)$ for all $j > i$ and from case (ii) we know $\Delta_i^P(D^-(\delta), D) > \Delta^i(D^-(\delta), D)$. Hence, we have completed the proof for the Aligned Thinning Lemma (Lemma 14.1). □

Now we construct $\hat{D}_i$ and I_i that satisfy equations (8.1)-(8.6).

14.2 Construction of $\hat{D}_i$

First, we rule out the pathological cases.

Assume that D_i is not redundant. Thus, for all $d \in D_i$, there is an s such that $x_i^{D_i}(s) = d$. Second, if there is an $l \in D_i$ such that $l < k_i$ or an $h \in D_i$ such that $h > 1 + k_i$ such that h or l are played with zero probability (for a single state, 0 or 1) than remove l and h from D_i to achieve $\hat{D}_i$. From here on, q will denote the only point (by nonredundancy) less than k_i (if such q exists) and w will denote the only point (by nonredundancy) greater than $1 + k_i$.

Now if $D_i \subseteq (1 + k_i, \infty)$ (in words, D_i is too high), let $\alpha = w - 1$. Replace D_i with $\{k_i - \alpha\}$. This satisfies equations (8.4)-(8.6).

Assume there exists a w in D_i and that $D_i \cap (-\infty, 1 + k_i] \neq \emptyset$. Thus, there is a gap of the form (l, w) in D_1 (notice that it is possible for $l < k_i$). By nonredundancy, we know that there is an $s' < 1$ such that $x_i^{D_i}(s) = w$ for all s such that $1 \geq s \geq s'$ (also notice that $s' = \frac{w+l}{2} - k_i$). Let $\lambda_i = 1 - s'$ (which means that $1 - \lambda_i = s' = \frac{w+l}{2} - k_i$).

Let

$$D'_i = (D_i \cap (-\infty, 1 + k_i]) \cup [l, l + \lambda_i]. \tag{14.45}$$

In words, we are tossing out the point above $1 + k_i$, w, and thickening D_i on the right.

This clearly raises the expected payoff to the principal.

$$\Delta_i^P(D'_i, D_i) := \mathbb{E}_i^P D'_i - \mathbb{E}_i^P D_i \tag{14.46}$$

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(k_i)ds + \int_{\max\{0,l+\lambda_i-k_i\}}^{1} U(l+\lambda_i-s)ds - \int_{\max\{0,l-k_i\}}^{\frac{w+l}{2}-k_i} U(l-s)ds - \int_{\frac{w+l}{2}-k_i}^{1} U(w-s)ds \tag{14.47}$$

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(k_i)ds + \int_{\max\{-l-\lambda_i,-k_i\}}^{1-l-\lambda_i} U(s)ds - \int_{\max\{-l,-k_i\}}^{\frac{w-l}{2}-k_i} U(s)ds - \int_{-\frac{w-l}{2}-k_i}^{1-w} U(s)ds \tag{14.48}$$

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(k_i)ds + \int_{\max\{-l-\lambda_i,-k_i\}}^{\frac{w-l}{2}-k_i} U(s)ds - \int_{\max\{-l,-k_i\}}^{\frac{w-l}{2}-k_i} U(s)ds - \int_{-\frac{w-l}{2}-k_i}^{1-w} U(s)ds \tag{14.49}$$

(since $1 - \lambda_i = \frac{w+l}{2} - k_i$)

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(k_i)ds + \int_{k_i-\frac{w-l}{2}}^{\min\{l+\lambda_i,k_i\}} U(s)ds - \int_{k_i-\frac{w-l}{2}}^{\min\{l,k_i\}} U(s)ds - \int_{w-1}^{k_i+\frac{w-l}{2}} U(s)ds \tag{14.50}$$

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(k_i)ds + \int_{k_i-\frac{w-l}{2}}^{\min\{l_i,k_i\}} U(s)ds + \int_{\min\{l,k_i\}}^{\min\{l+\lambda_i,k_i\}} U(s)ds \tag{14.51}$$

$$- \int_{k_i-\frac{w-l}{2}}^{\min\{l,k_i\}} U(s)ds - \int_{w-1}^{k_i+\frac{w-l}{2}} U(s)ds$$

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(k_i)ds + \int_{\min\{l,k_i\}}^{\min\{l+\lambda_i,k_i\}} U(s)ds - \int_{w-1}^{k_i+\frac{w-l}{2}} U(s)ds \tag{14.52}$$

$$\geq \lambda_i U(k_i) - \int_{w-1}^{k_i+\frac{w-l}{2}} U(s)ds \tag{14.53}$$

$$> \lambda_i U(k_i) - \lambda_i U(k_i) \tag{14.54}$$

since $1 - w + \frac{w-l}{2} + k_i = 1 - \frac{w+l}{2} + k_i = \lambda_i$.

Hence, from equation (14.54) we see that:

$$\Delta_i^P(D_i', D_i) = \mathbb{E}_i^P D_i' - \mathbb{E}_i^P D_i > \lambda_i U(k_i) - \lambda_i U(k_i) = 0. \tag{14.55}$$

In addition, we can also bound the change in expected payoff to the principal from below. Since $\int_{w-1-k_i}^{\frac{w-l}{2}} U(s+k_i)ds = \int_{w-1}^{k_i+\frac{w-l}{2}} U(s)ds$ and since $U(\cdot)$ is nonpositive, and strictly concave, we have:

$$-U(s+k_i) + U(k_i) > -U(s) \tag{14.56}$$

$$\Delta_i^P(D_i', D_i) \geq \lambda_i U(k_i) - \int_{w-1}^{k_i+\frac{w-l}{2}} U(s)ds. \tag{14.57}$$

$$= \int_{w-1-k_i}^{\frac{w-l}{2}} U(k_i)ds - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s+k_i)ds > \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds, \tag{14.58}$$

by equation (14.56).

We now show that the change in expected payoff for type i is also positive and bound it from above by the change in expected payoff to the principal. For agent type i, the gain is:

$$\Delta^i(D_i', D_i) := \mathbb{E}^i D_i' - \mathbb{E}^i D_i \tag{14.59}$$

$$= \int_{\max\{0,l-k_i\}}^{\max\{0,l+\lambda_i-k_i\}} U(0)ds + \int_{\max\{0,l+\lambda_i-k_i\}}^{1} U(l+\lambda_i-s-k_i)ds \tag{14.60}$$

$$- \int_{\max\{0,l-k_i\}}^{\frac{w+l}{2}-k_i} U(l-s-k_i)ds - \int_{\frac{w+l}{2}-k_i}^{1} U(w-s-k_i)ds$$

$$= \int_{\max\{k_i-l-\lambda_i,0\}}^{1+k_i-l-\lambda_i} U(s)ds - \int_{\max\{k_i-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{-\frac{w-l}{2}}^{1+k_i-w} U(s)ds \tag{14.61}$$

$$= \int_{\max\{k_i-l-\lambda_i,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{\max\{k_i-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{-\frac{w-l}{2}}^{1+k_i-w} U(s)ds \tag{14.62}$$

(since $1 - \lambda_i = \frac{w+l}{2} - k_i$)

$$= \int_{\max\{k_i-l-\lambda_i,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{\max\{k_i-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds. \tag{14.63}$$

From equation (14.63) we see that

$$0 < \Delta^i(D_i', D_i) = \mathbb{E}^i D_i' - \mathbb{E}^i D_i \leq - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds. \tag{14.64}$$

Thus, by equations (14.58) and (14.64) we have:

$$\Delta^P(D_i', D_i) = \mathbb{E}^P D_i' - \mathbb{E}^P D_i > - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds \geq \mathbb{E}^i D_i' - \mathbb{E}^i D_i = \Delta^i(D_i', D_i) > 0. \tag{14.65}$$

Thus, the expected payoff gain to the principal from replacing D_i with D_i' is higher than the expected payoff gain of type i. We also know that the gain to type i is higher than the gain to types j for all $j > i$:

For agent type j, note that $j > i$ so $k_j > k_i$. Hence, the expected gain to type j is:

$$\Delta^j(D_i', D_i) := \mathbb{E}^j D_i' - \mathbb{E}^j D_i \tag{14.66}$$

$$= \int_{\max\{0,l-k_j\}}^{\max\{0,l+\lambda_i-k_j\}} U(0)ds + \int_{\max\{0,l+\lambda_i-k_j\}}^{1} U(l + \lambda_i - s - k_j)ds \tag{14.67}$$

$$- \int_{\max\{0,l-k_j\}}^{\frac{w+l}{2}-k_j} U(l - s - k_j)ds - \int_{\frac{w+l}{2}-k_j}^{1} U(w - s - k_j)ds$$

$$= \int_{\max\{k_j-l-\lambda_i,0\}}^{1+k_j-l-\lambda_i} U(s)ds - \int_{\max\{k_j-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{-\frac{w-l}{2}}^{1+k_j-w} U(s)ds \tag{14.68}$$

$$= \int_{\max\{k_j-l-\lambda_i,0\}}^{\frac{w-l}{2}+k_j-k_i} U(s)ds - \int_{\max\{k_j-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{-\frac{w-l}{2}}^{1+k_j-w} U(s)ds \tag{14.69}$$

(since $1 - \lambda_i = \frac{w+l}{2} - k_i$)

$$= \int_{\max\{k_j-l-\lambda_i,0\}}^{\frac{w-l}{2}+k_j-k_i} U(s)ds - \int_{\max\{k_j-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{w-1-k_j}^{\frac{w-l}{2}} U(s)ds \tag{14.70}$$

$$= \int_{\max\{k_j-l-\lambda_i,0\}}^{\max\{k_j-l,0\}} U(s)ds + \int_{\max\{k_j-l,0\}}^{\frac{w-l}{2}} U(s)ds + \int_{\frac{w-l}{2}}^{\frac{w-l}{2}+k_j-k_i} U(s)ds \tag{14.71}$$

$$- \int_{\max\{k_j-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{w-1-k_j}^{\frac{w-l}{2}} U(s)ds$$

$$< \int_{\max\{k_j-l-\lambda_i,0\}}^{\max\{k_j-l,0\}} U(s)ds - \int_{w-1-k_i}^{\min\{\frac{w-l}{2},w-k_j\}} U(s)ds \tag{14.72}$$

$\Big($where we broke the integral $\int_{\max\{k_j-l-\lambda_i,0\}}^{\frac{w-l}{2}+k_j-k_i} U(s)ds$ into a sum of three integrals $\int_{\max\{k_j-l-\lambda_i,0\}}^{\max\{k_j-l,0\}} U(s)ds + \int_{\max\{k_j-l,0\}}^{\frac{w-l}{2}} U(s)ds + \int_{\frac{w-l}{2}}^{\frac{w-l}{2}+k_j-k_i} U(s)ds\Big)$

which holds since

$$\int_{\frac{w-l}{2}}^{\frac{w-l}{2}+k_j-k_i} U(s)ds - \int_{w-1-k_j}^{\frac{w-l}{2}} U(s)ds < -\int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds.$$

This strict inequality holds since either $(i): w > 1+k_j$ or $(ii): w \leq 1+k_j$.
(i): If $w > 1+k_j$ holds, then:

$$\frac{w-l}{2} > w-1-k_j \iff 1 > \frac{w+l}{2} - k_j. \tag{14.73}$$

(ii): If $1+k_j > w$ holds, recalling that $w > 1+k_i$, we see that:

$$1+k_j-w < 1+k_j-(1+k_i) = k_j-k_i < \frac{w-l}{2}+k_j-k_i. \tag{14.74}$$

Recall from equation (14.63) that

$$\Delta^i(D_i', D_i) = \int_{\max\{k_i-l-\lambda_i,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{\max\{k_i-l,0\}}^{\frac{w-l}{2}} U(s)ds - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds \tag{14.75}$$

$$= \int_{\max\{k_i-l-\lambda_i,0\}}^{\max\{k_i-l,0\}} U(s)ds - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds. \tag{14.76}$$

Thus, by equation (14.72) we have:

$$\Delta^j(D_i', D_i) = \mathbb{E}^j D_i' - \mathbb{E}^j D_i < \int_{\max\{k_j-l-\lambda_i,0\}}^{\max\{k_j-l,0\}} U(s)ds - \int_{w-1-k_i}^{\min\{\frac{w-l}{2}, w-k_j\}} U(s)ds \quad (14.77)$$

$$\leq \int_{\max\{k_i-l-\lambda_i,0\}}^{\max\{k_i-l,0\}} U(s)ds - \int_{w-1-k_i}^{\frac{w-l}{2}} U(s)ds = \mathbb{E}^i D_i' - \mathbb{E}^i D_i = \Delta^i(D_i', D_i).$$

Thus, the gain to type i from replacing D_i with D_i' is strictly higher than that of type j for all $j > i$.

Recalling from equation (14.64) that $\Delta^i(D_i', D_i) = \mathbb{E}^i D_i' - \mathbb{E}^i D_i > 0$ we now find a $\hat{D}_i$ such that $\Delta^i(\hat{D}_i, D_i) = 0$. We do this by first noticing that D_i' is thick at the top.

Recalling equation (14.45) that $D_i' = (D_i \cap (-\infty, 1+k_i]) \cup [l, l+\lambda_i]$ we denote

$$D(\epsilon)_i := (D_i \cap (-\infty, 1+k_i]) \cup [l, l+\lambda_i - \epsilon]. \quad (14.78)$$

Thus, $D(0)_i = D_i'$ and $D(\lambda_i) = D_i \cap (-\infty, 1+k_i]$ From equation (14.64) we know that $\mathbb{E}^i D(0)_i - \mathbb{E}^i D_i > 0$. In addition, since the agent chooses w (the point above $1+k_i$) with positive probability the agent is worse off when w is removed (and nothing else is added). Thus, we know that $\mathbb{E}^i D(\lambda_i)_i - \mathbb{E}^i D_i < 0$. Hence, by the Intermediate Value Theorem, there is a $\hat{\delta} > 0$ such that $\mathbb{E}^i D_i(\hat{\delta}) - \mathbb{E}^i D_i = 0$. Let

$$D_i(\hat{\delta}) = \hat{D}_i. \quad (14.79)$$

Notice that $\Delta_i^P(D_i, D_i') > \Delta^i(D, D_i') > \Delta^j(D_i, D_i')$, for all $j > i$. In addition $\hat{D}_i$ is a δ-thinning of D_i'. Thus, by the Aligned Thinning Lemma (Lemma 14.1) since $\Delta_i^P(\hat{D}_i, D_i') > \Delta^i(\hat{D}_i, D_i') > \Delta^j i(\hat{D}_i, D_i')$,

So

$$\Delta_i^P(\hat{D}_i, D_i) = \Delta_i^P(\hat{D}_i, D_i') + \Delta_i^P(D_i', D_i) > \Delta^i(\hat{D}_i, D_i') + \Delta^i(D_i', D_i) = 0 = \Delta^i(\hat{D}_i, D_i) \quad (14.80)$$

$$> \Delta^j(\hat{D}_i, D_i') + \Delta^j(D_i', D_i) = \Delta^j(\hat{D}_i, D_i).$$

Hence, $\mathbb{E}^P \hat{D}_i > \mathbb{E}^P D_i$. Thus, $\hat{D}_i$ has the desired incentive compatibility properties for Step 1: equations (8.1)- (8.3) are satisfied.

14.3 Construction of I_i

From Step 1, we know that we can restrict attention to $\hat{D}_i$ such that $\hat{D}_i \subseteq (-\infty, 1+k_i]$. Let $a_i = \max_{d \in \hat{D}_i} d$. Let $D_i'' = [k_i, a_i]$. In words, we filled in all of the gaps in $\hat{D}_i$. This replacement strictly raises agent i's expected payoff. We will then replace D_i''

with the interval, $I_i = [k_i, a_i^*]$ such that agent i is indifferent between I_i and D_i (thus, $a_i^* < a_i$). Such an a_i^* exists by the Intermediate Value Theorem. Hence, I_i satisfies equation (8.4). It remains to show that it satisfies equations (8.5) and (8.6).

Equation (8.6) is the easier relation to prove. First, note that D_i'' fills in all gaps in D_i. $\mathbb{E}^j D_i'' - \mathbb{E}^j D_i$ is just the expected gain to type $j > i$ from filling in all the gaps in D_i. The gain from a gap $G \subseteq [k_j, 1 + k_j]$ is the same to type j as it is to type i since $U(\cdot)$ is a function of the absolute value of distance from action d to $s + k_j$. Thus, only case where the change in expected payoff from filling in a gap is different is for the case when $G = (l, h)$, $h > k_j$, and $l < k_j$. In this case, the loss to the agent j over the gap is

$$\int_0^{\frac{h+l}{2}-k_j} U(l - s - k_j)ds + \int_{\frac{h+l}{2}-k_j}^{h-k_j} U(h - s - k_j)ds \tag{14.81}$$

$$= \int_{k_j-l}^{\frac{h-l}{2}} U(s)ds + \int_0^{\frac{h-l}{2}} U(s)ds. \tag{14.82}$$

For type i, the loss over the gap is

$$\int_{\max\{0,l-k_i\}}^{\frac{h+l}{2}-k_i} U(l - s - k_i)ds + \int_{\frac{h+l}{2}-k_i}^{h-k_i} U(h - s - k_i)ds \tag{14.83}$$

$$= \int_{\max\{k_i-l,0\}}^{\frac{h-l}{2}} U(s)ds + \int_0^{\frac{h-l}{2}} U(s)ds < \int_{k_j-l}^{\frac{h-l}{2}} U(s)ds + \int_0^{\frac{h-l}{2}} U(s)ds. \tag{14.84}$$

Thus, the loss to type i is greater from this kind of gap. Hence, filling it in will increase the expected payoff of type i by more than that of type $j > i$. Hence, we have that

$$\Delta^i(D_i'', D_i) > \Delta^j(D_i'', D_i). \tag{14.85}$$

In addition, since

$$\Delta^i(I_i, D_i'') = \mathbb{E}^i I_i - \mathbb{E}^i D_i'' = \int_0^{1+k_i-a_i^*} U(s)ds - \int_0^{1+k_i-a_i} U(s)ds = \int_{1+k_i-a_i}^{1+k_i-a_i^*} U(s)ds \tag{14.86}$$

and, since $\int_{1+k_i-a_i}^{1+k_i-a_i^*} U(s)ds > \int_{1+k_j-a_i}^{1+k_j-a_i^*} U(s)ds$ for $k_j > k_i$, then we know that

$$\Delta^j(I_i, D_i'') = \mathbb{E}^j I_i - \mathbb{E}^j D_i'' = \int_{1+k_j-a_i}^{1+k_j-a_i^*} U(s)ds \tag{14.87}$$

$$< \int_{1+k_i-a_i}^{1+k_i-a_i^*} U(s)ds = \mathbb{E}^i I_i - \mathbb{E}^i D_i'' = \Delta^i(I_i, D_i'').$$

(Notice that this argument also shows that $\Delta^P(I_i, D_i'') \geq \Delta^i(I_i, D_i'')$, since $k_P = 0 < k_i$.) Thus,

$$\mathbb{E}^j I_i - \mathbb{E}^j D_i = \Delta^j(I_i, D_i'') + \Delta^j(D_i'', D_i) < \Delta^i(I_i, D_i'') + \Delta^i(D_i'', D_i) = \mathbb{E}^i I_i - \mathbb{E}^i D_i = 0. \tag{14.88}$$

So $\mathbb{E}^j I_i < \mathbb{E}^j D_i$ for all $j > i$. This shows that equation (8.6) is satisfied by I_i. It remains to show that I_i satisfies equation (8.5).

In order to show equation (8.5), we break it into two cases:

Nice Gap: $G \subseteq [k_i, 1 + k_i]$

Let $L_i^j(G)$ denote the loss over a gap, $G = (l, h) \subseteq [k_i, 1 + k_i]$, to an agent j or a principal with agent i. Thus, $j \in \{P, 1, 2, \ldots, N\}$ and if $j \neq P$, then $j = i$:

$$L_i^P(G) = \int_{l-k_i}^{\frac{h+l}{2}-k_i} U(l-s)ds + \int_{\frac{h+l}{2}-k_i}^{h-k_i} U(h-s)ds = \int_{-k_i}^{-k_i+\frac{h-l}{2}} U(s)ds + \int_{-k_i-\frac{h-l}{2}}^{-k_i} U(s)ds \tag{14.89}$$

$$= \int_{-k_i-\frac{h-l}{2}}^{-k_i+\frac{h-l}{2}} U(s)ds = \int_{k_i-\frac{h-l}{2}}^{k_i+\frac{h-l}{2}} U(s)ds. \tag{14.90}$$

The loss to an agent with bias k_i is:

$$L_i^i(G) = \int_{l-k_i}^{\frac{h+l}{2}-k_i} U(l-s-k_i)ds + \int_{\frac{h+l}{2}-k_i}^{h-k_i} U(h-s-k_i)ds = \int_0^{\frac{h-l}{2}} U(s)ds + \int_{-\frac{h-l}{2}}^{0} U(s)ds \tag{14.91}$$

$$L_i^i(G) = \int_{-\frac{h-l}{2}}^{\frac{h-l}{2}} U(s)ds > \int_{k_i-\frac{h-l}{2}}^{k_i+\frac{h-l}{2}} U(s)ds = L_i^P(G), \tag{14.92}$$

for all $k_i > 0$. Notice that $U(\cdot)$ is negative, so even though the *value* of the loss is higher for the agent, the *absolute value* of the loss from the gap is higher for the principal.

When the gap is filled in, the loss to the principal is $F_i^P(G) := (h - l)U(k_i)$ and the loss to the agent is $F_i^i(G) := (h - l)U(k_i) = 0$. Thus, the gain from filling in the gap to the principal, $\Delta_i^P(G)$ is

$$\Delta_i^P(G) = F_i^P(G) - L_i^P(G) = (h - l)U(k_i) - \int_{k_i-\frac{h-l}{2}}^{k_i+\frac{h-l}{2}} U(s)ds. \tag{14.93}$$

Notice that by the Gap Filling Lemma (Lemma 13.2), $\Delta_i^P(G) > 0$.

For the type i agent, the gain from filling in the gap $\Delta^P i_i(G)$ is

$$\Delta_i^i(G) = F_i^P(G) - L_i^P(G) = -\int_{-\frac{h-l}{2}}^{\frac{h-l}{2}} U(s)ds. \tag{14.94}$$

Thus, gain is clearly positive since $U(\cdot) \leq 0$. For quadratic functions ($U(s) = -s^2$), it turns out that these two gains are equal since, (letting $\epsilon = \frac{h-l}{2}$):

$$\Delta_i^i(G) = \int_{-\epsilon}^{\epsilon} s^2 ds = \frac{2}{3}\epsilon^3. \tag{14.95}$$

In contrast,

$$\Delta_i^P(G) = -(2\epsilon)k_i^2 + \int_{k_i-\epsilon}^{k_i+\epsilon} s^2 ds \tag{14.96}$$

$$= -(2\epsilon)k_i^2 + \frac{1}{3}(k_i+\epsilon)^3 - \frac{1}{3}(k_i-\epsilon)^3 = -(2\epsilon)k_i^2 + \frac{1}{3}\Big((k_i+\epsilon)^3 - (k_i-\epsilon)^3\Big) \tag{14.97}$$

$$= -(2\epsilon)k_i^2 + \frac{1}{3}\Big((k_i+\epsilon) - (k_i-\epsilon)\Big)\Big((k_i+\epsilon)^2 + (k_i+\epsilon)(k_i-\epsilon) + (k_i-\epsilon)^2\Big), \tag{14.98}$$

(by the identity $a^3 - b^3 = (a-b)(a^2+ab+b^2)$).

$$= -(2\epsilon)k_i^2 + \frac{1}{3}(2\epsilon)\Big(k_i^2+\epsilon^2+k_i^2-\epsilon^2+k_i^2+\epsilon^2\Big) = -(2\epsilon)k_i^2 + \frac{1}{3}(2\epsilon)(3k_i^2+\epsilon^2) \tag{14.99}$$

$$= -(2\epsilon)k_i^2 + \frac{1}{3}(2\epsilon)(3k_i^2+\epsilon^2) = \frac{2}{3}\epsilon^3 = \Delta_i^i(G), \tag{14.100}$$

by equation (14.95). Hence, to summarize, we have (for quadratic loss functions) $\Delta_i^i(G) = \Delta_i^P(G)$. In addition, if we would only partially fill in the gap, G, add $[l, l+\epsilon] \cup [h-\epsilon, h]$ to the set) then the gain would also be equal to both the principal and agent type i (since partially filling the gap and then filling in the remainder is equivalent to completely filling in the gap). Thus, the expected gain to the principal from filling in a gap is the *exact same* expected payoff as type i.

Low Gap Case: $G = (l, h)$, $l < k_i < h \leq 1 + k_i$

If the $l < k_i < h$ and $\epsilon = k_i - l$. Notice that ϵ was chosen to turn the low gap case into a nice gap case. Once $[l, l+\epsilon] \cup [h-\epsilon, h]$ is added to the set, this gap is no longer a low gap and becomes a nice gap (the remaining gap is contained in $[k_i, 1+k_i]$).

Thus, if we can show that, for this chosen ϵ, the increase is higher for the principal than the agent type i, we are done.

Loss from gap G, Agent:

$$L_i^i(G) := L_i^i(G,0) = \int_0^{\frac{h+l}{2}-k_i} U(l-s-k_i)ds + \int_{\frac{h+l}{2}-k_i}^{h-k_i} U(h-s-k_i)ds \tag{14.101}$$

$$= \int_{k_i-l}^{\frac{h-l}{2}} U(s)ds + \int_0^{\frac{h-l}{2}} U(s)ds. \tag{14.102}$$

Loss from gap $G(\epsilon)$, Agent:

$$L_i^i(G,\epsilon) = \int_0^{\frac{h+l}{2}-k_i} U(l+\epsilon-s-k_i)ds + \int_{\frac{h+l}{2}-k_i}^{h-\epsilon-k_i} U(h-\epsilon-s-k_i)ds \tag{14.103}$$

$$= \int_{k_i-l-\epsilon}^{\frac{h-l}{2}-\epsilon} U(s)ds + \int_0^{\frac{h-l}{2}-\epsilon} U(s)ds. \tag{14.104}$$

If we subtract equation (14.102) from (14.104) we get the gain in expected payment to the agent from filling in the gap:

$$\Delta_i^i(G,\epsilon) = L_i^i(G,\epsilon) - L_i^i(G,0) \tag{14.105}$$

$$= \int_{k_i-l-\epsilon}^{\frac{h-l}{2}-\epsilon} U(s)ds + \int_0^{\frac{h-l}{2}-\epsilon} U(s)ds - \int_{k_i-l}^{\frac{h-l}{2}} U(s)ds - \int_0^{\frac{h-l}{2}} U(s)ds$$

$$= \int_{k_i-l-\epsilon}^{k_i-l} U(s)ds - \int_{\frac{h-l}{2}-\epsilon}^{\frac{h-l}{2}} U(s)ds - \int_{\frac{h-l}{2}-\epsilon}^{\frac{h-l}{2}} U(s)ds < -2\int_{\frac{h-l}{2}-\epsilon}^{\frac{h-l}{2}} U(s)ds \tag{14.106}$$

which is the benefit when $G \subseteq [k_i, 1+k_i]$.

The principal's benefit is higher. Loss from gap G, for the Principal:

$$L_i^P(G) := L_i^i(G,0) = \int_0^{\frac{h+l}{2}-k_i} U(l-s)ds + \int_{\frac{h+l}{2}-k_i}^{h-k_i} U(h-s)ds \tag{14.107}$$

$$= \int_{-l}^{\frac{h-l}{2}-k_i} U(s)ds + \int_{-k_i-\frac{h-l}{2}}^{-k_i} U(s)ds = \int_{k_i-\frac{h-l}{2}}^{l} U(s)ds + \int_{k_i}^{k_i+\frac{h-l}{2}} U(s)ds. \tag{14.108}$$

The loss from gap $G(\epsilon)$ to the principal is:

$$L_i^P(G,\epsilon) = \epsilon U(k_i) + \int_0^{\frac{h+l}{2}-k_i} U(l+\epsilon-s)ds + \int_{\frac{h+l}{2}-k_i}^{h-\epsilon-k_i} U(h-\epsilon-s)ds \quad (14.109)$$

$$= \epsilon U(k_i) + \int_{-l-\epsilon}^{-k_i+\frac{h-l}{2}-\epsilon} U(s)ds + \int_{-k_i-\frac{h-l}{2}+\epsilon}^{-k_i} U(s)ds = \epsilon U(k_i) + \int_{k_i-\frac{h-l}{2}+\epsilon}^{l+\epsilon} U(s)ds + \int_{k_i}^{k_i+\frac{h-l}{2}-\epsilon} U(s)ds. \quad (14.110)$$

If we subtract equation (14.108) from (14.110) we get the gain in expected payment to the principal from filling in the gap:

$$\Delta_i^P(G,\epsilon) = L_i^P(G,\epsilon) - L_i^P(G,0) \quad (14.111)$$

$$= \epsilon U(k_i) + \int_{k_i-\frac{h-l}{2}+\epsilon}^{l+\epsilon} U(s)ds + \int_{k_i}^{k_i+\frac{h-l}{2}-\epsilon} U(s)ds - \int_{k_i-\frac{h-l}{2}}^{l} U(s)ds - \int_{k_i}^{k_i+\frac{h-l}{2}} U(s)ds \quad (14.112)$$

$$= \epsilon U(k_i) + \int_{k_i-\frac{h-l}{2}+\epsilon}^{l} U(s)ds + \int_{l}^{l+\epsilon} U(s)ds - \int_{k_i-\frac{h-l}{2}}^{l} U(s)ds - \int_{k_i+\frac{h-l}{2}-\epsilon}^{k_i+\frac{h-l}{2}} U(s)ds \quad (14.113)$$

$$= \epsilon U(k_i) + \int_{l}^{l+\epsilon} U(s)ds - \int_{k_i-\frac{h-l}{2}}^{k_i-\frac{h-l}{2}+\epsilon} U(s)ds - \int_{k_i+\frac{h-l}{2}-\epsilon}^{k_i+\frac{h-l}{2}} U(s)ds \quad (14.114)$$

$$> 2\epsilon U(k_i) - \int_{k_i-\frac{h-l}{2}}^{k_i-\frac{h-l}{2}+\epsilon} U(s)ds - \int_{k_i+\frac{h-l}{2}-\epsilon}^{k_i+\frac{h-l}{2}} U(s)ds, \quad (14.115)$$

since $k_i \geq l+\epsilon$. But we know from the paragraph following equation (14.100)we know that that:

$$2\epsilon U(k_i) - \int_{k_i-\frac{h-l}{2}}^{k_i-\frac{h-l}{2}+\epsilon} U(s)ds - \int_{k_i+\frac{h-l}{2}-\epsilon}^{k_i+\frac{h-l}{2}} U(s)ds = -2\int_{\frac{h-l}{2}-\epsilon}^{\frac{h-l}{2}} U(s)ds. \quad (14.116)$$

Hence, from equations (14.106) and (14.115) that

$$\Delta_i^P(G,\epsilon) > \Delta_i^i(G,\epsilon). \quad (14.117)$$

Thus, the gain from filling in the low gap increases the expected payoff of the principal by an amount greater than that of agent type i.

These two cases show that:

$$\Delta^P(D_i'', D_i) \geq \Delta^i(D_i'', D_i). \tag{14.118}$$

We also know from the comment below equation (14.87) that

$$\Delta^P(I_i, D_i'') > \Delta^i(I_i, D_i''). \tag{14.119}$$

Hence by equations (14.118) and (14.119),

$$\begin{aligned} \mathbb{E}^P I_i - \mathbb{E}^P D_i &= \Delta^P(I_i, D_i'') + \Delta^P(D_i'', D_i) \\ &> \Delta^i(I_i, D_i'') + \Delta^i(D_i'', D_i) = \mathbb{E}^i I_i - \mathbb{E}^i D_i = 0. \end{aligned} \tag{14.120}$$

Thus, I_i satisfies equation (8.5) and the proof of Proposition 8.5 is complete.

15 Appendix E: Proof of Comparative Statics Result

Proof of Result 9.2:

Proof. Let k_L be the highest value in the support of $F_{(\mathbf{p}_L, \mathbf{k}_L)}(z)$. First-order stochastic dominance implies that there exist $k'_{N-j+1}, k'_{N-j+2}, \ldots, k'_N \geq k_L$ such that:

$$F_{(\mathbf{p}'_N, \mathbf{k}'_N)}(k'_{N-j+1}) \geq p_N. \tag{15.1}$$

(Notice that the rest of the points in the support of $F_{(\mathbf{p}'_N, \mathbf{k}'_N)}$ are $< k_L$.) Hold $\mathbf{p}'_N$ fixed, but replace $\mathbf{k}'_N$ with $\mathbf{k}^2_N$ (the 2 is a superscript not an exponent) such that $k^2_i = k'_i$ for $i < N - j + 1$ and $k^2_i = k_L$ for all $i \geq N - j + 1$. By Result 9.1 we know that $D^*(\mathbf{p}'_N, \mathbf{k}'_N) \subseteq D^*(\mathbf{p}'_N, \mathbf{k}^2_N)$.

Notice that $(\mathbf{p}'_N, \mathbf{k}^2_N) = (\mathbf{p}^2_{N-j+1}, \mathbf{k}^2_{N-j+1})$, where $p^2_i = p'_i$ if $i < N - j + 1$ and $p^2_{N-j+1} = \sum_{h=0}^{j-1} p_{N-h}$ and $k^2_i = k'_i$ if $i < N - j + 1$ and $k^2_{N-j+1} = k_L$.

Then fix $\mathbf{k}^2_{N-j+1}$ but replace $\mathbf{p}^2_{N-j+1}$ with $\hat{\mathbf{p}}^2_{N-j+1}$ such that $\hat{p}^2_{N-j+1} = p^2_{N-j+1}$ if $N - j + 1$ is the smallest index i such that $k^2_i \geq k_{L-1}$ (and denote this index by i^*_2). If not, let $\hat{p}^2_{N-j+1} = p_L$ and let $\hat{p}^2_{i^*_2} = p^2_{i^*_2} + \sum_{h=0}^{j-1} p_{N-h}$. For all other i, let $\hat{p}^2_i = p^2_i$. Thus, we just transferred probability from the highest value of of bias in the support $F_{(\mathbf{p}'_N, \mathbf{k}^2_N)}$ and transferred it to a lower value and left all other probabilities fixed. Remembering that $\frac{\partial V}{\partial u}(k_i, u)$ is nonincreasing we can conclude that:

$$D^*_P(\mathbf{p}^2_{N-j+1}, \mathbf{k}^2_{N-j+1}) \subseteq D^*_P(\hat{\mathbf{p}}^2_{N-j+1}, \mathbf{k}^2_{N-j+1}). \tag{15.2}$$

Continuing in this fashion, have a finite sequence of $(\hat{\mathbf{p}}^2_{N-j+1}, \mathbf{k}^2_{N-j+1}), (\hat{\mathbf{p}}^3_{N-j+1}, \mathbf{k}^3_{N-j+1}), \ldots, (\hat{\mathbf{p}}^q_{N-j+1}$ $(\mathbf{p}_L, \mathbf{k}_L)$, where

$$D^*_P(\mathbf{p}^r_{N-j+1}, \mathbf{k}^r_{N-j+1}) \subseteq D^*_P(\hat{\mathbf{p}}^s_{N-j+1}, \mathbf{k}^s_{N-j+1}), \tag{15.3}$$

if $s > r$.

□

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